Flood Studies Report

Volume I Hydrological Studies

Flood Studies Report

Volume I	Hydrological Studies
Volume II	Meteorological Studies
Volume III	Flood Routing Studies
Volume IV	Hydrological Data
Volume V	Maps

Flood Studies Report
in five volumes

Volume I

Hydrological Studies

Natural Environment Research Council
27 Charing Cross Road, London
1975

Printed in Great Britain by the Whitefriars Press Ltd
London and Tonbridge

Forewords by Mr R. J. H. Beverton and Sir Angus Paton

In 1967, the Committee on Floods of the Institution of Civil Engineers, under the Chairmanship of Sir Angus Paton, FRS, drew attention to the urgent need for improved techniques of flood prediction, and made proposals for the investigations that should be undertaken. The report of the Committee also stressed the need for a continuing programme of fundamental research related to flood hydrology.

In these circumstances it seemed appropriate that the study should be conducted under the auspices of the Institute of Hydrology, which the Council was at that time building up from the nucleus of the Hydrological Research Unit started under the former Department of Scientific & Industrial Research. Financial support for the project from other sources failed to materialise, but the Council was fortunate at that time to command funds with the flexibility to respond quickly and positively to the need.

The Floods Study Team, headed by a senior member of the Institute, Dr J. V. Sutcliffe, was assembled under the Director of the Institute, Dr J. S. G. McCulloch. The work was overseen by an Inter-Departmental Steering Committee chaired by the late Marshall Nixon. I am happy to place on record the appreciation of my Council for the work of the Steering Committee, for the diligence and enthusiasm with which it carried out its task, and in particular for the efforts of the late Mr Nixon, who took a personal and informed interest in the work of each member of the team.

With its task completed the Floods Team has been dismantled, as was originally intended. The majority of its members have been brought on to the permanent staff of the Council where they are putting their experience to good use either in commissioned research in applied hydrology or in following up fundamental hydrological problems generated during their floods work.

R. J. H. Beverton
Secretary, Natural Environment Research Council

As Chairman of the Institution of Civil Engineers' Committee appointed in 1965 to consider the review of the 1933 and 1960 Interim Reports on 'Floods in Relation to Reservoir Practice' it is very satisfactory that the recommendations of our Report of February 1967 on Flood Studies for the United Kingdom have been carried out so conscientiously and thoroughly.

Although practising engineers will still require to exercise judgement and make decisions in their designs for the control of floods the basic information on which such judgement depends is now available in the form of a comprehensive report and reliable records.

The lack of data has been a serious drawback in the past. We can now undertake our tasks with more confidence.

But a word of warning. We cannot predict future weather conditions. Considerable margins of safety are still necessary according to the location of any flood control works and the allowable risk. On the other hand, damage to life and property cannot be entirely eliminated in our densely crowded islands without incurring unwarranted expense.

On behalf of the Institution of Civil Engineers' members in general and the Floods Committee in particular may I express our thanks to the Natural Environment Research Council for undertaking this task, to their

Institute of Hydrology for directing the work under the guidance of an Inter-Departmental Steering Committee Chaired by the late Marshall Nixon, and to Dr J. V. Sutcliffe and his team for their most useful and thorough investigation and report.

Angus Paton

Preface

The investigations of methods of flood estimation for engineering design purposes, which are described in this report, were carried out at the Institute of Hydrology, the Meteorological Office and the Hydraulics Research Station, with the co-operation of the Irish Office of Public Works and Meteorological Service, the Soil Surveys and other organisations.

The Flood Studies Report consists of five volumes. Volume I contains the hydrological studies, Volume II the meteorological studies, Volume III the flood routing studies, Volume IV the hydrological data, and Volume V the maps.

Cross-references to sections, equations and figures are by chapter numbers, preceded by a volume number if necessary. Thus, Section I.3.5.2 is in Chapter 3 of Volume I. Equations are numbered consecutively within chapters, except in Chapters 1 and 2 of Volume I where it was necessary to number them within subsections. Figures are numbered consecutively within chapters; certain figures illustrating Volumes I and II are contained in Volume V.

The chapter titles illustrate the scope of the report.

Volume I—Hydrological studies

A Introduction
1 Statistics for flood hydrology
2 Statistical flood frequency analysis
3 Methods of extension of short records
4 Estimation of flood peaks from catchment characteristics
5 Estimation of flood volumes over different durations
6 Synthesis of the design flood hydrograph
7 Supplementary studies: snowmelt runoff, conceptual catchment model and flood routing
8 Future research and investigation needs

Volume II—Meteorological studies

1 A guide to procedures and contents of Volume II
2 Regional analysis of point rainfall extremes
3 Estimation and mapping of M5 (5 year) values for different durations
4 Estimated maximum falls of rain
5 Areal rainfall
6 Storm profiles
7 Snow cover and snowmelt
8 Examples of rainfall estimates for the Tyne and Wansbeck catchments
9 Some historic heavy rainfall events

Volume III—Flood routing studies

1 Choice of a flood routing method
2 Theory of flood routing
3 Comparison of flood routing methods
4 Strategy for flood routing
5 Appendices

Volume IV—Hydrological data

1 Collection of records
2 Data used in statistical analysis
3 Data used in unit hydrograph analysis
4 Historical flood records

5 Master list of gauging stations, catchment characteristics and flood statistics
6 Basic flood records

Volume V—Maps

The following maps illustrating Volumes I and II are contained in Volume V. (S indicates the southern part of Great Britain, N the northern part, and I indicates Ireland.)

I.4.18	(S, N and I)	Winter rain acceptance potential
I.4.19		Estimated mean soil moisture deficit
I.4.20		River gauging stations used in analysis
I.4.21		Mean annual flood (BESMAF) divided by area
I.4.22		Coefficient of variation of annual flood
I.4.23		Residuals from BESMAF prediction equations

II.3.1	(S, N, I and NI)	Average annual rainfall
II.3.2	(S, N and I)	2 day M5 rainfall
II.3.3	(S, N and I)	2 day M5 rainfall as % of AAR
II.3.4		25 day M5 rainfall
II.3.5	(S, N and I)	1 hour M5 expressed as % of 2 day M5
II.4.1		Estimated maximum 2 hour rainfall
II.4.2		Estimated maximum 24 hour rainfall

This volume, which forms Volume I of the Flood Studies Report, describes the hydrological studies carried out by the team at the Institute of Hydrology. The team consisted of Dr J. V. Sutcliffe (team leader), Mr M. A. Beran, Mr C. Cunnane, Mr R. C. Jones, Mr M. J. Lowing, Dr J. B. Miller and Dr M. D. Newson. They were assisted during the study by Mrs E. Bishop, Miss V. M. Black, Miss J. H. Ellis, Miss B. J. Glover, Mrs J. M. Haworth, Miss A. R. Herriotts, Mr J. L. Hill, Mrs I. M. Kent, Mr S. A. Mellanby, Mrs B. D. Newman, Mrs A. E. Sekulin, Mrs J. K. Travell, Miss A. M. Tucker, Mrs J. M. Tucker and Mrs E. A. Williams. Temporary assistance was provided by Mrs M. Henbest, Mr C. Johnson, Mr M. Patto, Mrs M. Sadler and Mrs M. E. Stevens. Secretarial assistance during the study was provided by Mrs S. Black, Miss C. Cremer-Evans, Mrs J. Fowler and Miss M. R. E. Kearsley.

Other contributors were Mr R. T. Clarke, Miss J. R. Frost, Mr P. W. Herbertson, Miss M. N. Leese and Mr A. N. Mandeville at the Institute of Hydrology, and Mr D. R. Archer and Mr P. Johnson at the University of Newcastle-upon-Tyne. Mr M. Mansell-Moullin and Professor T. O'Donnell acted as consultants.

The Office of Public Works, Dublin, co-operated in the investigation; staff included Mr P. Corish, Mr J. Howard, Mr F. McDonough and Mr G. Copeland under the direction of Mr M. A. Lynn. Professor J. E. Nash acted as a consultant.

The members of the Flood Studies Steering Committee were the late Mr M. Nixon (Chairman) who was succeeded as Chairman by Mr E. J. K. Chapman (ICE) in 1974, Mr G. Cole (MAFF), Mr V. K. Collinge (WRB), Mr D. Fiddes (TRRL), Mr J. Harding (MO) succeeded by Mr R. Murray and Mr J. F. Keers, Mr A. F. Jenkinson (MO), Mr M. A. Lynn (OPW, Dublin), Mr M. Mansell-Moullin, Dr J. S. G. McCulloch (Director, IH), Mr J. C. Munro (SDD), Dr J. V. Sutcliffe (IH), Mr J. I. Taylor (ARA), Mr S. F. White (DE) and Professor P. O. Wolf.

Acknowledgements

Thanks are due to the following organisations and individuals. All the many contributors of flow records and recording rainguage charts referred to in the lists in Chapters 3, 4 and 5 of Volume IV; the investigation would not have been possible without the generous help of organisations and individuals too numerous to list here. In particular, thanks are due to the River Authorities and River Purification Boards listed in Volume IV, Chapter 5, and their liaison officers who were closely involved with the investigation.

The Office of Public Works, Dublin (Messrs M. A. Lynn, P. Corish, J. Howard, F. McDonough, G. Copeland), who co-operated in the investigation; the Director and staff of the Irish Meteorological Service, especially Mr J. Logue, who analysed rainfall records, and Mr M. Connaughton, who analysed soil moisture deficit estimates; Dr M. J. Gardiner and Mr L. F. Galvin of the Agricultural Institute, Dublin, who provided a hydrological soil map.

The Soil Survey of England and Wales (Messrs K. E. Clare, D. Mackney, P. D. Smith and A. J. Thomasson) who provided a soil classification and the map of winter rain acceptance potential; Soil Survey of Scotland (Dr R. Glentworth); Dr S. McConaghy, Ministry of Agriculture, Northern Ireland; Mr B. S. Kear, Manchester University, who provided soil information for the Isle of Man.

The Water Resources Board, who provided much assistance with flow records and information about river gauging stations and who also gave permission for use of the shot noise model; Mr P. M. Stephenson, who provided a computer program for the derivation of stage–discharge relationships.

The Meteorological Office, in addition to the authors of Volume II, who provided a large volume of rainfall records; Mr J. Grindley and Miss D. J. T. Ayres, who provided estimates of soil moisture deficit for a large number of stations and dates.

Her Majesty's Stationery Office, Reprographic Division, Basildon (Mr C. Elliot), who carried out the microfilming programme on behalf of the Water Resources Board; the Co-ordinator of Government Reproduction Services for advice on microfilming; Mr W. R. Turner for design of the microfilm projection equipment.

The Forestry Commission (Dr W. O. Binns), for maps and statistics of forest cover; the Map Research and Library Division, Ministry of Defence; the Department of the Environment map library; the Scottish Development Department; Oxford University Geography Department map library; The Experimental Cartographic Unit; Mr A. D. Cooper, Luton College of Technology.

The Director General, Ordnance Survey for permission to reproduce map information. Crown copyright is reserved.

Professor E. M. Wilson, University of Salford, for permission to use an example from *Engineering Hydrology*, second edition, 1974.

Table 1.13 is taken from Table XX of Fisher and Yates' *Statistical Tables* (sixth edition) published by Longman Group Ltd, London (previously published by Oliver and Boyd, Edinburgh) by permission of the authors and publishers.

Useful discussions were held with a large number of individuals: particular mention must be made of the late Marshall Nixon, Chairman of the Steering Committee, and other members of the committee; MM J. Bernier, J. Jacquet, Electricité de France, and M. Roche, ORSTOM; Professor R. N. Curnow, Mr R. Stern, and Mr M. A. Moran, Department of Applied Statistics, Reading University; Mr G. Weiss, Dr P. E. O'Connell, and Professor T. O'Donnell, for help with the shot noise model; Mr M. Mansell-Moullin, Professor J. E. Nash and Professor T. O'Donnell, who acted as consultants; Mr F. M. Law, Binnie and Partners.

Contents

Notation

The following are the main symbols and abbreviations used in this volume. A single symbol has been used occasionally for different quantities but in any instance the meaning is clear from the context. Many other symbols having just a local meaning are defined where they occur. All units used are metric except where otherwise stated; flows are in cubic metres per second (cumecs).

A, AREA	catchment area (km^2)
AM	annual maximum
AMAF	mean of AM instantaneous flood series, synonymous with $\bar{Q}$ and MAF
ANSF	average nonseparated flow, cumecs/km^2
AR3	mean annual 3 day flood/mean annual calendar day flood = $\bar{Q}(3)$/CALMAF, and similarly for AR10.
BESMAF	EXTMAF where this exists; AMAF in other cases
CV	coefficient of variation (usually of AM flood series)
CALMAF	mean of AM calendar day flood series
CWI	catchment wetness index based on antecedent precipitation index (API5) and estimated soil moisture deficit
df	distribution function; also degrees of freedom
$E(.)$	expected value
EV	extreme value
EV1, EV2, EV3	extreme value Type 1, Type 2 and Type 3 distributions
EXTMAF	extended mean annual flood
F_i, $i = 1,2 \ldots N$	plotting positions expressed as probabilities
$F(Q)$	distribution function
$f(Q)$	probability density function
g	skewness
GEV	general extreme value distribution
$G(y)$	distribution function of reduced variate
$g(y)$	probability density function of reduced variate
ICE	Institution of Civil Engineers
IEM	isolated event model
k	shape parameter of general extreme value distribution; also a temperature index in Chapter 7.
$K(T)$, K_T	frequency factor (after Chow)
LAG	time in hours from the centroid of the rain profile to the peak runoff or to a 'centroid of peaks' if more than one peak
LAKE	fraction of catchment draining through a lake or reservoir
ln	natural logarithm
log	logarithm to base 10
MAF	mean of AM instantaneous flood series (= AMAF and $\bar{Q}$)
MARAIN	magnetic tape archive of daily rainfall data
MEDAF	median of AM instantaneous flood series (= Q_{med})
M52D	maximum 2 day rainfall of 5 year return period
MS	mean square (= ss/df)
N	length of record (years)
NMF	normal maximum flood
P	rainfall
pdf	probability density function
PERC	percentage runoff
POT	peaks over a threshold, e.g. POT series

PR(.)	probability e.g. PR ($Q < 10$)
Q,q	flood peak discharge (response runoff in Chapter 6)
q_o	threshold flow in POT model
$Q_{(i)}$	*i*th smallest in a sample of size N, the *i*th order statistic
Q_p	peak of T hour unit hydrograph expressed in cumecs/ 100 km^2
$Q(T)$, Q_T	flood peak of return period T, the T year flood
$\bar{Q}$, $\bar{Q}_i$	mean of AM instantaneous flood series, the mean annual flood (= AMAF and MAF)
$\bar{Q}(d)$	mean of AM series of floods of duration d
Q_{max}	maximum flood on record
Q_{med}	median of AM instantaneous flood series (= MEDAF)
r	ratio of mean annual flood of duration d to mean annual flood, $\bar{Q}(d)/\bar{Q}_i$, also sample correlation coefficient
R	coefficient of multiple correlation
RSMD	one day rainfall of 5 year return period less effective mean soil moisture deficit
S1085	10–85% stream slope (m/km) (S in Chapter 6)
S_{75}	soil moisture deficit estimated using 75 mm root constant
SAAR	standard (1916–50) annual average rainfall (mm)
se(.)	standard error
SMD	soil moisture deficit (estimated by Meteorological Office assuming standard catchment)
SMDBAR	effective mean soil moisture deficit
SOIL	soil index, being a weighted sum of SOIL1, SOIL2... SOIL5
SOIL1	fraction of catchment in soil class 1
SOIL2	fraction of catchment in soil class 2
SOIL3	fraction of catchment in soil class 3
SOIL4	fraction of catchment in soil class 4
SOIL5	fraction of catchment in soil class 5
SS	sum of squares
STMFRQ	stream frequency (junctions/km^2)
T	return period (also used in a different sense to define a unit hydrograph in Chapter 6 *viz.* the T hour unit hydrograph)
TAYSLO	Taylor-Schwarz slope (m/km)
TB	baselength of simplified unit hydrograph (hours)
T_p	time to peak of T hour unit hydrograph, measured from start of response runoff
u	location parameter in extreme value distributions
URBAN	fraction of catchment in urban development
W	width of simplified unit hydrograph measured at half height
$W(y_1; k)$	a standardised general extreme value variate
X, x	variate used in statistical exposition
$x_{(i)}$	*i*th order statistic, the *i*th smallest in a sample
Y, y	any reduced variate
Y_1, y_1	Gumbel or EV1 reduced variate
y_i $i = 1,2 \dots N$	plotting positions expressed as reduced variate
Z, z	variate used in statistical exposition, frequently $z = \ln X$
α	scale parameter in extreme value distributions
β	scale parameter of exponential distribution
θ	temperature

λ	average number of peaks exceeding a threshold, q_0, per year
$\mu\ (=\mu'_1)$	the mean of any distribution
μ_2, μ_3	second and third central moments
μ_z	mean of $z = \ln x$
ρ, $\rho(x,y)$	population value of correlation coefficient (also density in Chapter 7)
σ	standard deviation
σ_z	standard deviation of $z = \ln X$.

A Introduction

A.1 Present investigation

A.1.1 Introduction

Flood estimates are required for the design and economic appraisal of a variety of engineering works, including dam spillways, bridges and flood protection works. This report presents methods and results of analysis of flood records from the British Isles. The objective has been to develop and present methods of estimating the flood which will be exceeded at a given site on average once in T years and also the possible limit to the flood or the 'probable maximum flood'. Two main approaches have been used in the study; the first was based on statistical analysis of flood series at all gauging stations, and the second on investigation of rainfall and resulting runoff at selected stations. In practice estimates may be required at sites without records, so the results of each of these investigations have been related to catchment characteristics. Because the same sets of catchment characteristics have been used in both investigations, it is possible to relate the statistical and rainfall–runoff approaches.

A.1.2 Development of methods of flood estimation

Previous methods of flood estimation have necessarily been based on a rather small number of flood records which have been used to generalise on a regional basis. Wolf (1965) has described the incorporation of records into simple flood formulae of the type $Q_{max} = C\sqrt{A}$ by Jarvis and others. The exponent makes some allowance for the link between area A and rainfall, basin slope and the time scale of flood events, but the factor C is varied in order to allow for the effect of climate, soils and other variables. An early example of the rainfall–runoff approach is the 'rational method' which uses the expression $Q_{max} = CAi$ where C is the coefficient of runoff and i the mean intensity of rainfall during the period of concentration; here C can vary according to soil type and other factors.

The Interim Report of the Institution of Civil Engineers (ICE) Committee on Floods in relation to reservoir practice (Institution of Civil Engineers, 1933) has been used as a standard design method for a long period. This report was concerned primarily with upland areas of moderate extent (less than 100 km^2) and provides a table giving peak flood discharges recorded from various sites, together with an enveloping 'normal maximum curve' relating floods to area. In order to calculate attenuation by reservoir storage, simplified flood hydrographs were provided for various areas with times of rise calculated from average topographical variables. Besides the normal maximum flood, which was expected to be caused by about 3 inches of rain, larger 'catastrophic' floods were predicted following severe rainfalls. It was suggested that rainfalls of twice the intensity of those causing normal maximum floods could be expected, with peak discharges at least twice the normal maximum. No estimate of frequency was attached to these terms. Because the graphs of peak discharge depend entirely on catchment area, no specific allowance is made for variations in these small upland areas in rainfall intensity, losses or percentage runoff, or the effect of slope or channel network on the shape of the response and thus the peak flow. On the other hand, variations over the country are illustrated by Figure 4.21 in Volume V showing mean annual flood divided by area, which is more sensitive than the maximum to such differences.

However, this report was limited by lack of records; indeed, it was recorded that the Committee 'has been much impressed by the paucity of the records available in the British Isles'. The report was reprinted in 1960 with additional records and a request for further data. In this edition it was recommended that the time of concentration should be derived from observations. Further investigation of floods including aspects other than those relating to reservoir practice was recommended.

Morgan (1965) gives examples of how the ICE 1960 curve was applied to design problems in Scotland and Wales after slight adjustment to allow for the fact that this curve was set high to cover a single group of Lyn floods. Chapman & Buchanan (1965), after studying a number of catchments in the north and west of Britain, concluded that it is not practical to redefine the normal maximum flood as a flood of a definite frequency.

Since 1933, the development of unit hydrograph techniques and statistical methods in the United States and elsewhere has led to their use in specific projects; they have mainly been applied in overseas projects where guidance in the form of regional curves is rarely available. The application of the probable maximum precipitation and unit hydrograph approach as at Mangla (Binnie & Mansell-Moullin, 1965) has been followed by the application of similar techniques in this country with individual estimates of precipitation. A statistical approach to spillway flood design was considered necessary at Kariba (Reeve & Edmonds, 1965) but is generally applied in this country to problems involving lower return periods.

The introduction of these methods was followed by general countrywide investigations to allow their application to ungauged basins in the United Kingdom. The theory of these techniques and the generalised studies are well described by Nash (1966).

Nash (1960) carried out a general unit hydrograph study. He showed that the moments of the unit hydrograph could be derived directly from the records of surface runoff and net rainfall, after separating base flow and rainfall losses by certain rules. He then showed that the moments could be related to catchment characteristics and also related the moments to the parameters of a general equation. However, his investigation was not concerned with predicting rainfall losses which may be more important than unit hydrograph shape. No general guidance on losses has been available, though a number of forecasting investigations have shown that runoff volume could be predicted from antecedent conditions in particular cases.

Statistical investigations of flood records have led to a number of countrywide or regional equations relating floods to catchment characteristics. Nash & Shaw (1965) related the mean annual flood and the coefficient of variation of the annual flood to catchment characteristics which included area and mean annual rainfall. Cole (1965) divided the country into regions where the mean annual flood was adjusted to a common period and related to area; the coefficient of variation was assumed homogeneous in each region. Biswas & Fleming (1966) derived a regional equation for Scotland and Lynn (1971) derived similar equations for Ireland.

However, all but the most recent statistical studies have suffered from lack of records; Nash, for instance, was able to use only 57 stations. Recent steady progress in the establishment of gauging stations has made further investigation timely, especially as it has been possible to study the combined British and Irish records.

A.1.3 Recommendations for present study

The report of the ICE subcommittee on rainfall and runoff, *Floods in the British Isles*, was published in 1960. At the same time the ICE Council adopted the subcommittee's recommendations that (a) the 1933 Report should be reprinted with the additional tabulated data contained in the proposed Appendix; (b) the proposed Appendix (minus the tabulated data) should be published with an account of the discussion in the Proceedings, and that further contributions to the discussion should be invited; (c) consideration should be given to the desirability of preparing another report to deal with aspects of floods, other than those relating to reservoir practice, after any further contributions arising from (b) had been received. The subcommittee also recommended to the ICE Research Committee that papers should be invited describing current methods of flood estimation and a symposium on river flood hydrology was held in 1965. It was hoped that the papers and the comments during the discussion would provide a basis for a new investigation. Subsequently the report of the Committee on Floods in the United Kingdom, *Flood Studies for the U.K.*, was published by the ICE in February 1967. This recommended that a new investigation of floods in all regions should be undertaken and that all aspects of flood hydrology should be examined; meteorological records should be studied in order to understand the causes of floods, extend flood records, assist in flood frequency analyses and provide estimates of probable maximum precipitation. All available flood records should be assembled and reviewed; frequency analyses of flood peaks and volumes should be carried out; regional analyses and correlations with catchment characteristics were required to improve single station frequency distributions and to estimate flood frequencies at ungauged sites. Unit hydrographs, soil infiltration characteristics and snowmelt should be studied to derive precipitation–runoff models for use with the results of meteorological studies. Flood routing techniques should be reviewed and tested.

A.1.4 Plan of investigation

Following the appointment of staff to undertake these studies at the Institute of Hydrology and the Meteorological Office, programmes were drawn up in Autumn 1969 and approved by the Flood Studies Steering Committee. A summary of these programmes was published (Sutcliffe, 1971) in an appeal for records. The programme was planned to give information on the limit of the floods which might occur and the relationship of flood magnitude to frequency at either a gauged or an ungauged site. The hydrological programme was split into three broad topics—catchment response, flood frequency analysis and record extension. Each topic was allocated to a small group which was also responsible for collecting flow records from one third of the country.

The Meteorological Office team planned to provide estimates of the design storm of a given frequency for a given basin, and it would be necessary to convert these to a flood at a site by estimates of catchment response. These were to be obtained by selecting a number of basins with adequate hourly rainfall records and simultaneous flow data and deriving the unit hydrograph, or the response to a unit of effective rainfall. By studying a number of events on each catchment, the variability of the response was to be examined so that the response to a large storm could be estimated from the response to recorded storms. It was planned to relate the volume and the shape of the response to antecedent conditions and to catchment characteristics like area, slope and soils.

Flood frequency analysis can only be based on records at a single site if a long period of records exists. The choice of statistical distribution or of certain parameters of a distribution could best be based on analysis of a large number of records. These could be supplemented by records of historic floods. It was intended to compare the methods of analysis which use the maximum flood for each year of record with those based on all the flood events above a threshold value.

Record extension could be used to improve the estimate of floods at a site with short records, by extending the record through comparison with long term flow or rainfall records. Estimates could also be extended to ungauged sites by comparing indices of the flood regime, for instance the mean annual flood, with mapped catchment characteristics including some index of soils and of urban development to represent catchment cover and an index of net daily rainfall to represent flood producing rainfall.

A.1.5 Account of investigation

The timetable and staffing of the investigation followed the recommendations of the ICE Report. It was planned as a three year investigation following a period from October 1969 to March 1970 for technical planning and recruitment of staff. The main investigation was carried out in about three years from April 1970 and was followed by the production of the report. The team of fifteen was supplemented by temporary staff when required, for example, for extraction of peak flows from microfilm. Progress has been recorded in successive annual reports (Institute of Hydrology, Research, 1969–73).

The visits to some 1200 gauging stations which took place mainly between March 1970 and March 1971 were followed by preparation of station reports and grading of rating curves as described in Volume IV. The collection and microfilming of river level charts between April 1970 and September 1971 resulted in an archive of about half a million charts which was supported by station files containing reports and rating curves. The extraction of peak flow records for statistical analysis followed between September 1971 and April 1972; however, checking was required before the processed data were ready for analysis and publication.

Discussions with the Irish authorities led to a co-operative investigation with the Office of Public Works. A team in Dublin carried out an identical appraisal of gauging stations, extracted flood records for statistical analysis from charts and derived catchment characteristics for each station.

The extraction of data for unit hydrograph analysis for 140 catchments was also based on the microfilmed charts. After events had been selected from the runoff record and the relevant recording raingauge charts obtained, hourly falls were extracted and processed with daily rainfall records obtained from the Meteorological Office.

The processing and checking of the records required for analysis was completed by December 1972. The technical planning of the investigation and development of programs had proceeded at the same time as the data collection and processing, and has been described in a series of internal technical notes. Thus, the statistical analysis of the records of individual stations and unit hydrograph analysis of individual storm/runoff events could immediately follow processing. These analyses were followed by comparison of flood statistics with catchment characteristics, and of losses and unit hydrograph dimensions with catchment conditions and characteristics.

Numerical values of catchment characteristics were required for extrapolation to the ungauged site. Some physical variables, including area, channel slope, stream frequency and the fractions of the basin affected by urban development or lake storage, were directly measured from topographic maps. An index of soil cover required interpretation of soil maps from a hydrological viewpoint; the appropriate Soil Surveys were asked to provide maps indicating the infiltration at field capacity, from which a basin soil index could be obtained. Measures of climate were provided by mean annual rainfall and also by estimates of net short term rainfall based on rainfall maps and soil moisture deficit estimates, all provided by the Meteorological Office and the Irish Meteorological Service.

The investigation by the Meteorological Office team (Volume II) was largely based on the statistical analysis of rainfall records from daily read and recording gauges. These analyses were used to produce maps of point rainfall of 5 year return period for 2 day and for 60 minute durations. Falls of other durations and return periods could be related to these estimates and maximum rainfalls for different durations were also estimated. The ratios of areal to point rainfall and the time profiles of storms were investigated. Information on snow cover and possible snowmelt rates was collected and studied.

A survey of flood routing methods for British rivers (Volume III) was produced by the Hydraulics Research Station at the request of the Steering Committee. A theoretical analysis was supported by numerical examples.

Snowmelt runoff was investigated at the University of Newcastle-upon-Tyne (Chapter 7.2). Runoff records were related to temperature for a number of events on 16 basins.

At the request of the Steering Committee a nonlinear conceptual model dealing with isolated storm events was developed at the Institute of Hydrology (Chapter 7.3); this model was tested and applied to 500 events on 21 catchments.

The whole investigation was supervised by the Steering Committee which met regularly to discuss progress and liaison between organisations.

A.2 Summary of conclusions

A brief account is given of the main conclusions of the investigation on which the suggested methods of flood estimation are based. It is also hoped that this summary of the whole report will enable the reader to turn to individual chapters and volumes with a knowledge of where they fit into a framework. The order follows that of the report, except that the meteorological studies are described first as the results are required for an understanding of the hydrological studies.

A.2.1 Volume II Meteorological studies

The meteorological study provided estimates of the rainfall depth corresponding to a given duration and return period, both at a point and over an area, together with a profile or time distribution of this rainfall. The rainfall records analysed in this study comprised daily falls from 600 long term stations with an average record of 60 years, 6000 additional stations for the decade 1961–70, and also records from some 200 autographic raingauge stations. Durations of 2 days and of 60 minutes were used as a basis for analysis and other durations were related to these basic periods.

For each station the highest 2 day rainfalls recorded in each year provided an annual maximum series. The 2 day rainfall of 5 year return period (2 day M5), or the rainfall exceeded on average once in five years, was estimated from the mean of the upper two quartiles of the annual maxima. Stations within various ranges of M5 were grouped to give curves of M*T*/M5 or 'growth factor'. Similar growth curves were deduced for other durations and ranges of M5; and it was found that for a given M5 a single growth curve could be used for England and Wales to give the ratio M*T*/M5, regardless of duration. A second curve was derived for Scotland and Northern Ireland.

Because the rainfall M*T* of any return period *T* years could be deduced from an estimate of M5, the latter was mapped for durations of 2 days and of 60 minutes. Rainfalls of other durations can be deduced from these estimates to give a corresponding M5; rainfall estimates for other return periods can be deduced from the growth curve as a function of M5.

Estimates of maximum 2 hour rainfall were based on major storms adjusted for maximum observed storm efficiency together with maps of precipitable water corresponding to dew point estimates. Estimated maximum falls for 24 hours based on maximum storm efficiencies were related to the 2 day rainfall map to provide a map of estimated maximum 24 hour rainfall.

The areal reduction factor is the ratio between the areal rainfall for a catchment of a given size and the point rainfall of the same duration and return period. This factor was found to be related to duration and area but was found not to vary with location.

Storm profiles were extracted for a large number of storms and were centred on the most intense part of the storm. These standardised profiles were found not to vary with duration, return period or location to a significant extent. Summer and winter profiles were classified by peak intensity and the probability of various profiles was determined.

A preliminary attempt to estimate rare snowmelt rates was based on the collection and analysis of records of snow depth, snow density and temperatures over snow.

A practical guide is given to the estimation of the catchment rainfall for a given duration and return period, a likely time profile for this rainfall and also the maximum rainfall in the same duration.

A.2.2 Volume I Hydrological studies

A.2.2.1 Chapters 1–2: Statistics and flood frequency analysis

In order to provide a statistical framework within which floods can be analysed, the properties of the various distributions which have been proposed in flood hydrology are described; by assembling these descriptions together, the relationships between different distributions are stressed. Methods of fitting distributions to sample data are described including graphical fitting and fitting by moments and maximum likelihood. The choice of correct plotting positions affects some methods of fitting and the correct choice for different distributions is discussed. Standard errors of estimates are derived for certain distributions.

Certain distributions have been used for annual maximum floods on empirical grounds—for example lognormal and Pearson Type 3; the extreme value distributions applied by Gumbel have some theoretical basis but still require testing. Because there is no firm theoretical basis for choice between distributions, goodness of fit tests are often used for comparison. However, even this comparison depends on the index of fit and on the plotting positions used. As in earlier investigations, the three parameter distributions were found to be better or more flexible than the two parameter distributions. However, no clear cut choice resulted from this comparison.

The question of sampling error was considered and theoretical estimates were checked against empirical estimates derived from long term records analysed by decades. A single formula is suggested for the standard error of $Q(T)$ regardless of distribution fitted.

On the assumption that the records of an area provide an average picture of the variation of floods, region curves were built up by combining sets of stations in regions and countrywide. Historical records were included in these curves which show an increase in CV and skewness from the north west to the south east of the country.

An alternative to the estimation of floods from the annual maximum series is the use of the series of peaks over a threshold with an assumed distribution. The relative efficiencies of the two methods are discussed.

Methods of including historical records or missing peaks in flood estimation are described; these methods are based on the statistical theory of censored samples.

The time series approach to flood frequency analysis is illustrated by an application of the 'shot noise' model. The relative merits of the three types of model are discussed.

The relationship between design life, risk of failure and design return period is explained. Recommendations are made for estimating the flood of a given return period in different circumstances.

A.2.2.2 Chapter 3: Extension of short records

In many practical cases estimates of the flood regime must be made from short periods of records but the sampling error of a short record is high. The estimates may therefore be improved by adding further information from adjacent long term records through extending the short term record by correlation.

A detailed study was made of this technique and a method was developed and used for adjusting flood estimates. The criteria for extension may be expressed in terms of the correlation between short term and long term stations and a pilot study showed that sufficient correlation could be obtained by using adjacent flow and rainfall stations. It was decided to concentrate on improving the estimate of the mean annual flood and correlation of monthly maxima was used for this purpose. Programs for handling these data on a large scale provided information on the records available within 100 km of the short term station. Correlation maps based on this information enabled long term stations to be chosen to extend batches of short term records.

Although the improvement using extended data in regression of flood estimates on catchment characteristics was small, this may be due to the limitations of the regression model. When an individual short term record is to be extended, more detailed attention can be given; an example is presented of the technique which should be adopted in practice, particularly when a short term record covers a period which is known to be biassed.

A method of extending the peaks over a threshold series is presented with a numerical example. The extension of records directly from rainfall by means of a conceptual model is discussed, although the application of such methods is likely to be limited by lack of recording raingauge information.

Methods of combining information from various sources are discussed in terms of information from catchment characteristics supplemented by records, but are generally applicable to different sources of information.

A.2.2.3 Chapter 4: Flood estimation from catchment characteristics

Where no records are available at a site, a preliminary estimate may be made from relations between floods and catchment characteristics. A number of these characteristics were chosen for testing and were measured for those catchments where mean annual flood estimates were available. These characteristics were chosen to be hydrologically relevant, to be as uncorrelated as possible, and to be capable of being measured simply for a large number of catchments.

Physical characteristics which were measured from maps included area and main stream length, two measures of channel slope, and stream frequency or the number of junctions per unit area.

A soil variable was developed by classifying soils according to winter rain acceptance. Maps showing this classification were used to measure the fractions of catchments in each soil class and these were used as the basis of a single soil index. The fractions of each catchment draining through lakes or under urban development also provided useful indices.

While annual average rainfall has been used previously as an index of climate, it was considered desirable to test short term rainfall and, if possible, rainfall excess or RSMD, defined as the 5 year return period value of the daily rainfall minus soil moisture deficit. It was found that the difference between rainfall and rainfall excess did not vary greatly with return period and this difference could be estimated as an effective mean soil moisture deficit. This was mapped and provided RSMD from short term rainfall maps.

The mean annual flood and the coefficient of variation were chosen for regression on catchment characteristics; these were estimated from the annual maximum series. Alternative estimates based on the series of peaks over a threshold were also available and both catchment characteristics and flood statistics are given in the Master List in Volume IV, Chapter 5.

Preliminary regression tests were carried out in order to answer certain questions. Which estimate of the mean annual flood is best related to catchment characteristics? Should all available records be used or should they be selected according to length of record or grade? Should the whole set of records be treated together or can they usefully be divided according to size of catchment or, alternatively, according to region?

The estimates of the mean annual flood derived from the annual maximum series and from the peaks over a threshold series proved interchangeable and the two sets of estimates were equally well related to catchment characteristics. Because the annual maximum series had been derived for some stations where the peaks over a threshold series could not be extracted, the annual maximum estimates were chosen for regression.

The length of record appeared to improve the fit but the regression coefficients remained stable as shorter records were added; hence all records over 5 years were used although the standard error of estimate was increased. Where estimates of the mean annual flood based on extension of short term records were available (Chapter 3) these were used in the regressions. No significant improvement in the regressions resulted when the stations of lower grade were dropped from the analysis. Therefore, all stations of grades A–D were used.

It was found that little would be gained by dividing the records into ranges of catchment size; on the other hand, significant improvements could be made by dividing the country into geographical regions. The best preliminary estimates of the mean annual flood could be obtained by deriving common regression coefficients and different intercepts for groups of regions. The Thames, Lee and Essex region proved an exception in that different variables and coefficients were required.

It proved impossible to relate much of the variation of CV to catchment characteristics so that region curves have to be used for ungauged sites. A small proportion of the variation of CV could be attributed to climate and this is consistent with the growth curves of the meteorological study and with the regional mean CV values.

A.2.2.4 Chapter 5: Estimation of flood volumes

It was found that the relation of the mean annual flood to duration can be described by a reduction curve which can be fitted to actual records and is particularly useful for durations over one day. However, the chosen parameters of these reduction curves were not well related to catchment characteristics.

On the other hand, simple ratios of 3 day and 10 day floods to 1 day floods were well related to channel slope and thus can be used to estimate flood volumes of various durations from calendar day floods. The daily floods can in turn be estimated from catchment characteristics or deduced from records which are easily available. The relationship between calendar day floods and catchment characteristics is similar to that of the instantaneous flood, and depends on the same characteristics except that slope is omitted.

A.2.2.5 Chapter 6: Synthesis of the design flood hydrograph

In parallel with the statistical analysis of flood records, an investigation of catchment response to rainfall was carried out. If an estimated rainfall of a given duration and return period can be converted into runoff, an alternative estimate of the flood of a given return period can be deduced with not only the peak but also the shape of the flood specified. Unit hydrograph techniques allow the problems of predicting runoff volume and timing to be separated by isolating the quick response component of the runoff hydrograph.

The investigation of these prediction methods was based on records of about 1500 rainfall/runoff events on 140 catchments. Consistent rules for hydrograph separation provided estimates of response runoff and made it possible to compare runoff volumes with rainfall amounts and antecedent conditions on these catchments. These comparisons were used in a combined statistical analysis to provide a means of predicting the percentage response runoff in terms of soils and urban fraction, and of antecedent conditions and rainfall amount.

The loss, or the difference between rainfall and response runoff, was distributed through the storm on the assumption that the loss varied inversely with a catchment wetness index (CWI). A comparison of the resulting net rainfall and the response runoff was used to derive a least squares estimate of the unit hydrograph, defined as the response to 10 mm of net rainfall falling in 1 hour. A linear approximation to this unit hydrograph was specified by the time to peak, peak flow, and the width at half the peak, where these three dimensions were highly correlated. No apparent tendency was found within the data for these to vary consistently with runoff volume or rainfall intensity; therefore the average for each dimension was used to represent the catchment in subsequent statistical analysis.

The time to peak was found to be related to channel slope and other factors, with evidence of the effect of urban development. Once the time to peak has been estimated, the remaining dimensions of the unit hydrograph can be directly deduced from it.

Prediction of the runoff volume, together with a prediction of its time distribution, does not provide a direct means of estimating the flood of a given return period; the flood of a given magnitude could result from a number of combinations of different rainfall depths, durations and profiles with different antecedent conditions. A study has shown that it is possible to derive the flood distribution by sampling these various combinations. This approach can be reduced to the choice of a single value for each variable—rainfall (depth, duration and profile) and antecedent condition—to derive the design flood.

This technique offers a gradation of prediction methods according to the information available. If no records exist, the hydrograph can be estimated from catchment characteristics alone. If some records exist, the time to peak can be deduced from the measured lag time and the remaining unit hydrograph dimensions can be derived from this. With more evidence or further analysis, the actual unit hydrograph can be derived and substituted for a synthetic one. From evidence of actual rainfall/runoff events, the runoff volume prediction could be adjusted for any systematic error in the prediction equation for the particular catchment. Either a standard CWI, storm duration and profile could be used, or a simulation study could be carried out to deduce the flood frequency relationship from the rainfall frequency.

The application of this technique to estimating the probable maximum flood requires more conservative assumptions about the antecedent condition, storm profile and unit hydrograph. It is suggested that the profile and catchment wetness index at the start of the design duration should be based on the assumption that the estimated maximum rainfall occurs in all durations centred on the storm peak. The unit hydrograph should be adjusted for exceptional conditions by reducing its time to peak and increasing its peak.

A.2.2.6 Chapter 7: Supplementary studies

As a result of an investigation of snowmelt, typical features of snowmelt runoff in Britain are described by means of an example. It is suggested that rapid snowmelt is usually caused by an influx of warm moist air so that air temperatures provide a reasonable index of the energy available for snowmelt in this country.

Given the water depth corresponding to the initial snowpack and a subsequent temperature sequence, a reasonable forecast of snowmelt runoff can be obtained by simple temperature index methods. The snowmelt index or the ratio of runoff to temperature was found to vary between events on a single basin; it increased with total runoff or initial snowpack content and was related either to catchment area or to channel slope. Because direct measurements are limited, snowpack contents can at present only be deduced from study of snowmelt runoff records. Temperature and snow content frequencies can be combined to give an estimate of snowmelt runoff frequencies which corresponds to direct evidence.

An approximate assessment of snowmelt runoff for an ungauged catchment can be made by deducing a snowmelt coefficient from estimates of snow depth and density and either basin area or channel slope and then multiplying this coefficient by the temperature of a given return period when snow is lying. However, it is suggested that both rainfall and snowmelt floods should be combined in frequency analysis and that separate allowance need only be made for snowmelt in estimating maximum floods.

A new type of nonlinear conceptual model was examined as an alternative to the unit hydrograph method as a means of estimating the flood resulting from a given rainfall on a gauged catchment. The model dealt with isolated storm and runoff events and a selection of the records collected for the unit hydrograph study was used in this research. A simple four parameter model was developed to relate the storm rainfall and initial catchment conditions to the outflow hydrograph. The optimum model parameters for a given catchment are determined by optimisation. The model was applied to 500 events on 21 catchments.

A computer program for routing a flood through a reservoir is presented.

A.2.3 Volume III Flood routing studies

Most flood routing methods have been developed for application to inbank floods: the extension of these methods to the routing of overbank floods in long reaches usually produces erroneous results. The Hydraulics Research Station carried out research into the relevance of existing flood routing methods to British rivers, together with the development of a new method for routing floods in rivers with extensive flood plains.

The simple storage–routing methods, such as the Muskingum method, have been confirmed as being sufficiently accurate to route inbank floods; the diffusion method also gives accurate results. In addition the formula developed by Forchheimer for the attenuation along the reach of the peak level or discharge for a flood can similarly be used with confidence if the predicted attenuation is less than 10% of the original peak discharge.

When a flood inundates a flood plain associated with the river, the values of the parameters used by these methods for an overbank flood can be considerably different from the values of the same parameters for an inbank flood in the same river. A new flood routing method presented in this volume uses a parameter which allows the speed of the flood wave to vary with discharge and a second parameter which describes the effect on a flood wave of the irregularities in the channel geometry.

When the inundated area for a particular flood is known, as is usually the case for the largest recorded flood in the river, the value of the second parameter can readily be calculated from a survey map. Similarly this parameter can be found for a bankfull flood. However, it is usually more difficult to determine the intermediate values of this parameter and these have then to be estimated.

This flood routing method has a number of practical applications. For example, if an onstream reservoir is to be sited well downstream in a river system, the method can readily be used to route a flood hydrograph from an upstream section to the reservoir site to assist in the hydraulic design of the spillway. The method could also be used to investigate the effect of flood alleviation schemes on the propagation of a flood, or could possibly be adapted for inclusion in an online flood warning scheme.

A.2.4 Volume IV Hydrological data

The basic hydrological data collected and used during the investigation are presented or summarised. These comprise lists of gauging stations and their gradings and catchment characteristics, flood statistics, the basic peak flow records for 530 stations, with historical records where these were found, and summaries of 1500 events used in catchment response studies.

An account of the work involved in assembling these records is presented as a guide to the available data, to enable the user to judge its reliability, and as a background to the analysis presented in Volume I. This account is divided into the collection and appraisal of records, the extraction and processing of flow records used in statistical analysis, and the selection, extraction and processing of the data used in unit hydrograph analysis. This account is followed by the basic data.

A.3 Choice of methods of estimation

The specification of the design flood is outside the terms of reference of this study, but must be determined by the design engineer in the light of policy or of a code of practice, or following an economic appraisal. The return period T of the design flood $Q(T)$ is determined by the risk r which is accepted of the design flood being exceeded during the life L of the structure or project. They are related by $r = 1-(1-1/T)^L$, (Equation 2.2.2.1), where T and L are in years. The cost of designing for the flood of a given return period or frequency may be compared with the risk and cost of failure. Economic criteria may be overridden by policy decisions in some circumstances; if no appreciable risk of exceedance of the design flood can be accepted, then an estimate of the maximum flood may be required. The designer may also wish to allow for errors of estimate of $Q(T)$ due to uncertainty of choice of model or of its parameters.

Methods of estimating a design flood, based on the current investigation, are presented in this report. There are a number of routes by which an estimate may be reached, and an attempt is made to show in what circumstances one has advantages over another. It is not always possible, or even desirable, to eliminate alternative methods of estimation and so methods of combining estimates are presented. While this study has taken advantage of the large number of continuous flood records now available, the reliability of the results would benefit from continued analysis as record lengths increase; although accepted techniques of analysis were used in order to provide estimates for the ungauged site, research will continue to provide new methods of flood estimation to supplement these design methods.

The two main routes of estimation are through statistical analysis of peak flows and through unit hydrograph synthesis of the flood corresponding to a design storm. The choice between these two depends largely on two questions:

a Is the hydrograph, or the detailed shape of the flood, required in addition to the instantaneous peak flow, as in the case where the flood will need routing through a reservoir?

b Is an estimate of the maximum flood required, rather than an estimate of the flood of a given frequency or return period?

If the answers to these questions are that neither the hydrograph nor the maximum flood is needed, then a choice of methods is available. If the answer to either question is yes, then unit hydrograph methods are necessary. These choices may be subdivided according to whether there are long, short, or no records at the design site or nearby on the same river. These are summarised in Figure A.1, on which the following discussion is based.

If the flood $Q(T)$ of a finite return period T is required, and a detailed hydrograph is not needed, the statistical approach provides the simpler technique. The choice of statistical distribution as well as the estimation of the constants or parameters of that distribution could be based on the flood record at the design site if this were sufficiently long. However, in practice the choice of distribution should be based on the study of a number of long term stations. The distributions which have been used in hydrology are described in Chapter 1, and it is shown in Chapter 2 that there is little to choose between the three parameter distributions on goodness of fit tests. Because of its consistency in these tests and because there is some theoretical basis for the choice, the general extreme value distribution is recommended for use. Region curves based on this distribution have been derived from all the annual maximum records in each region; these may be used to derive $Q(T)$ from the mean annual flood $\bar{Q}$ for a site.

Where no records exist, a preliminary feasibility estimate of $\bar{Q}$ and thus $Q(T)$ may be made from catchment characteristics. However, a gauge should be installed as soon as a project is proposed in order to improve this estimate.

Where less than 10 years of record exist, the record could be extended by correlating the monthly maximum flows with adjacent records and the arithmetic mean of the extended annual maximum series should be compared with the mean annual flood derived from the series of peak flows over a threshold.

Where there are 10–25 years of record, the arithmetic mean should be derived from the annual maximum series of the highest flows in each year. The appropriate region curve should be used to derive the flood frequency curve for the site.

If there are more than 25 years of record, the general extreme value distribution may be fitted to the annual maximum series by methods des-

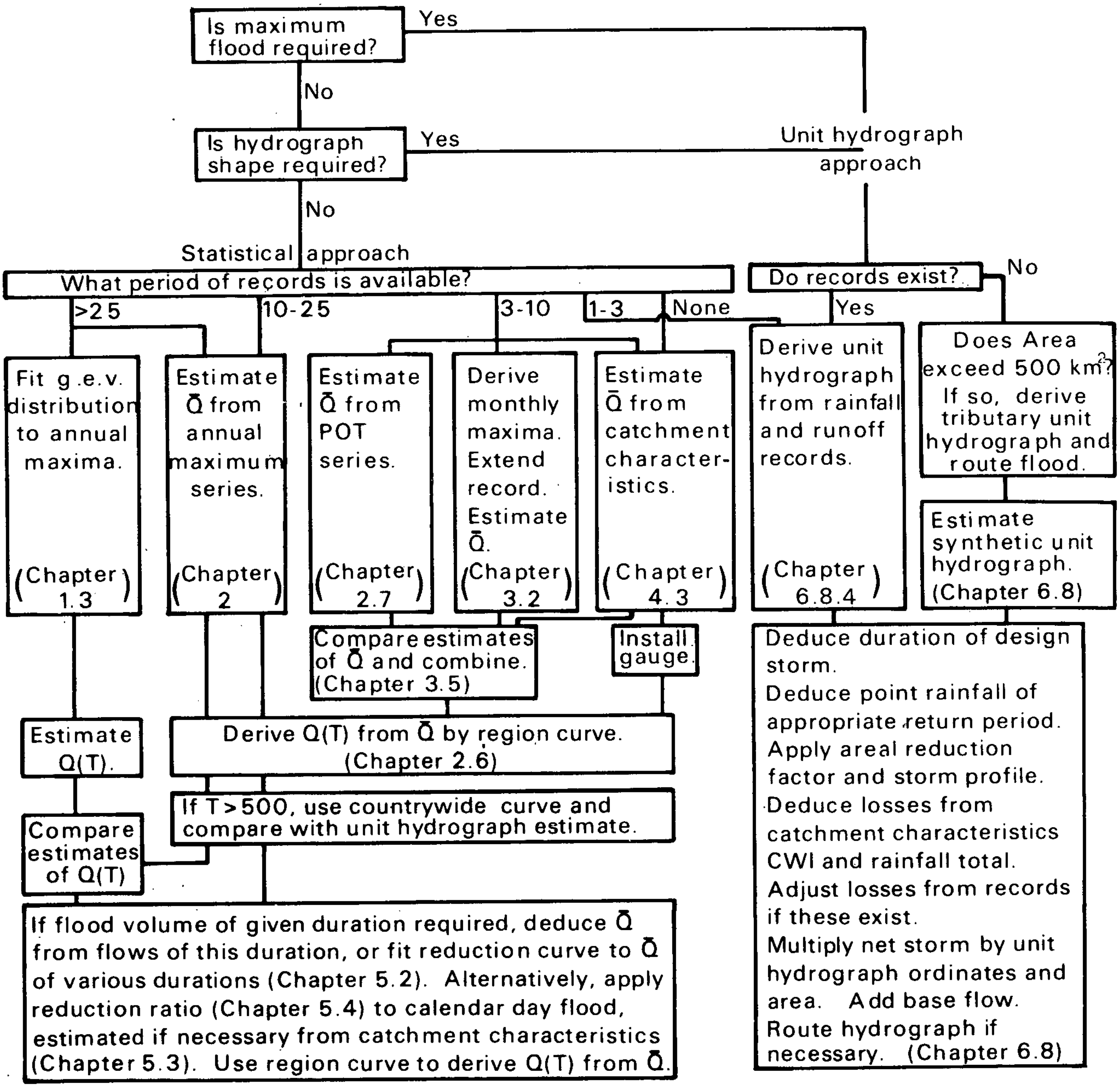


Fig A.1 Estimation of design flood.

cribed in Chapter 2. The result should be compared with the estimates obtained by estimating the mean annual flood and using a region curve.

If a flood volume over one or more days is required rather than the instantaneous flood, this may be obtained by analysing the annual maximum flows over the given duration, or by analysing several durations and fitting a reduction curve to these results. If no records exist, a mean annual calendar day flood can be estimated from catchment characteristics together with the ratio of longer duration floods to the calendar day flood. The same region curve may be used to extend the estimate to a given return period.

Where there is a need to specify the shape of the flood or to estimate the maximum flood required then the unit hydrograph approach is appropriate; the procedure is rather more complex but the number of alternatives is less. The unit hydrograph for the design site should be derived if possible from rainfall and runoff records but may be estimated from catchment characteristics. The duration of the design storm is given by the unit hydrograph dimensions and the point rainfall is estimated for a return period which depends on the return period of the design flood. An areal reduction factor is applied to give the catchment rainfall total, and a time profile is provided.

The proportion of this storm which provides immediate runoff is calculated from an equation involving soils, an appropriate antecedent condition and the rainfall total. This estimate can be adjusted to take account of records at the site. The net storm is multiplied by the ordinates of the unit hydrograph to give the design flood, with base flow added.

This technique may be supplemented by a simulation procedure which gives estimates based on sampling the various conditions rather than using a single choice. Because unit hydrographs may be derived from a short period of record, these techniques provide a comparison with the statistical approach for short records. For return periods over 500 years, where region curves become increasingly ill-defined, unit hydrograph techniques should be used to supplement statistical estimates based on the countrywide curve.

An estimate of the maximum flood follows similar lines except that more conservative assumptions should be made and an allowance is required for snowmelt.

If the catchment greatly exceeds 500 km^2, extra care should be taken; the unit hydrograph should be derived from events with reasonably uniform rainfall. Where this is not possible, or where the records do not exist and a synthetic unit hydrograph is required, it will be necessary to estimate design floods at sites on tributaries and to route these down to the design site by appropriate flood routing methods.

A.4 Summary of methods of estimation

A brief summary of the main suggested methods of flood estimation is presented as an introduction to the general reader. This is not intended to replace the recommendations and examples given in individual volumes and chapters, to which the user should turn for guidance about the techniques and their limitations.

A.4.1 Derivation of $\bar{Q}$ from catchment characteristics (Chapter 4)

Where no records exist at a site, then the mean annual flood $\bar{Q}$ (in cumecs) can be roughly estimated from catchment characteristics defined in Table A.1 on p.22.
The average countrywide equation is

$$\bar{Q} = 0.0201\ \text{AREA}^{0.94}\ \text{STMFRQ}^{0.27}\ \text{S1085}^{0.16}\ \text{SOIL}^{1.23}\ \text{RSMD}^{1.03}\ (1+\text{LAKE})^{-0.85}$$

but an improvement in the estimate will result from the use of regional multipliers instead of 0.0201 as follows:

Region	Region no.	Hydrometric areas	Regional multiplier to replace 0.0201 above
Northern Scotland	1	1–16, 88–97, 104–108	0.0186
East Anglia	5	29–35	0.0153
South Coast	7	40–44, 101	0.0234
South west England	8	45–53	0.0315
Central region	2, 3, 4, 9, 10	17–28, 54–87, 102	0.0213
Ireland			0.0172

For the Thames, Lee and Essex areas (Region 6, hydrometric areas 36–39) the appropriate equation is

$$\bar{Q} = 0.302\ \text{AREA}^{0.70}\ \text{STMFRQ}^{0.52}\ (1+\text{URBAN})^{2.5}.$$

It is recommended that, when a design flood is required for an ungauged site, a gauging station and raingauges should be installed so that a direct estimate can be made by unit hydrograph or statistical methods before a final figure is required.

A.4.2 Estimation of $\bar{Q}$ by extension of short record (Chapter 3)

Given a short record (say 3–10 years) of flows at a site, this may be adjusted for short term fluctuations as follows. Extract the monthly maximum instantaneous flows at the short term station and at nearby long term stations; daily rainfall totals and daily flows or even levels may be included in the long term set. Normalise the data where necessary by taking logarithms, and derive a regression equation for the short term station using the useful long term stations. The choice of regression equation should be made on statistical criteria, supplemented by the use of correlation maps. It will probably be necessary to derive several regression equations for different parts of the extension period in order to use all the information available.

These regression equations are used to extend the short term monthly maxima backwards until the number of closely related long term stations has fallen to make the prediction equation unreliable, as shown by multiple correlation coefficients. From each complete year of extended record, the highest of the monthly maxima provides an estimate of the annual maximum and the complete series of annual maxima is used to derive the mean annual flood $\bar{Q}$. It is important to note that this extended series would provide a biassed estimate of the variability of annual maxima.

A.4.3 Estimation of $\bar{Q}$ from POT series (Chapter 2)

Choose a threshold flow q_0 such that on average about three to five peaks a year exceed this threshold. Extract all independent instantaneous peak flows above the threshold, using as a rule for independence that peaks should be separated in time by three times the time to peak and that the flow should decrease between peaks to two thirds of the first peak. List the magnitudes q_i of the M exceedances in N years of record.

A simple model treats the number of exceedances per year as a Poisson variate whose parameter λ is given by

$$\hat{\lambda} = M/N,$$

and their magnitudes are treated as an exponential distribution whose parameter β is estimated by

$$\hat{\beta} = \bar{q} - q_0 = \sum_{i=1}^{M} (q_i - q_0)/M.$$

Then the T year flood $Q(T)$ may be estimated from

$$Q(T) = q_0 + \hat{\beta}\ln\hat{\lambda} + \hat{\beta}\ln T$$

and specifically $\bar{Q}$ may be estimated from

$$\bar{Q} = q_0 + \hat{\beta}\ln\hat{\lambda} + 0.5772\hat{\beta}.$$

It is shown in Section 2.7 that these models are equivalent to the double exponential distribution of the annual maxima. This distribution is not always appropriate but the method may be used in all cases for estimating the mean annual flood.

A.4.4 Estimation of $\bar{Q}$ from annual maxima (Chapter 2)

Where an adequate period of records exists, say over 10 years, the mean annual flood may be estimated directly from the annual maxima. The maximum instantaneous flows in each year of record provide the annual maximum series and its arithmetic mean may be used as an estimate of the mean annual flood.

Where this series includes an outlier, such that $Q_{max} > 3Q_{med}$, then the mean annual flood should be estimated (see 2.3.5) from the median annual maximum Q_{med} using

$$\bar{Q} = 1.07Q_{med}.$$

A.4.5 Estimation of mean annual flood of various durations (Chapter 5)

The corresponding methods of statistical analysis described for instantaneous flood records may be applied to daily flows or flows of several days' duration to give estimates of the mean annual flood for these durations. Alternatively, analyses of flows of several durations could be used to derive a reduction curve for a station from which flood volumes corresponding to different durations could be interpolated. The method of analysis should depend on the period of records available in the same way as the method of analysis of instantaneous flows.

Where no records exist at a site the mean annual calendar day flood may be estimated in cumecs from

$$\text{CALMAF} = m\ \text{AREA}^{0.9475}\ \text{STMFRQ}^{0.4068}\ \text{RSMD}^{0.6280}\ \text{SOIL}^{0.7102}$$

with regional multipliers m as follows:

Region	Hydrometric areas	Multiplier
1	1–16, 88–97, 104–108	0.0395
2	17–21, 77–87	0.0417
3	22–27	0.0410
4	28, 54	0.0360
5	29–35	0.0279
6	36–39	0.0250
7	40–44, 101	0.0428
8	45–53	0.0585
9	55–67, 102	0.0539
10	68–76	0.0459

The mean annual flood of longer duration may be estimated from CALMAF by reference to the two ratios of 3 day and 10 day flood to daily flood, each defined as a flow over the given duration, and to families of reduction curves; AR3 and AR10 are given by

$$\log_{10}(\text{AR3}) = -0.101 - 0.081 \log_{10}(\text{S1085})$$
$$\log_{10}(\text{AR10}) = -0.269 - 0.127 \log_{10}(\text{S1085}).$$

A.4.6 Derivation of flood of return period T from mean annual flood (Chapter 2)

Given an estimate of the mean annual flood $\bar{Q}$, the flood of return period T may be estimated from a set of growth curves based on all the records of a region. A countrywide curve has also been derived, but it is considered that account should be taken of regional differences by using region curves for values of T below 500. These curves are given in full in Table 2.39, but the table below gives summaries of $Q(T)/\bar{Q}$ for different regions and values of T.

Region	T (years)		
	10	100	200
1	1.45	2.48	2.89
2	1.42	2.63	3.18
3	1.45	2.08	2.27
4	1.49	2.57	2.98
5	1.65	3.56	4.46
6/7	1.62	3.19	3.86
8	1.49	2.42	2.74
9	1.42	2.18	2.45
10	1.38	2.08	2.32
Ireland	1.37	1.96	2.14

These curves were derived from instantaneous flows, but may also be applied to floods of longer durations where they may be somewhat conservative.

A.4.7 Estimation of Q(T) from annual maxima (Chapter 2)

Where ample records exist, it is possible to estimate $Q(T)$ directly from annual maximum flows. Given the set of annual maxima comprising the highest instantaneous flows in each year of record, it is suggested that a general extreme value distribution should be fitted to this set. The three parameters, u, α and k of this distribution may be estimated graphically or by moments, sextiles or maximum likelihood as described in Section 1.3.4. However, it will be extremely rare for records to be adequate to provide reasonable estimates of three parameters. It is therefore suggested that, for records between 10 years and 25 years, an extreme value Type 1 distribution may be used for return periods up to $2N$, where N is the number of years of record. For records over 25 years, a general extreme value distribution may be fitted for return periods up to $2N$. In both cases region or countrywide curves should be used for higher return periods.

An alternative is to fit a general extreme value distribution with shape factor k corresponding to the region curve and estimate the other two parameters from the records by moments.

A.4.8 Estimation of Q(T) using unit hydrograph methods (Chapter 6)

The detailed procedure is given in Section 6.8 with a numerical example, and is only briefly summarised here.

The time to peak in hours of the unit hydrograph may be estimated from

$$T_p = 46.6(\text{MSL})^{0.14}(\text{S1085})^{-0.38}(1+\text{URBAN})^{-1.99}(\text{RSMD})^{-0.4}$$

where there are no records. Where some records exist, T_p may be estimated more reliably from the lag of the catchment, or the time from the centroid of rainfall to the peak runoff or centroid of peaks, using

$$T_p = 0.9\text{LAG}$$

or by deriving a unit hydrograph from records of rainfall and runoff. The peak of the unit hydrograph Q_p in cumecs /100 km² is estimated from $Q_p = 220/T_p$ and its time base as $2.525T_p$.

The duration D of the design storm depends on T_p and the mean annual rainfall SAAR, by

$$D = (1+\text{SAAR}/1000)T_p.$$

The return period of the design storm is deduced from the return period of the design flood using Figure 6.61. The mean point rainfall of duration D and return period 5 years is derived for a given catchment from the mean point rainfall of 2 day duration and 5 year return period (2 day M5) given by Figure II.3.2 and the ratio M5(D)/2 day M5 (Table II.3.10). Then the growth factor MT/M5 (Table II.2.7) is used to estimate the mean point rainfall of appropriate return period. This is reduced to the catchment rainfall P using an areal reduction factor (Figure II.5.1), and the catchment storm of duration D is distributed in time by the standard profile (Figure 6.65).

The appropriate catchment wetness index CWI is estimated from Figure 6.62 and the percentage runoff is estimated from

$$\text{PR} = 95.5\ \text{SOIL} + 12\ \text{URBAN} + 0.22\ (\text{CWI} - 125) + 0.1(P - 10).$$

This percentage runoff may be applied to the design storm, and the resulting ordinates are multiplied by the unit hydrograph, adjusted for catchment area. The nonseparated or base flow in cumecs/km² may be added from

$$\text{ANSF/AREA} = 0.00033\ \text{CWI} + 0.00074\ \text{RSMD} - 0.038.$$

Where records exist not only can the unit hydrograph be deduced but also the PR may be modified from comparisons of rainfall and response runoff. A simulation study may be carried out to overcome the oversimplification of a single choice of each variable; a large number of hydrographs are generated by sampling from known distributions of the storm and catchment condition variables, and these hydrographs are then analysed as though they represented a sample of peak flows.

When maximum flood estimates are required, this approach is modified through the rainfall total and profile, while T_p should be reduced and thus Q_p increased, and a CWI of 125 is selected for the start of the storm. An allowance for snowmelt runoff is added.

Where the design catchment exceeds 500 km² the unit hydrograph method should be used with care. Where records exist, the unit hydrograph must be derived from events with reasonably uniform rainfall. If this is not possible, design floods may be estimated for sites on tributaries and routed down to the design site. The choice of flood routing method depends on an estimate of the attenuation of the peak flow (Volume III, Section 5.1) and this estimate, together with the design requirements, makes it possible to decide (Figure III.4.1) whether the Muskingum–Cunge method (III.5.2) is adequate or whether the variable parameter diffusion method (III.5.3) is necessary.

A.5 Compatibility of two main approaches

Methods of combining estimates from different sources are described in Chapter 3. Although estimates of the mean annual flood from catchment characteristics and from a small sample provide an example, the principle can be extended to other types of estimate including unit hydrograph predictions.

The methods suggested for estimating floods by combining rainfall frequency analysis with unit hydrograph techniques are consistent with the direct statistical analysis of floods. Because the same set of catchment characteristics was used, the similarity between the two main approaches is best illustrated by their extension to ungauged sites.

The mean annual flood of 1 day duration is predicted as a product of area, net short term rainfall, soil index; of stream frequency with a small exponent; and of an index for lakes where they exist. An equation involving channel slope relates flood volumes to instantaneous floods, which are in turn predicted by an equation whose main difference from the daily flood equation is the inclusion of a channel slope term. Likewise, the runoff volume estimated from the rainfall/runoff approach is a product involving area, short term rainfall and percentage immediate runoff, which itself depends mainly on the soil index but with adjustment for mean catchment condition during the storm. Runoff volume is transformed to instantaneous peak by means of the unit hydrograph and this, like other measures of response shape, is largely determined by channel slope.

The fraction of urban development was found to increase the instantaneous flood where sufficient urban catchments provide evidence, and has a similar effect on the unit hydrograph peak. It appears to have some effect on the percentage immediate runoff but not on daily floods.

These relationships may be illustrated by equations from various chapters.

Mean annual flood peak:

$$\text{BESMAF} = 0.0201\ \text{AREA}^{0.94}\text{RSMD}^{1.03}\text{SOIL}^{1.23}\text{STMFRQ}^{0.27}\text{S1085}^{0.16}(1+\text{LAKE})^{-0.85}$$

Mean annual calendar day flood:

$$\text{CALMAF} = 0.0056\ \text{AREA}^{0.97}\text{RSMD}^{1.09}\text{SOIL}^{0.49}\text{STMFRQ}^{0.46}$$

Ratio of 10 day to calendar day flood:

$$\text{AR10} = 0.54\ \text{S1085}^{-0.13}$$

Peak of unit hydrograph:

$$Q_p = 220/T_p = 4.71\ \text{S1085}^{0.38}\text{RSMD}^{0.42}\text{MSL}^{-0.14}(1+\text{URBAN})^{2.00}$$

or alternatively

$$Q_p = 9.71\ \text{S1085}^{0.60}(1+\text{URBAN})^{2.03}$$

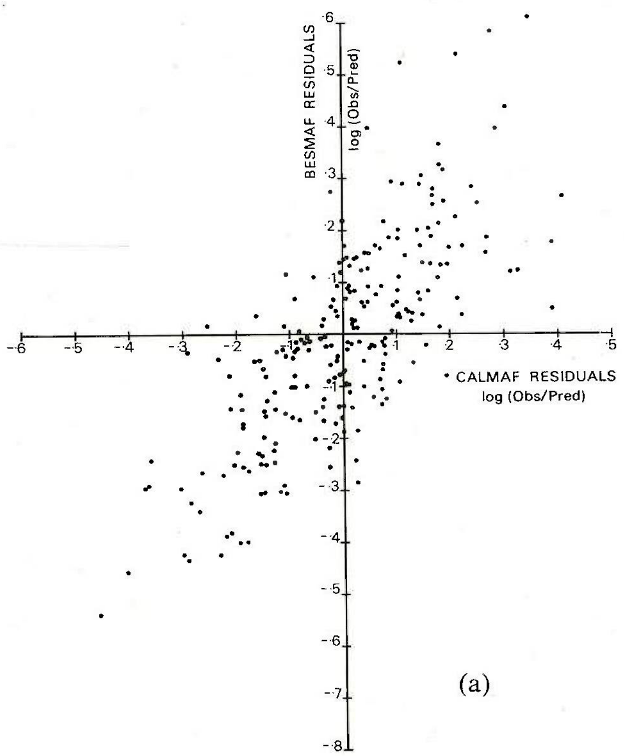
Standard percentage immediate runoff:

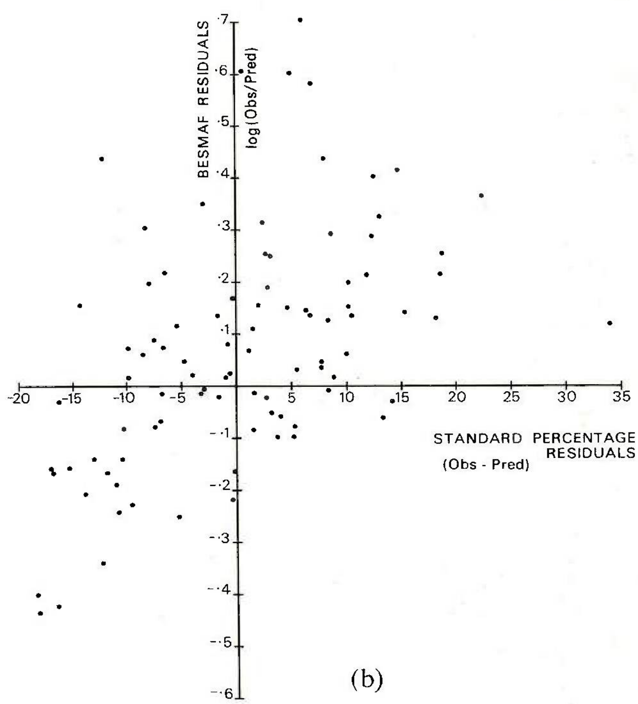
$$\text{SPR} = 95.5(\text{SOIL}) + 12(\text{URBAN})$$

Where there is correlation between catchment characteristics, as between stream frequency and soil, or between slope and rainfall, it must be stressed that these are regression equations rather than functional relationships. However, these somewhat subjective points of similarity between the statistical and rainfall/runoff approaches are supported by comparison between regressions using observed and synthetic floods in Section 6.7.

The residual errors from these equations are fairly large, and indirect evidence suggests that these are not mainly due to sampling errors because the elimination of short records or their indirect extension do not greatly reduce the residual error of the prediction of mean annual flood. Similarly, it seems unlikely that errors of measurement are mainly responsible, because use of the better graded stations alone does not much improve the predictions. The regional patterns of the residuals suggest that some further real factor affects the mean annual flood, and in fact that the residual errors from the statistical approach are linked with those from the unit hydrograph approach. The residuals from this equation for the mean annual instantaneous flood (BESMAF) are compared in Figure A.2(a) and (b) with those from the equations for the mean annual daily flood (CALMAF) and the standard percentage immediate runoff (SPR), using all the common gauging stations. The residuals are of the same order of magnitude.

Fig. A.2 Residuals from equation predicting mean annual instantaneous flood (BESMAF) related to (a) residuals from equation predicting mean annual calendar day flood (CALMAF), and (b) residuals from equation predicting standard percentage runoff (SPR).





The BESMAF and CALMAF residuals are closely linked, and the BESMAF residuals are related to the SPR residuals; most of the outliers in Figure

A.2(b) are from urban catchments. The residuals from predicted unit hydrograph dimensions were not well related to the other residuals. This suggests that the main source of the errors lies with the prediction of runoff volume and specifically of percentage response runoff, as found in the testing of unit hydrograph prediction (6.6.2). Channel storage is unlikely to be responsible. Although some errors in all three may be caused by rating curve errors, or even by errors in catchment area (the area of 28/801 was found to be 9.1 km^2 rather than the accepted 13 km^2 after the end of the investigation), such errors are unlikely to display the geographical pattern of the residual maps. The conclusions must be that prediction errors in adjacent or similar catchments may be taken into account when flood estimates are required for ungauged sites, that a short period of flood record will greatly improve estimates, and that further investigation should concentrate on the prediction of percentage runoff.

In both the statistical approach and the unit hydrograph approach the increase of flood magnitude with return period is related to a growth curve based on records. The growth curves in the statistical analysis are based on recorded ratios of $Q(T)/\bar{Q}$ by compiling a mean growth curve from the measured instantaneous flows of a region. Rainfall growth curves based on countrywide records are used in estimating a flood frequency curve by unit hydrograph methods, though they are subsequently modified by prediction of the response of the individual catchment. These two sets of curves have similar shapes and both vary consistently from one region to another; in other words regions with high rainfall growth factors tend to have high flood growth factors.

Table A.1 Brief definitions of catchment characteristics.

MSL	km	Length of main stream measured on 1/25000 map in 0.1 km steps
S1085	m/km	Main stream slope between 10 and 85% upstream from gauge
URBAN	fraction	Fraction of catchment shown as urban on topographical map; usually used in form 1+URBAN
AREA	km^2	Area of catchment
SAAR	mm	Annual average rainfall
LAG	hours	Lag between centroid of rain and peak of runoff or centroid of peaks
SOIL	fraction	Weighted soil fraction, derived from fractions of soil 1–5 by SOIL = (0.15 SOIL 1+0.30 SOIL 2+0.40 SOIL 3+0.45 SOIL 4+0.50 SOIL 5)/(SOIL 1+SOIL 2+SOIL 3+SOIL 4+SOIL 5)
RSMD	mm	1 day rainfall of 5 year return period, derived from 2 day M5×ARF (areal reduction factor), with SMD subtracted
STMFRQ	number/km^2	Number of stream junctions (including outfall)÷AREA

A.6 References

BINNIE G.M. & MANSELL-MOULLIN M. (1965) The estimated probable maximum storm and flood on the Jhelum River—a tributary of the Indus. *ICE Symposium, River Flood Hydrology*, publ. 1966, 189–210.

BISWAS A.K. & FLEMING G. (1966) Floods in Scotland: magnitude and frequency. *Water and Water Engineering*, **70**, (844), 246–252.

CHAPMAN E.J.K. & BUCHANAN R.W. (1965) Frequency of floods of 'normal maximum' intensity in upland areas of Great Britain. *ICE Symposium, River Flood Hydrology*, publ. 1966, 65–86.

COLE G. (1965) An application of the regional analysis of flood flows. *ICE Symposium, River Flood Hydrology*, publ. 1966, 39–57.

INSTITUTION OF CIVIL ENGINEERS (1933) *Interim Report of the Committee on Floods in Relation to Reservoir Practice*, 40 pp.

INSTITUTION OF CIVIL ENGINEERS (1960) *Floods in Relation to Reservoir Practice* (Reprint of the Interim Report of the Committee on Floods, 1933, with additional data on floods recorded in the British Isles between 1932 and 1957), 66 pp.

INSTITUTION OF CIVIL ENGINEERS (1960) Subcommittee on Rainfall and Runoff. Floods in the British Isles. *Proceedings of the Institution of Civil Engineers*, **15**, 119–144.

INSTITUTION OF CIVIL ENGINEERS (1967) Flood Studies for the United Kingdom. *Report of the Committee on Floods in the United Kingdom*, February, 38 pp.

LYNN M.A. (1971) Flood estimation for ungauged catchments. *Transactions of Institution of Engineers of Ireland*, **96**, 154–176.

MORGAN H.D. (1965) Estimation of design floods in Scotland and Wales. *ICE Symposium, River Flood Hydrology*, publ. 1966, 59–64.

NASH J.E. (1960) A unit hydrograph study, with particular reference to British catchments. *Proceedings of the Institution of Civil Engineers*, **17**, 249–282.

NASH J.E. & SHAW B.L. (1965) Flood frequency as a function of catchment characteristics. *ICE Symposium, River Flood Hydrology*, publ. 1966, 115–136.

NASH J.E. (1966) Applied flood hydrology. *River Engineering and Water Conservation Works*, Ed. by R.B. Thorn, Chap. 6. Butterworth, London.

REEVE W.T.N. & EDMONDS D.T. (1965) Zambezi River flood hydrology and its effect on design and operation of Kariba Dam. *ICE Symposium, River Flood Hydrology*, publ. 1966, 169–187.

SUTCLIFFE J.V. (1971) Flood Studies at the Institute of Hydrology, Wallingford. *Journal of the Institution of Water Engineers*, **25**, 160–162.

WOLF P.O. (1965) Comparison of methods of flood estimation. *ICE Symposium, River Flood Hydrology*, publ. 1966, 1–23.

1 Statistics for flood hydrology

1.1 Statistical terms and definitions

1.1.1 Introduction

The statistical method is well established among the various approaches towards the specification of a design flood. The method is familiar to many hydrologists but the occasional user may still be confused by the variety of notations used as well as the difficulties of the subject matter. In this chapter one aspect of statistics, namely the specification of distributions and estimation of parameters, is treated in a consistent notation so that relations between the hydrologically relevant distributions can be seen easily.

Section 1.1 describes or defines the major statistical features with which familiarity is assumed in the remainder of the chapter. Section 1.2 outlines the distributions of use in flood hydrology; Normal, exponential, gamma, Pearson Type 3, extreme value Types 1, 2 and 3, lognormal, log Gumbel and log Pearson Type 3. Methods of estimation of parameters and quantiles are described in Section 1.3 and applied to each distribution in turn. Finally, in Section 1.4 standard errors of parameter and quantile estimates are discussed for a few well known cases.

This chapter is intended to complement rather than replace the formal study of statistical methods from appropriate statistical courses and text books. It may be treated as a reference; the results of applying these methods are presented not in this chapter but in Chapter 2.

1.1.2 Events and probability of events

The occurrence of a flood peak of 100 cumecs on a 10 km^2 catchment in Britain is rare but to say this alone is insufficient for hydrological purposes. It is necessary to know during the design and planning of hydraulic works whether it is worthwhile designing against 100 cumecs or whether a design figure of 80 or 120 cumecs would be appropriate. The study of the value of any course of action requires a scale of rarity for flood peak magnitudes. The decision maker would want to know whether a peak of 120 cumecs could not occur during the next 50 years or whether it could occur once, twice or three times. A simple yes or no cannot be given to these enquiries because empirical knowledge shows that there might be no occurrence of such a peak during one 50 year period while there might be four occurrences during another such period. The information has to be given as the probability of none, one, two . . . or more occurrences during the 50 years.

The definition of probability can be considered from the point of view of psychology, philosophy or mathematics but the mathematical rules to be followed once the foundation has been laid are the same regardless of the approach followed. It is sufficient to draw a distinction between two types of probability, namely degree of belief probability (also known as personal or subjective probability) and relative frequency probability. It is customary to refer to the probability of one event A relative to another basic or conditioning event B, but the conditioning event is often omitted. In hydrology the conditioning event is often the passage of a period of time, for instance a year. If A is the event that a flow exceeding 120 cumecs occurs and B is the passage of a year the *relative frequency probability* of A given B is

$$PR(A/B) = \lim_{M \to \infty} m(A)/M \tag{1.1.2.1}$$

where $m(A)$ is the number of years in which A occurs.

In some cases the event B is such that it cannot be repeated but it is still legitimate to speak of the probability that the event A accompanies it. This is not a relative frequency probability because, since there are not several B events, Equation (1.1.2.1) is inapplicable. The truth is that A either does or does not occur with B but before one of these cases is proved there can be varying degrees of opinion supporting either proposition. If these are expressed as numbers between 0 and 1 they are called *subjective* or *degree of belief probabilities*. Such a probability is dependent on the information available to the subject who expresses it. For example, let B be the moon and A the proposition that there are oceans on it. In 1970 an informed man would say that $PR(A/B)$ is small whereas in 1770 his counterpart would have said that it was large.

Returning to earth the two types of probability can be viewed side by side at some site on a river. In one example let B be a year and A the event that the average flow during a year is >50 cumecs; then $PR(A/B)$ has a relative frequency interpretation because B may be observed repeatedly. In another let B be the next 100 years and A the proposition that the average flow during B is >60; here $PR(A/B)$ is necessarily a subjective probability because B may not be observed repeatedly. However, the information on which $PR(A/B)$ is based could, by the application of statistical theory, be obtained from relative frequency probabilities estimated from observed yearly flows. Another example of the distinction between subjective and relative frequency probabilities is given in Section 3.5.3.

The algebraic manipulation of probabilities and indeed of all statistical mathematics rests on three axioms

i $0 \leqslant PR(A/B) \leqslant 1$

ii If $A_1, A_2, \ldots A_n$ are exclusive given B then $PR(A_1 \text{ or } A_2 \text{ or } \ldots A_n/B) = PR(A_1/B)+PR(A_2/B)+\ldots PR(A_n/B)$

iii $PR(C/AB)PR(A/B) = PR(AC/B)$.

In *ii* the fact that A_i and A_j are exclusive means that they cannot both occur on the same population unit. Axiom *ii* is often called the addition law of probabilities while *iii* is called the multiplication law of probabilities; each occurrence of B may be accompanied by either of the events A or C or by both of them simultaneously. If $PR(C/AB) = PR(C/B)$ so that $PR(AC/B) = PR(A/B).PR(C/B)$ then A and C are said to be *statistically independent*.

1.1.3 Statistical populations

In flood frequency analysis a basic idea is that of a statistical population. The population consists of units. A unit in this context is usually a unit of time such as a day, month, year, decade or century. More exactly perhaps a unit is the streamflow process over a unit of time. With each population unit may be associated a variable such as maximum instantaneous flood during that period of time or mean flow during the period. A streamflow record is considered to be a collection of such population units drawn randomly from the entire population. When a unit is understood to be drawn randomly from the population its variable value is called a random variable or variate. A random variable is characterised by its probability distribution and this may usually be described in two ways:

a By its distribution function (df) $F(x)$ which is the probability that the variate value of a unit drawn at random from the population is less than x,

$$F(x) = PR(X \leqslant x) \tag{1.1.3.1}$$

and whose derivative $F(x)$ is called the probability density function (pdf),

$$f(x) = \frac{dF(x)}{dx}. \tag{1.1.3.2}$$

b By expressing the variate x as a function of another variate y

$$x = H(y) \tag{1.1.3.3}$$

where the df $F(y)$ of y is known.

The most common use of *b* is when

$$x = a + by \tag{1.1.3.4}$$

where a and b are the location and scale parameters (see Section 1.1.4 below) of the x distribution, and the y distribution has location and scale parameters which are 0 and 1 respectively; in this case y is called a *standardised or reduced variate* with respect to x.

Method *a* is the most convenient in theoretical statistics but method *b* forms the basis of probability plotting of observed statistical data and therefore plays a part in practical if not in theoretical statistical manipulations.

A population may be finite or infinite and a random variable may be discrete or continuous. The finiteness of a population refers only to the number of units in it. The population of rivers in a country is finite but the population of cross-sections along any reach of river is infinite. A random variable value occurs with each unit regardless of how many units there are in the population. Values of river flow are treated as continuous variates whereas the number of flood peaks exceeding a threshold in a year is treated as a discrete variate. The number of values which a continuous variate may assume is infinite while the number of values which a discrete variate may assume is usually finite in practice.

If on each population unit there are two variables of interest x and z, in addition to speaking of distribution functions $F(x)$ and $G(z)$ it is possible to speak of their joint distribution function $\Psi(x,y)$ defined by

$$\Psi(x, y) = PR(X \leqslant x \text{ and } Y \leqslant y). \tag{1.1.3.5}$$

This is a *bivariate* or *joint distribution.*

1.1.4 Features of distributions

Parameters. Many of the distribution functions commonly used in hydrology are not specified uniquely by the functional form. For instance the extreme value distribution

$$F(x) = \exp\left[-e^{-(x-u)/\alpha}\right] \tag{1.1.4.1}$$

depends on the constants u and α as well as on the form of the function. These constants u and α are called *parameters* of the distribution. In this example u may have any value, positive or negative, while α may have positive values only.

Every possible pair of parameter values (u,α) specifies the distribution of a particular population. There is thus a family of distributions having the distributional form (1.1.4.1). The majority (99.85%) of variate values lie between $x = u - 2\alpha$ and $x = u + 7\alpha$. The parameter u locates the position of the population variate on the x axis and hence it is often referred to as a *location parameter*. The parameter α controls the spread of the variate values on either side of $x = u$ and is often referred to as a *scale parameter*.

Mean, median and mode. The *mean* is the average variate value on all the population units. It is also called the *expected value* of the variate. If the variate values on all the population units are arranged into order from smallest to largest the middle value is the *median*; 50% of all values are less than it. The most frequently occurring variate value is called the *mode*. These three quantities are called measures of location or central tendency because the variate values group around them on the variate axis. The mean and median are widely used in practice. The mode is useful occasionally because of its mathematical convenience in a particular case, for instance in Equation (1.1.4.1) where the parameter u is the mode of the distribution.

Quartiles and sextiles. The *quartiles* are the variate values which divide the population into four equal portions. If x_1, x_2 and x_3 are the quartiles then

$$F(x_1) = 0.25$$
$$F(x_2) = 0.50 \qquad (1.1.4.2)$$
$$F(x_3) = 0.75,$$

a quarter of the population having x values less than x_1, half of them having values less than x_2 and a quarter having values greater than x_3. A quartile mean is the average value of the x values in the population lying between two quartile values, thus

$$\text{2nd quartile mean} = 4\int_{x_1}^{x_2} x\, f(x)\, dx. \qquad (1.1.4.3)$$

The factor 4 enters because when the variate values between x_1 and x_2 are considered as a population in themselves the pdf $f(x)$ must be divided by 0.25 so that its integral from x_1 to x_2 is unity. Quartile means are used extensively in the derivation of the rainfall growth curves contained in Volume II and are also mentioned in Section 1.3 below.

Sextiles which divide the population into six equally sized classes and sextile means are defined in a similar fashion to quartiles.

Quantiles and return period. A *quantile* is a variate value having a stated probability of being exceeded; thus the 5% quantile has a probability of being exceeded of 0.05. It can also be stated that it would appear on average among every 20 units of the population. This number 20 is called also the *return period T* and the corresponding quantile value by $x(T)$ or x_T. The return period of any quantile value x is formally defined as

$$T = \frac{1}{1 - F(x)}. \qquad (1.1.4.4)$$

Moments. The mean value μ of x is also called the first moment about the origin by analogy with the moment of mass or area as used in mechanics.

The mean value of x^r is called the rth *moment about the origin*, denoted μ'_r. The mean value of $(x-\mu)^r$ is called the rth *moment about the mean*, μ_r, or the rth *central moment*. Moments about the origin are distinguished from the central moments by the apostrophe. The second central moment μ_2 is also called the *variance*, σ^2, or mean square deviation

$$\mu_2 = \sigma^2 = E(x-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,\mathrm{d}x. \tag{1.1.4.5}$$

The square root of μ_2 is called the *standard deviation* σ, or, less frequently, the root mean square deviation, and it determines the extent of the spread of the variate values over the x axis. It is a scale parameter.

If a distribution such as the extreme value distribution of Equation (1.1.4.1) has just two parameters associated respectively with location and scale, the mean and standard deviation are completely specified by them. For instance, in the distribution (1.1.4.1)

$$\mu'_1 = u + 0.5772\alpha$$

$$\sigma = \frac{\pi}{\sqrt{6}}\alpha. \tag{1.1.4.6}$$

The entire family of possible populations could therefore be described by a set of every possible pair of (μ'_1, σ) values.

The third moment about the mean is

$$\mu_3 = \int_{-\infty}^{\infty} (x-\mu'_1)^3 f(x)\,\mathrm{d}x \tag{1.1.4.7}$$

and the dimensionless ratio

$$g = \mu_3/\sigma^3 \tag{1.1.4.8}$$

is called the skewness of the distribution. A distribution is symmetrical about its mean if μ_3 is zero. Generally g may be considered as a standardised measure of asymmetry. The distributions appropriate to flood hydrology have positive skewness. The qualitative relation between the position and shape of the probability density function on the x axis and the moments μ_1, μ_2 and μ_3 is shown in Figures 1.1–1.3.

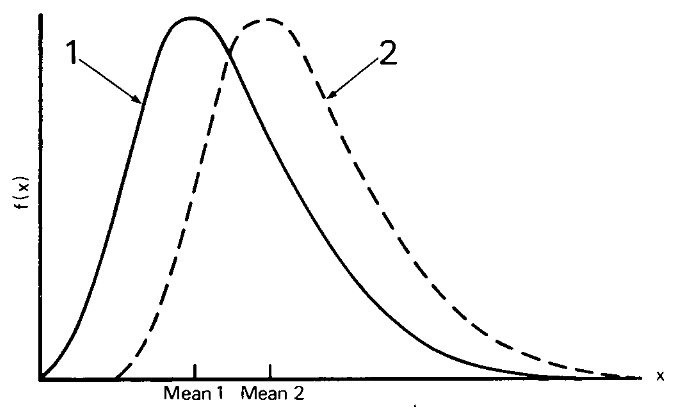


Fig 1.1 Probability density functions of two distributions equal in all respects except the mean.

In Figure 1.1 populations (1) and (2) have equal values of μ_2 and μ_3 but they have different values of μ'_1.

In Figure 1.2 populations (1) and (2) have equal values of μ'_1 but population (2) has a larger value of μ_2 than population (1). The dimensionless quantity

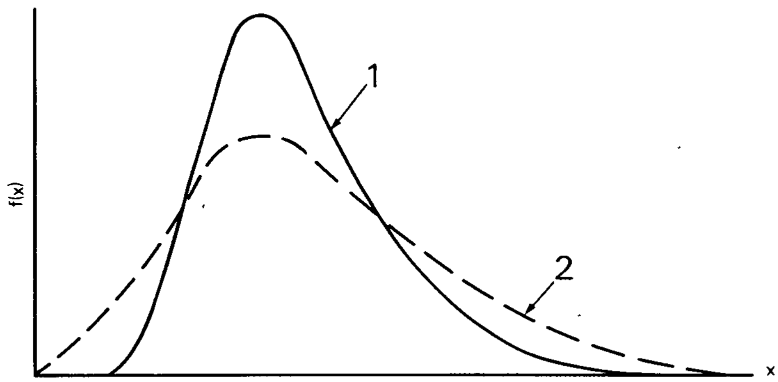


Fig 1.2 Probability density functions of two distributions equal in all respects except the variance.

$$\text{CV} = \frac{\sigma}{\mu'_1} = \frac{\sqrt{\mu_2}}{\mu'_1} \tag{1.1.4.9}$$

known as the *coefficient of variation* is useful in distinguishing between populations. Population (2) has a greater value of CV than population (1).

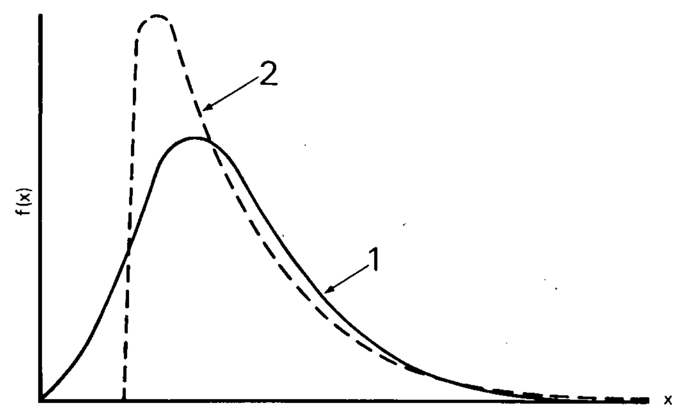


Fig 1.3 Probability density functions of two distributions equal in mean and variance but differing in skewness.

In Figure 1.3 population (1) is distributed more symmetrically about its middle than population (2). The latter has a higher value of skewness.

1.1.5 Samples, estimation and sampling distributions

Random samples and sequences. A random sample of N units from a population is most easily understood if it is assumed temporarily that there are a finite number M units in the population. Then there are $\binom{M}{N}$ different samples that could be drawn and one of these selected at random, by a lottery method say, is a random sample from the population. During this procedure variate values play no part; they are merely observed as values on the units that constitute the sample. In a random sample individual variate values are mutually independent in the statistical sense. A record of annual maximum peak flows can be considered to be a random sample but a record of total annual flows is not a random sample because there is a tendency, especially on large catchments, for a given year's flow to be influenced by the previous year's flow. In this case the sequence of annual flows may be considered to be randomly drawn from all possible sequences of equal length but no such statement can be made about individual values in the sequence.

Estimation. The question of estimation arises when a random sample of data is available from a population and it is wished to make some statement about the population which is itself unknown. If the form of the population is known then only the parameter values are unknown. For instance if the distribution is given by Equation (1.1.4.1) then only the values of u and α are unknown. In a colloquial sense the distribution of variate values in the sample is expected to bear a resemblance to the distribution of variate values in the population so that sample parameter values resemble or estimate the corresponding population quantities.

Statistic values and sampling distributions. Consider a random sample of N variate values from the population. Denote these values $x_1, x_2, \ldots x_i \ldots x_N$. The arithmetic mean of these values is

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i. \tag{1.1.5.1}$$

$\bar{x}$ is called a *statistic*. The value of $\bar{x}$ so computed from random samples generally differs from sample to sample. Just as there are many possible different samples of size N so are there many possible values of $\bar{x}$. The probability that $\bar{x}$ is less than x in some arbitrary random sample of size N may be written

$$PR(\bar{x} \leqslant x) = G(x). \tag{1.1.5.2}$$

$G(x)$ is a distribution function. The population with which it is associated is the set of all possible $\bar{x}$ values. This distribution is called the *sampling distribution* of the statistic $\bar{x}$. Its form may be deduced from that of the parent distribution. All other sample quantities are also given the title statistic. Thus, the standard deviation, largest value, smallest value, range and median of a sample are examples of statistics. Each one of these has its own sampling distribution.

1.1.6 Mean and variance of a linear function of random variables

Let x and y be random variables related by

$$x = a + by \tag{1.1.6.1}$$

where a and b are constants. Then the mean and variance of x in terms of those of y are

$$E(x) = a + b\,E(y) \tag{1.1.6.2}$$

$$\text{var}\ x = b^2\ \text{var}\ y. \tag{1.1.6.3}$$

An extension of this is if

$$x = a + b_1 y_1 + b_2 y_2 + b_3 y_3 \tag{1.1.6.4}$$

where b_1, b_2 and b_3 are constants then

$$E(x) = a + b_1 E(y_1) + b_2 E(y_2) + b_3 E(y_3) \tag{1.1.6.5}$$

$$\begin{aligned}\text{var}\ x = {} & b_1^2\ \text{var}\ y_1 + b_2^2\ \text{var}\ y_2 + b_3^2\ \text{var}\ y_3 + 2b_1 b_2\ \text{cov}(y_1, y_2) \\ & + 2b_2 b_3\ \text{cov}(y_2, y_3) + 2b_3 b_1\ \text{cov}(y_3, y_1)\end{aligned} \tag{1.1.6.6}$$

and this result may be extended to include any number of random variables $y_1, y_2, \ldots$ on the right hand side. These results are easily proved and are contained in many statistical text books. Widespread use is made of them in many applications of statistical methods. For instance, if $x_1, x_2 \ldots x_N$ is a random sample of size N from a population with variance σ^2 then

$$\text{var } \bar{x} = \text{var} \sum_{i=1}^{N} \frac{x_i}{N} = \Sigma \text{ var} \frac{x_i}{N} = \frac{1}{N^2} \Sigma \text{ var } x_i = \frac{1}{N^2} N\sigma^2 = \frac{\sigma^2}{N}, \quad (1.1.6.7)$$

the covariance terms vanishing because x_i is independent of x_j, $i \neq j$, in random sampling.

Expressions like Equation (1.1.6.1) appear frequently in statistical hydrology where a variate x is related to the corresponding standardised or reduced variate y by

$$x = \mu + \sigma y \quad (1.1.6.8)$$

where μ and σ are the mean and standard deviation of x. Then

$$E(x) = \mu + \sigma E(y) \quad (1.1.6.9)$$

and

$$\text{var } x = \sigma^2 \text{ var } y. \quad (1.1.6.10)$$

When μ and σ are estimated from a sample let the estimates be $\hat{\mu}$ and $\hat{\sigma}$. These are random variables and so for a fixed y

$$\hat{x} = \hat{\mu} + \hat{\sigma} y \quad (1.1.6.11)$$

is a random variable with sampling variance

$$\text{var } \hat{x} = \text{var } \hat{\mu} + 2y \text{ cov}(\hat{\mu}, \hat{\sigma}) + y^2 \text{ var } \hat{\sigma}. \quad (1.1.6.12)$$

Expressions such as these appear in Section 1.4 below.

1.2 Statistical distributions and their properties

1.2.1 Introduction

The distributions which are most frequently used in flood frequency analysis are outlined in this section, but the section is confined to the properties of the distributions and how quantile values of given return periods are calculated when the parameter values are specified. When a distribution is to be applied suitable parameter values have to be used. The question of how to obtain such parameter values both when there is and is not a flow record available is discussed later in Sections 1.3, 2.6, 2.7 and 2.11.

The Normal distribution is treated first because it is well known and the other distributions can be studied by analogy with it. The distributions are presented in a uniform style and the use of each is illustrated by an example. The choice of parameters to describe a distribution are those which give the simplest mathematical expressions. Thus, although the mean, variance and skewness are universally familiar they are used to define a distribution only if they also give the simplest formulae for the probability density and distribution functions. The parameters used to describe the extreme value (EV) Type 1, 2 and 3 distributions are those which show the relationships between these distributions clearly and in particular that the EV Type 1 is a special limiting case of both the EV Type 2 and EV Type 3. The parameters used also allow the region curves derived in Section 2.6 to be expressed in a succinct form. In addition they are the same as those used by Jenkinson (1969) and are therefore compatible with the theoretical basis of Chapter 2 of Volume II (Meteorological Studies).

Symbols which are not so general as to be included or defined in Section 1.1 are defined wherever they occur.

1.2.2 Normal (or Gaussian) distribution

The probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}[(x-\mu)/\sigma]^2}, \quad -\infty < x < \infty. \qquad (1.2.2.1)$$

The distribution function

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}[(x-\mu)/\sigma]^2} \, dx \qquad (1.2.2.2)$$

can be calculated numerically for any μ and σ.

The parameters μ and σ are location and scale parameters respectively. The mean and variance are

$$\text{Mean} = E(x) = \mu_1' = \mu \qquad (1.2.2.3)$$

$$\text{Variance} = E(x-E(x))^2 = \mu_2 = \sigma^2. \qquad (1.2.2.4)$$

The standardised or reduced Normal variate is

$$y = (x-\mu)/\sigma \qquad (1.2.2.5)$$

where x is distributed as in Equation (1.2.2.1).

The probability density function of y is

$$g(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \qquad (1.2.2.6)$$

and the distribution function is

$$G(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \, dy. \qquad (1.2.2.7)$$

Values of x and y for which $F(x) = G(y)$ are related by Equation (1.2.2.5) and by its inverse

$$x = \mu + \sigma y \qquad (1.2.2.8)$$

Since Equation (1.2.2.7) contains no unknown parameters a simple one way table can be calculated showing the value of $G(y)$ for given y, Table 1.1(a), or conversely the value of y for a given $G(y)$, Table 1.1(b). In

Table 1.1 Normal distribution.

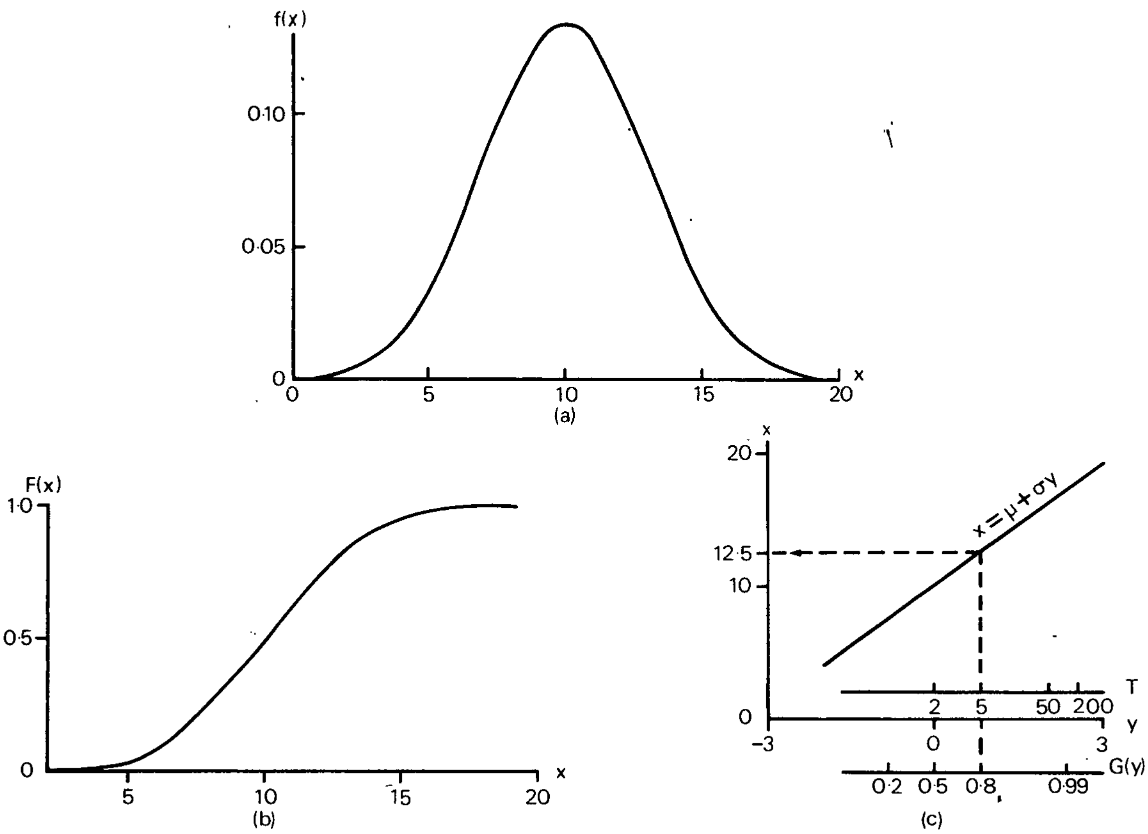
(a)		(b)		(c)	
$G(y)$	y	y	$G(y)$	T	y
0.10	−1.2816	−2.0	0.0228	2	0.00
0.20	−0.8416	−1.5	0.0668	5	0.84
0.30	−0.5244	−1.0	0.1587	10	1.28
0.40	−0.2533	−0.5	0.3085	25	1.75
0.50	0.0000	0.0	0.5000	50	2.05
0.60	0.2533	0.5	0.6915	100	2.33
0.70	0.5244	1.0	0.8413	1000	3.09
0.80	0.8416	1.5	0.9332		
0.90	1.2816	2.0	0.9772		

addition, values of y for a selection of values of return period $T = 1/(1-G(y))$ are given in Table 1.1(c). A more extensive version of Table 1.1(c) is included in Table 1.12.

Tables 1.1(a) and 1.1(b) are used with Equations (1.2.2.5) and (1.2.2.7) to calculate $F(x)$ and its inverse.

In Figure 1.4(a) and (b) the pdf and df of a Normal variate x with mean $\mu = 10$ and standard deviation $\sigma = 3$ are shown, while Figure 1.4(c) shows the line which relates values of x and reduced variate y which have equal return period.

Fig 1.4 Normal distribution with mean = 10 and standard deviation = 3 showing (a) the pdf, (b) the df and (c) the variate–reduced variate relation.



Example 1. The variate x is Normally distributed with mean $\mu = 10$ and standard deviation $\sigma = 3$. What value of x corresponds to $F(x) = 0.80$, i.e. $T = 1/(1-F(x)) = 5$?

From Table 1.1(a) first get y corresponding to $G(y) = F(x) = 0.80$. It is $y = 0.8416$. Then from Equation (1.2.2.8)

$$x = \mu+\sigma y = 10+3(0.8416) = 12.5248.$$

These steps can be followed graphically on Figure 1.4(c) by following the dashed vertical line from $T = 5$ to $G(y) = 0.80$ to $y = 0.84$; thence to $x = \mu+\sigma y$ and horizontally to the required value of x.

A Normal variate with mean μ and standard deviation σ is often referred to as an $N(\mu,\sigma)$ variate while the standardised Normal variate y ($\mu = 0$, $\sigma = 1$) is in this notation an $N(0,1)$ variate.

1.2.3 The exponential, gamma and Pearson Type 3 distributions

The Pearson Type 3 distribution has three parameters which can be denoted by x_0, β and γ. One special case occurs when $x_0 = 0$ and gives the gamma distribution; another arises when $\gamma = 1$ giving the exponential distribution.

a Exponential distribution. The probability density function (pdf) is

$$f(x) = \frac{1}{\beta} e^{-(x-x_0)/\beta} \qquad (1.2.3.1)$$

and the distribution function (df) is

$$F(x) = 1 - e^{-(x-x_0)/\beta}. \qquad (1.2.3.2)$$

The parameter x_0 is a location parameter since it determines a lower bound to variate values; the parameter β is a scale parameter since it determines the spread of the values.

The mean, variance and skewness are

$$\text{Mean} = E(x) = \mu_1' = x_0 + \beta \qquad (1.2.3.3)$$

$$\text{Variance} = E(x - E(x))^2 = \mu_2 = \beta^2 \qquad (1.2.3.4)$$

$$\text{Skewness} = g = 2. \qquad (1.2.3.5)$$

The standardised exponential variate y is related to the variate of Equation (1.2.3.1) by

$$y = (x - x_0)/\beta \qquad (1.2.3.6)$$

and has pdf and df given by

$$g(y) = e^{-y} \qquad (1.2.3.7)$$

and

$$G(y) = 1 - e^{-y}. \qquad (1.2.3.8)$$

Values of x and y for which $F(x) = G(y)$ are related by Equation (1.2.3.6) and its inverse

$$x = x_0 + \beta y. \qquad (1.2.3.9)$$

Equation (1.2.3.8) contains no parameters, so that $G(y)$ can be tabulated in a simple one way table as in Table 1.2(a) and (b). Values of return period $T = 1/(1 - G(y))$ and the corresponding values of y for a selection of T values are given in Table 1.2(c). A larger selection is included in Table 1.12. The tabulated relation is $y = \ln T$.

Figure 1.5(a) and (b) show the pdf and df of an exponential variate x with $x_0 = 15$ and $\beta = 5$, while Figure 1.5(c) shows the relation of x values to the standardised exponential variate y values having equal return periods, that is values of x and y for which $F(x) = G(y)$ given by Equation (1.2.3.9).

Example 1. The variate x is exponentially distributed with mean 20 and standard deviation 5. What value of x corresponds to $F(x) = 0.90$, i.e. $T = 1/(1 - F(x)) = 10$?

From Equations (1.2.3.3) and (1.2.3.4)

Mean $= x_0 + \beta = 20$

Variance $= \beta^2 = 5^2 = 25$ giving $\beta = 5$. Therefore, $x_0 = 20 - \beta = 15$.

Table 1.2 Exponential distribution.

(a)		(b)		(c)	
$G(y)$	y	y	$G(y)$	T	y
0.00	0.000	0.0	0.000	2	0.69
0.10	0.105	0.5	0.393	5	1.61
0.30	0.357	1.0	0.632	10	2.30
0.50	0.693	1.5	0.777	25	3.22
0.70	1.204	2.0	0.865	50	3.91
0.90	2.303	2.5	0.918	100	4.61
				1000	6.91

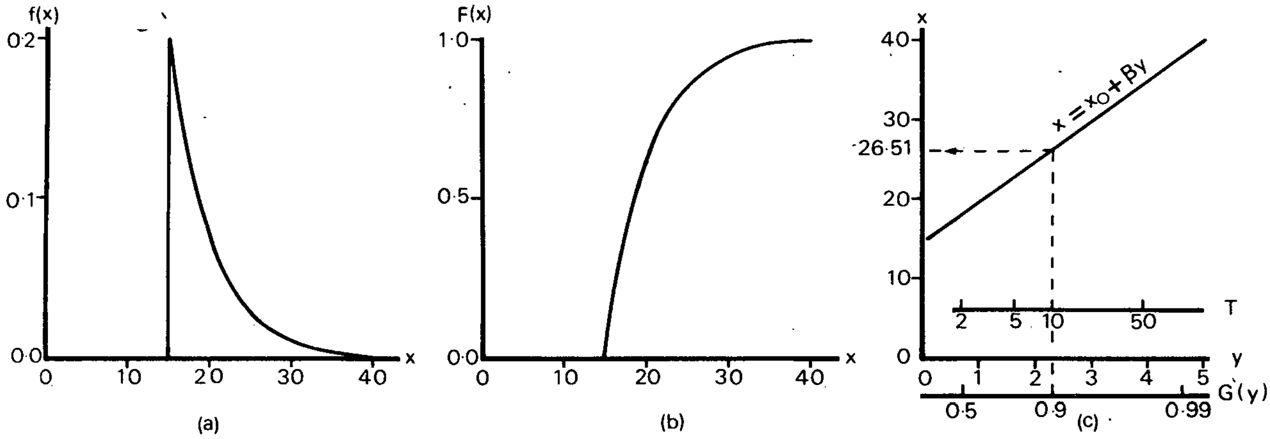


Fig 1.5 A two parameter exponential distribution with $x_0 = 15, \beta = 5$ showing (a) the pdf, (b) the df and (c) the variate–reduced variate relation.

From Table 1.2(a) first get y corresponding to $G(y) = F(x) = 0.90$. This is 2.303.

Then from Equation (1.2.3.9),

$$x = x_0 + \beta y = 15 + 5(2.303) = 26.515,$$

the required x value. These steps can also be followed graphically along the dashed lines in Figure 1.5(c).

b Gamma distribution. The probability density function is

$$f(x) = \frac{x^{\gamma-1}\, e^{-x/\beta}}{\beta^{\gamma}\Gamma(\gamma)}, \quad \gamma > 0, \qquad (1.2.3.10)$$

and the distribution function is

$$F(x) = \int_0^x \frac{x^{\gamma-1}\, e^{-x/\beta}}{\beta^{\gamma}\Gamma(\gamma)}\, dx, \qquad (1.2.3.11)$$

where $\Gamma(\gamma)$ is the complete gamma function while the integral in Equation (1.2.3.11) is known as the incomplete gamma function. For given β and γ this integral must be evaluated by numerical integration. The distribution has no location parameter while β and γ are scale and shape parameters respectively. The mean, variance and skewness are

$$\text{Mean} = E(x) = \mu_1' = \beta\gamma \qquad (1.2.3.12)$$

$$\text{Variance} = E(x-E(x))^2 = \mu_2 = \beta^2\gamma \qquad (1.2.3.13)$$

$$\text{Skewness} = g = 2/\sqrt{\gamma}. \qquad (1.2.3.14)$$

The standardised gamma variate y is related to this x by

$$y = x/\beta \qquad (1.2.3.15)$$

with pdf and df given by

$$g(y) = \frac{y^{\gamma-1} e^{-y}}{\Gamma(\gamma)} \tag{1.2.3.16}$$

and

$$G(y) = \int_0^y \frac{y^{\gamma-1} e^{-y}}{\Gamma(\gamma)} \mathrm{d}y. \tag{1.2.3.17}$$

If the substitution given by Equation (1.2.3.15) is made in the right hand side of Equation (1.2.3.11) the latter becomes

$$F(x) = \int_0^{x/\beta} \frac{y^{\gamma-1} e^{-y}}{\Gamma(\gamma)} \mathrm{d}y \tag{1.2.3.18}$$

which is, by comparison with Equation (1.2.3.17), $G(y)$ evaluated at $y = x/\beta$, i.e.

$$F(x) = G(x/\beta). \tag{1.2.3.19}$$

Thus, values of x and y for which $F(x) = G(y)$ are related by Equation (1.2.3.15) and its inverse

$$x = \beta y. \tag{1.2.3.20}$$

Equation (1.2.3.17) contains the parameter γ; therefore each given positive value of γ determines a different pdf and tabulation of $G(y)$ requires a separate one way table for each γ. A selection of such tables is given in Table 1.3; these are a subset of those given by Wilk, Gnanadesikan & Huyett (1962). They could also be obtained indirectly from the Pearson Type 3 tables given by Harter (1969).

Table 1.3 Percentage points of standardised gamma variates.

$G(y)$	$\gamma = 1$	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
0.10	0.105	0.532	2.433	6.221	14.53
0.20	0.223	0.824	3.090	7.289	16.17
0.30	0.357	1.097	3.634	8.133	17.44
0.40	0.511	1.376	4.148	8.904	18.57
0.50	0.693	1.678	4.671	9.669	19.67
0.60	0.916	2.022	5.237	10.476	20.81
0.70	1.204	2.439	5.890	11.387	22.08
0.80	1.609	2.994	6.721	12.519	23.63
0.90	2.303	3.890	7.994	14.206	25.90
0.95	2.996	4.744	9.154	15.705	27.88
0.99	4.605	6.638	11.605	18.783	31.85

Fig 1.6 A gamma distribution with $\gamma = 5$, $\beta = 5.63$ showing (a) the pdf, (b) the df and (c) the variate as a function of the exponential reduced variate.

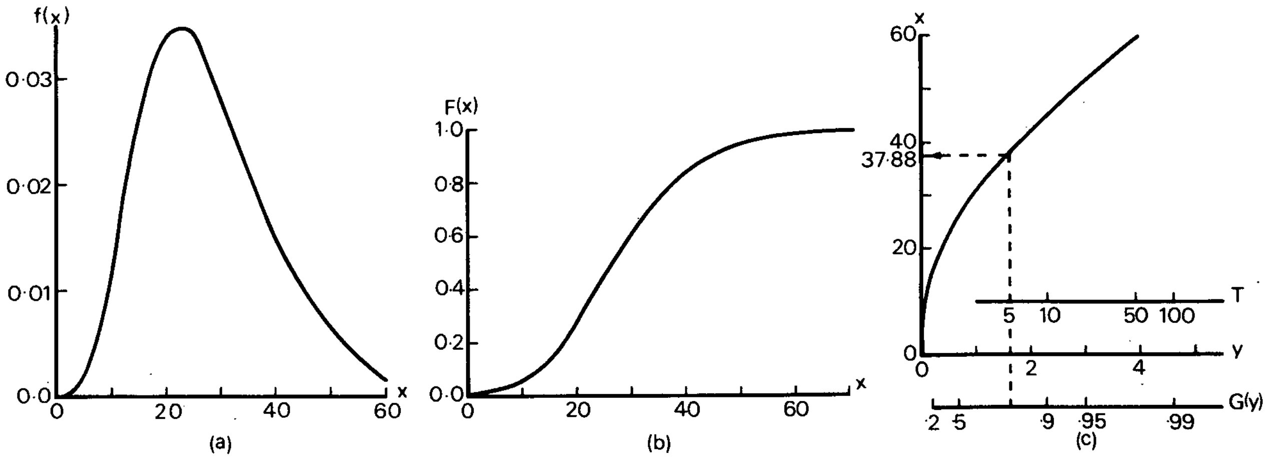


Figure 1.6(a) and (b) show the pdf and df of a gamma variate x with $\gamma = 5$ and $\beta = 5.63$ while Figure 1.6(c) shows the relation between values of x and standardised exponential variate y for which $F(x) = G(y)$.

Example 2. Variate x is gamma distributed with mean 30 and standard deviation 13. What value of x corresponds to $F(x) = 0.80$, i.e. $T = 1/(1-F(x)) = 5$?

First derive values for β and γ. Using Equations (1.2.3.12) and (1.2.3.13)

Mean $= \beta\gamma = 30$

Variance $= \beta^2\gamma = (13)^2$.

Therefore, $\beta = 13 \times 13/30 = 5.633$ and $\gamma = 30/\beta = 5.326$.

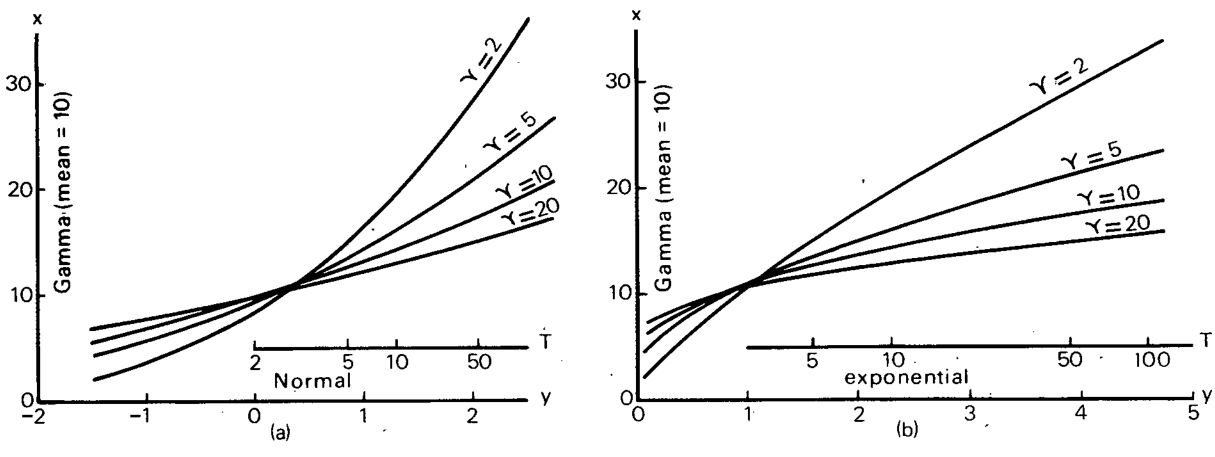
The value of y for which $G(y) = F(x) = 0.80$ in a standardised gamma distribution (Equation (1.2.3.17)) with $\gamma = 5.326$ is required. This may be determined either by use of a more extensive table than Table 1.3 or by numerical integration. If $\gamma = 5$ is accepted as a sufficiently good approximation for illustrative purposes the value $y = 6.721$ at $G(y) = 0.8$ is obtained. The value of x for which $F(x) = G(y)$ is given by Equation (1.2.3.20)

$x = \beta y = 5.633(6.721) = 37.86.$

The dashed lines on Figure 1.6(c) illustrate this result graphically.

Figure 1.7(a) and (b) show some gamma variates each having mean = 10, and differing γ values related to Normal and exponential standardised variates. It can be seen that when γ is large the curve is almost a straight line when the abscissa is Normal and when γ is close to 1 the curve is almost a straight line when the abscissa is exponential.

Fig 1.7 Gamma variates having fixed mean but varying values of shape parameter γ shown as functions of (a) Normal reduced variate and (b) exponential reduced variate.



Notes on gamma distribution

1 When $\gamma = 1$, (note that $\Gamma(1) = 1$), it can be seen that the probability density function is

$$f(x) = \frac{1}{\beta} e^{-x/\beta}$$

which is an exponential pdf with lower bound at zero, i.e. $x_0 = 0$.

2 When γ is very large the skewness $g = 2/\sqrt{\gamma}$ is very small and the distribution is almost Normal.

c The Pearson Type 3 distribution. The pdf and df are respectively

$$f(x) = \frac{(x-x_0)^{\gamma-1} e^{-(x-x_0)/\beta}}{\beta^\gamma \Gamma(\gamma)} \tag{1.2.3.21}$$

and

$$F(x) = \int_{x_0}^{x} \frac{(x-x_0)^{\gamma-1} e^{-(x-x_0)/\beta}}{\beta^\gamma \Gamma(\gamma)} \, dx. \tag{1.2.3.22}$$

If on the right hand side of Equation (1.2.3.22) the variable $z = x - x_0$ is introduced this becomes

$$\int_0^z \frac{z^{\gamma-1} e^{-z/\beta}}{\beta^\gamma \Gamma(\gamma)} \, dz$$

showing that this z has a gamma distribution with parameters β and γ. This means that a Pearson Type 3 distribution with parameters (β,γ,x_0) can be considered to be a gamma distribution with parameters β and γ and whose variate has origin at $x = x_0$.

The mean, variance and skewness are

$$\text{Mean} = \mu_1' = E(x) = x_0 + \beta\gamma \tag{1.2.3.23}$$

$$\text{Variance} = \mu_2 = E(x-E(x))^2 = \beta^2\gamma \tag{1.2.3.24}$$

$$\text{Skewness} = g = 2/\sqrt{\gamma}. \tag{1.2.3.25}$$

Only the mean differs from that for a two parameter gamma distribution, and that by the constant x_0.

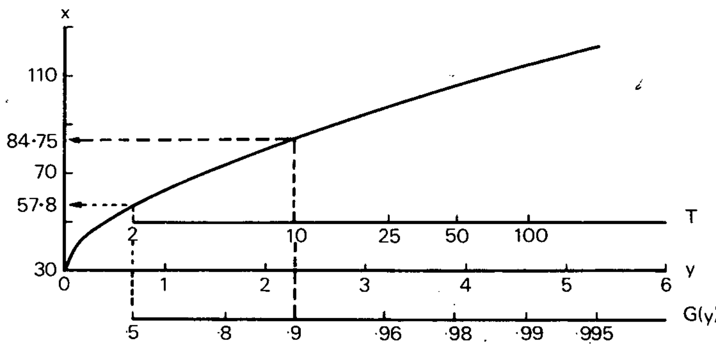


Fig 1.8 A Pearson Type 3 variate having $x_0 = 20.002$, $\beta = 8.1$ and $\gamma = 5$ shown as a function of exponential reduced variate.

The reduced variate y of this distribution is that of the gamma distribution with the same value of γ. Values of x and y such that $F(x) = G(y)$ are related by

$$x = x_0 + \beta y \tag{1.2.3.26}$$

$$y = (x - x_0)/\beta. \tag{1.2.3.27}$$

Example 3. Given the mean, standard deviation and skewness of a Pearson Type 3 variate as 60, 18 and 0.9 calculate approximately (a) the median and (b) the variate value of return period 10.

First, Equations (1.2.3.23)–(1.2.3.25) are used to calculate β,γ and x_0 from the mean, variance and skewness.

From Equation (1.2.3.25) $\gamma = (2/g)^2 = 4/(0.9)^2 = 4.938$. From Equation (1.2.3.24) $\mu_2 = \beta^2\gamma$ and therefore $\beta = \sqrt{\mu_2}/\sqrt{\gamma} = 18/\sqrt{4.938} = \pm 8.100$, the positive value being the correct value to take since β is defined to be positive. From Equation (1.2.3.23) $x_0 = \mu_1' - \beta\gamma = 60 - 8.100(4.938) = 20.002$.

As an approximation $\gamma = 5$ is taken instead of 4.938. For (a) the value of y corresponding to $G(y) = 0.5$ is required which from Table 1.3 under $\gamma = 5$ is $y = 4.671$.

Then using Equation (1.2.3.26)

$$x = x_0 + \beta y = 20.002 + (8.1)(4.671) = 57.837.$$

For (b) the value of y for which $G(y) = 1 - 1/T = 0.90$ is required. From Table 1.3 under $\gamma = 5$, $y = 7.994$ is found at $G(y) = 0.90$ and then from Equation (1.2.3.26)

$$x = x_0 + \beta y = 20.002 + 8.1(7.994) = 84.753.$$

The Pearson Type 3 variate with $x_0 = 20.002$, $\beta = 8.1$, and $\gamma = 5$ is shown related to the standardised exponential variate y in Figure 1.8 where the derivation of this result is shown by the dashed lines.

Notes

1 When $\gamma = 1$ the Pearson Type 3 distribution is identical with the two parameter exponential distribution.

2 The notation used for the exponential, gamma and Pearson Type 3 distributions and the use of (x_0, β, γ) as parameters mean that the reduced or standardised variate has its distribution expressed in its simplest form. The parameters x_0, β and γ fall into the general categories of location, scale and shape parameters respectively. But the mean, standard deviation and coefficient of skewness fall into these categories also, so why not use these? The reason will be evident when they are used. In Equation (1.2.3.21) the pdf is written in terms of x_0, β and γ. Using Equations (1.2.3.25), (1.2.3.24) and (1.2.3.23) these may be expressed in terms of μ, σ and g thus

$$\gamma = 4/g^2 \qquad (1.2.3.28)$$

$$\beta = \sigma/\sqrt{\gamma} = \sigma g/2 \qquad (1.2.3.29)$$

$$x_0 = \mu - \beta\gamma = \mu - (4/g^2).(\sigma g/2) = \mu - 2\sigma/g. \qquad (1.2.3.30)$$

Substituting these expressions into Equation (1.2.3.21) gives

$$f(x) = \frac{(x-\mu+2\sigma/g)^{(4-g^2)/g^2}\, e^{-2(x-\mu+2\sigma/g)/\sigma g}}{(\tfrac{1}{2}\sigma g)^{4/g^2}\Gamma(4/g^2)}. \qquad (1.2.3.31)$$

However forbidding Equation (1.2.3.21) may look, Equation (1.2.3.31) looks even more forbidding; it is therefore more convenient to use the parameters x_0, β, and γ than μ, σ and g despite the wider statistical usage of the latter.

3 Quantiles expressed in terms of moments.

So far the expression for a quantile has involved the reduced gamma variate y, thus

$$x(T) = x_0 + \beta y(T) \qquad (1.2.3.32)$$

where $x(T)$ and $y(T)$ are x and y values having $F(x) = G(y) = 1 - 1/T$.

In the Normal distribution the expression is

$$x(T) = \mu + \sigma y(T) \qquad (1.2.3.33)$$

where now y is the Normal reduced variate. In hydrology, Equation

(1.2.3.33) has been considered as the prototype to be copied for use with all distributions. The essential aspect of it is that it involves the familiar mean and standard deviation. Equation (1.2.3.32) contains x_0 and β which have no such specific and familiar meaning. Hydrologists have preferred the system where instead of using a reduced variate $y(T)$ a frequency factor, $K(T)$, is used instead in the manner of

$$x(T) = \mu+\sigma K(T). \tag{1.2.3.34}$$

The real meaning of $K(T)$ is that it is the variate value of return period T in a distribution having the same form as that of x but which has zero mean and unit variance. Thus in the case of the Normal distribution $K(T)$ is exactly the same as $y(T)$ but this is not so in any other distribution.

To get an expression like Equation (1.2.3.34) for the Pearson Type 3 distribution, replace x_0 and β in Equation (1.2.3.32) by the expressions for β and x_0 in Equations (1.2.3.29) and (1.2.3.30) to give

$$\begin{aligned} x(T) &= \mu - \frac{2\sigma}{g} + \frac{\sigma g}{2}\, y(T) \\ &= \mu+\sigma\left(\frac{g}{2}\, y(T) - \frac{2}{g}\right). \end{aligned} \tag{1.2.3.35}$$

This is of the same form as Equation (1.2.3.34) if, and only if,

$$K(T) = \frac{g}{2}\, y(T) - \frac{2}{g}. \tag{1.2.3.36}$$

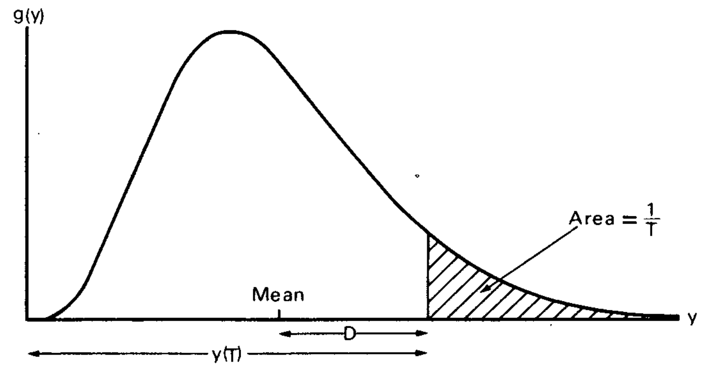


Fig 1.9 Illustration of relation between $y(T)$ and $K(T)$ in a gamma distribution. The mean is γ and standard deviation is $\sqrt{\gamma}$. The distance between the mean and $y(T)$ measured in units of standard deviations is $K(T) = D/\sqrt{\gamma}$.

Figure 1.9 illustrates the meaning of $K(T)$ for a standardised gamma variate with parameter γ; $y(T)$ is the distance from the origin to the variate value of return period T. The mean is γ and the standard deviation is $\sqrt{\gamma}$. $y(T)$ is a distance $D = y(T)-\gamma$ from the mean and this distance is $D/\sqrt{\gamma}$ standard deviations. That is, $K(T)$ is the distance in standard deviations of the variate value of return period T from the mean. $K(T) = D/\sqrt{\gamma} = (y(T)-\gamma)/\sqrt{\gamma} = y(T)g/2-2/g$ as in Equation (1.2.3.36) since $g = 2/\sqrt{\gamma}$.

Chow (1951) wrote about the factor $K(T)$ and derived its form for the lognormal and extreme value Type 1 distributions. Its use was also proposed for the Pearson Type 3 distribution by Foster (1924). Harter (1969) tabulated the function $K(T)$ for this distribution for different values of return period T and skewness g and his tables are summarised in the report of the United States Water Resources Council's committee on flow frequency methods (1967) and also by Benson (1968). Its equivalent was also tabulated by Pearson (1934) although not in such a convenient form as Harter's.

1.2.4 Extreme value (EV) distributions

The extreme value distributions feature prominently in attempts to deduce on purely theoretical grounds how annual maximum floods are distributed. In this section however the discussion will be restricted to their properties.

Just as there is a family of Pearson Type 3 distributions, each member being characterised by a value of γ, so there is also a family of EV distributions, each member of which is characterised by the value of a parameter denoted by k. The family can be divided into three classes, corresponding to different ranges of k values. The three classes are referred to as Fisher–Tippett Type 1, Type 2 and Type 3 (Fisher & Tippett, 1928). They will sometimes be referred to as EV1, EV2 and EV3 for short (EV for extreme value). In practice, k values lie in the range -0.6 to $+0.6$. This range is divided among the three classes as follows: k negative corresponds to Type 2, k zero corresponds to Type 1, k positive corresponds to Type 3.

Figure 1.10 shows in one way how these variates are related to each other. The abscissa is the standardised EV1 variate y_1 which will be described below. The values of EV1 variate x_1 and y_1 having the same return period are linearly related while the values of EV2 x_2 variate and y_1 having the same return period are related by a curve which is convex to the y_1 axis. The corresponding relation between the EV3 variate x_3 and y_1 is also curved, being concave towards the y_1 axis. The EV1 variate has neither a lower nor upper limit while the EV2 variate has a lower limit and the EV3 variate has an upper limit.

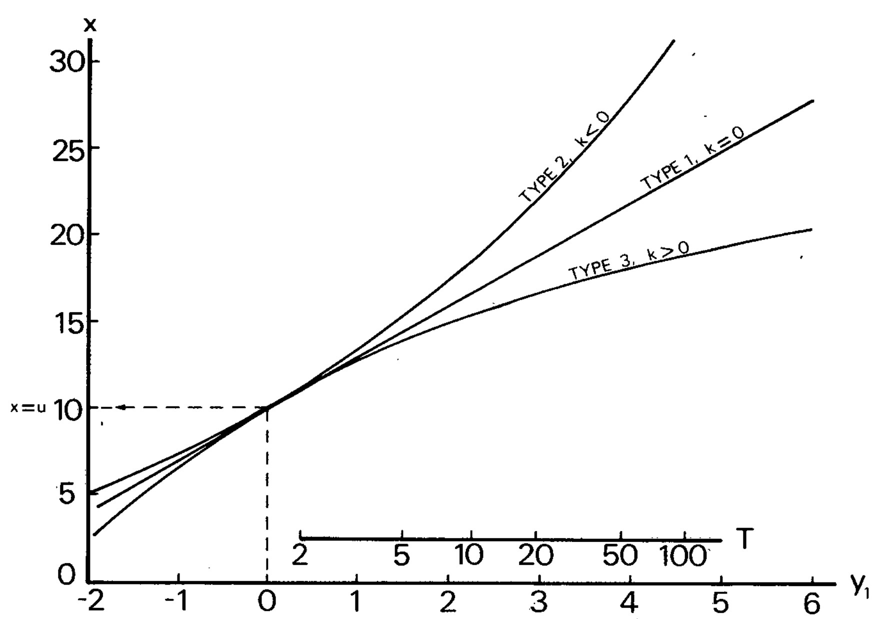


Fig 1.10 The three types of extreme value variate shown as functions of the Type 1 reduced variate by the relation $x = u+\alpha(1-e^{-ky_1})/k$.

Distributions of Types 1 and 2 are also known as Gumbel and Fréchet distributions. Further, if x has the Fréchet distribution $\log x$ has the Gumbel distribution. If a variate x has an EV3 distribution, then $-x$ is said to have a Weibull distribution.

The EV distributions have another property. Suppose that x is an EV variate, and let Z be the maximum (or extreme) value in a random sample of size N from this distribution. Because the sample is random, Z is also random with the special property that the distribution of Z has the same *form* as that of x although the mean and possibly the standard deviation of Z differ from those of x. This is the major characteristic of the EV distributions, namely that maxima of random samples drawn from such a population have the same form of distribution as the parent.

The Type 1 or Gumbel distribution (*EV1*). There is only one member in the Type 1 family. The distribution function is

$$F(x_1) = \exp(-e^{-(x_1-u)/\alpha}) \quad (1.2.4.1)$$

and this form also gives rise to the name double exponential distribution. The pdf is

$$f(x_1) = \frac{1}{\alpha}\exp[-(x_1-u)/\alpha - e^{-(x_1-u)/\alpha}]. \quad (1.2.4.2)$$

The parameter u is a location parameter and α is a scale parameter. The k referred to above is zero and does not appear in these expressions.

The mode is at $x_1 = u$, the mean, variance and skewness are

$$\text{Mean} = \mu'_1 = E(x_1) = u+\alpha C = u+0.5772\alpha \quad (1.2.4.3)$$

$$\text{Variance} = \sigma^2 = E(x_1-E(x_1))^2 = \frac{\pi^2\alpha^2}{6} \quad (1.2.4.4)$$

$$\text{Skewness} = g = 1.14. \quad (1.2.4.5)$$

The standardised or reduced variate y_1 is related to this x_1 variate by

$$y_1 = (x_1-u)/\alpha \quad (1.2.4.6)$$

and has pdf and df given by

$$g(y_1) = \exp(-y_1-e^{-y_1}) \quad (1.2.4.7)$$

and

$$G(y_1) = \exp(-e^{-y_1}). \quad (1.2.4.8)$$

Note: Many of the earliest papers on extreme value theory and applications were written in the French language, for instance Gumbel (1937a, 1937b, 1940) and Von Mises (1936). When Gumbel began to write in English, he adapted and translated the French 'variable réduite' to 'reduced variate', which is equivalent to a 'standardised variate'.

Values of x_1 and y_1 for which $F(x_1) = G(y_1)$ are related by Equation (1.2.4.6) and its inverse

$$x_1 = u+\alpha y_1. \quad (1.2.4.9)$$

Since Equations (1.2.4.7) and (1.2.4.8) contain no parameters, the relations between y_1 and $G(y_1)$ can be tabulated in simple one way tables, Table 1.4(a) and (b). Table 1.4(c) gives values of y_1 corresponding to a selection of T values. The relationship is $y_1 = -\ln-\ln(1-1/T)$ and for $T>5$ this is $y_1 = \ln(T-\frac{1}{2})$ to 3 significant digits. This relation is more extensively tabulated in Table 1.12.

Example 1. x_1 is an EV1 variate with mean = 81 and standard deviation = 23. What is the return period T of the variate value $x_1 = 91$?

First get u and α from μ and σ.

Using Equation (1.2.4.4)

$$\alpha = \sigma\sqrt{6}/\pi = 23\sqrt{6}/3.1416 = 17.94.$$

From Equation (1.2.4.3)

Table 1.4 Extreme value Type 1 distribution.

(a)		(b)		(c)		
$G(y_1)$	y_1	y_1	$G(y_1)$	T	y_1	$K(T)$
0.00	$-\infty$	−2.0	0.0006	2	0.37	−0.17
0.10	−0.83	−1.0	0.066	5	1.50	0.72
0.20	−0.48	0.0	0.367	10	2.25	1.31
0.30	−0.19	0.5	0.545	25	3.20	2.04
0.40	0.09	1.0	0.692	50	3.90	2.59
0.50	0.37	1.5	0.800	100	4.60	3.14
0.60	0.67	2.0	0.873	1000	6.91	4.94
0.70	1.03	2.5	0.921			
0.80	1.50	3.0	0.951			
0.90	2.25	4.0	0.982			
1.00	$+\infty$	5.0	0.993			

$u = \mu - 0.5772\alpha = 81 - 0.5772(17.94) = 70.65.$

The probability value $F(x_1)$ associated with $x = 91$ is required. The value of y_1 for which $G(y_1) = F(x_1)$ is obtained from Equation (1.2.4.6) giving

$y_1 = (x_1 - u)/\alpha = (91 - 70.65)/17.94 = 1.134.$

The value of $G(y_1)$ for this value of y_1 is obtained from Table 1.4(a). The nearest argument in Table 1.4(a) is $y_1 = 1.00$ for which $G(y_1) = 0.692$. But $F(x_1) = G(y_1)$ and therefore

$T = 1/(1 - F(x_1)) \simeq 1/(1 - 0.692) = 3.25.$

The pdf and df of this distribution are shown in Figure 1.11(a) and (b) and the calculations are demonstrated graphically in Figure 1.11(c).

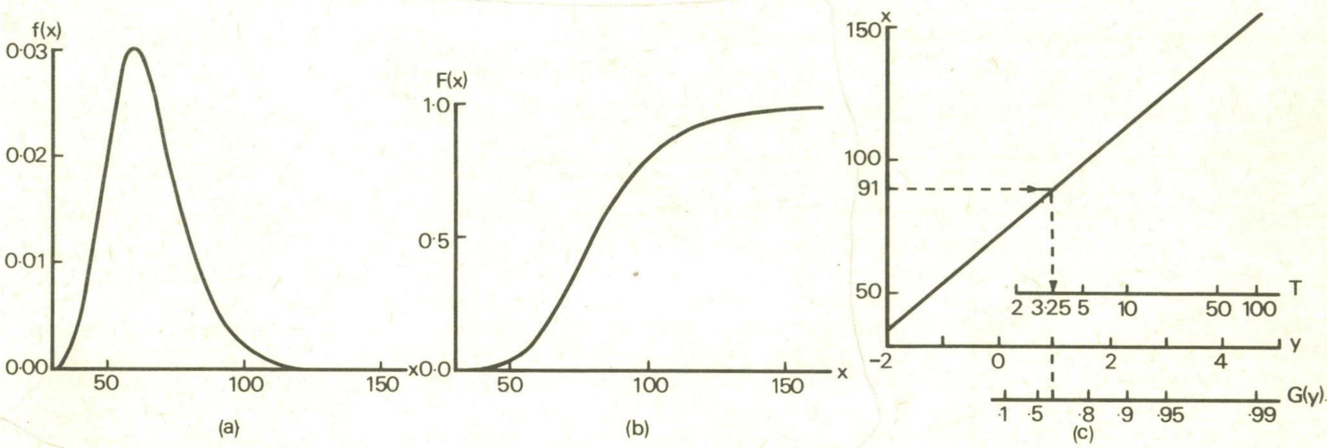


Fig 1.11 Extreme value Type 1 distribution with $u = 72.65$ and $\alpha = 17.93$ showing (a) the pdf, (b) the df and (c) the variate–reduced variate relationship.

Notes

1 It was shown earlier that a distribution may be specified by expressing the variate x_1 as a function of another variate y_1 the distribution of which is given (Method *b*, Section 1.1.3). Applied to the EV1 distribution the function $x = H(y)$ takes the form

$$x_1 = u + \alpha y_1 \qquad (1.2.4.10)$$

where the distribution of y_1 is given by $PR(Y_1 \leqslant y_1) = G(y_1) = \exp(-e^{-y_1})$.

2 The variate value of return period T may also be written as a linear function of μ and σ instead of u and α

$$x(T) = \mu + K(T)\sigma$$

as in Equation (1.2.3.34) for the Pearson Type 3 distribution. The frequency factor $K(T)$ is related to the EV1 reduced variate $y(T)$ by $K(T) = -0.45+0.78y(T)$ and the values of $K(T)$ for a selection of T values are given in Table 1.4(c).

3 If the more familiar mean μ and standard deviation σ instead of the parameters u and α specify the distribution, the df would be written using Equations (1.2.4.3) and (1.2.4.4) in Equation (1.2.4.1)

$$F(x_1) = \exp(-e^{-(x_1-\mu+0.45\sigma)/0.78\sigma}). \tag{1.2.4.11}$$

The Type 2 or Fréchet distribution (EV2). This distribution is also known as the log Gumbel distribution and will be briefly mentioned later in Section 1.2.5 on log distributions.

The df is

$$F(x_2) = e^{-[1-k(x_2-u)/\alpha]^{1/k}} \tag{1.2.4.12}$$

where

$$k < 0, \quad \alpha > 0, \quad u + \frac{\alpha}{k} \leqslant x_2 \leqslant \infty.$$

This distribution has three parameters; u and α are location and scale parameters respectively while k is a shape parameter which must be negative. The variate values have a lower bound $u+\alpha/k$. Since α and k are of opposite sign this lower bound is less than u. The pdf corresponding to Equation (1.2.4.12) is

$$f(x_2) = \frac{1}{\alpha}\left(1 - \frac{x_2-u}{\alpha}k\right)^{1/k-1} . e^{-[1-k(x_2-u)/\alpha]^{1/k}}. \tag{1.2.4.13}$$

If the EV2 reduced variate y_2 is written

$$y_2 = 1 - \frac{x_2-u}{\alpha}k, \quad 0 \leqslant y_2 \leqslant \infty \tag{1.2.4.14}$$

its df and pdf are

$$G(y_2) = e^{-y_2^{1/k}} \tag{1.2.4.15}$$

and

$$g(y_2) = \frac{-y_2^{1/k-1}}{k} e^{-y_2^{1/k}}. \tag{1.2.4.16}$$

The inverse of Equation (1.2.4.14) is

$$x_2 = u + \frac{\alpha}{k} - \frac{\alpha}{k} y_2 \tag{1.2.4.17}$$

which is of the form

$$x_2 = A + By_2 \tag{1.2.4.17a}$$

where $A = u+\alpha/k<u$ is a location parameter, the lower bound of x_2, and $B = -\alpha/k>0$ is a scale parameter. This is exactly analogous to Equation (1.2.3.26), $x = x_0+\beta y$, in the Pearson Type 3 distribution.

Since y_2 in Equation (1.2.4.17) depends on k, tabulation of $G(y)$ for given y requires a separate one way table for each value of k. The following is a selection. Values of x_2 and y_2 for which $F(x_2) = G(y_2)$, are related by

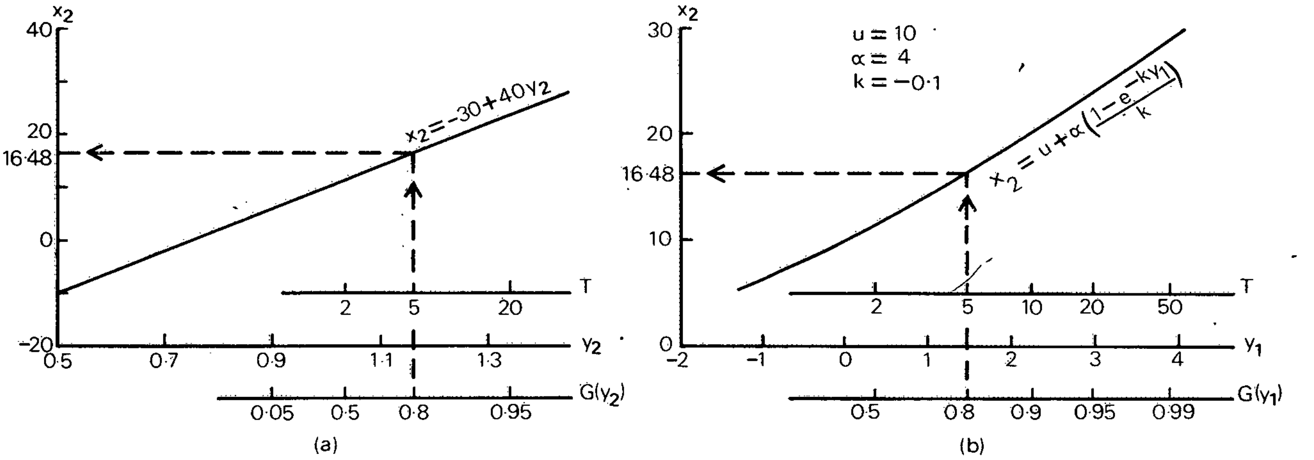


Fig 1.12 Extreme value Type 2 distribution with $u = 10$, $\alpha = 4$ and $k = -0.10$ shown as a function of (a) the EV2 reduced variate and (b) the EV1 reduced variate.

Table 1.5 y_2 as a function of $G(y_2)$ and k.

$G(y_2)$	$k = -0.05$	$k = -0.10$	$k = -0.15$	$k = -0.20$	$k = -0.25$
0.00	0.000	0.000	0.000	0.000	0.000
0.05	0.947	0.896	0.848	0.803	0.760
0.10	0.959	0.920	0.882	0.846	0.812
0.20	0.976	0.954	0.931	0.909	0.888
0.30	0.991	0.982	0.973	0.964	0.955
0.40	1.004	1.009	1.013	1.018	1.022
0.50	1.018	1.037	1.057	1.076	1.096
0.60	1.034	1.069	1.106	1.143	1.183
0.70	1.053	1.109	1.167	1.229	1.294
0.80	1.078	1.162	1.252	1.350	1.455
0.90	1.119	1.252	1.401	1.568	1.755

Equations (1.2.4.14) and (1.2.4.17).

Example 2. x_2 is an EV2 variate with $u = 10$, $\alpha = 4$ and $k = -0.1$; what is the event of return period $T = 5$?

$T = 5$ implies $F(x_2) = 0.8$ and in Table 1.5 $y_2 = 1.162$ corresponds to $G(y_2) = F(x_2) = 0.80$ when $k = -0.1$. Hence, from Equation (1.2.4.17)

$$x_2 = u + \alpha/k - (\alpha/k)y_2 = 10 - 40 + 40y_2 = -30 + 40(1.162) = 16.48.$$

This result is presented graphically in Figure 1.12(a), the abscissa y_2 being calibrated by using $G(y_2)$ from Table 1.5 and the corresponding T scale obtained by calculating $T = 1/(1 - G(y_2))$.

Alternative method of expressing the relation between x_2 and $F(x_2)$

It can be shown that if y_2 is an EV2 reduced variate with parameter k then the relation

$$y_2 = e^{-ky_1} \qquad (1.2.4.18)$$

holds, where y_1 is an EV1 reduced variate; and conversely. Consequently in Equation (1.2.4.17) y_2 may be replaced by $\exp(-ky_1)$ to give

$$x_2 = u + \frac{\alpha}{k} - \frac{\alpha}{k} e^{-ky_1} = u + \alpha\left(\frac{1 - e^{-ky_1}}{k}\right) \qquad (1.2.4.19)$$

a form first used by Jenkinson (1955). Writing

$$W(y_1; k) = \frac{1 - e^{-ky_1}}{k} \qquad (1.2.4.20)$$

Equation (1.2.4.19) can be written

$$x_2 = u+\alpha\, W(y_1; k). \tag{1.2.4.21}$$

These Equations (1.2.4.20) and (1.2.4.21) correspond to the second of the two methods mentioned earlier, Equation (1.1.3.3), for representing the distribution of a variate. Table 1.6 gives values of W for a selection of negative values of k and for some frequently used return periods. A more extensive version of Table 1.6 is contained in Table 1.12.

Table 1.6 Quantiles of W in the EV2 distribution.

T	y_1 reduced EV1 variate	$W(y_1;k) = \{1-\exp(-ky_1)\}/k$				
		$k = -0.05$	$k = -0.10$	$k = -0.15$	$k = -0.20$	$k = -0.25$
2	0.366	0.37	0.37	0.38	0.38	0.38
5	1.50	1.56	1.62	1.68	1.75	1.80
10	2.25	2.38	2.52	2.68	2.84	3.02
20	2.97	3.20	3.46	3.74	4.06	4.40
25	3.20	3.47	3.77	4.11	4.48	4.90
50	3.90	4.31	4.77	5.30	5.91	6.60
100	4.60	5.17	5.84	6.62	7.55	8.63

Example 2 may now be repeated using Equation (1.2.4.21) and Table 1.6, where, at $T = 5$ and $k = -0.10$, $W(y_1;k)$ has the value 1.62. Hence, Equation (1.2.4.21) gives

$$x_2 = u+\alpha W(y_1;k) = 10+4(1.62) = 16.48$$

as in the earlier calculation. This method is demonstrated in Figure 1.12(b). This is the easier of the two methods as only the distribution of y_1 need be remembered in order to compute W values.

Notes

1 When $k = 0$ the expression in Equation (1.2.4.12) reduces to $F(x) = \exp[-\exp\{-(x-u)/\alpha\}]$ showing that the EV1 variate is a special limiting case of the EV2 variate.

2 Mode and median. The median reduced variate value y_2^* is given by $G(y_2^*) = 0.5$ and from Equation (1.2.4.15) this gives

$$y_2^* = (\ln 2)^k = (0.69315)^k. \tag{1.2.4.22}$$

The modal value is

$$y_2^\# = (1-k)^k. \tag{1.2.4.23}$$

In terms of u, α and k the median and modal x_2 values are from Equations (1.2.4.17), (1.2.4.22) and (1.2.4.23)

$$x_2^* = u+\frac{\alpha}{k}-\frac{\alpha}{k}(0.69315)^k \tag{1.2.4.24}$$

$$x_2^\# = u+\frac{\alpha}{k}-\frac{\alpha}{k}(1-k)^k \tag{1.2.4.25}$$

and the inequality

$$x_2^\# < u < x_2^* \tag{1.2.4.26}$$

also holds.

3 Moments. In Section 1.3 below different methods of estimating parameters from sample data are discussed. One of these methods, the method of moments, requires expressions relating the parameters to the moments of the distribution. The moments of y_2 are first given and those of x_2 are then derived from these. The moments of y_2 about $y_2 = 0$ are

$$E(y_2) = \Gamma(1+k), \quad k < 0 \tag{1.2.4.27}$$

$$E(y_2^2) = \Gamma(1+2k), \quad k < 0 \tag{1.2.4.28}$$

and in general

$$E(y_2^r) = \Gamma(1+rk), \quad k < 0. \tag{1.2.4.29}$$

Hence,

$$\text{var } y_2 = \Gamma(1+2k) - \Gamma^2(1+k). \tag{1.2.4.30}$$

The relation between the coefficient of variation and k is

$$\text{cv}^2 = \frac{\Gamma(1+2k)}{\Gamma^2(1+k)} - 1 \tag{1.2.4.31}$$

and a graph of cv against k may be used to find a moments estimate of k when the lower bound of the variate $\varepsilon = u+\alpha/k$ is known to be zero (i.e. in the two parameter case) or equivalently when it is known that $u = -\alpha/k$.

4 Skewness. This dimensionless quantity g is independent of location and scale parameters, and therefore depends only on k. It is

$$g = \mu_3/\mu_2^{3/2}$$

where μ_2 = var y_2 of Equation (1.2.4.30) and

$$\mu_3 = E[y_2 - E(y_2)]^3 = E(y_2^3) - 3E(y_2^2)E(y_2) + 2[E(y_2)]^3$$

which on using Equation (1.2.4.29) with r = 1, 2 and 3 gives

$$\mu_3 = \Gamma(1+3k) - 3\Gamma(1+2k)\Gamma(1+k) + 2\Gamma^3(1+k). \tag{1.2.4.32}$$

The variance μ_2, coefficient of variation cv and skewness g of y_2 are tabulated in Table 1.7 for a series of practical values of k. The skewness as a function of k is shown in Figure 1.13.

Table 1.7 Moments and skewness of y_2 as a function of k.

k	$E(y_2) = \mu_1$	$\mu_2 = \text{var } y_2$	$\sigma = \sqrt{\mu}$	$CV = \frac{\sigma}{\mu_1}$	g
−0.05	1.031453	0.004727	0.0687	0.0666	1.532
−0.10	1.068622	0.022272	0.1492	0.1396	1.903
−0.15	1.112482	0.060426	0.2458	0.2209	2.532
−0.20	1.164225	0.133763	0.3657	0.3141	3.535
−0.25	1.225413	0.270803	0.5204	0.4247	5.605

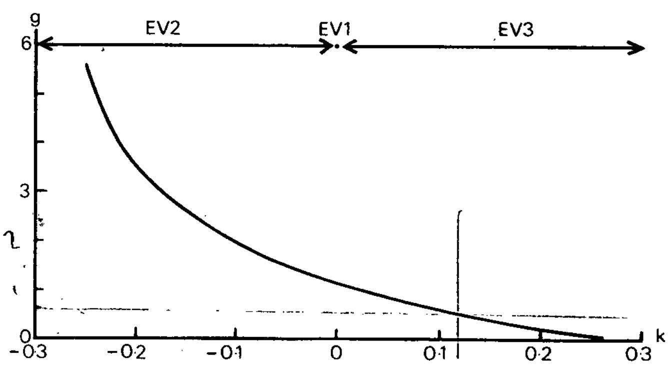


Fig 1.13 Skewness g, of extreme value variates as a function of the shape parameter k.

The mean and variance of x_2 are obtained from those of y_2 by using the results given by Equations (1.1.6.5) and (1.1.6.6). From Equation (1.2.4.17)

$$x_2 = \left(u + \frac{\alpha}{k}\right) + \left(\frac{-\alpha}{k}\right) y_2.$$

Then Equation (1.1.6.5) gives

$$\text{Mean } x_2 = E(x_2) = u + \frac{\alpha}{k} - \frac{\alpha}{k} E(y_2) \tag{1.2.4.33}$$

and Equation (1.1.6.6) gives

$$\text{var } x_2 = E(x_2 - E(x_2))^2 = \left(\frac{\alpha}{k}\right)^2 \text{var } y_2. \tag{1.2.4.34}$$

For a given k, $E(y_2)$ and var y_2 are obtained from Table 1.7 and inserted in these expressions. The skewness of x_2 is the same as that of y_2 with the same k.

The Type 3 distribution (EV3). The distribution function is

$$F(x_3) = e^{-[1-k(x_3-u)/\alpha]^{1/k}} \tag{1.2.4.35}$$

where $k > 0$, $\alpha > 0$, $-\infty \leqslant x_3 \leqslant u + \frac{\alpha}{k}$.

Equation (1.2.4.35) is of exactly the same form as Equation (1.2.4.12) for $F(x_2)$ but with the sign of k changed. x_3 has an upper limit of $u + \alpha/k$ which is greater than u since both α and k are positive. The probability density function is

$$f(x_3) = \frac{1}{\alpha}\left(1 - \frac{x_3 - u}{\alpha} k\right)^{1/k-1} e^{-[1-k(x_3-u)/\alpha]^{1/k}}. \tag{1.2.4.36}$$

Write the EV3 reduced variate y_3 as

$$-y_3 = 1 - \left(\frac{x_3 - u}{\alpha}\right) k, \quad -\infty \leqslant y_3 \leqslant 0, \tag{1.2.4.37}$$

or

$$x_3 = u + \frac{\alpha}{k} + \frac{\alpha}{k} y_3. \tag{1.2.4.38}$$

The df and pdf of y_3 are

$$G(y_3) = e^{-(-y_3)^{1/k}} \tag{1.2.4.39}$$

$$g(y_3) = \frac{1}{k}(-y_3)^{1/k-1} e^{-(-y_3)^{1/k}}. \tag{1.2.4.40}$$

Note that the relation (1.2.4.38) is of the form

$$x_3 = A + By_3 \tag{1.2.4.41}$$

where $A = u + \alpha/k > u$ is a location parameter, the upper bound of x, and $B = \alpha/k > 0$ is a scale parameter.

The distribution of y_3 depends on k; tabulation of the relation between y_3 and $G(y_3)$ therefore requires a separate one way table for each value of k. Values of x_3 and y_3 for which $F(x_3) = G(y_3)$ are related by Equations (1.2.4.37) and (1.2.4.38).

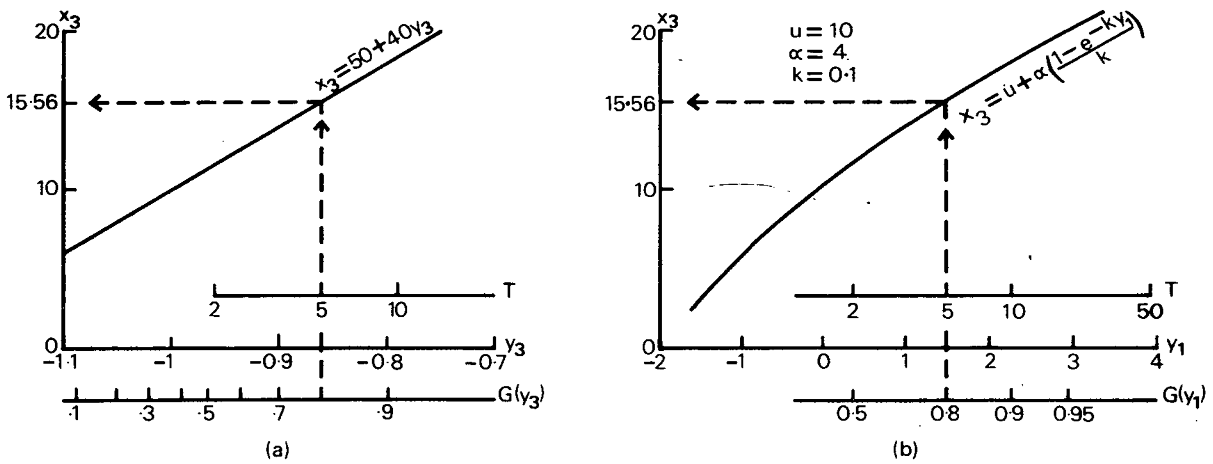


Fig 1.14 Extreme value Type 3 distribution with $u = 10$, $\alpha = 4$ and $k = 0.10$ shown as a function of (a) the EV3 reduced variate and (b) the EV1 reduced variate.

Table 1.8 y_3 as a function of $G(y_3)$ and k.

$G(y_3)$	$k = 0.05$	$k = 0.10$	$k = 0.15$	$k = 0.20$	$k = 0.25$
0.00	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
0.05	−1.506	−1.116	−1.179	−1.245	−1.316
0.10	−1.043	−1.087	−1.133	−1.182	−1.232
0.20	−1.024	−1.049	−1.074	−1.100	−1.126
0.30	−1.009	−1.019	−1.028	−1.038	−1.048
0.40	−0.997	−0.991	−0.987	−0.983	−0.978
0.50	−0.982	−0.964	−0.947	−0.929	−0.912
0.60	−0.967	−0.935	−0.904	−0.874	−0.845
0.70	−0.950	−0.902	−0.857	−0.814	−0.773
0.80	−0.928	−0.861	−0.799	−0.799	−0.687
0.90	−0.894	−0.798	−0.714	−0.638	−0.570
1.00	0.000	0.000	0.000	0.000	0.000

Example 3. x_3 is an EV3 variate with $u = 10$, $\alpha = 4$ and $k = 0.10$. What is the event of return period $T = 5$?

Return period $T = 5$ implies $F(x_3) = 1 - 1/T = 0.80$. Therefore, find the value of y_3 for which $G(y_3) = F(x_3) = 0.80$ and $k = 0.10$ in Table 1.8. This is $y_3 = -0.861$ and inserting this in Equation (1.2.4.38) gives

$$x_3 = 10 + \frac{4}{0.1} + \frac{4}{0.1}(-0.861) = 15.56.$$

This example is illustrated in Figure 1.14(a) where the EV3 reduced variate y_3 is used as abscissa. Table 1.8 is used to mark out the $G(y_3)$ and T scales on the diagram.

Alternative method of expressing the relation between x_3 and $F(x_3)$

It can be shown that if y_3 is an EV3 reduced variate then the following relation holds;

$$y_3 = -e^{-ky_1} \tag{1.2.4.42}$$

where $k > 0$ and y_1 is an EV1 reduced variate. Consequently substituting for y from Equation (1.2.4.42) in Equation (1.2.4.38) gives

$$x_3 = u + \frac{\alpha}{k} - \frac{\alpha}{k}e^{-ky_1} = u + \alpha\left(\frac{1 - e^{-ky_1}}{k}\right). \tag{1.2.4.43}$$

This is the same form as Equation (1.2.4.19) which relates the EV2 variate x_2 to y_1. However, in Equation (1.2.4.19) k is negative whereas in Equation (1.2.4.43) it is positive. Following Jenkinson (1969) this may be written

$$x_3 = u+\alpha W(y_1;k) \quad (1.2.4.44)$$

where $W(y_1;k)$ is the bracketed part of Equation (1.2.4.43), defined explicitly in Equation (1.2.4.20).

Table 1.9 gives values of $W(y_1;k)$ for $k = 0.05(0.05)0.25$ and for a selection of frequently used return periods. A larger selection is given in Table 1.12.

Example 3 may now be repeated using Equation (1.2.4.44) and Table 1.9. The value of y_1 for $T = 5$ is 1.50. From Table 1.9 at $T = 5$ and $k = 0.10$, $W(y_1;k) = 1.39$. Then Equation (1.2.4.44) gives

$$x_3 = u+\alpha W(y_1;k) = 10+4(1.39) = 15.56$$

in agreement with the earlier result. This is illustrated in Figure 1.14(b) in which the abscissa is the EV1 reduced variate y_1. In this report diagrams of the type of Figure 1.14(b) are used rather than the type of Figure 1.14(a).

Table 1.9 Quantiles of W in the EV3 distribution.

T		$W(y_1;k)$				
	y_1	$k = 0.05$	$k = 0.10$	$k = 0.15$	$k = 0.20$	$k = 0.25$
2	0.366	0.36	0.36	0.36	0.35	0.35
5	1.50	1.45	1.39	1.34	1.30	1.25
10	2.25	2.13	2.01	1.91	1.81	1.72
20	2.97	2.76	2.57	2.40	2.24	2.10
25	3.20	2.96	2.74	2.54	3.36	2.20
50	3.90	3.54	3.23	2.95	2.71	2.49
100	4.60	4.11	3.69	3.32	3.01	2.73

Notes

1 When $k = 0$ the expression (1.2.5.35) for $F(x_3)$ reduces to $\exp(-\exp -(x-u)/\alpha)$ showing that the EV1 variate is a special limiting case of the EV3 variate.

2 Mode and median. The median reduced variate y_3^* is given by $G(y_3^*) = 0.5$ and from Equation (1.2.4.39) this gives

$$y_3^* = -(\ln 2)^k = -(0.69315)^k. \quad (1.2.4.45)$$

The modal reduced variate $y_3^\#$ is

$$y_3^\# = -(1-k)^k \quad (1.2.4.46)$$

provided $k<1$; otherwise it does not exist.

The modal and median x values are therefore, from Equation (1.2.4.38)

$$x_3^\# = u+\frac{\alpha}{k}-\frac{\alpha}{k}(1-k)^k \quad (1.2.4.47)$$

$$x_3^* = u+\frac{\alpha}{k}-\frac{\alpha}{k}(0.69315)^k. \quad (1.2.4.48)$$

3 Moments. The moments of the distribution expressed in terms of the parameters u, α and k will be needed in Section 1.3 of this chapter on estimation. The moments of y_3 are first obtained; those of x_3 can then be easily expressed in terms of these.

The rth moment of y_3 about $y_3 = 0$ is

$$E(y_3^r) = (-1)^r\,\Gamma(1+rk). \tag{1.2.4.49}$$

Hence,

$$\text{Mean} = E(y_3) = -\Gamma(1+k) \tag{1.2.4.50}$$

$$\text{var } y_3 = \mu_2 = \Gamma(1+2k) - \Gamma^2(1+k) \tag{1.2.4.51}$$

$$\begin{aligned}\mu_3 &= E[y_3 - E(y_3)]^3 \\ &= -\Gamma(1+3k) + 3\Gamma(1+2k)\Gamma(1+k) - 2\Gamma^3(1+k).\end{aligned} \tag{1.2.4.52}$$

The skewness $g = \mu_3/\mu_2{}^{3/2}$ and is tabulated in the following table for various values of k.

Table 1.10 Moments, coefficient of variation and skewness of EV3 reduced variate for $k = 0.05(0.05)0.25$.

k	$E(y_3) = \mu_1$	$\mu_2 = \text{var } y_3$	$\sigma = \sqrt{\mu_2}$	$CV = \left\lvert\frac{\sigma}{\mu_1}\right\rvert$	g
0.05	−0.97350	0.003650	0.06042	0.06206	0.911500
0.10	−0.95135	0.013093	0.11442	0.12028	0.623041
0.15	−0.93304	0.026906	0.16403	0.17580	0.436171
0.20	−0.91816	0.044242	0.21034	0.22909	0.255755
0.25	−0.90640	0.064659	0.25428	0.28054	0.086610

The mean and variance of x_3 are by analogy with Equations (1.1.6.5) and (1.1.6.6),

$$\text{Mean } x_3 = E(x_3) = u + \frac{\alpha}{k} + \frac{\alpha}{k}\,E(y_3) \tag{1.2.4.53}$$

$$\text{var } x_3 = E(x_3 - E(x_3))^2 = \left(\frac{\alpha}{k}\right)^2 \text{var } y_3. \tag{1.2.4.54}$$

The skewness of x_3 is independent of the location and scale parameters u and α and equals the skewness of the y_3 variate which has the same k value.

General extreme value distribution (*GEV*). It was seen that the distribution function for the EV2 and EV3 variates (Equations (1.2.4.12) and (1.2.4.35)) is the same for each with the provision that $k<0$ in the former and $k>0$ in the latter. Furthermore the limit of each of these expressions as k tends to zero is Equation (1.2.4.1) which is the EV1 distribution function. Thus, one equation contains the df for the three extreme value variates,

$$F(x) = e^{-[1-k(x-u)/\alpha]^{1/k}}. \tag{1.2.4.55}$$

The variate x in Equation (1.2.4.55) is referred to as the general extreme value, GEV, variate and the expression is due to Jenkinson (1955).

The concept of a GEV distribution is useful if it is wished to use an extreme value distribution but the type is unknown. When sample data are available the GEV distribution may be fitted to it by methods to be described in Section 1.3. The fitting procedure estimates the type automatically by the value of k. In the event of an estimated k value being close to zero it would be advisable to re-analyse the data by fitting the EV1 distribution because the parameters u and α of this distribution would thus be more efficiently estimated than the u and α values estimated for the GEV distribution.

The GEV distribution is used in Section 2.6 to describe the average distribution over a region of standardised annual maximum floods. The rainfall growth curves described in Volume II of this Report are also GEV distributions.

Values of the GEV variate x and of the EV1 variate y_1 for which $F(x) = G(y_1)$ are related by

$$x = u + \alpha\left(\frac{1-e^{-ky_1}}{k}\right) \qquad (1.2.4.56)$$

which is the same expression as Equations (1.2.4.19) and (1.2.4.43). When this is written as

$$x = u + \alpha W(y_1;k) \qquad (1.2.4.57)$$

as in Equations (1.2.4.21) and (1.2.4.44) the quantity $W(y_1;k)$ may be regarded as the general extreme value reduced variate. This is tabulated in Tables 1.6 and 1.9 for negative and positive k values over a small section of return period values. It is more extensively tabulated in Table 1.12.

1.2.5 Log distributions

The best known log distributions in hydrology are the lognormal (Chow, 1954), log Pearson Type 3 and log Gumbel distributions. If a variate x has a lognormal distribution, for instance, this means that $Z = \ln x$ has a Normal distribution. Similarly if x is a log Pearson Type 3 variate $Z = \ln x$ is a Pearson Type 3 variate and if x is a log Gumbel variate $Z = \ln x$ is a Gumbel or EV1 variate. A generalisation is possible where a lower bound x_0 is introduced, in which case $\ln x$ above is replaced by $\ln(x-x_0)$. The words log domain will often be used with reference to Z values.

The two parameter lognormal distribution. The variate x is positive and the bound x_0 mentioned above does not appear. The variate

$$z = \ln x \qquad (1.2.5.1)$$

is Normally distributed with mean μ_z and variance σ_z^2. These parameters are used here to specify the distribution although the mean and variance of x could also be used but at the expense of simplicity.

The pdf of z is

$$\phi(z) = \frac{1}{\sigma_z\sqrt{2\pi}}\, e^{-\frac{1}{2}[(z-\mu_z)/\sigma_z]^2}. \qquad (1.2.5.2)$$

The df of z is referred to as $\Phi(z)$ below.

Then

$$f(x) = \phi\{z(x)\}\left|\frac{dz}{dx}\right| \qquad (1.2.5.3)$$

where $z(x)$ is z considered as a function of x, as in Equation (1.2.5.1). Therefore, the pdf of x is

$$f(x) = \frac{1}{x\sigma_z\sqrt{2\pi}}\, e^{-\frac{1}{2}[(\ln x-\mu_z)/\sigma_z]^2}. \qquad (1.2.5.4)$$

The necessity for tabulating the df $F(x)$ by integrating this expression is obviated by the fact that $F(x)$ may be derived from $\Phi(z)$, the $N(\mu_z,\sigma_z)$ distribution function. The mean and variance are

$$\text{Mean} = E(x) = e^{\mu_z+\frac{1}{2}\sigma_z^2} \qquad (1.2.5.5)$$

$$\text{Variance} = E[x - E(x)]^2 = e^{(2\mu_z + \sigma_z^2)}.(e^{\sigma_z^2} - 1). \quad (1.2.5.6)$$

In later calculations the coefficient of variation, which is a function of σ_z^2 only, will be useful. It is

$$\text{cv} = (e^{\sigma_z^2} - 1)^{\frac{1}{2}} \quad (1.2.5.7)$$

and consequently

$$\sigma_z^2 = \ln(1 + \text{cv}^2). \quad (1.2.5.8)$$

The skewness is

$$g = \mu_3/\mu_2^{3/2} = (e^{\sigma_z^2} - 1)^{\frac{1}{2}}(e^{\sigma_z^2} + 2). \quad (1.2.5.9)$$

This relation will be referred to again in Section 1.3; for practical values of σ_z^2, $0.1 < \sigma_z^2 < 0.6$, the relation is almost linear

$$g \simeq 0.52 + 4.85\sigma_z^2 \quad (1.2.5.9a)$$

which is correct to within 2% in the stated range.

Of the three, only the mean depends on the location parameter x_0. The variance and skewness never depend on a location parameter. Further, since g depends only on σ_z^2 the latter, which is a scale parameter in the log domain, may be regarded as the shape parameter of the distribution. The parameter μ_z controls the scale of the distribution because $E(x)$ may be written

$$E(x) = e^{\mu_z}.e^{\frac{1}{2}\sigma_z^2} \quad (1.2.5.10)$$

which is reminiscent of the corresponding expression $E(x) = \beta\gamma$ in the gamma distribution where β is a scale parameter and γ is the mean of the reduced gamma variate whose shape parameter is γ. Similarly in Equation (1.2.5.10), $e^{\frac{1}{2}\sigma_z^2}$ is the mean of the lognormal distribution in which μ_z is zero, i.e. $e^{\mu_z} = 1$.

Hence, in summary, the location parameter in the log domain controls the scale in the x domain while the scale parameter in the log domain controls the skewness and hence shape in the x domain.

A reduced variate may be obtained for which the relation

$$x = A + By \quad (1.2.5.11)$$

holds when $F(x) = G(y)$, but it is not as useful as the reduced variates in the distributions described earlier. In this case $A = 0$ and $B = e^{\mu_z}$ = scale parameter and y is such that $\ln y$ is Normally distributed with mean zero and variance σ_z^2. It could be used in the computation of x values of given return period but by far the simplest procedure makes use of the fact that if $z_1 = \ln x_1$ then

$$PR(Z < z_1) = PR(X < x_1). \quad (1.2.5.12)$$

This is so because the log transformation is single valued and monotonic. Therefore, the z value corresponding to T is found in the $N(\mu_z, \sigma_z)$ distribution as described in Section 1.2.2 and the required x value is then

$$x = e^z. \quad (1.2.5.13)$$

Example 1. X is a two parameter lognormal variate with mean 10 and variance 9. Find the variate value x which has return period $T = 1/(1 - F(x)) = 10$. First compute the log domain parameters μ_z and σ_z from the mean and variance. The coefficient of variation

$$\text{cv} = \sqrt{9}/10 = 0.30$$

and then from Equation (1.2.5.8)

$\sigma_z^2 = \ln(1+\text{cv}^2) = \ln(1.09) = 0.0862$ and $\sigma_z = +\sqrt{0.0862} = 0.2936$.

From Equation (1.2.5.5)

$\mu_z = \ln E(x) - \frac{1}{2}\sigma_z^2 = 2.3026 - \frac{1}{2}(0.0862) = 2.260.$

The value of $F(x)$ corresponding to $T = 10$ is $F(x) = 1-1/T = 0.90$. The method described earlier is used to get an $N(\mu_z,\sigma_z)$ value with this $\Phi(z) = 0.90$. The corresponding $N(0,1)$ variate value is $y = 1.282$ and hence

$z = \mu_z + 1.282\sigma_z = 2.260 + 1.282(0.294) \doteq 2.637$

and by Equation (1.2.5.13) the required x is

$x = e^z = e^{2.637} = 13.97.$

The three parameter lognormal distribution. This differs from the two parameter lognormal distribution by the introduction of a lower bound x_0 so that

$$z = \ln(x-x_0) \tag{1.2.5.14}$$

(and not $z = \ln(x)$) is Normally distributed with parameters μ_z and σ_z^2.

Thus, it is a three parameter distribution; x_0 being a location parameter while μ_z and σ_z^2 control the scale and skewness (shape) respectively. It can be shown that the pdf of x is

$$f(x) = \frac{1}{(x-x_0)\sigma_z\sqrt{2\pi}} e^{-\frac{1}{2}[(\ln(x-x_0)-\mu_z)/\sigma_z]^2}. \tag{1.2.5.15}$$

The moments of x may be obtained from the corresponding moments in the two parameter distribution because the two variates differ only in the location parameter x_0. Therefore, writing the two parameter variate temporarily as ξ the three parameter variate x is

$$x = x_0 + 1.\xi. \tag{1.2.5.16}$$

Then using Equations (1.1.6.2) and (1.1.6.3)

$$\text{Mean} = E(x) = x_0 + 1.E(\xi) = x_0 + e^{\mu_z + \frac{1}{2}\sigma_z^2} \tag{1.2.5.17}$$

$$\text{Variance} = \text{var}\, x = (1)^2 \,\text{var}\,(\xi) = e^{(2\mu_z+\sigma_z^2)}.(e^{\sigma_z^2}-1). \tag{1.2.5.18}$$

In this case the coefficient of variation, cv, is not as succinct as its counterpart, Equation (1.2.5.7), in the two parameter case and does not prove as helpful as it did there. Since Equation (1.2.5.16) is just a linear transformation x and ξ have the same skewness, g, which is given in Equations (1.2.5.9) and (1.2.5.9a).

The distribution may be specified by the mean, variance and skewness but it is most convenient when computing quantile values or return periods to convert to the parameters x_0, μ_z and σ_z^2 and carry out the probability calculations in terms of $N(0,1)$ and $N(\mu_z,\sigma_z^2)$ variates as described in the next example. The connection between the results of such calculations and the x domain is given by

$$x = x_0 + e^z \tag{1.2.5.19}$$

which is the counterpart of Equation (1.2.5.13).

Example 2. The variate x is a three parameter lognormal variate with mean 20, standard deviation 6 and skewness 1.5. Find the value of x which has return period $T = 1/(1-F(x)) = 5$.

First convert the parameters to x_0, μ_z and σ_z^2 beginning with the skewness. From Equation (1.2.5.9a) a value of skewness $g = 1.5$ corresponds to $\sigma_z^2 = 0.20$ and therefore $\sigma_z = 0.447$. Rearranging Equation (1.2.5.18) gives

$$\mu_z = \ln(\text{standard deviation}) - \tfrac{1}{2}\sigma_z^2 - \tfrac{1}{2}\ln(e^{\sigma_z^2} - 1)$$

$$\mu_z = \ln 6 - \tfrac{1}{2}(0.20) - \tfrac{1}{2}\ln(1.2214 - 1)$$
$$= 1.79176 - 0.10 + 0.7539 = 2.4457$$

and from Equation (1.2.5.17)

$$x_0 = \text{Mean} - e^{\mu_z + \frac{1}{2}\sigma_z^2}$$
$$= 20 - e^{2.5457} = 20.0 - 12.75 = 7.25.$$

Having obtained the parameter values x_0, μ_z and σ_z the next step is to obtain the z value of return period $T = 5$ in the $N(\mu_z, \sigma_z)$ distribution. The value of $N(0,1)$ variate for which $\Phi(z) = F(x) = 1 - 1/T = 0.80$ is 0.84. Therefore $z = \mu_z + \sigma_z(0.84) = 2.4457 + 0.447(0.84) = 2.821$ and $x = x_0 + e^z = 7.25 + 16.80 = 24.05$.

The log Pearson Type 3 distribution. The traditional hydrological method of dealing with this distribution is to consider only the distribution of the logarithmically transformed variate, $Z = \ln x$. The distribution of Z is taken as Pearson Type 3 with three parameters. A more general approach would be to consider $Z = \ln(x - x_0)$ as a three parameter Pearson Type 3 variate, x then having a four parameter distribution.

Let the location, scale and shape parameters of the Z variate be z_0, λ and η respectively. The density function is

$$\phi(z) = \frac{e^{-(z-z_0)/\lambda}(z-z_0)^{\eta-1}}{\lambda^\eta \Gamma(\eta)}. \qquad (1.2.5.20)$$

If $z = \ln x$ the density function of x is

$$f(x) = \phi(z(x)) \left|\frac{dz}{dx}\right|$$
$$= \frac{e^{-(\ln x - z_0)/\lambda}(\ln x - z_0)^{\eta-1}}{x\lambda^\eta \Gamma(\eta)}. \qquad (1.2.5.21)$$

A quantile is most easily computed if the parameters are specified as z_0, λ and η, the log domain parameters. In that case the corresponding quantile y in a reduced gamma distribution with parameter η is found as described earlier for instance, by using Table 1.3. Then

$$z_p = z_0 + \lambda y \qquad (1.2.5.22)$$

is the corresponding quantity in the Pearson Type 3 distribution and the corresponding log Pearson Type 3 variate is

$$x_p = e^{z_p} \qquad (1.2.5.23)$$

or

$$x_p = x_0 + e^{z_p}$$

in the four parameter case.

This procedure obviates the need to integrate $f(x)$ of Equation (1.2.5.21) numerically. The log Pearson Type 3 distribution differs from the lognormal distribution in that it is customary to define the former in the log domain only, because simple expressions linking the log domain parameters z_0, λ and η with the moments in the x domain are not available whereas such expressions are available for the latter.

Example 3. x is a log Pearson Type 3 variate such that $\ln x$ has mean = 5, variance = 0.08 and skewness = 0.61. What is the quantile of return period $T = 10$?

First compute z_0, λ and η from the mean, variance and skewness by the methods described for the Pearson Type 3 variate in Section 1.2.3. In that case x_0, β and γ correspond to z_0, λ and η of the present variate z. Hence, from Equations (1.2.3.23)–(1.2.3.25) the relations between moments and parameters are

$$\text{Mean} = z_0 + \lambda\eta$$
$$\text{Variance} = \lambda^2\eta$$
$$\text{Skewness} = \frac{2}{\sqrt{\eta}}.$$

Therefore, $\eta = 10.75$, $\lambda = 0.0863$ and $z_0 = 4.0723$. Table 1.3 does not have an entry for $\eta = \gamma = 10.75$ but on interpolation in fuller tables given by Wilk, Gnanadesikan & Huyett (1962) the reduced gamma variate y with $G(y) = 1-1/T = 0.90$ is 15.1. Then from Equation (1.2.5.22)

$$z = z_0 + \lambda y = 4.0723 + 0.0863(15.1) = 5.3754.$$

This is the required quantile value in the log domain. In the x domain this is given by Equation (1.2.5.23)

$$x = e^z = e^{5.3754} = 216.03.$$

The log Gumbel distribution. The log Gumbel variate x is such that $z = \ln x$ is a Gumbel or EV1 variate and it will be seen below that it is equivalent to a Fréchet or EV2 distribution. Since the Gumbel variate depends on two parameters this is a two parameter distribution when specified like this. A third parameter x_0 may be introduced by specifying that $z = \ln(x-x_0)$ is a Gumbel variate.

Let the Gumbel variate z have location and scale parameters δ and λ respectively. Then the df of z is

$$\Phi(z) = \exp(-e^{-(z-\delta)/\lambda}). \qquad (1.2.5.24)$$

In this case it is algebraically neater to specify the df of x than the pdf of x as was done in the lognormal case.

Since the logarithmic function is single valued and monotonic

$$PR(X<x) = PR(Z<\ln x) \quad \text{or} \quad PR(Z<\ln(x-x_0)) \qquad (1.2.5.25)$$

depending on whether x_0 is to be included or not. For the two parameter case this gives

$$F(x) = PR(X < x) = \exp(-e^{-(\ln x-\delta)/\lambda}) \qquad (1.2.5.26)$$

while in the three parameter case

$$F(x) = \exp(-e^{-(\ln(x-x_0)-\delta)/\lambda}). \qquad (1.2.5.27)$$

Equation (1.2.5.27) may be rewritten, noting that

$$e^{-\ln(x-x_0)/\lambda} = (x-x_0)^{-1/\lambda}$$

$$F(x) = \exp\left[-(x-x_0)^{-1/\lambda}.e^{\delta/\lambda}\right] \quad (1.2.5.27a)$$

and substituting for x_0, δ and λ by

$$\lambda = -k \quad (1.2.5.28)$$

$$\delta = -\ln\left(-\frac{k}{\alpha}\right) \quad (1.2.5.29)$$

$$x_0 = u+\frac{\alpha}{k} \quad (1.2.5.30)$$

gives

$$F(x) = e^{-[1-k(x-u)/\alpha]^{1/k}} \quad (1.2.5.31)$$

which shows that x has the Fréchet or EV2 distribution of Equation (1.2.4.12).

Similarly Equation (1.2.5.26) may be rewritten

$$F(x) = \exp\left(-x^{-1/\lambda}.e^{\delta/\lambda}\right). \quad (1.2.5.26a)$$

Since $x = e^z$ ranges from 0 to ∞, this places a constraint on the parameters of the EV2 variate, namely that the lower bound $u+\alpha/k$ must be zero. If Equations (1.2.5.28) and (1.2.5.29) are used in Equation (1.2.5.26a) to substitute for λ and δ the resulting equation is

$$F(x) = e^{-(xk/\alpha)^{1/k}} \quad (1.2.5.32)$$

which is the same as Equation (1.2.4.12) with the proviso that $u = -\alpha/k$. Hence, the two parameter log Gumbel variate is also a special case of the Fréchet or EV2 variate.

Therefore, whenever it is appropriate to use the log Gumbel distribution the methods described in Section 1.2.4 should be used bearing in mind the relations between the log domain parameters and the x domain parameters given in Equations (1.2.5.28)–(1.2.5.30).

One important point to remember is that if x is the three parameter log Gumbel variate, values of ln x and y_1 for which $F(x) = G(y_1)$ are not exactly linearly related.

It is ln $(x-x_0)$ and y_1, for which $F(x) = G(y_1)$, which are linearly related. This point is important in graphical estimation and hypothesis testing.

1.2.6 Mixtures of distributions

In the statistical populations considered thus far the variate values associated with the population units come from a single distribution $F(x)$. It is conceivable that a certain proportion, p, of population units have variate values which come from a distribution $F_1(x)$ while the remainder have variate values which come from a different distribution $F_2(x)$. If the units are considered to be thoroughly mixed the probability is p that a single unit drawn at random from the population is from the first distribution and $1-p$ that it is from the second. The unconditional probability that the variate value on such a randomly drawn unit is less than x is

$$PR(X \leqslant x) = F(x) = pF_1(x)+(1-p)F_2(x). \quad (1.2.6.1)$$

A distribution defined by such a relation is said to be a mixture of distributions. In general, a mixture may consist of a number of component

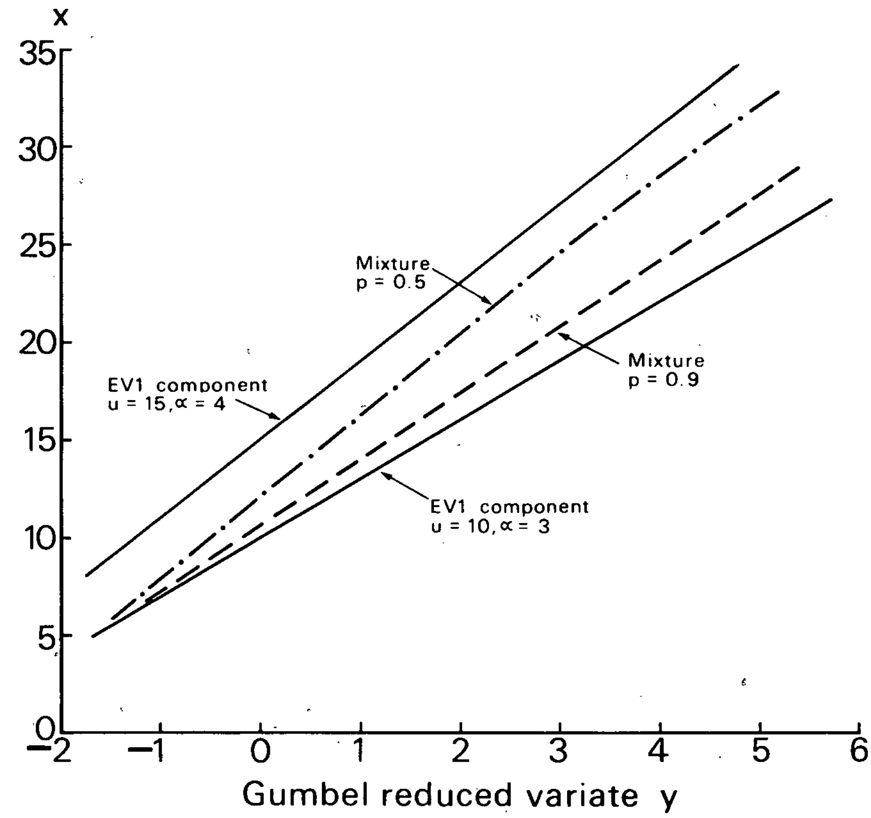


Fig 1.15 Two distributions formed as mixtures from the same two EV1 distributions, one with mixture parameter $p = 0.5$, the other with $p = 0.9$.

distributions, a proportion p_i of the entire distribution being from the ith distribution. $F(x)$ is then given by

$$F(x) = p_1F_1(x)+p_2F_2(x)+ \ldots +p_iF_i(x)+ \ldots p_mF_m(x) \qquad (1.2.6.2)$$

subject to $\sum_{i=1}^{m} p_i = 1$ where m is the number of component distributions in the mixture.

As an example, suppose $p = 0.90$ and the two component distributions are extreme value Type I (Gumbel) with $(u_1,\alpha_1) = (10,3)$ and $(u_2,\alpha_2) = (15,4)$. The mean and standard deviation in the first distribution are therefore 11.73 and 3.84 while in the second distribution they are 17.31 and 5.12. These distributions are shown as straight lines on Figure 1.15 where the abscissa is Gumbel reduced variate. The dashed curve is the mixture of these two with $p = 0.90$. This x–y relation is obtained as follows. For a given x, $F_1(x)$ and $F_2(x)$ are computed as

$$F_1(x) = \exp(-e^{-(x-u_1)/\alpha_1})$$
$$F_2(x) = \exp(-e^{-(x-u_2)/\alpha_2})$$

and then $F(x) = pF_1(x)+(1-p)F_2(x)$ is computed. The value of y in the reduced Gumbel distribution having non exceedance probability $G(y) = F(x)$ is obtained from the inverse of

$$F(x) = G(y) = \exp(-e^{-y})$$

which is $y = -\ln-\ln F(x)$. This is demonstrated in Table 1.11 where the values of $F(x)$ in the mixture of distributions corresponding to the x values in column 1 are given in column 6; column 7 gives the corresponding values of reduced variate y against which the x values of column 1 are plotted to give the dashed curve in Figure 1.15.

x	$(x-u_1)/\alpha_1$	$F_1(x)$	$(x-u_2)/\alpha_2$	$F_2(x)$	$F(x)$	y	$F(x)$	y
8	−0.666	0.143	−1.750	0.003	0.129	−0.72	0.073	−0.96
12	+0.666	0.598	−0.750	0.120	0.550	0.52	0.359	−0.02
16	2.000	0.873	0.250	0.459	0.832	1.69	0.666	0.90
20	3.333	0.965	1.250	0.751	0.944	2.85	0.858	1.88
24	4.666	0.991	2.250	0.900	0.982	3.98	0.946	2.89
28	6.000	0.998	3.250	0.962	0.994	5.11	0.980	3.90
32	7.333	0.999	4.250	0.986	0.998	6.21	0.993	4.96

Table 1.11 Calculation of a mixture of distributions.

The values of x, $F_1(x)$ and $F_2(x)$ were also used with a mixture parameter $p = 0.5$ to give the results in columns 8 and 9. The corresponding x–y relation is shown as a chain curve in Figure 1.15.

1.2.7 Tabulated reduced variate values

Reduced variate values y corresponding to convenient and practical values of return period T and distribution function $G(y)$ are required both in the numerical calculation of quantiles of variates which are functions of y and in the construction of curves showing how x and y values, for which $F(x) = G(y)$, are related. Short tables of Normal, exponential and Gumbel (EV1) reduced variates y are presented in Tables 1.1(c), 1.2(c) and 1.4(c) while short tables of $W(y_1;k)$ the general extreme value reduced variate are presented in Table 1.6 for $k<0$ and in Table 1.9 for $k>0$.

These quantities are more extensively tabulated in Table 1.12. The range of df values and return periods ought to be sufficient for most practical purposes. At the foot of the table the formula for y in terms of T is given so that additional values may be computed if required.

1.3 Estimation and inference

1.3.1 Introduction

This section deals with the statistical methods available for estimating location and scale parameters (and shape parameters where applicable) when stream flow records are available at the site under consideration. This process will be demonstrated by reference to the annual maximum distribution throughout because this distribution is given most attention in this report. These methods will also be used in Section 2.7 where the peaks over a threshold (POT) model is used.

Various theoretically specified distributions have been described earlier; Normal, exponential, gamma, Pearson Type 3, extreme value and log distributions. Since no deductive method exists for deciding on the form of a distribution, the suitability of each possible candidate distribution must be examined. Each distribution is considered in turn to be the correct distribution and some numerical index is calculated expressing the agreement (or lack of it) between the assumption and the information about the distribution contained in flow records. In this case the information in the flow record is that contained in the appropriate series alone. First the parameters of the distribution must be estimated from the data. A numerical measure of the difference between the two distributions, fitted

Table 1.12 Reduced variate values.

Return period T†	Nonexceedance probability $G(y)$‡	Normal $N(0,1)$ y§	Exponential y‖	Gumbel or EV1 y_1¶	General extreme value $W(y_1;k)$††									
					$k=-0.25$	$k=-0.20$	$k=-0.15$	$k=-0.10$	$k=-0.05$	$k=0.05$	$k=0.10$	$k=0.15$	$k=0.20$	$k=0.25$
1.0101	0.01	−2.33	0.01	−1.53	−1.27	−1.32	−1.36	−1.42	−1.47	−1.59	−1.65	−1.72	−1.79	−1.86
1.0526	0.05	−1.64	0.05	−1.10	−0.96	−0.99	−1.01	−1.04	−1.07	−1.13	−1.16	−1.19	−1.23	−1.26
1.1111	0.10	−1.28	0.11	−0.83	−0.75	−0.77	−0.78	−0.80	−0.82	−0.85	−0.87	−0.89	−0.91	−0.93
1.250	0.20	−0.84	0.22	−0.48	−0.45	−0.45	−0.46	−0.46	−0.47	−0.48	−0.49	−0.49	−0.50	−0.51
1.428	0.30	−0.52	0.36	−0.19	−0.18	−0.18	−0.18	−0.18	−0.18	−0.19	−0.19	−0.19	−0.19	−0.19
1.667	0.40	−0.25	0.51	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
2.00	0.50	0.00	0.69	0.37	0.38	0.38	0.38	0.37	0.37	0.36	0.36	0.36	0.35	0.35
2.50	0.60	0.25	0.92	0.67	0.73	0.72	0.71	0.69	0.68	0.66	0.65	0.64	0.63	0.62
3.33	0.70	0.52	1.20	1.03	1.18	1.14	1.11	1.09	1.06	1.00	0.98	0.96	0.93	0.91
5.00	0.80	0.84	1.61	1.50	1.82	1.75	1.68	1.62	1.56	1.45	1.39	1.34	1.30	1.25
10.00	0.90	1.28	2.30	2.25	3.02	2.84	2.68	2.52	2.38	2.13	2.02	1.91	1.81	1.72
20.00	0.95	1.64	3.00	2.97	4.41	4.06	3.74	3.46	3.20	2.76	2.57	2.40	2.24	2.10
25.00	0.96	1.75	3.22	3.20	4.90	4.48	4.10	3.77	3.47	2.96	2.74	2.54	2.36	2.20
40.00	0.975	1.96	3.69	3.68	6.03	5.43	4.90	4.44	4.04	3.36	3.08	2.83	2.60	2.40
50.00	0.98	2.05	3.91	3.90	6.61	5.91	5.30	4.77	4.31	3.54	3.23	2.95	2.71	2.49
75.00	0.9866	2.21	4.32	4.31	7.75	6.84	6.06	5.39	4.81	3.88	3.50	3.17	2.89	2.64
100.00	0.99	2.33	4.61	4.60	8.63	7.55	6.63	5.84	5.17	4.11	3.69	3.32	3.01	2.73
200.00	0.995	2.58	5.30	5.30	11.03	9.42	8.09	6.98	6.06	4.65	4.11	3.65	3.27	2.94
500.00	0.998	2.88	6.21	6.21	14.91	12.33	10.26	8.61	7.29	5.34	4.63	4.04	3.56	3.15
1000.00	0.999	3.09	6.91	6.91	18.49	14.90	12.12	9.95	8.25	5.84	4.99	4.30	3.74	3.29
5000.00	0.9998	3.54	8.52	8.52	29.64	22.46	17.25	13.44	10.62	6.94	5.73	4.81	4.09	3.52
10000.00	0.9999	3.72	9.21	9.21	36.00	26.55	19.87	15.12	11.70	7.38	6.02	4.99	4.21	3.60

†$T = \dfrac{1}{1-G(y)}$.

‡$G(y) = 1-\dfrac{1}{T}$.

§Refer to published tables.

‖$y = \ln T$.

¶$y_1 = -\ln-\ln\dfrac{T-1}{T}$.

††$W(y_1;k) = \dfrac{1-e^{-ky_1}}{k}$.

theoretical and observed, should be used to make a decision between different forms of distribution; however with the small samples available these measures are not alone sufficient as will be discussed in Section 2.4.

Thus, in choosing between the different forms of distribution at a given section of river having historical data there are two steps;

a estimation of parameters and

b calculation of the numerical measure of agreement.

As will be shown the process of estimation can be regarded as drawing a line or curve through a series of points plotted on a graph which is also referred to as fitting a line or curve to the plotted points. Hence, the words 'to fit' and 'fitting' are used synonymously with 'to estimate' and 'estimation'. Further, the agreement between theoretical (in most practical cases a fitted one) and observed distributions is referred to as goodness of fit and any numerical measure of goodness of fit is referred to as a goodness of fit index.

In practice, the choice of a distribution for flood frequency work at a site is not as clearcut and straightforward as it may seem superficially. In the first place there is more than one method of estimating or fitting and further there are several possible goodness of fit indices. Thus, the form of distribution chosen is dependent not only on the method of estimation but also on the choice of goodness of fit index and therefore the final choice is not unique except in the context of a stated method of fitting and a stated goodness of fit index. The question of estimation which forms a large part of statistical theory is dealt with here; the question of goodness of fit indices is discussed in Section 2.4.

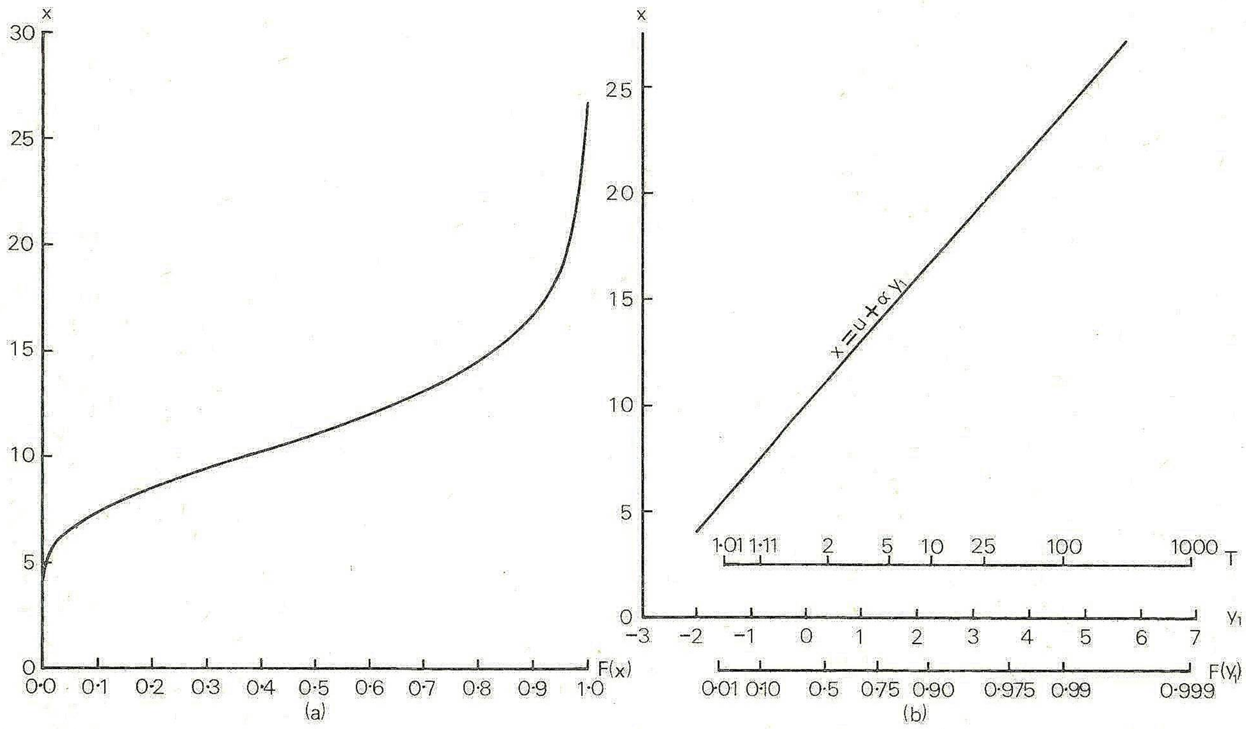


Fig 1.16 An EV1 variate x shown as a function of (a) its own df which itself may be considered as a variate distributed between 0 and 1 and (b) an EV1 reduced variate y.

Estimation. From the hydrological point of view what is required is a procedure which, for instance, makes use of the observed sample of annual maxima to produce an estimate of the T year flood, $Q(T)$. This involves theoretical and observed, should be used to make a decision between different forms of distribution; however with the small samples available these measures are not alone sufficient as will be discussed in Section 2.4.

Thus, in choosing between the different forms of distribution at a given section of river having historical data there are two steps;

a estimation of parameters and

b calculation of the numerical measure of agreement.

As will be shown the process of estimation can be regarded as drawing a line or curve through a series of points plotted on a graph which is also referred to as fitting a line or curve to the plotted points. Hence, the words 'to fit' and 'fitting' are used synonymously with 'to estimate' and 'estimation'. Further, the agreement between theoretical (in most practical cases a fitted one) and observed distributions is referred to as goodness of fit and any numerical measure of goodness of fit is referred to as a goodness of fit index.

In practice, the choice of a distribution for flood frequency work at a site is not as clearcut and straightforward as it may seem superficially. In the first place there is more than one method of estimating or fitting and further there are several possible goodness of fit indices. Thus, the form of distribution chosen is dependent not only on the method of estimation but also on the choice of goodness of fit index and therefore the final choice is not unique except in the context of a stated method of fitting and a stated goodness of fit index. The question of estimation which forms a large part of statistical theory is dealt with here; the question of goodness of fit indices is discussed in Section 2.4.

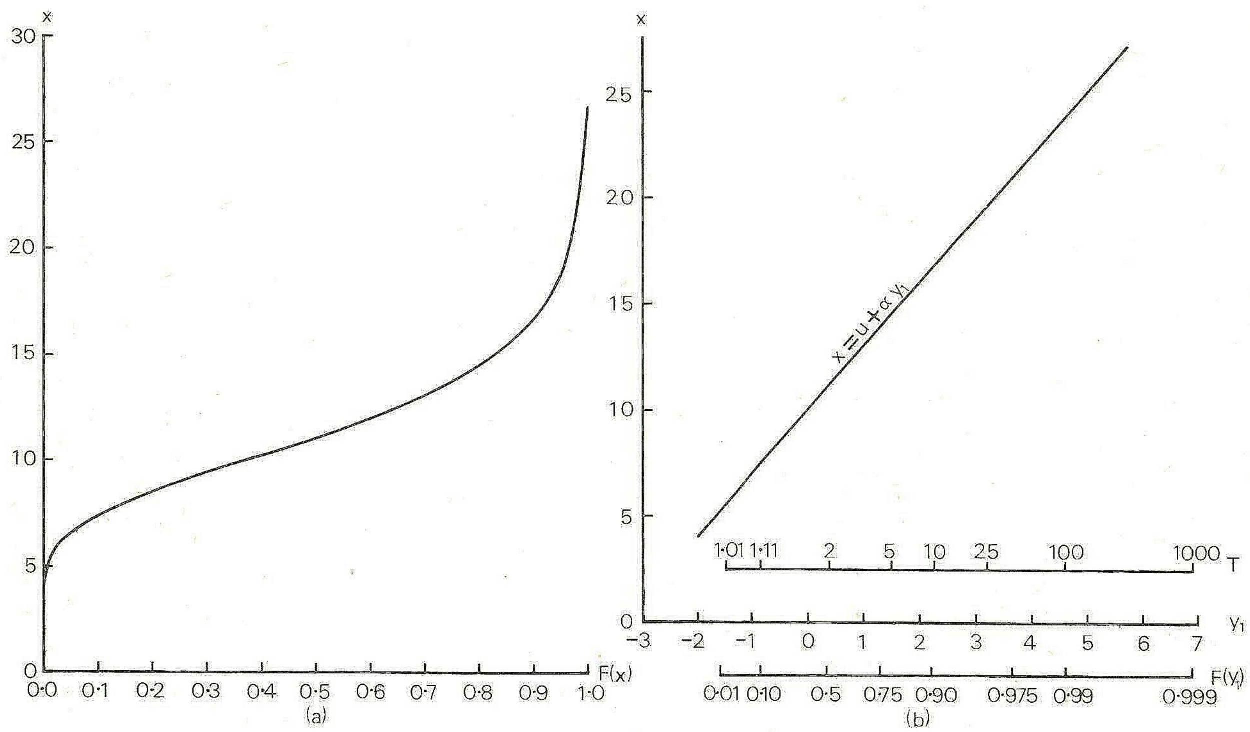


Fig 1.16 An EV1 variate x shown as a function of (a) its own df which itself may be considered as a variate distributed between 0 and 1 and (b) an EV1 reduced variate y.

Estimation. From the hydrological point of view what is required is a procedure which, for instance, makes use of the observed sample of annual maxima to produce an estimate of the T year flood, $Q(T)$. This involves

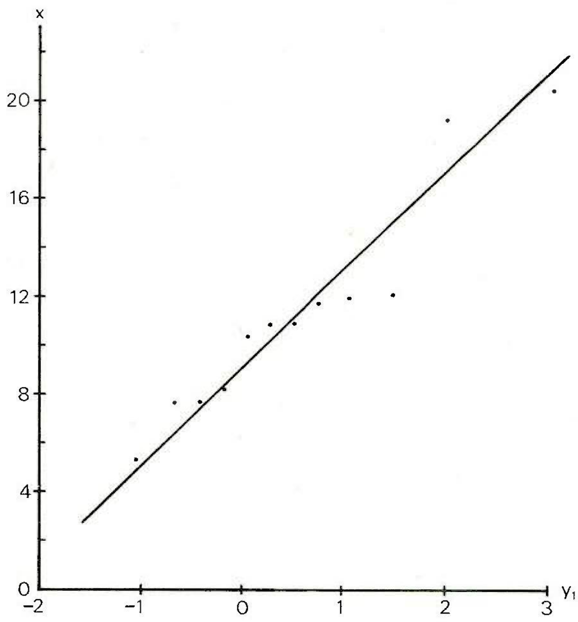


Fig 1.17 A random sample of size 12 from an EV1 distribution with $u = 10$, $\alpha = 3$ shown plotted against EV1 reduced variate. This is synonymous with a probability plot.

estimating the parameters of the distribution from the sample and using them in a separate step as shown in the examples in Section 1.2, to calculate the corresponding $Q(T)$, but there would be no loss incurred if the parameters themselves were not estimated explicitly provided that $Q(T)$ were estimated explicitly. Therefore, *ab initio*, such methods of estimation are not to be ruled out.

There follows a discussion of graphical estimation because it introduces probability plots which are used throughout the report for the display of both data and fitted distributions.

1.3.2 Graphical estimation

In graphical estimation, the variate under consideration is regarded as a function of a standardised or reduced variate with known distribution. For instance, Figure 1.16(a) shows an extreme value Type 1 (Gumbel) variate as a function of probability $F(x)$ (which can be considered as a variate which is uniformly distributed between 0 and 1) and as a linear function of the standardised variate y in Figure 1.16(b). In graphical estimation the sample of data is plotted as a series of N discrete points on an x–y plot. These points represent the sample distribution and a line drawn through these is taken as an estimate of the population x–y relation. Figure 1.17 is an example of a sample of size 12 from the Gumbel distribution, ($u = 10$, $\alpha = 3$) with an eye guided straight line shown as an estimate of the population x–y relation.

In graphical estimation the line is subjectively placed and could vary with analyst and even with occasion. This subjectivity is regarded as a major drawback by most modern writers.

If it is known that the sample came from the EV1 population then a straight line is required and consequently the subjectivity of the graphical estimate is based on two degrees of freedom; otherwise the analyst exercises judgement as to whether he should draw a straight line or a curve. In either case he could make a mistake by drawing a curve where a line represents the real population and vice versa.

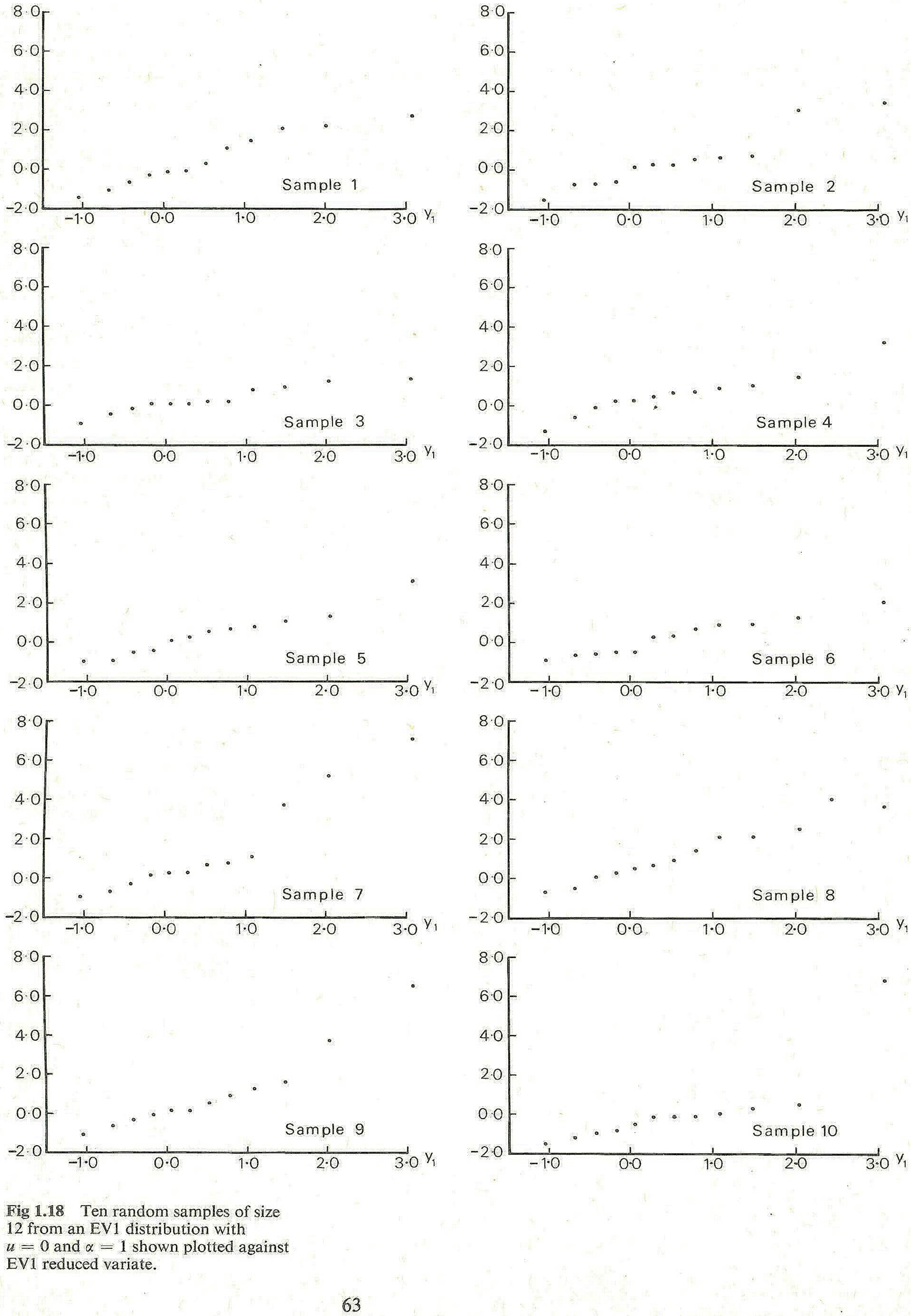


Fig 1.18 Ten random samples of size 12 from an EV1 distribution with $u = 0$ and $\alpha = 1$ shown plotted against EV1 reduced variate.

Figure 1.18 shows plots of ten samples of size 12 from the EV1 distribution with parameters $u = 0$, $\alpha = 1$. These diagrams show that some samples are well represented by straight lines but others display definite curvature or changes of slope despite the fact that they come from a straight line population, while one contains an 'outlier'. These samples are sufficient to show that even when the population distribution is known it is quite possible to find a sample which displays curvature which would convince an analyst, if he did not know otherwise, that the distribution relation was curved.

These samples of size 12 are typical of the type found in hydrology and in this study; therefore it can never be assumed that the choice of a straight line or curve is correct. In other words, it is very easy to make a mistake of type of distribution with the sample sizes that are available in hydrology. This is, of course, a drawback in the graphical method but it should be remembered that this drawback is implicit in more objective methods where a choice of some particular distribution to be fitted is analogous to the choice between a straight line and a curved distribution on the probability plot.

The advantage of objective methods is that under the assumptions stated in their use the best method may be sought by theoretical means and the goodness of fit may be assessed probabilistically. However, with the small samples available in hydrology these goodness of fit indices are not sufficiently sensitive to change in the assumed distribution and consequently are not of much use (see Section 2.4).

It is also evident from these plots that the largest values display far more variation between samples than do the smaller values. These large values make it difficult to choose between straight lines and curves. Because of this large sampling variation, a curve or line drawn through a set of plotted points should not necessarily be made to pass through the highest point. In graphical estimation the curve or line drawn should be allowed to deviate from a plotted point by an amount which is proportional to the sampling standard error of that plotted point. In the present distribution, EV1, the sampling variance of the largest value equals the variance of the parent distribution. This should be sufficient to convince graphical analysts that there is no need to insist on drawing curves through the highest points.

Example. The 35 years of annual maxima for 27/1 the Nidd at Hunsingore are shown plotted on Figure 1.19. The horizontal axis is the EV1 (Gumbel) standardised variate. This axis is used not because it is asserted that the data are distributed as EV1 but because of its general suitability, as the plotted data from every station can be represented on it without serious curvature.

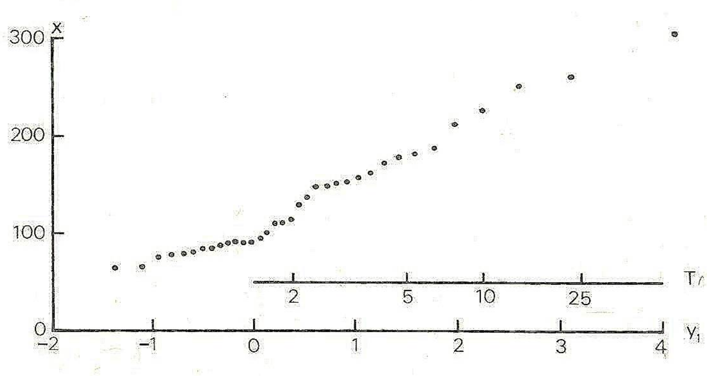


Fig 1.19 Probability plot of the 35 annual maxima at 27/1, Nidd at Hunsingore, with the return period scale replacing the probability scale. The x values are paired with y values for plotting. Here y is EV1 reduced variate.

Plotting positions

The production of a plot such as that in Figure 1.19 needs N pairs of co-ordinates (x,y). The N values of x are the ordered set of sampled data $x_{(1)} \leqslant x_{(2)} \leqslant \ldots x_{(i)} \ldots \leqslant x_{(N)}$. The corresponding y values are not observed but values $y_1 < y_2 < \ldots y_i \ldots < y_N$ called the plotting positions are chosen as follows. Assume that the population is EV1 so that the population x–y relation is the straight line

$$x = u + \alpha y. \tag{1.3.2.1}$$

The observed $x_{(i)}$ is to be plotted at $y = y_i'$, say, where y_i' should be such that the average value of $x_{(i)}$, namely $E(x_{(i)})$, when plotted at y_i' would lie on the population line. Such a plotting position is said to be *unbiassed.* This condition written algebraically is

$$E(x_{(i)}) = u + \alpha y_i'$$

giving

$$y_i' = \frac{E(x_{(i)}) - u}{\alpha} = E(y_{(i)}). \tag{1.3.2.2}$$

The right hand side of Equation (1.3.2.2) is the mean value of $y_{(i;N)}$, the ith smallest value in a random sample of size N from the standardised distribution. This quantity of course depends on the order i, the sample size N and the form of the distribution. It is not easily computed exactly but an excellent approximation (more than sufficient for plotting purposes) is given for the EV1 case by Gringorten (1963). First the value of probability F_i corresponding to $E(y_{(i)})$ is computed from Gringorten's formula

$$F_i = \frac{i - 0.44}{N + 0.12} \tag{1.3.2.3}$$

and then the value of y corresponding to this is computed from the inverse of

$$F(y) = e^{-e^{-y}} \tag{1.3.2.4.}$$

namely

$$y_i = -\ln - \ln F_i \tag{1.3.2.5}$$

and this is the ith plotting position. If the graph paper has an extreme value probability scale marked on it the values F_i of Equation (1.3.2.3) can be used as plotting positions but in practice tables of y_i can be produced which allow the use of linear paper for the plot (see Table 1.16). When large samples are plotted the middle values are plotted very close together. It is valid to divide the middle of the ordered data and corresponding plotting positions into groups and plot only the group averages. These averaged plotting positions retain the unbiassed property of the original plotting positions.

1.3.3 Analytical methods of fitting

There are two main methods of fitting which are entirely objective. These are the methods of moments and maximum likelihood. Other objective methods less frequently used are the methods of least squares (regression), minimum chi square and sextiles.

a The method of moments makes use of the fact that if all the moments of a distribution are known then everything about the distribution is known. In all the distributions in common usage four moments or fewer are sufficient to specify all the moments. For instance, two moments, the first together with any moment of even order, are sufficient to specify all the moments of the Normal distribution and therefore the entire distribution. Similarly, in the EV1 distribution the first two moments are sufficient to specify all the moments and hence the distribution. In the Pearson Type 3 distribution three moments—always taken as the first three—are required to specify all the moments. In these cases the number of moments needed to specify all the moments and hence the distribution equals the number of parameters; in general this is so because a location parameter is analogous to the first moment, a scale parameter to the square root of the second moment and a shape parameter is dependent on the third moment.

Estimation is dependent on the assumption that the distribution of variate values in the sample is representative of the population distribution. Therefore, a representation of the former provides an estimate of the latter. Given that the form of the distribution is known or assumed, the distribution of the sample is specified by its first two or three moments calculated from the data and this is the estimate of the population distribution by the method of moments. As an example, suppose the moments estimate of an EV1 population is required. The sample moments are denoted by m_1 and m_2. The parameters u and α of the EV1 distribution are related to the population moments μ_1' and μ_2 by the equations

$$\mu_1' = u + 0.5772\alpha \quad \text{and} \quad \mu_2 = \frac{\pi^2\alpha^2}{6}.$$

The moments method of estimation replaces μ_1' and μ_2 by the sample values m_1 and m_2 and the corresponding estimates of u and α are

$$\hat{\alpha} = \frac{\sqrt{6}}{\pi}\sqrt{m_2}$$

$$\hat{u} = m_1 - (0.5772)\frac{\sqrt{6}}{\pi}\sqrt{m_2}.$$

The ^ sign over α and u indicates that they are estimated values rather than the true population values. The mean value of m_2 over all possible samples is $(N-1)\mu_2/N$ and is called a biassed estimate of μ_2. Unbiassed estimates are to be preferred and the following estimates of μ_1', μ_2, standard deviation σ, and the skewness, g, are to be used.

$$\begin{aligned}
\hat{\mu}_1' &= \sum_{i=1}^{N} x_i/N \\
\hat{\mu}_2 &= \sum_{i=1}^{N} (x_i-\bar{x})^2/(N-1) \\
\hat{\sigma} &= (\hat{\mu}_2)^{\frac{1}{2}} \\
\hat{g} &= \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} (x_i-\bar{x})^3/(\hat{\mu}_2)^{3/2}.
\end{aligned} \qquad (1.3.3.1)$$

b Maximum likelihood (ML) estimation

Let the set of parameters of the distribution to be estimated be **A** and let the sample values $x_1, x_2, \ldots, x_N$ be denoted collectively as **X**. Let $PR(\mathbf{X}/\mathbf{A})$ be the probability of drawing the observed random sample **X** from a population with parameters **A**. Obviously for a given observed sample **X** the value of $PR(\mathbf{X}/\mathbf{A})$ depends on the parameter values **A**. Of all possible sets of **A** values that set **A**′ which is such that $PR(\mathbf{X}/\mathbf{A}')$ is greater than $PR(\mathbf{X}/\mathbf{A})$ for any other set of **A** values is taken as the estimate of the population parameters. In the expression $PR(\mathbf{X}/\mathbf{A})$ the variable is **A** whereas in similar expressions in the conditional probability context **X** is the variable.

Define $L(\mathbf{X}/\mathbf{A})$ by

$$L(\mathbf{X}/\mathbf{A}) = \prod_{i=1}^{N} f(x_i/\mathbf{A}) \tag{1.3.3.2}$$

where $f(x/\mathbf{A})$ is the pdf and call $L(\mathbf{X}/\mathbf{A})$ the *likelihood* of **A** given the observed sample **X**. When **X** is a continuous variate $PR(\mathbf{X}/\mathbf{A})$ is formally zero. Because $\hat{P}R(\mathbf{X}-\mathrm{d}\mathbf{X}<\mathbf{X}<\mathbf{X}+\mathrm{d}\mathbf{X}/\mathbf{A})$ is proportional to $L(\mathbf{X}/\mathbf{A})$ the estimates are found by attempting from the outset to find **A** which maximises the likelihood, $L(\mathbf{X}/\mathbf{A})$; hence the name of maximum likelihood. Broadly speaking the maximum likelihood estimates of the parameters are those which make the given sample most likely or probable.

c The method of least squares can be used to estimate location and scale parameters such as (u,α) in the extreme value Type 1 distribution. The $x_{(i)}$ values are regressed on the y_i values (the plotting positions) and the parameter estimates $\hat{u}$ and $\hat{\alpha}$ in the relation $x = u+\alpha y$ are

$$\hat{\alpha} = \frac{\sum_{i=1}^{N} (x_{(i)}-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{N} (y_i-\bar{y})^2} \tag{1.3.3.3}$$

$$\hat{u} = \bar{x}-\hat{\alpha}\bar{y} \tag{1.3.3.4}$$

where $\bar{x}$ is the arithmetic mean of the sample values and $\bar{y}$ is the arithmetic mean of the N plotting positions.

This method was proposed by Chow (1953). In his usage the y_i values correspond to the Weibull plotting position based on a probability $F_i = i/(N+1)$ and the resulting estimates are biassed. If $E(y_{(i)})$ is used as plotting position the estimates are unbiassed (Kimball, 1960).

d For estimation by the minimum χ^2 method consider Figure 1.20 in which sample data are represented by a histogram of m blocks and a theoretical pdf is sketched over it. The form of the function is assumed known and the parameters are unknown. Let the jth block of the histogram be contained between $X = x_{j-1}$ and $X = x_j$. Let the number of sample members in this block be O_j and for some values of population parameters, **A**, let E_j be the mean number out of N occurring between x_{j-1} and x_j. E_j is

$$E_j = NP_j(\mathbf{A})$$

where $P_j(\mathbf{A})$ is the area under the probability density curve between x_{j-1} and x_j when the parameters are **A**. The quantity

$$\chi^2 = \sum_{j=1}^{m} (O_j-E_j)^2/E_j \tag{1.3.3.5}$$

is a measure of how closely the theoretical pdf with parameters **A** fits the sample histogram. That set of parameters **A**′ for which χ^2 is least is the minimum χ^2 estimate for the parameters. The system of equations to be solved to obtain **A**′ is rarely simple. The equations are

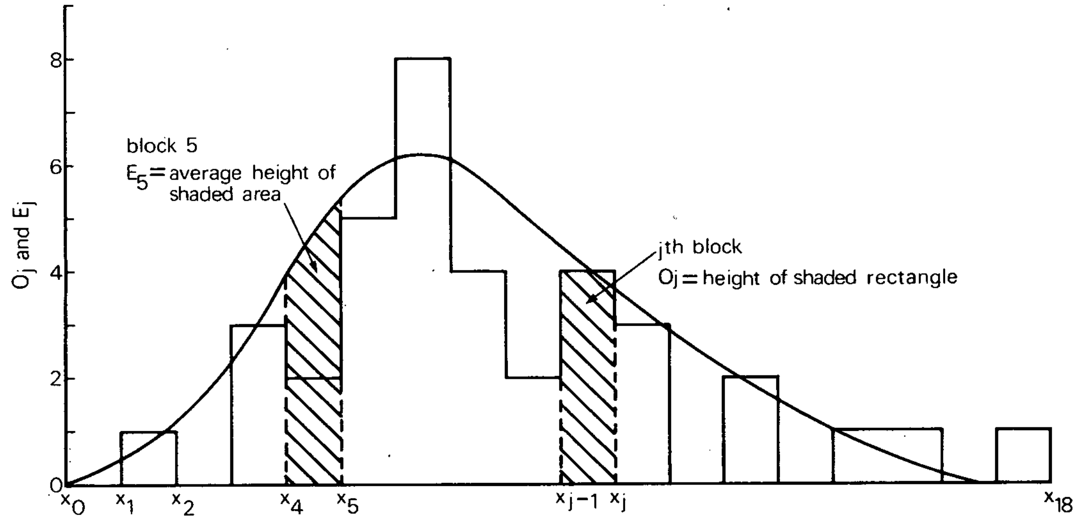


Fig 1.20 A sample histogram with a theoretical pdf superimposed on it and illustrating the notation E_j and O_j.

$$\sum_{j=1}^{m}\left\{\frac{O_j-NP_j(\mathbf{A})}{P_j(\mathbf{A})}+\frac{(O_j-NP_j(\mathbf{A}))^2}{2NP_j^2(\mathbf{A})}\right\}\frac{\partial P_j(\mathbf{A})}{\partial a_k}=0 \qquad (1.3.3.6)$$

for $k = 1, 2, \ldots s$, where a_k is the kth of the s parameters which constitute the parameter set **A**.

If N is large the second term has only a small effect and the solution of the equations which omit this term, namely

$$\sum_{j=1}^{m}\frac{O_j-NP_j(\mathbf{A})}{P_j(\mathbf{A})}\cdot\frac{\partial P_j(\mathbf{A})}{\partial a_k}=0 \qquad (1.3.3.7)$$

(for $k = 1, 2, \ldots s$), is called the modified χ^2 minimum method by Cramér (1946). Neither of these methods is used in this report but the quantity defined in Equation (1.3.3.5) will be referred to in Section 2.4 as a goodness of fit index.

e The method of sextiles was developed by Jenkinson (1969) for the general extreme value distribution which has location parameter u, scale parameter α, and shape parameter k. The range of the variate is considered to be divided into six intervals such that the cumulative probability in each interval is one sixth. If the variate values bounding the jth interval are x_{j-1} and x_j and the pdf and df are $f(x)$ and $F(x)$ respectively, then

$$F(x_{j-1}) = (j-1)/6$$

$$F(x_j)-F(x_{j-1}) = \int_{x_{j-1}}^{x_j} f(x)\,dx = 1/6.$$

Denote the mean value of variate in the jth interval by w_j

$$w_j = \int_{x_{j-1}}^{x_j} 6xf(x)\,dx.$$

These six means $w_1, w_2 \ldots w_6$, called by Jenkinson the sextile means, characterise the distribution. Their mean μ_w, standard deviation σ_w and the ratio $\ell = (w_2-w_1)/(w_6-w_5)$ are expressible in terms of the population parameters u, α and k. For estimation purposes the sample data are split up into six equal groups and the mean of each group is calculated. From these $\hat{\mu}_w$, $\hat{\sigma}_w$ and $\hat{\ell}$ are calculated and these are taken as estimates of the population values. Changes in the dimensionless quantity ℓ reflect changes in the shape parameter k. Jenkinson gives a table relating ℓ to k from which an estimate $\hat{k}$ can be obtained from $\hat{\ell}$.

In the reduced variate population where $u = 0$, $\alpha = 1$, the quantities μ_ω and σ_ω corresponding to μ_w and σ_w depend only on k and can be tabulated. The relations linking μ_w and σ_w to μ_ω and σ_ω contain u and α: by rearranging, u and α can be expressed in terms of μ_w, σ_w, μ_ω and σ_ω, each of which can be estimated from the sample.

It is conceivable that such a method could be used for other distributions with a shape parameter such as the Pearson Type 3 but the preparation of the tables mentioned above might be even more difficult than in the GEV case.

The main features of the sextiles method are that the sextile means have smaller sampling variance than individual sample observations and that the effect of outliers is largely removed.

Sampling distributions and properties of estimators. The word 'estimator' is used to describe the procedure followed in performing some type of estimation. The estimator is the collection of rules or series of calculating steps which can be stated in symbolic form quite independently of any particular sample data. We speak of a moments estimator and of a maximum likelihood estimator when we mean a method of estimation. The result of using an estimator is an estimate—a numerical value.

Three classes of distribution must be distinguished. These are

a the distribution of the variate in the population,

b the distribution of variate values in a sample considered as a population of finite size, and

c the sampling distribution between samples of some quantity, a statistic, calculated from a sample.

An estimate of a population quantity obtained from a sample is therefore a sample statistic and its sampling distribution contains all the information on the suitability of the estimator in all respects except the ease with which it can be obtained. In practice, reference is made only to the mean and variance of the sampling distribution. The square root of the variance is, in this context, not often called a standard deviation but a *standard error*. The mean and standard error of the distribution of many simple statistics can be expressed algebraically in terms of the sample size and population parameters, the form of the expression depending on the form of the population distribution.

If the mean or expected value of the sampling distribution of an estimate equals the value of the parameter in the population then the corresponding estimator is said to be unbiassed. If not, the difference between the expected value and the population value is called the bias. Obviously an estimator having small or zero bias is desirable.

The range or spread of all possible estimate values is proportional to the standard error. If two methods of estimation A and B have sampling variances V_A and V_B the relative efficiency of method A relative to method B is

$$\text{Efficiency of A to B} = \frac{1/V_A}{1/V_B} = \frac{V_B}{V_A}.$$

Thus, if method B gives $V_B = 9$ and method A gives $V_A = 7$ then the relative efficiency of A to B is 1.28 and method A is said to be more efficient than method B.

At first sight when choosing between estimators, one with zero or negligible bias and having least sampling variance should be selected. This is the obvious statistical choice but this might not be the ultimate criterion. A primary criterion should perhaps be that an estimator should always give hydrologically sensible results; failure to meet this criterion has brought objective methods of fitting and even the use of algebraically specified distributions into disrepute in the past.

From a theoretical point of view, maximum likelihood (ML) estimators have certain desirable properties, particularly for large samples. They are consistent (as the sample size increases, the estimator converges in probability to the population value); they are approximately Normally distributed when samples are large; they have sampling variances which are certainly not greater than those of any other estimators when samples are large. In addition, for a certain class of problems, estimators exist which summarise all the information in the sample; these are called sufficient estimators, and when they exist, ML estimators are functions of them. This property of sufficiency depends on the distribution whose parameters are being estimated and not on the sample size; when it applies, therefore, ML estimators are to be preferred even in small samples.

1.3.4 Estimates for particular distributions

Up to now, Section 1.3 has given an outline of what statistical estimation is, a summary of the main methods and an introduction to the sampling distribution of statistics calculated from samples.

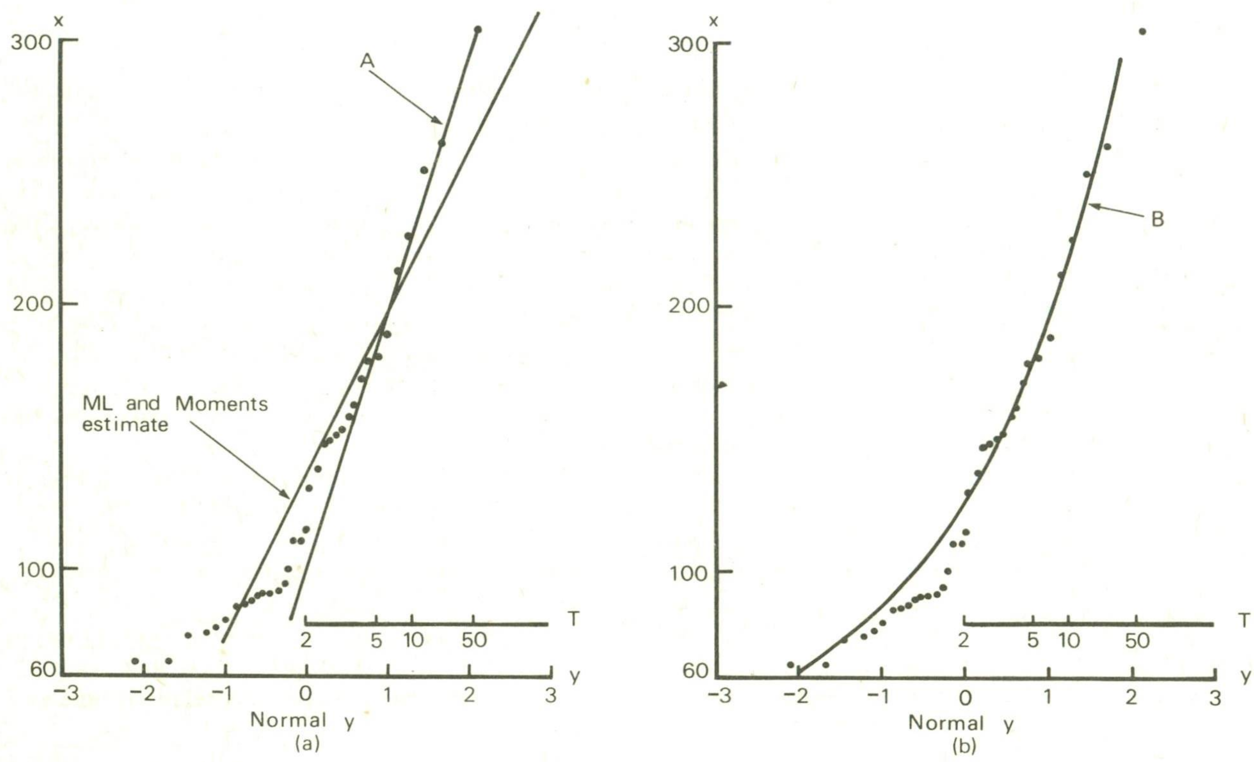


Fig 1.21 27/1, Nidd at Hunsingore, annual maxima plotted against Normal reduced variate showing (a) a theoretically fitted Normal distribution and a straight line A which represents the upper end only and (b) a curve B which represents the entire sample.

In the remainder of Section 1.3 the methods of estimation already described are developed for each of the distributions set out in Section 1.2. Estimates derived by these methods applied to series of annual maxima are used in Section 2.4. Estimators are applied to POT series in Section 2.7 and to the impulses in time series analysis in Section 2.9 and also in many practical hydrological situations.

The Normal distribution

The plotting positions for graphical estimation and display of the data, expressed as probabilities, are

$$F_i = (i-3/8)/(N+1/4), \quad i = 1, 2 \ldots N. \tag{1.3.4.1}$$

This formula is due to Blom (1958). The value of reduced variate y_i corresponding to this F_i is a sufficiently good approximation to the unbiassed plotting position $E(y_{(i)})$ which is the mean value of the ith smallest in a sample from the $N(0,1)$ distribution. If Normal probability paper is being used on which a probability scale is graduated on the horizontal axis the F_i values are used but if linear paper is used the F_i values are first converted to the corresponding values of Normal reduced or standardised variate y_i and these are used as plotting positions. The plotting positions $E(y_{(i)})$ for sample sizes up to $N = 50$ are given in Table 1.13 which is taken from Table XX of Fisher & Yates (1963). For larger sample sizes y_i values may be obtained from the plotting probabilities F_i of Equation (1.3.4.1) or from Harter (1961). When using y_i as plotting position on linear paper a return period scale may be marked over the y scale as follows

T	2	5	10	25	50	100	200
y	0.00	0.84	1.28	1.75	2.05	2.33	2.58

As early as 1924 Foster had found that peak flows were not Normally distributed and this conclusion is generally accepted. Hence, the use of annual maximum flows to demonstrate the practice of estimation for the Normal distribution is not realistic but will be useful when the lognormal distribution is discussed. The annual maximum data for 27/1, Nidd at Hunsingore are given in abridged form in Table 1.14 in column 2 and the ranked values are given in column 3. The plotting positions y_i are given in

Table 1.13 Plotting positions, $E(y_{(i)})$, for Normal samples.

i	N=2	3	4	5	6	7	8	9	10
1	0.56	0.85	1.03	1.16	1.27	1.35	1.42	1.49	1.54
2	—	—	0.30	0.50	0.64	0.76	0.85	0.93	1.00
3	—	—	—	—	0.20	0.35	0.47	0.57	0.66
4	—	—	—	—	—	—	0.15	0.27	0.38
5	—	—	—	—	—	—	—	—	0.12

i	N=11	12	13	14	15	16	17	18	19	20
1	1.59	1.63	1.67	1.70	1.74	1.76	1.79	1.82	1.84	1.87
2	1.06	1.12	1.16	1.21	1.25	1.28	1.32	1.35	1.38	1.41
3	0.73	0.79	0.85	0.90	0.95	0.99	1.03	1.07	1.10	1.13
4	0.46	0.54	0.60	0.66	0.71	0.76	0.81	0.85	0.89	0.92
5	0.22	0.31	0.39	0.46	0.52	0.57	0.62	0.67	0.71	0.75
6	—	0.10	0.19	0.27	0.34	0.39	0.45	0.50	0.55	0.59
7	—	—	—	0.09	0.17	0.23	0.30	0.35	0.40	0.45
8	—	—	—	—	—	0.08	0.15	0.21	0.26	0.31
9	—	—	—	—	—	—	—	0.07	0.13	0.19
10	—	—	—	—	—	—	—	—	—	0.06

The lower half of the data is plotted against the negative of these values. If N is odd the value of rank $(N+1)/2$ is plotted at 0.0 (from Fisher & Yates, 1963).

Table 1.13 continued

i	*N*=21	22	23	24	25	26	27	28	29	30
1	1.89	1.91	1.93	1.95	1.97	1.98	2.00	2.01	2.03	2.04
2	1.43	1.46	1.48	1.50	1.52	1.54	1.56	1.58	1.60	1.62
3	1.16	1.19	1.21	1.24	1.26	1.29	1.31	1.33	1.35	1.36
4	0.95	0.98	1.01	1.04	1.07	1.09	1.11	1.14	1.16	1.18
5	0.78	0.82	0.85	0.88	0.91	0.93	0.96	0.98	1.00	1.03
6	0.63	0.67	0.70	0.73	0.76	0.79	0.82	0.85	0.87	0.89
7	0.49	0.53	0.57	0.60	0.64	0.67	0.70	0.73	0.75	0.78
8	0.36	0.41	0.45	0.48	0.52	0.55	0.58	0.61	0.64	0.67
9	0.24	0.29	0.33	0.37	0.41	0.44	0.48	0.51	0.54	0.57
10	0.12	0.17	0.22	0.26	0.30	0.34	0.38	0.41	0.44	0.47
11	—	0.06	0.11	0.16	0.20	0.24	0.28	0.32	0.35	0.38
12	—	—	—	0.05	0.10	0.14	0.19	0.22	0.26	0.29
13	—	—	—	—	—	0.05	0.09	0.13	0.17	0.21
14	—	—	—	—	—	—	—	0.04	0.09	0.12
15	—	—	—	—	—	—	—	—	—	0.04

i	*N*=31	32	33	34	35	36	37	38	39	40
1	2.06	2.07	2.08	2.09	2.11	2.12	2.13	2.14	2.15	2.16
2	1.63	1.65	1.66	1.68	1.69	1.70	1.72	1.73	1.74	1.75
3	1.38	1.40	1.42	1.43	1.45	1.46	1.48	1.49	1.50	1.52
4	1.20	1.22	1.23	1.25	1.27	1.28	1.30	1.32	1.33	1.34
5	1.05	1.07	1.09	1.11	1.12	1.14	1.16	1.17	1.19	1.20
6	0.92	0.94	0.96	0.98	1.00	1.02	1.03	1.05	1.07	1.08
7	0.80	0.82	0.85	0.87	0.89	0.91	0.92	0.94	0.96	0.98
8	0.69	0.72	0.74	0.76	0.79	0.81	0.83	0.85	0.86	0.88
9	0.60	0.62	0.65	0.67	0.69	0.72	0.73	0.75	0.77	0.79
10	0.50	0.53	0.56	0.58	0.60	0.63	0.65	0.67	0.69	0.71
11	0.41	0.44	0.47	0.50	0.52	0.54	0.57	0.59	0.61	0.63
12	0.33	0.36	0.39	0.41	0.44	0.47	0.49	0.51	0.54	0.56
13	0.24	0.28	0.31	0.34	0.36	0.39	0.42	0.44	0.46	0.49
14	0.16	0.20	0.23	0.26	0.29	0.32	0.34	0.37	0.39	0.42
15	0.08	0.12	0.15	0.18	0.22	0.24	0.27	0.30	0.33	0.35
16	—	0.04	0.08	0.11	0.14	0.17	0.20	0.23	0.26	0.28
17	—	—	—	0.04	0.07	0.10	0.14	0.16	0.19	0.22
18	—	—	—	—	—	0.03	0.07	0.10	0.13	0.16
19	—	—	—	—	—	—	—	0.03	0.06	0.09
20	—	—	—	—	—	—	—	—	—	0.03

i	*N*=41	42	43	44	45	46	47	48	49	50
1	2.17	2.18	2.19	2.20	2.21	2.22	2.22	2.23	2.24	2.25
2	1.76	1.78	1.79	1.80	1.81	1.82	1.83	1.84	1.85	1.85
3	1.53	1.54	1.55	1.57	1.58	1.59	1.60	1.61	1.62	1.63
4	1.36	1.37	1.38	1.40	1.41	1.42	1.43	1.44	1.45	1.46
5	1.22	1.23	1.25	1.26	1.27	1.28	1.30	1.31	1.32	1.33
6	1.10	1.11	1.13	1.14	1.16	1.17	1.18	1.19	1.21	1.22
7	0.99	1.01	1.02	1.04	1.05	1.07	1.08	1.09	1.11	1.12
8	0.90	0.91	0.93	0.95	0.96	0.98	0.99	1.00	1.02	1.03
9	0.81	0.83	0.84	0.86	0.88	0.89	0.91	0.92	0.94	0.95
10	0.73	0.75	0.76	0.78	0.80	0.81	0.83	0.84	0.86	0.87
11	0.65	0.67	0.69	0.71	0.72	0.74	0.76	0.77	0.79	0.80
12	0.58	0.60	0.62	0.64	0.65	0.67	0.69	0.70	0.72	0.74
13	0.51	0.53	0.55	0.57	0.59	0.60	0.62	0.64	0.66	0.67
14	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.59	0.61
15	0.37	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.53	0.55
16	0.31	0.33	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.49
17	0.25	0.27	0.29	0.32	0.34	0.36	0.38	0.40	0.42	0.44
18	0.18	0.21	0.23	0.26	0.28	0.30	0.32	0.34	0.36	0.38
19	0.12	0.15	0.17	0.20	0.22	0.25	0.27	0.29	0.31	0.33
20	0.06	0.09	0.12	0.14	0.17	0.19	0.21	0.24	0.26	0.28
21	—	0.03	0.06	0.09	0.11	0.14	0.16	0.18	0.21	0.23
22	—	—	—	0.03	0.06	0.08	0.11	0.13	0.15	0.18
23	—	—	—	—	—	0.03	0.05	0.08	0.10	0.13
24	—	—	—	—	—	—	—	0.03	0.05	0.08
25	—	—	—	—	—	—	—	—	—	0.03

i	x	$x_{(i)}$	Normal y	EV1 y
1	189.02	65.08	−2.11	−1.38
2	91.80	65.60	−1.69	−1.12
3	115.52	75.06	−1.45	−0.95
4	78.55	76.22	−1.27	−0.82
5	162.99	78.55	−1.12	−0.70
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
31	87.76	213.70	1.12	1.99
32	251.96	226.48	1.27	2.25
33	138.72	251.96	1.45	2.60
34	305.75	261.82	1.69	3.12
35	226.48	305.75	2.11	4.13

$\sum_{i=1}^{N} x = 4783.1$, $\sum(x_i-\bar{x})^2 = 125437.82$, $\bar{x} = 136.66$, $\hat{\sigma}^2 = 3689.35$, $\hat{\sigma} = 60.74$.

Table 1.14 Summary of annual maxima at 27/1, Nidd at Hunsingore, with Normal and EV1 plotting positions.

column 4. The ordered flow values are shown plotted against these y_i values in Figure 1.21. There is a distinct bend in the line of the plotted points just to the left of $y = 0$, the median. This is so definite that it can hardly be attributed to random sampling variation and it is concluded that the flows are not Normally distributed. This is borne out by similar plots of data from other rivers.

To see if the hypothesis of Normality is reasonable a straight line must be drawn through the points. In Figure 1.21 a line which attempts to represent the pattern exhibited by both the low and high values must necessarily be a compromise which is not a good fit at either end. The straight line, A, is a valid estimate at the upper end but is unreasonable over the entire range. Therefore, a curve such as B may be drawn which is tantamount to asserting that the distribution is not Normal.

Moments estimate. The moments estimate of the Normal distribution parameters are

$$\left.\begin{aligned}\hat{\mu} &= \text{mean of sample} = \bar{x}\\ \hat{\sigma}^2 &= \text{sample variance} = \Sigma(x_i-\bar{x})^2/(N-1)\end{aligned}\right\} \qquad (1.3.4.2)$$

because the population parameters μ and σ^2 are themselves the first and second moments of the population; in this case they are $\hat{\mu} = 136.66$ and $\hat{\sigma}^2 = 3689.35$ and hence $\hat{\sigma} = 60.74$. The corresponding population is represented on Figure 1.21 by the line $x = \hat{\mu}+\hat{\sigma}y = 136.66+60.74y$.

Maximum likelihood estimate. The likelihood of any arbitrary values of μ and σ^2 given the observed sample $\mathbf{X} = \{x_1, x_2, \ldots, x_N\}$ is

$$L(\mathbf{X}/\mu,\sigma^2) = \prod_{i=1}^{N} f(x_i) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}[(x_i-\mu)/\sigma]^2}$$

$$= \frac{1}{(\sqrt{2\pi}\sigma)^N} e^{-\frac{1}{2}\Sigma[(x_i-\mu)/\sigma]^2}. \qquad (1.3.4.3)$$

Setting $\partial L/\partial\hat{\mu}$ and $\partial L/\partial\hat{\sigma}^2$ to zero and solving yields the maximum likelihood estimates

$$\left.\begin{aligned}\hat{\mu} &= \frac{1}{N}\Sigma x_i &&= m_1\\ \hat{\sigma}^2 &= \frac{1}{N}\Sigma(x_i-\bar{x})^2 &&= m_2.\end{aligned}\right\} \qquad (1.3.4.4)$$

The estimate of $\hat{\sigma}^2$ is m_2, the second moment of the sample which itself has sampling distribution with mean $E(m_2) = ((N-1)/N)\sigma^2$. Hence, it is a biassed estimate of σ^2. Any function of m_2 retains the important property of sufficiency so that $(N/(N-1))m_2$, which is an unbiassed estimate of σ^2, can be taken and the ML estimates are revised to

$$\left.\begin{aligned} \hat{\mu} &= \frac{1}{N}\Sigma x_i \\ \hat{\sigma}^2 &= \frac{\Sigma(x_i-\bar{x})^2}{N-1} \end{aligned}\right\} \qquad (1.3.4.5)$$

Strictly speaking the ML estimates are given by Equation (1.3.4.4) and these alone maximise L of Equation (1.3.4.3). The estimates of Equation (1.3.4.5) however are sufficient and are the most efficient among unbiassed estimators. Since they do not maximise L they are not strictly ML estimators but in practice are usually allowed to retain the ML title. For the Normal distribution, therefore, the moments estimates of μ and σ^2 coincide with the ML estimates.

The exponential distribution

This distribution has a location parameter x_0 and a scale parameter β. The variate–reduced variate relationship is

$$x = x_0+\beta y \qquad (1.3.4.6)$$

where y is the standardised or reduced exponential variate with df

$$F(y) = PR(Y<y) = 1-e^{-y}. \qquad (1.3.4.7)$$

In some cases the location parameter x_0 is known and does not need to be estimated. In this case the estimation of β is simpler than the joint estimate of β and x_0 together.

Graphical estimation. The plotting probabilities may be taken to be the same as those specially derived for the extreme value Type 1 distribution, namely, those given by the Gringorten formula,

$$F_i = \frac{i-0.44}{N+0.12}. \qquad (1.3.4.8)$$

The corresponding reduced variate values

$$y_i = -\ln(1-F_i) \qquad (1.3.4.9)$$

are used as plotting positions on linear paper. These are very good approximations to the exact values of the expected order statistics which are the unbiassed plotting positions. In this case these can be easily computed and are

$$E(y_{(i)}) = \sum_{j=1}^{i}\frac{1}{N+1-j}. \qquad (1.3.4.10)$$

The values obtained from either Equation (1.3.4.9) or (1.3.4.10), which differ slightly, may be used as plotting positions. This distribution is exceptional in that Equation (1.3.4.10) is a simple, easily computed expression; usually only the approximation (1.3.4.9) is available.

Whether x_0 is known or not the sample value $x_{(i)}$ is plotted against y_i from Equation (1.3.4.9) or (1.3.4.10) as the first step. Then, if x_0 is known *a priori* an eye guided straight line is drawn through the plotted points and is made to pass through the point ($x = x_0$, $y = 0$). The slope of this line is the estimate of β. If x_0 is not known *a priori* the eye guided straight line is drawn through the data and the intercept at $y = 0$ is the estimate of x_0 and again the slope of the line is the estimate of β.

A return period scale may be marked in over the y scale as a guide. The following points are sufficient for exploratory work. The relation is $y = \ln T$.

T	2	5	10	20	25	50	100	200
y	0.69	1.61	2.30	3.00	3.22	3.91	4.61	5.30

Estimation by moments. The mean and standard deviation μ and σ are related to x_0 and β by

$$\mu = x_0+\beta \qquad (1.3.4.11)$$

$$\sigma = \beta. \qquad (1.3.4.12)$$

Therefore, if x_0 is known and the sample mean $\bar{x}$ is the estimate of μ, $\hat{\beta}$ is got from

$$\hat{\beta} = \bar{x}-x_0. \qquad (1.3.4.13)$$

If both x_0 and β are unknown both Equations (1.3.4.11) and (1.3.4.12) must be used. First

$$\hat{\beta} = \hat{\sigma} \qquad (1.3.4.14)$$

where $\hat{\sigma}$ is the sample estimate of σ and then

$$\hat{x}_0 = \bar{x}-\hat{\beta} = \bar{x}-\hat{\sigma}. \qquad (1.3.4.14a)$$

Estimation by maximum likelihood. The likelihood function is

$$L(x_1, x_2, \ldots, x_N/x_0, \beta) = \prod_{i=1}^{N} \frac{1}{\beta} e^{-(x_i-x_0)/\beta}. \qquad (1.3.4.15)$$

The values of x_0 and β which maximise this quantity also maximise the log likelihood

$$LL(x_1, x_2, \ldots, x_N/x_0, \beta) = -N \ln \beta - \sum_{i=1}^{N} (x_i-x_0)/\beta. \qquad (1.3.4.16)$$

β alone unknown. When x_0 is known the value of β alone which maximises Equation (1.3.4.15) or (1.3.4.16) is required. Setting $\partial LL/\partial\hat{\beta}$ to zero gives

$$-\frac{N}{\hat{\beta}}+\sum_{i=1}^{N}\frac{(x_i-x_0)}{\hat{\beta}^2} = 0$$

which on solving for $\hat{\beta}$ gives

$$\hat{\beta} = \frac{1}{N}\Sigma(x_i-x_0) = \bar{x}-x_0. \qquad (1.3.4.17)$$

Thus, when β alone is unknown the moments and maximum likelihood estimates take the same form.

If both x_0 and β are unknown and have to be estimated the procedure is not so straightforward because equating the derivatives $\partial LL/\partial\beta$ and $\partial LL/\partial x_0$ to zero does not afford two independent equations in x_0 and β. However, from statistical theory it is known that if sufficient statistics exist for x_0 and β then the maximum likelihood estimates are functions of them. Now, if β is known, $t_1 = x_{(1)} = \min(x_1, x_2, \ldots x_i, \ldots x_N)$ is a sufficient statistic for x_0; while if x_0 is known $t_2 = \bar{x}-x_0$ is a sufficient statistic for β. But, t_1 and t_2 being individually sufficient for x_0 and β

implies that they are a pair of jointly sufficient statistics for x_0 and β. Hence, the maximum likelihood estimates are functions of t_1 and t_2. Indeed, t_1 and t_2 are themselves maximum likelihood estimates but they are biassed; hence some functions of them which are unbiassed are desirable.

Since $x_{(1)}$ is always greater than or equal to x_0 it follows that its mean must exceed x_0, i.e. $E(x_{(1)}) > x_0$ and consequently $x_{(1)}$ is a biassed estimate of x_0. For the same reason the expected value of t_2 is less than β.

The corrections to be applied to t_1 and t_2 to remove the bias are obtained from the distributional properties of $x_{(1)}$. For

$$\begin{aligned} PR(x_{(1)} < x) &= 1 - PR(x_1 > x, x_2 > x, \ldots, x_i > x, \ldots, x_N > x) \\ &= 1 - [e^{-(x-x_0)/\beta}]^N \\ &= 1 - e^{-(x-x_0)/(\beta/N)} \end{aligned} \tag{1.3.4.18}$$

showing that $x_{(1)}$ has an exponential distribution with mean

$$E(x_{(1)}) = x_0 + \frac{\beta}{N} \tag{1.3.4.19}$$

and variance

$$\text{var}(x_{(1)}) = \frac{\beta^2}{N^2}.$$

Using Equation (1.3.4.19) it can be shown that

$$\hat{\beta} = \frac{N}{N-1}(\bar{x} - x_{(1)}) = \frac{\sum_{i=1}^{N}(x_i - x_{(1)})}{N-1} \tag{1.3.4.20}$$

and

$$\hat{x}_0 = x_{(1)} - \frac{\hat{\beta}}{N} = x_{(1)} - \frac{1}{N}\frac{\Sigma(x_i - x_{(1)})}{N-1} \tag{1.3.4.21}$$

are unbiassed in that $E(\hat{\beta}) = \beta$ and $E(\hat{x}_0) = x_0$ and since they are functions of t_1 and t_2 above they are unbiassed, maximum likelihood estimators for β and x_0.

The two parameter gamma distribution

This distribution has no location parameter; its two parameters are a scale and a shape parameter respectively. The variate–reduced variate relationship is

$$x = \beta y \tag{1.3.4.22}$$

which is a special case of the general relation in Equation (1.1.3.4)

$$x = a + by \tag{1.3.4.23}$$

with $a = 0$, $b = \beta$. The second parameter γ in the gamma distribution is contained in the distribution of the reduced gamma variate, y. In order that Equation (1.3.4.22) should hold, the shape parameter of the y variate should equal that of the x variate. Therefore, if a sample from the x population is to be plotted to estimate β of Equation (1.3.4.22) by drawing a straight line through the plot, the value of γ on which the horizontal probability scale depends must be that of the x variate. If any other value of γ is used the theoretical x–y relation is no longer linear and the drawing of a straight line in that situation necessarily introduces bias in the estimate of β.

Graphical estimation. Suppose it were known that all annual maximum populations were gamma distributed with the same value of gamma, say $\gamma = 3$, at every station but that β varied from station to station. Then probability paper could be constructed for this value of γ and a straight line drawn through the plotted points would then give a valid estimate of β. The plotting positions strictly speaking depend on the value of γ. In the example cited $\gamma = 3$ which is close to the γ value of 1 in the exponential distribution. Consequently, the plotting probabilities $F_i = (i-0.44)/(N+0.12)$ appropriate to the exponential distribution may be used. If γ is very large then the distribution is close to Normal and then the plotting probabilities $F_i = (i-3/8)(N+1/4)$, Equation (1.3.4.1), should be used.

In practice, even if it is known that the distribution is gamma the value of γ is not known and consequently the use of a formula which is apparently exact gives an illusion of accuracy and a compromise plotting probability

$$F_i = (i-2/5)/(N+1/5) \tag{1.3.4.24}$$

could be used regardless of the value of γ.

If γ is known, β may be estimated graphically but when γ is unknown there is no straightforward method of estimating both β and γ graphically. A practical graphical method which estimates quantiles but not the individual parameters is to plot the data against both exponential reduced variates and Normal reduced variates and to draw a smooth curve through the plotted points on the plot which appears least curved. The plot showing least curvature should be adopted because (a) the plotting position bias is kept to a minimum and (b) extrapolation is easier and safer.

Estimation by method of moments. The mean and standard deviation in terms of β and γ are

$$\mu = \beta\gamma$$
$$\sigma = \beta\sqrt{\gamma} \tag{1.3.4.25}$$

which on rearrangement gives

$$\gamma = \frac{\mu^2}{\sigma^2} = \frac{1}{\text{CV}^2}$$
$$\beta = \mu/\gamma = \sigma^2/\mu. \tag{1.3.4.26}$$

The sample estimates of μ and σ namely $\bar{x}$ and $\hat{\sigma}$ are substituted to give the moments estimates of β and γ

$$\hat{\gamma} = \bar{x}^2/\hat{\sigma}^2$$
$$\hat{\beta} = \hat{\sigma}^2/\bar{x}. \tag{1.3.4.27}$$

Estimation by maximum likelihood. The likelihood function is

$$L(\mathbf{X}/\beta,\gamma) = \prod_{i=1}^{N} f(x_i/\beta,\gamma) = \prod_{i=1}^{N} \frac{e^{-x_i/\beta} x_i^{\gamma-1}}{\beta^{\gamma}\Gamma(\gamma)}. \tag{1.3.4.28}$$

The log likelihood is

$$LL(\mathbf{X}/\beta,\gamma) = \sum_{i=1}^{N} \ln f(x_i/\beta,\gamma)$$

$$= -N\gamma \ln \beta - N \ln \Gamma(\gamma) - \Sigma \frac{x_i}{\beta} + (\gamma - 1)\Sigma \ln x_i. \quad (1.3.4.29)$$

The maximum likelihood estimates must satisfy

$$\frac{\partial LL}{\partial \hat{\beta}} = 0 \quad \text{and} \quad \frac{\partial LL}{\partial \hat{\gamma}} = 0. \quad (1.3.4.30)$$

Differentiating Equation (1.3.4.29) with respect to $\hat{\beta}$ and $\hat{\gamma}$ gives

$$\bar{x}/\hat{\beta} = \hat{\gamma} \quad (1.3.4.31)$$

$$\ln\hat{\beta} + \frac{\partial}{\partial \hat{\gamma}} \ln\Gamma(\hat{\gamma}) - \frac{1}{N} \Sigma \ln x_i = 0. \quad (1.3.4.32)$$

From Equation (1.3.4.31), $\ln\hat{\beta} = \ln\bar{x} - \ln\hat{\gamma}$ and inserting this in Equation (1.3.4.32) together with the digamma function $\Psi(\hat{\gamma})$ for $\partial \ln\Gamma(\hat{\gamma})/\partial\hat{\gamma}$ gives

$$\ln\bar{x} - \ln\hat{\gamma} + \Psi(\hat{\gamma}) - \frac{1}{N} \Sigma \ln x_i = 0. \quad (1.3.4.33)$$

The method of solving Equation (1.3.4.33) is that described by Thom (1958). For values of $\gamma = 1$ or larger the approximation

$$\Psi(\hat{\gamma}) \simeq \ln\hat{\gamma} - \frac{1}{2\hat{\gamma}} - \frac{1}{12\hat{\gamma}^2} \quad (1.3.4.34)$$

is correct to within 2%. Substituting this in Equation (1.3.4.33) yields

$$12\left(\ln\bar{x} - \frac{1}{N} \Sigma \ln x\right)\hat{\gamma}^2 - 6\hat{\gamma} - 1 = 0.$$

Letting

$$A = \ln\bar{x} - \frac{1}{N} \Sigma \ln\bar{x} \quad (1.3.4.35)$$

this is simplified to

$$12A\hat{\gamma}^2 - 6\hat{\gamma} - 1 = 0 \quad (1.3.4.36)$$

which is a quadratic in the unknown $\hat{\gamma}$ and the root

$$\hat{\gamma} = \frac{1 + \sqrt{1 + 4A/3}}{4A} \quad (1.3.4.37)$$

is used. Because of the approximation in Equation (1.3.4.34) this value of $\hat{\gamma}$ is not the exact solution of Equation (1.3.4.33). Thom gives the following corrections $\Delta\hat{\gamma}$ to be subtracted from the result of Equation (1.3.4.37) to correct the result of using the approximation to $\Psi(\hat{\gamma})$. Thus, if Equation (1.3.4.37) gives a value $\hat{\gamma} = 1.3$, the corrected value of $\hat{\gamma}$ should be taken as $1.3 - 0.006 = 1.294$. The ML estimate of β is then from Equation (1.3.4.31)

$$\hat{\beta} = \bar{x}/\hat{\gamma}. \quad (1.3.4.38)$$

Table 1.15 Corrections to $\hat{\gamma}$ estimated by Thom's approximation.

$\hat{\gamma}$	$\Delta\hat{\gamma}$	$\hat{\gamma}$	$\Delta\hat{\gamma}$	$\hat{\gamma}$	$\Delta\hat{\gamma}$	$\hat{\gamma}$	$\Delta\hat{\gamma}$
0.2	0.034	0.8	0.012	1.4	0.006	2.2	0.003
0.3	0.029	0.9	0.011	1.5	0.005	2.3	0.002
0.4	0.025	1.0	0.009	1.6	0.005	3.1	0.002
0.5	0.021	1.1	0.008	1.7	0.004	3.2	0.001
0.6	0.017	1.2	0.007	1.8	0.004	5.5	0.001
0.7	0.014	1.3	0.006	1.9	0.003	5.6	0.000

The Pearson Type 3 distribution

This distribution may be regarded as a form of the gamma distribution generalised by the addition of a location parameter. The variate–reduced variate relationship is

$$x = x_0 + \beta y \qquad (1.3.4.39)$$

where y is the reduced gamma variate with parameter γ. All the remarks made about graphical estimation in the case of the gamma distribution apply here also. The parameter values cannot be estimated explicitly but quantile values can be obtained by using the compromise plotting probabilities

$$F_i = \frac{i-2/5}{N+1/5} \qquad (1.3.4.40)$$

on either Normal or exponential probability papers. If rectangular or linear paper is used instead of ready made probability paper, plotting positions y_i corresponding to the F_i of Equation (1.3.4.40) are used: $y_i = \Phi^{-1}(F_i)$ in the Normal case where $\Phi^{-1}(.)$ is the inverse standardised Normal probability integral and $y_i = -\ln(1-F_i)$ in the exponential case. Return period scales may be marked alongside the y axis as described under the headings of exponential and Normal distributions in this section.

Estimation by moments. Because there are three parameters to be estimated three relations between parameters and moments must be used. These are

$$\left.\begin{aligned} \mu &= x_0+\beta\gamma \\ \sigma &= \beta\sqrt{\gamma} \\ g &= 2/\sqrt{\gamma} \end{aligned}\right\} \qquad (1.3.4.41)$$

where g is the skewness, $\mu_3/\mu_2^{3/2}$.

If $\bar{x}$, $\hat{\sigma}$ and $\hat{g}$ are the sample estimates of μ, σ and g (see Equation (1.3.3.1)) Equation (1.3.4.41) gives on rearranging

$$\left.\begin{aligned} \hat{\gamma} &= 4/\hat{g}^2 \\ \hat{\beta} &= \hat{g}\hat{\sigma}/2 \\ \hat{x}_0 &= \bar{x}-\hat{\beta}\hat{\gamma} = \bar{x}-2\hat{\sigma}/\hat{g} \end{aligned}\right\} \qquad (1.3.4.42)$$

Estimation by maximum likelihood. The likelihood function is

$$L(x_1, x_2, \ldots, x_N/x_0, \beta, \gamma) = \prod_{i=1}^{N} \frac{e^{-(x_i-x_0)/\beta}(x_i-x_0)^{\gamma-1}}{\beta^{\gamma}\Gamma(\gamma)} \qquad (1.3.4.43)$$

and the log likelihood is

$$LL(x_1,x_2,\ldots,x_N/x_0,\beta,\gamma) = -N\gamma\ln\beta - N\ln\Gamma(\gamma) - \frac{\Sigma(x_i-x_0)}{\beta} + (\gamma-1)\Sigma\ln(x_i-x_0). \qquad (1.3.4.44)$$

The maximum likelihood estimates must satisfy

$$\partial LL/\partial\hat{x}_0 = 0, \quad \partial LL/\partial\hat{\beta} = 0, \quad \text{and} \quad \partial LL/\partial\hat{\gamma} = 0. \qquad (1.3.4.45)$$

Differentiating Equation (1.3.4.44) with respect to $\hat{x}_0$, $\hat{\beta}$ and $\hat{\gamma}$ gives

$$\begin{aligned} &\frac{N}{\hat{\beta}} - (\hat{\gamma}-1)\Sigma\frac{1}{x_i-\hat{x}_0} = 0 \\ &\frac{-N\hat{\gamma}}{\hat{\beta}} + \frac{\Sigma(x_i-\hat{x}_0)}{\hat{\beta}^2} = 0 \end{aligned} \qquad (1.3.4.46)$$

or

$$-\hat{\beta}\hat{\gamma}+\frac{\Sigma(x_i-\hat{x}_0)}{N}=0 \qquad (1.3.4.47)$$

and

$$-N\ \ln\hat{\beta}-N\frac{\partial}{\partial\hat{\gamma}}\ln\Gamma(\hat{\gamma})+\Sigma\ \ln(x_i-\hat{x}_0)=0$$

or writing the digamma function $\Psi(\gamma)$ for $\partial \ln\Gamma(\gamma)/\partial\gamma$

$$\ln\hat{\beta}+\Psi(\hat{\gamma})-\frac{\Sigma\ \ln(x_i-\hat{x}_0)}{N}=0. \qquad (1.3.4.48)$$

Equations (1.3.4.46), (1.3.4.47) and (1.3.4.48) are nonlinear in the unknowns $\hat{x}_0$, $\hat{\beta}$ and $\hat{\gamma}$, and a completely automatic iterative solution would be both time-consuming in operation and difficult to program.

The following procedure suggested by R. T. Clarke and quoted by Wallis & Matalas (1973) is to assume a value of $\hat{x}_0$ and use that to solve Equations (1.3.4.46) and (1.3.4.47) for β and γ giving

$$\hat{\gamma}=D/(D-1) \qquad (1.3.4.49)$$

where

$$D=\frac{1}{N^2}\Sigma(x_i-x_0).\Sigma\left(\frac{1}{x_i-x_0}\right)$$

and

$$\hat{\beta}=\Sigma(x_i-\hat{x}_0)/(N\hat{\gamma})=\frac{1}{N}\Sigma(x_i-\hat{x}_0)-\frac{N}{\Sigma\left(\frac{1}{x_i-\hat{x}_0}\right)}. \qquad (1.3.4.50)$$

These three values $\hat{x}_0$, $\hat{\beta}$ and $\hat{\gamma}$ are inserted into Equation (1.3.4.48) which when evaluated yields a number R, say. For these three values $\hat{x}_0$, $\hat{\beta}$, $\hat{\gamma}$ to be maximum likelihood estimates this R should be zero but if $|R|$ is less than some small arbitrary amount, Equation (1.3.4.48) is considered to be satisfied. If not, a new value of $\hat{x}_0$ is chosen and the procedure repeated. After a few trials the choice may be guided by plotting R against $\hat{x}_0$. In small samples particularly, the likelihood surface may be very flat and it may be almost impossible to arrive at a solution.

Another method in the same vein is to assume a value of $\hat{x}_0$ and calculate ($z_i = x_i-\hat{x}_0$, $i = 1, 2, \ldots, N$) and then estimate β and γ by fitting a gamma distribution by Thom's method to this sample of z values. The log likelihood LL is then computed from Equation (1.3.4.44) with these values of $\hat{x}_0$, $\hat{\beta}$ and $\hat{\gamma}$ and after this has been done with a few values of $\hat{x}_0$ a plot of LL against $\hat{x}_0$ is prepared. A smooth curve is drawn through the plotted points and the value of $\hat{x}_0$ for which LL is a maximum is estimated and new values of β and γ obtained from the resulting sequence of z values. It should be noted that when Equation (1.3.4.44) is being evaluated, the quantities $\Sigma(x_i-\hat{x}_0)$ and $\Sigma\ln(x_i-\hat{x}_0)$ have already been calculated in the course of estimating $\hat{\beta}$ and $\hat{\gamma}$ by Thom's procedure. If the log likelihood surface is flat this method may also be unsatisfactory in some cases.

The log Pearson Type 3 distribution

Estimation for this distribution is usually carried out in the log domain. This is a consequence of the fact that the distribution is much easier to define in the log domain. With the exception of a possible fourth parameter in the form of a location parameter in the real domain the parameters of the distribution are log domain parameters. It was remarked in Section 1.2 that the moments of the distribution in the real domain may not be expressible in terms of the parameters and this means that moments estimates cannot be sought from the sample moments.

The likelihood function in the three parameter case is

$$L(x_1, x_2, \ldots, x_N/z_0, \lambda, \eta) = \prod_{i=1}^{N} \frac{e^{-\{\ln(x_i)-z_0\}/\lambda}\{\ln(x_i)-z_0\}^{\eta-1}}{x_i \lambda^{\eta}\Gamma(\eta)} \quad (1.3.4.51)$$

where z_0, λ and η are location, scale and shape parameters of the Pearson Type 3 distribution of $\ln x$. The log likelihood is

$$\begin{aligned} LL(x_1, x_2, \ldots, x_N/z_0, \lambda, \eta) \neq & -\Sigma \left(\frac{\ln x_i - z_0}{\lambda}\right) \\ & +(\eta-1)\Sigma \ln\{\ln(x_i)-z_0\} \\ & -\Sigma \ln x_i - N(\eta \ln \lambda + \ln \Gamma(\eta)). \end{aligned} \quad (1.3.4.52)$$

Solution of the equations $\partial LL/\partial \hat{z}_0 = 0$, $LL/\partial \hat{\lambda} = 0$, $\partial LL/\partial \hat{\eta} = 0$ has not been attempted nor is it known whether it is feasible.

It is seen therefore that direct estimation in the real domain either by moments or maximum likelihood is far from easy and as yet unexplored. For this reason the procedure used in practice is to fit a Pearson Type 3 distribution to the logarithms of the original values, $z_i = \ln x_i$, $i = 1, 2, \ldots N$, compute quantiles, z_p, of the fitted distribution and then compute the required x quantiles by the transformation, $x_p = e^{z_p}$. While this is a valid method of estimation it should be remembered that the estimates are by no means the same as those which would be obtained by the corresponding methods without transformation of the data. Indeed there is no guarantee that unbiassed, minimum variance estimators in the log domain exhibit the same desirable qualities after transformation back into the real domain.

The extreme value Type 1 (EV1) or Gumbel distribution

The plotting positions for graphical estimation and display of the data, expressed as probabilities are

$$F_i = (i-0.44)/(N+0.12) \quad (1.3.4.53)$$

due to Gringorten (1963). The reduced or standardised variate values y_i corresponding to F_i are

$$y_i = -\ln - \ln F_i \quad (1.3.4.54)$$

which is the inverse of the df $\exp(-e^{-y})$. These y_i values are a very good approximation to the unbiassed positions $E(y_{(i)})$ the expected values of the reduced variate order statistic $y_{(i;N)}$. When data are being plotted on A4 size paper the difference between using the approximate values y_i and the exact values $E(y_{(i)})$ is so small as to be negligible. The exact values for sample sizes up to 35 are given in Table 1.16 and the approximate values y_i are given for sample sizes from 36 to 50 in the same table.

If extreme value probability paper (Gumbel paper) is used the ordered sample values are plotted against F_i and if not they are plotted against y_i values on linear paper; a return period scale may be quickly marked alongside the horizontal y axis using the following figures.

T	2	5	10	20	50	100	200	1000
y	0.37	1.50	2.25	2.97	3.90	4.60	5.30	6.91

(For $T > 10$ the relation $y = \ln(T-\frac{1}{2})$ holds to 3 significant digits in y and for $T \geqslant 50$ it holds to 5 significant digits in y.)

i	*N*=2	3	4	5	6	7	8	9	10
1	−0.11	−0.40	−0.57	−0.69	−0.78	−0.85	−0.90	−0.95	−0.99
2	+1.27	0.46	0.16	−0.11	−0.25	−0.36	−0.45	−0.52	−0.58
3	—	1.67	0.81	0.43	0.19	0.22	−0.10	−0.20	−0.28
4	—	—	1.96	1.07	0.66	0.41	0.23	0.10	−0.01
5	—	—	—	2.19	1.27	0.85	0.59	0.40	0.26
6	—	—	—	—	2.37	1.44	1.01	0.74	0.54
7	—	—	—	—	—	2.52	1.59	1.15	0.87
8	—	—	—	—	—	—	2.66	1.71	1.27
9	—	—	—	—	—	—	—	2.77	1.83
10	—	—	—	—	—	—	—	—	2.88

i	*N*=11	12	13	14	15	16	17	18	19	20
1	−1.02	−1.05	−1.08	−1.11	−1.13	−1.15	−1.17	−1.19	−1.21	−1.22
2	−0.64	−0.68	−0.72	−0.76	−0.79	−0.82	−0.84	−0.87	−0.89	−0.91
3	−0.35	−0.41	−0.46	−0.51	−0.55	−0.58	−0.62	−0.65	−0.67	−0.70
4	−0.10	−0.18	−0.24	−0.30	−0.34	−0.39	−0.43	−0.47	−0.50	−0.53
5	0.14	0.05	−0.03	−0.10	−0.16	−0.21	−0.26	−0.30	−0.34	−0.38
6	0.39	0.27	0.17	0.09	0.02	−0.04	−0.10	−0.15	−0.19	−0.23
7	0.67	0.51	0.39	0.29	0.20	0.13	0.06	0.00	−0.05	−0.10
8	0.98	0.78	0.62	0.49	0.39	0.30	0.22	0.15	0.09	0.04
9	1.37	1.08	0.88	0.72	0.58	0.48	0.38	0.30	0.24	0.17
10	1.93	1.47	1.18	0.97	0.80	0.67	0.56	0.46	0.38	0.31
11	2.98	2.02	1.56	1.26	1.05	0.88	0.75	0.63	0.54	0.45
12	—	3.06	2.10	1.64	1.34	1.12	0.94	0.82	0.70	0.61
13	—	—	3.14	2.18	1.71	1.41	1.19	1.02	0.88	0.77
14	—	—	—	3.22	2.25	1.78	1.48	1.26	1.09	0.95
15	—	—	—	—	3.29	2.32	1.85	1.54	1.32	1.15
16	—	—	—	—	—	3.35	2.38	1.91	1.60	1.38
17	—	—	—	—	—	—	3.41	2.44	1.96	1.66
18	—	—	—	—	—	—	—	3.47	2.49	2.02
19	—	—	—	—	—	—	—	—	3.52	2.55
20	—	—	—	—	—	—	—	—	—	3.57

i	*N*=21	22	23	24	25	26	27	28	29	30
1	−1.24	−1.25	−1.26	−1.28	−1.29	−1.30	−1.31	−1.32	−1.33	−1.34
2	−0.93	−0.95	−0.97	−0.98	−1.00	−1.01	−1.03	−1.04	−1.05	−1.06
3	−0.73	−0.75	−0.77	−0.79	−0.81	−0.82	−0.84	−0.86	−0.87	−0.89
4	−0.56	−0.58	−0.61	−0.63	−0.65	−0.67	−0.69	−0.71	−0.73	−0.74
5	−0.41	−0.44	−0.47	−0.49	−0.52	−0.54	−0.56	−0.58	−0.60	−0.62
6	−0.27	−0.31	−0.34	−0.37	−0.40	−0.42	−0.45	−0.47	−0.49	−0.51
7	−0.14	−0.18	−0.22	−0.25	−0.28	−0.31	−0.34	−0.36	−0.38	−0.41
8	−0.01	−0.05	−0.10	−0.13	−0.17	−0.20	−0.23	−0.26	−0.29	−0.31
9	0.12	0.06	0.02	−0.02	−0.06	−0.10	−0.13	−0.16	−0.19	−0.22
10	0.25	0.19	0.14	0.09	0.05	0.01	−0.03	−0.06	−0.10	−0.13
11	0.38	0.32	0.26	0.20	0.16	0.11	0.07	0.03	0.00	−0.04
12	0.52	0.45	0.38	0.32	0.27	0.22	0.17	0.13	0.09	0.05
13	0.67	0.58	0.51	0.44	0.38	0.32	0.27	0.23	0.18	0.14
14	0.83	0.73	0.64	0.56	0.50	0.43	0.38	0.33	0.28	0.24
15	1.00	0.89	0.79	0.70	0.62	0.55	0.49	0.43	0.38	0.33
16	1.20	1.06	0.94	0.84	0.75	0.67	0.60	0.54	0.48	0.43
17	1.43	1.26	1.11	0.99	0.89	0.80	0.72	0.65	0.58	0.52
18	1.71	1.49	1.31	1.16	1.04	0.94	0.85	0.77	0.69	0.63
19	2.07	1.76	1.53	1.36	1.21	1.09	0.98	0.89	0.81	0.74
20	2.60	2.12	1.81	1.58	1.40	1.26	1.13	1.03	0.93	0.85
21	3.62	2.64	2.17	1.85	1.63	1.44	1.30	1.17	1.07	0.97
22	—	3.67	2.69	2.21	1.90	1.67	1.49	1.34	1.22	1.11
23	—	—	3.71	2.73	2.25	1.94	1.71	1.53	1.38	1.25
24	—	—	—	3.75	2.78	2.29	1.98	1.75	1.57	1.42
25	—	—	—	—	3.80	2.81	2.33	2.02	1.79	1.60
26	—	—	—	—	—	3.83	2.85	2.37	2.06	1.82
27	—	—	—	—	—	—	3.87	2.89	2.41	2.09
28	—	—	—	—	—	—	—	3.91	2.93	2.44
29	—	—	—	—	—	—	—	—	3.94	2.96
30	—	—	—	—	—	—	—	—	—	3.98

Table 1.16 Plotting position, $E(y_{(i)})$, for extreme value Type 1 distribution.

i	N=31	32	33	34	35	36	37	38	39	40
1	−1.35	−1.36	−1.36	−1.37	−1.38	−1.43	−1.43	−1.44	−1.45	−1.45
2	−1.08	−1.09	−1.10	−1.11	−1.12	−1.14	−1.15	−1.16	−1.17	−1.18
3	−0.90	−0.91	−0.92	−0.94	−0.95	−0.97	−0.98	−0.99	−1.00	−1.01
4	−0.76	−0.77	−0.79	−0.80	−0.82	−0.84	−0.85	−0.86	−0.87	−0.88
5	−0.64	−0.66	−0.67	−0.69	−0.70	−0.73	−0.74	−0.75	−0.77	−0.78
6	−0.53	−0.55	−0.57	−0.58	−0.60	−0.63	−0.64	−0.66	−0.67	−0.68
7	−0.43	−0.45	−0.47	−0.49	−0.51	−0.53	−0.55	−0.57	−0.58	−0.59
8	−0.34	−0.36	−0.38	−0.40	−0.42	−0.45	−0.46	−0.48	−0.50	−0.51
9	−0.24	−0.27	−0.29	−0.31	−0.34	−0.36	−0.38	−0.40	−0.42	−0.43
10	−0.16	−0.18	−0.21	−0.23	−0.25	−0.28	−0.31	−0.32	−0.34	−0.36
11	−0.07	−0.10	−0.12	−0.15	−0.17	−0.21	−0.23	−0.25	−0.27	−0.29
12	0.02	−0.01	−0.04	−0.07	−0.10	−0.13	−0.15	−0.18	−0.20	−0.22
13	0.11	0.07	0.04	0.01	−0.02	−0.05	−0.08	−0.10	−0.13	−0.15
14	0.20	0.16	0.12	0.09	0.06	0.02	−0.01	−0.03	−0.06	−0.08
15	0.29	0.24	0.21	0.17	0.14	0.10	0.07	0.04	0.01	−0.01
16	0.38	0.33	0.29	0.25	0.21	0.17	0.14	0.11	0.08	0.05
17	0.47	0.42	0.38	0.33	0.29	0.25	0.21	0.18	0.15	0.12
18	0.57	0.51	0.46	0.42	0.38	0.33	0.29	0.25	0.22	0.19
19	0.67	0.61	0.56	0.51	0.46	0.41	0.37	0.33	0.29	0.26
20	0.78	0.71	0.65	0.60	0.54	0.49	0.45	0.40	0.37	0.33
21	0.89	0.82	0.75	0.69	0.63	0.57	0.53	0.48	0.44	0.40
22	1.01	0.93	0.86	0.79	0.73	0.66	0.61	0.56	0.52	0.48
23	1.15	1.05	0.97	0.89	0.82	0.75	0.70	0.65	0.60	0.55
24	1.29	1.18	1.09	1.00	0.93	0.85	0.79	0.73	0.68	0.63
25	1.45	1.33	1.22	1.12	1.04	0.95	0.88	0.82	0.76	0.71
26	1.64	1.49	1.36	1.25	1.16	1.06	0.99	0.92	0.85	0.80
27	1.86	1.68	1.52	1.40	1.29	1.18	1.09	1.02	0.95	0.88
28	2.13	1.89	1.71	1.56	1.43	1.31	1.21	1.13	1.05	0.98
29	2.48	2.16	1.93	1.74	1.59	1.45	1.34	1.24	1.16	1.08
30	2.99	2.51	2.19	1.96	1.77	1.61	1.48	1.37	1.27	1.19
31	4.01	3.03	2.54	2.22	1.99	1.79	1.64	1.51	1.40	1.30
32	—	4.04	3.06	2.57	2.25	2.00	1.82	1.67	1.54	1.43
33	—	—	4.07	3.09	2.60	2.27	2.03	1.85	1.70	1.57
34	—	—	—	4.09	3.10	2.61	2.29	2.06	1.88	1.72
35	—	—	—	—	4.13	3.12	2.64	2.32	2.09	1.90
36	—	—	—	—	—	4.16	3.15	2.67	2.35	2.11
37	—	—	—	—	—	—	4.19	3.18	2.69	2.38
38	—	—	—	—	—	—	—	4.21	3.20	2.72
39	—	—	—	—	—	—	—	—	4.24	3.23
40	—	—	—	—	—	—	—	—	—	4.26

i	N=41	42	43	44	45	46	47	48	49	50
1	−1.46	−1.46	−1.47	−1.47	−1.48	−1.48	−1.49	−1.49	−1.50	−1.50
2	−1.19	−1.19	−1.20	−1.21	−1.21	−1.22	−1.23	−1.23	−1.24	−1.24
3	−1.02	−1.03	−1.04	−1.05	−1.05	−1.06	−1.07	−1.08	−1.08	−1.09
4	−0.89	−0.90	−0.91	−0.92	−0.93	−0.94	−0.95	−0.96	−0.96	−0.97
5	−0.79	−0.80	−0.81	−0.82	−0.83	−0.84	−0.85	−0.86	−0.87	−0.87
6	−0.69	−0.71	−0.72	−0.73	−0.74	−0.75	−0.76	−0.77	−0.78	−0.79
7	−0.61	−0.62	−0.63	−0.65	−0.66	−0.67	−0.68	−0.69	−0.70	−0.71
8	−0.53	−0.54	−0.55	−0.57	−0.58	−0.59	−0.60	−0.62	−0.63	−0.64
9	−0.45	−0.47	−0.48	−0.49	−0.51	−0.52	−0.53	−0.55	−0.56	−0.57
10	−0.38	−0.39	−0.41	−0.42	−0.44	−0.45	−0.47	−0.48	−0.49	−0.50
11	−0.31	−0.32	−0.34	−0.36	−0.37	−0.39	−0.40	−0.42	−0.43	−0.44
12	−0.24	−0.26	−0.27	−0.29	−0.31	−0.32	−0.34	−0.36	−0.37	−0.38
13	−0.17	−0.19	−0.21	−0.23	−0.25	−0.26	−0.28	−0.30	−0.31	−0.32
14	−0.10	−0.13	−0.15	−0.17	−0.18	−0.20	−0.22	−0.24	−0.25	−0.27
15	−0.04	−0.06	−0.08	−0.10	−0.12	−0.14	−0.16	−0.18	−0.20	−0.21
16	0.03	0.00	−0.02	−0.04	−0.06	−0.08	−0.10	−0.12	−0.14	−0.16
17	0.09	0.07	0.04	0.02	0.00	−0.02	−0.04	−0.06	−0.08	−0.10
18	0.16	0.13	0.11	0.08	0.06	0.04	0.01	−0.01	−0.03	−0.05
19	0.23	0.20	0.17	0.14	0.12	0.09	0.07	0.05	0.03	0.01
20	0.30	0.27	0.24	0.21	0.18	0.15	0.13	0.11	0.08	0.06
21	0.37	0.33	0.30	0.27	0.24	0.21	0.19	0.16	0.14	0.12
22	0.44	0.40	0.37	0.33	0.30	0.27	0.25	0.22	0.19	0.17
23	0.51	0.47	0.43	0.40	0.37	0.34	0.31	0.28	0.25	0.23

Table 1.16 Plotting position, $E(y_{(i)})$, for extreme value Type 1 distribution.

i	N=41	42	43	44	45	46	47	48	49	50
24	0.59	0.54	0.50	0.47	0.43	0.40	0.37	0.34	0.31	0.28
25	0.66	0.62	0.57	0.53	0.50	0.46	0.43	0.40	0.37	0.34
26	0.74	0.69	0.65	0.61	0.57	0.53	0.49	0.46	0.42	0.40
27	0.83	0.77	0.72	0.68	0.64	0.59	0.57	0.52	0.49	0.45
28	0.92	0.86	0.80	0.75	0.71	0.66	0.62	0.58	0.55	0.51
29	1.01	0.95	0.89	0.83	0.78	0.74	0.69	0.65	0.61	0.58
30	1.11	1.04	0.97	0.92	0.86	0.81	0.76	0.72	0.68	0.64
31	1.21	1.14	1.07	1.00	0.94	0.89	0.84	0.79	0.74	0.70
32	1.33	1.24	1.16	1.09	1.03	0.97	0.91	0.86	0.82	0.77
33	1.46	1.36	1.27	1.19	1.12	1.06	1.00	0.94	0.89	0.84
34	1.59	1.48	1.38	1.30	1.22	1.15	1.08	1.02	0.97	0.91
35	1.75	1.62	1.51	1.41	1.32	1.24	1.17	1.11	1.05	0.99
36	1.93	1.78	1.65	1.53	1.44	1.35	1.27	1.20	1.13	1.07
37	2.14	1.96	1.80	1.67	1.56	1.46	1.37	1.29	1.22	1.15
38	2.40	2.17	1.98	1.83	1.70	1.58	1.48	1.40	1.32	1.24
39	2.74	2.43	2.19	2.00	1.85	1.72	1.61	1.51	1.42	1.34
40	3.25	2.77	2.45	2.22	2.03	1.87	1.74	1.63	1.53	1.44
41	4.29	3.28	2.79	2.48	2.24	2.05	1.90	1.77	1.65	1.55
42	—	4.31	3.30	2.82	2.50	2.26	2.08	1.92	1.79	1.68
43	—	—	4.34	3.32	2.84	2.52	2.28	2.10	1.94	1.81
44	—	—	—	4.36	3.35	2.86	2.54	2.31	2.12	1.96
45	—	—	—	—	4.38	3.37	2.88	2.57	2.33	2.14
46	—	—	—	—	—	4.41	3.39	2.91	2.59	2.35
47	—	—	—	—	—	—	4.43	3.41	2.93	2.61
48	—	—	—	—	—	—	—	4.45	3.43	2.95
49	—	—	—	—	—	—	—	—	4.47	3.45
50	—	—	—	—	—	—	—	—	—	4.49

Table 1.16 Plotting position, $E(y_{(i)})$, for extreme value Type 1 distribution.

In Table 1.14 the annual maximum data for station 27/1, Nidd at Hunsingore, are given in column 2 and the ordered sample in column 3. The exact EV1 plotting positions $E(y_{(i)})$ are given in column 5. The ordered sample values, column 3, are plotted against the y values in Figure 1.22. The slope of the plot of the lowest third of the data differs from that of the upper two thirds. A straight line is not a perfect representation of the data points at the lower end but it is better than any straight line through the data points plotted against Normal reduced variate in Figure 1.21.

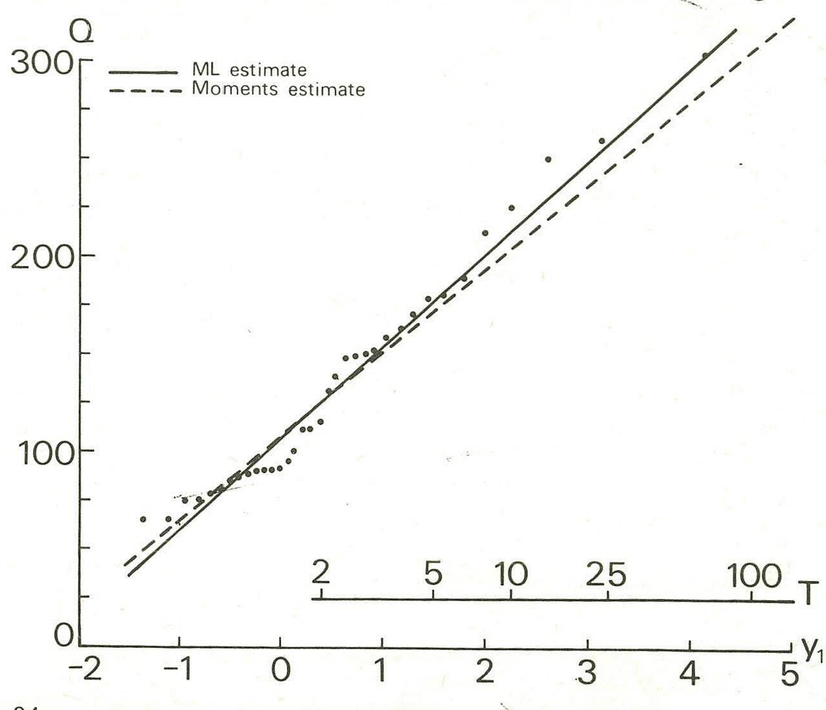


Fig 1.22 27/1, Nidd at Hunsingore, annual maxima plotted against EV1 reduced variate showing straight lines which are EV1 variates fitted to the sample by different methods.

The six highest plotted points on Figure 1.22 show remarkably little departure from collinearity in the light of the large sampling variance of the highest values. Any curve would offer a slight improvement on a straight line but the evidence is slight that a straight line is inadequate.

Moments estimates. The mean and the standard deviation in terms of the parameters u and α are

$$\mu = u+0.577\alpha \tag{1.3.4.55}$$

$$\sigma = \frac{\pi\alpha}{6}\cdot \tag{1.3.4.56}$$

The population quantities μ and σ are estimated by $\bar{x}$ and $\hat{\sigma}$ and the moments estimates of u and α in terms of these are

$$\hat{\alpha} = \frac{\sqrt{6}}{\pi}\hat{\sigma} = 0.78\hat{\sigma} \tag{1.3.4.57}$$

$$\hat{u} = \bar{x}-(0.577)\hat{\alpha} = \bar{x}-0.45\hat{\sigma}. \tag{1.3.4.58}$$

In this example, $\bar{x} = 136.66$ and $\hat{\sigma} = 60.74$ which give $\hat{u} = 109.33$ and $\hat{\alpha} = 47.36$.

Maximum likelihood estimates. The likelihood function $L(\mathbf{X}/u,\alpha)$, where $\mathbf{X}$ represents the sample collectively, is

$$\begin{aligned} L(\mathbf{X}/u,\alpha) &= \prod_{i=1}^{N} f(x_i/u,\alpha) \\ &= \prod_{i=1}^{N} \frac{1}{\alpha}\exp[-(x_i-u)/\alpha - e^{-(x_i-u)/\alpha}] \\ &= \frac{1}{\alpha^N}\exp[-\Sigma(x_i-u)/\alpha - \Sigma\, e^{-(x_i-u)/\alpha}]. \end{aligned} \tag{1.3.4.59}$$

The values of u and α which maximise this quantity, given the observed sample values $x_1, x_2, \ldots, x_N$, are the maximum likelihood estimates. Those values which maximise the likelihood also maximise the logarithm of the likelihood since the latter is a monotonically increasing function of the former. Since the log likelihood is easier to work with, the maximum likelihood estimates will be sought as those values which maximise the log likelihood.

The log likelihood is

$$\begin{aligned} LL(\mathbf{X}/u,\alpha) &= \ln L(\mathbf{X}/u,\alpha) \\ &= -N\ln\alpha - \Sigma\left(\frac{x_i-u}{\alpha}\right) - \Sigma\, e^{-(x_i-u)/\alpha} \\ &= -N\ln\alpha - \Sigma y_i - \Sigma\, e^{-y_i} \end{aligned} \tag{1.3.4.60}$$

where

$$y_i = \frac{(x_i-u)}{\alpha}. \tag{1.3.4.61}$$

The partial derivatives with respect to u and α are

$$-\frac{\partial LL}{\partial u} = \frac{-N+\Sigma\, e^{-y}}{\alpha} = -\frac{P}{\alpha} \tag{1.3.4.62}$$

$$-\frac{\partial LL}{\partial \alpha} = \frac{N-\Sigma y+\Sigma y\, e^{-y}}{\alpha} = \frac{R}{\alpha} \tag{1.3.4.63}$$

the notation P and R being introduced for algebraic convenience later.

The maximum likelihood estimates $(\hat{u},\hat{\alpha})$ must satisfy

$$\frac{\partial LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial \hat{u}} = 0 \quad \text{and} \quad \frac{\partial LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial \hat{\alpha}} = 0. \tag{1.3.4.64}$$

These equations do not have explicit solutions but an iterative method of solution has been given by Jenkinson (1969). Let α_i and u_i be the estimates after the ith step of the iteration; the initial estimates α_1 and u_1 may be taken as the moments estimates. Let $\hat{u}$ and $\hat{\alpha}$ be the true maximum likelihood estimates and let

$$\begin{aligned} \hat{u} &= u_i + \delta u_i \\ \hat{\alpha} &= \alpha_i + \delta\alpha_i. \end{aligned} \tag{1.3.4.65}$$

That is, δu_i and $\delta\alpha_i$ are the differences between the current estimate and the required estimate. The function $-\partial LL/\partial u$ and $-\partial LL/\partial \alpha$ may be expanded in a Taylor series about the maximum likelihood estimates $(\hat{u},\hat{\alpha})$ (all derivatives on the right hand side being evaluated at $(\hat{u},\hat{\alpha})$).

$$\begin{aligned} \frac{-\partial LL(\mathbf{X}/u_i,\alpha_i)}{\partial u} &= \frac{-\partial LL(\mathbf{X}/\hat{u}-\delta u_i,\hat{\alpha}-\delta\alpha_i)}{\partial u} \\ &= \frac{-\partial LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u} - (u_i-\hat{u})\frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u^2} \\ &\qquad -(\alpha_i-\hat{\alpha})\frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u\,\partial\alpha} \\ &\qquad + \text{terms containing higher powers of } \delta u \text{ and } \delta\alpha. \end{aligned}$$

Since $\partial LL(\mathbf{X}/\hat{u},\hat{\alpha})/\partial u$ is zero by Equation (1.3.4.64)

$$-\frac{\partial LL(\mathbf{X}/u_i,\alpha_i)}{\partial u} \simeq -\frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u^2}(u_i-\hat{u}) - \frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u\,\partial\alpha}(\alpha_i-\hat{\alpha}) \tag{1.3.4.66}$$

the approximation being due to the omission of terms containing powers and cross products of δu and $\delta\alpha$.

A similar expression may be derived for $-\partial LL(\mathbf{X}/u,\alpha)/\partial\alpha$ and is

$$-\frac{\partial LL(\mathbf{X}/u_i,\alpha_i)}{\partial \alpha} \simeq -\frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u\,\partial\alpha}(u_i-\hat{u}) - \frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial\alpha^2}(\alpha_i-\hat{\alpha}). \tag{1.3.4.67}$$

Equations (1.3.4.66) and (1.3.4.67) may be rewritten in matrix notation and noting from Equation (1.3.4.65) that $u_i-\hat{u} = -\delta u_i$ and $\alpha_i-\hat{\alpha} = -\delta\alpha_i$ they yield

$$\begin{pmatrix} -\dfrac{\partial LL(\mathbf{X}/u_i,\alpha_i)}{\partial u} \\ -\dfrac{\partial LL(\mathbf{X}/u_i,\alpha_i)}{\partial \alpha} \end{pmatrix} = \begin{pmatrix} -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u^2} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u\,\partial\alpha} \\ -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u\,\partial\alpha} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial\alpha^2} \end{pmatrix} \cdot \begin{pmatrix} -\delta u_i \\ -\delta\alpha_i \end{pmatrix}. \tag{1.3.4.68}$$

On the left hand side (u_i,α_i) are the current estimates and using these in Equation (1.3.4.61), y_i, $(i = 1, 2, \ldots, N)$ may be calculated. These in turn are used in Equations (1.3.4.62) and (1.3.4.63) to calculate the elements of the left hand side of Equation (1.3.4.68). Then

$$\begin{pmatrix} -\delta u_i \\ -\delta\alpha_i \end{pmatrix} = \begin{pmatrix} -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u^2} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u\,\partial\alpha} \\ -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial u\,\partial\alpha} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha})}{\partial\alpha^2} \end{pmatrix}^{-1} \cdot \begin{pmatrix} -\dfrac{\partial LL(\mathbf{X}/u_i,\alpha_i)}{\partial u} \\ -\dfrac{\partial LL(\mathbf{X}/u_i,\alpha_i)}{\partial \alpha} \end{pmatrix}. \tag{1.3.4.69}$$

The elements of the square matrix on the right hand side need to be evaluated at the maximum likelihood values $(\hat{u},\hat{\alpha})$ which are unknown. The best alternative is to evaluate them at the current estimates (u_i,α_i). However, the rate of convergence of the iteration procedure to the final values is not affected too much if these quantities are computed anew at every third rather than at each step of the iteration. Jenkinson (1969) suggests using the expected value of the inverted matrix which is the large sample maximum likelihood variance covariance matrix (Kendall & Stuart, 1961). The use of the same matrix at all steps of the iteration must reduce the speed of convergence but reduces the amount of calculation during each iteration. The variance–covariance matrix (Kimball, 1949) is

$$\begin{pmatrix} \operatorname{var}\hat{u} & \operatorname{cov}(\hat{u},\hat{\alpha}) \\ \operatorname{cov}(\hat{u},\hat{\alpha}) & \operatorname{var}\hat{\alpha} \end{pmatrix} = \frac{\alpha^2}{N}\begin{pmatrix} 1+\frac{6}{\pi^2}(1-\gamma)^2 & \frac{6}{\pi^2}(1-\gamma) \\ \frac{6}{\pi^2}(1-\gamma) & \frac{6}{\pi^2} \end{pmatrix}$$

$$= \frac{\alpha^2}{N}\begin{pmatrix} 1.11 & 0.26 \\ 0.26 & 0.61 \end{pmatrix}. \tag{1.3.4.70}$$

Inserting this matrix in Equation (1.3.4.69) and making use of Equations (1.3.4.62) and (1.3.4.63) gives

$$\begin{pmatrix} -\delta u_i \\ -\delta\alpha_i \end{pmatrix} = \frac{\alpha_i^2}{N}\begin{pmatrix} 1.11 & 0.26 \\ 0.26 & 0.61 \end{pmatrix}\cdot\begin{pmatrix} -P_i/\alpha_i \\ R_i/\alpha_i \end{pmatrix}$$

yielding

$$\begin{aligned} \delta u_i &= (1.11P_i - 0.26R_i)\frac{\alpha_i}{N} \\ \delta\alpha_i &= (0.26P_i - 0.61R_i)\frac{\alpha_i}{N} \end{aligned} \tag{1.3.4.71}$$

where the subscript i on P and R mean that these quantities are computed using u_i and α_i, the current estimates of u and α. The values δu_i and $\delta\alpha_i$ are obtained by inserting α_i, P_i and R_i into Equation (1.3.4.71); the ith step of the iteration is completed by computing the revised estimates

$$u_{i+1} = u_i + \delta u_i \quad \text{and} \quad \alpha_{i+1} = \alpha_i + \delta\alpha_i.$$

Example. The 35 annual maxima at 27/1, Nidd at Hunsingore, are used to illustrate the computations which are shown in Table 1.17. Moments estimates are used as the starting values u_1 and α_1. After the second iteration $\partial LL/\partial u = P/\alpha \simeq 0.006$ and $\partial LL/\partial\alpha = -R/\alpha = -0.022$. If these are considered to be sufficiently close to zero the values (u_2,α_2) may be considered to satisfy Equation (1.3.4.64) and be taken as the maximum likelihood estimates.

The general extreme value distribution (including the extreme value Types 2 and 3)

Graphical estimation. Conventionally graphical estimation is used only when the data are considered to be a random sample from a population whose variate–reduced variate relationship is a straight line, for instance from the Normal or extreme value Type 1 (Gumbel) distributions. In the general extreme value distribution this relationship is curved if the parameter-free Gumbel reduced variate y_1 is used. The relation is

Sample values	First iteration			Second iteration		
	$u_1 = 109.33$	$\alpha_1 = 47.36$	$N = 35$	$u_1 = 109.89$	$\alpha_2 = 43.75$	
x	$y = \frac{x-u_1}{\alpha_1}$	e^{-y}	ye^{-y}	$y = \frac{x-u_2}{\alpha_2}$	e^{-y}	ye^{-y}
189.02	1.6826	0.1859	0.3128	1.8105	0.1636	0.2962
91.80	−0.3701	1.4479	−0.5359	−0.4140	1.5129	−0.6263
115.52	0.1307	0.8775	0.1147	0.1287	0.8792	0.1132
78.55	−0.6499	1.9153	−1.2448	−0.7171	2.0485	−1.4690
162.99	1.1330	0.3221	0.3649	1.2149	0.2967	0.3605
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
87.76	−0.4554	1.5768	−0.7181	−0.5064	1.6593	−0.8403
251.96	3.0116	0.0492	0.1482	3.2506	0.0388	0.1261
138.72	0.6206	0.5376	0.3336	0.6596	0.5171	0.3411
305.75	4.1474	0.0158	0.0655	4.4813	0.0113	0.0506
226.48	2.4736	0.0843	0.2085	2.6676	0.0694	0.1851
Total	20.1995	33.4308	−9.7029	21.4376	34.7448	−12.5956

$P_1 = N - \sum e^{-y} = 1.5692$

$R_1 = N - \sum y + \sum ye^{-y} = 5.0976$

$\delta u_1 = (1.11P_1 - 0.26R_1)\frac{\alpha_1}{N} = 0.5634$

$\delta\alpha_1 = (0.26P_1 - 0.61R_1)\frac{\alpha_1}{N} = -3.655$

$u_2 = u_1 + \delta u_1 = 109.89$

$\alpha_2 = \alpha_1 + \delta\alpha_1 = 43.75$

$P_2 = 0.2552$

$R_2 = 0.9668$

$\delta u_2 = 0.0398$

$\delta\alpha_2 = -0.6534$

$u_3 = 109.9328$

$\alpha_3 = 43.0516$

Table 1.17 Calculation of maximum likelihood estimates of EV1 parameters.

$$x = u + \frac{\alpha}{k}(1 - e^{-ky_1}) \tag{1.3.4.72}$$

which is concave upward when $k<0$ and convex upward when $k>0$ (Figure 1.10). There are two possible approaches to graphical estimation in these circumstances. One is to use the Gumbel reduced variate y_1 and accept that, since the population x–y_1 relationship is curved, a curve must be drawn through the plotted points. This suffers from a disadvantage if the population relation is very curved as the estimated curve is very sensitive to changes in the operator's subjectivity and to plotting position formula. If the population x–y_1 curve is mild then the extreme value Type 1 plotting positions $F_i = (i-0.44)/N+0.12)$ may be used; if it is strongly concave upwards this plotting position introduces at the upper end bias which leads to consistent overestimation. In such a case the use of the Hazen formula would be justified although it might still be biassed, and conversely if the x–y_1 relation is strongly convex upward the median plotting position would be justified.

Another method can be used which avoids a disadvantage mentioned above. When the x–y_1 relation is strongly curved, instead of y_1 the variate W to which x is linearly related may be used. The x–y_1 relation in Equation (1.3.4.72) may be rewritten

$$x = u + \alpha W \tag{1.3.4.73}$$

where

$$W = (1 - e^{ky_1})/k \tag{1.3.4.74}$$

depends on the shape parameter k. If the value of k is known *a priori*, the plotting probabilities F_i can be converted to W_i values by first converting to y_i values

$$y_i = -\ln - \ln F_i \tag{1.3.4.75}$$

and thence to W_i values *via* Equation (1.3.4.74). Alternatively, a special probability scale may be constructed using the relations in Equations (1.3.4.75) and (1.3.4.74) between F, y and W and the data plotted against F_i values directly. Any bias in plotting mentioned above will still be present but the sensitivity of the estimation to operator subjectivity is lessened. This procedure also holds good if the value of k is unknown but an estimate based perhaps on prior knowledge is closer to reality than $k = 0$.

Estimation by moments. Consider the case when the population is of Type 2, $k<0$. Let y_2 be the EV2 reduced variate the distribution of which depends only on k. The EV2 variate x_2 is related to this by

$$x_2 = u + \frac{\alpha}{k} - \frac{\alpha}{k} y_2 \tag{1.3.4.76}$$

or

$$x_2 = A + By_2 \tag{1.3.4.77}$$

where $A = u + \alpha/k$ is a location parameter—the lower bound of x_2 and $B = -\alpha/k > 0$ is a scale parameter.

The moments of y_2 depend on k only and are tabulated in Table 1.7. Since x_2 is linearly related to y_2 their skewnesses are equal provided they both have the same value of k. Therefore, this table can be entered with the sample skewness computed from the sample moments of x_2 to give $\hat{k}$, the moments estimate of k. From the same table the mean and variance of y_2 are obtained. Then from Equation (1.3.4.77) and the theorem on the mean and variance of a linear function of a random variable, the mean and variance of x_2 are

$$E(x_2) = A + BE(y_2) \tag{1.3.4.78}$$

$$\text{var } x_2 = B^2 \text{ var } y_2. \tag{1.3.4.79}$$

$E(x_2)$ and var x_2 are estimated by the sample mean and variance $\bar{x}$ and $\hat{\sigma}^2$. Then from Equation (1.3.4.79)

$$\hat{B} = \left(\frac{\hat{\sigma}^2}{\text{var } y_2}\right)^{\frac{1}{2}} \tag{1.3.4.80}$$

and from Equation (1.3.4.78)

$$\hat{A} = \bar{x} - \hat{B}E(y_2). \tag{1.3.4.81}$$

Since $\hat{k}$ is known and $B = -\alpha/k$

$$\hat{\alpha} = -\hat{k}\hat{B} \tag{1.3.4.82}$$

and since $A = u + \alpha/k = u - B$, u equals $A + B$ and

$$\hat{u} = \hat{A} + \hat{B}. \tag{1.3.4.83}$$

Equation (1.3.4.82) and (1.3.4.83) are the moments estimates of α and u while the moments estimate of k have been obtained by entering Table 1.7 with the sample skewness.

The skewness of the extreme value Type 1 distribution is 1.14. For $k<0$ the skewness is greater than 1.14 while if $k>0$ the skewness is less than 1.14. Thus, the size of the sample skewness relative to 1.14 determines whether the sample indicates a Type 2 or Type 3 distribution in moments estimation.

Considering now the Type 3

$$x_3 = u+\frac{\alpha}{k}+\frac{\alpha}{k}y_3 \tag{1.3.4.84}$$

where y_3 is the EV3 reduced variate, $-\infty<y_3<0$. This is of the form

$$x_3 = A+By_3 \tag{1.3.4.85}$$

where $A = u+\alpha/k$ is a location parameter—the upper bound of x_3 and $B = \alpha/k$ is a scale parameter.

The skewness of x_3 and y_3 are equal and depend only on k. Hence, the sample skewness can be used to enter Table 1.10 from which $\hat{k}$ may be read. Then the corresponding values of $E(y_3)$ and var y_3 can be obtained in the same table and expressions similar to Equations (1.3.4.80) and (1.3.4.81) give

$$\hat{B} = \left(\frac{\hat{\sigma}^2}{\text{var } y_3}\right)^{\frac{1}{2}} \tag{1.3.4.86}$$

$$\hat{A} = \bar{x}-\hat{B}E(y_3). \tag{1.3.4.87}$$

Then, since

$$B = \alpha/k$$

$$\hat{\alpha} = \hat{B}\hat{k} \tag{1.3.4.88}$$

and since $A = u+\alpha/k = u+B$

$$\hat{u} = \hat{A}-\hat{B}. \tag{1.3.4.89}$$

Equations (1.3.4.88) and (1.3.4.89) give the moments estimates of α and u when $\hat{k}$ is positive. The terms involving $\hat{B}$ differ in sign between these and the corresponding expressions, Equations (1.3.4.82) and (1.3.4.83), for negative $\hat{k}$.

Estimation by sextiles. This method was developed by Jenkinson (1969) for this distribution although in theory it is possible to extend its use to other distributions. The general extreme value variate x is related to the reduced variate W by

$$x = u+\alpha W \tag{1.3.4.90}$$

where

$$W = (1-e^{ky_1})/k \tag{1.3.4.91}$$

is the general extreme value reduced variate which depends on the shape parameter k in the same way for instance as the Pearson Type 3 reduced variate depends on the shape parameter γ. In Equation (1.3.4.91), y_1 is the extreme value Type 1 reduced variate.

The variates x and W share the same value of shape parameter k. Hence, a sample estimate of the shape parameter for x is also an estimate of the same quantity for W. When k is known for W, a location and scale parameter which depend only on k, are also known for W. These are related to the corresponding quantities in the x population by relations involving u and α which are simple and easily derived because of the linearity of the relation in Equation (1.3.4.90). The sample estimates of these quantities are inserted in these relations and the equations are solved for u and α.

The sample quantities and the location and scale quantities of the W variate mentioned in the last paragraph are all computed from sextile means. The range of the variate is considered to be divided into six intervals each containing one sixth of the variate values. Denote the interval boundaries of the ith sextile by W_{i-1} and W_i. With W_0 being the lower bound of the variate and $h(w)$ being its pdf these are such that

$$\int_{W_0}^{W_i} h(w)\,\mathrm{d}w = \frac{i}{6} \qquad (1.3.4.92)$$

and

$$\int_{W_{i-1}}^{W_i} h(w)\,\mathrm{d}w = \frac{1}{6}. \qquad (1.3.4.93)$$

Denote the mean of the variate values in the ith sextile by v_i,

$$v_i = 6\int_{W_{i-1}}^{W_i} w\,h(w)\,\mathrm{d}w. \qquad (1.3.4.94)$$

The six values $v_1, v_2, \ldots, v_6$ are used to define location, scale and shape parameters of the W distribution as follows:
location:

$$v = \text{mean of } v_i = \frac{1}{6}\sum_{i=1}^{6} v_i \qquad (1.3.4.95)$$

scale:

$$\sigma_v = \text{standard deviation of } v_i = \sqrt{\frac{1}{6}\Sigma(v_i - v)^2} \qquad (1.3.4.96)$$

shape:

$$\ell = \frac{v_2 - v_1}{v_6 - v_5}. \qquad (1.3.4.97)$$

These quantities depend on k only and they have been tabulated by Jenkinson (1969), and are reproduced here in Table 1.18.

k	v	σ_v	ℓ
−0.5	1.54	2.85	0.08
−0.4	1.22	2.24	0.11
−0.3	0.99	1.83	0.16
−0.2	0.82	1.55	0.23
−0.1	0.69	1.34	0.32
0.0	0.58	1.20	0.43
0.1	0.49	1.09	0.58
0.2	0.41	1.01	0.79
0.3	0.34	0.96	1.05
0.4	0.28	0.92	1.39
0.5	0.23	0.89	1.82

Table 1.18 Mean, standard deviation and shape ratio of sextile means which define the GEV distribution with parameter k (after Jenkinson, 1969).

The corresponding sextile means in the x population are denoted by $\omega_1, \omega_2, \ldots, \omega_6$. Because of the relation in Equation (1.3.4.90) ω_i and v_i are related by

$$\omega_i = u + \alpha v_i \qquad (1.3.4.98)$$

so that

$$\mu_\omega = \text{mean of } \omega_1, \omega_2, \ldots, \omega_6 = u + \alpha v \qquad (1.3.4.99)$$

$$\sigma_\omega = \text{standard deviation of } \omega_1, \omega_2, \ldots, \omega_6 = \alpha\sigma_v \qquad (1.3.4.100)$$

and the shape parameter

$$\frac{\omega_2-\omega_1}{\omega_6-\omega_5} = \frac{u+\alpha v_2-(u+\alpha v_1)}{u+\alpha v_6-(u+\alpha v_5)} = \frac{v_2-v_1}{v_6-v_5} = \ell. \quad (1.3.4.101)$$

Equation (1.3.4.100) gives

$$\alpha = \sigma_\omega/\sigma_\nu \quad (1.3.4.102)$$

and Equations (1.3.4.99) and (1.3.4.102) give

$$u = \mu_\omega - \alpha v$$
$$= \mu_\omega - v\sigma_\omega/\sigma_\nu. \quad (1.3.4.103)$$

The quantities ℓ, u_ω and σ_ω are estimated from the sample data as follows. The data are ranked into order and divided into six groups of equal size. If the sample size is N and M is the integral part of $N/6$ then there should be M sample members in some groups and $M+1$ in others. Thus, if $N = 40$ there would be $M = 6$ items in two groups and seven items in the remaining four groups. The adoption of an arbitrary rule of deciding which groups receive an extra item does of course introduce a degree of subjectivity but the effect on the final outcome is small. The mean of each group is computed. Let these be denoted $w_1, w_2, \ldots, w_6$. Then the quantities

$$\bar{w} = \frac{1}{6}\sum_{i=1}^{6} w_i \quad (1.3.4.104)$$

$$s_w = \sqrt{\frac{1}{6}\Sigma(w_i-\bar{w})^2} \quad (1.3.4.105)$$

$$\hat{\ell} = \frac{w_2-w_1}{w_6-w_5} \quad (1.3.4.106)$$

are used as estimates of μ_ω, σ_ω and ℓ respectively. Table 1.18 is entered with $\hat{\ell}$ and the corresponding $\hat{k}$, $\hat{v}$ and $\hat{\sigma}_\nu$ obtained by interpolation. This $\hat{k}$ is the sextile estimate of k; $\bar{w}$ and s_w from Equations (1.3.4.104) and (1.3.4.105) are used as estimates of μ_ω and σ_ω in Equations (1.3.4.102) and (1.3.4.103) so that $\hat{\alpha}$ and $\hat{u}$ are

$$\hat{\alpha} = s_w/\hat{\sigma}_\nu \quad (1.3.4.107)$$

$$\hat{u} = \bar{w} - \hat{v}\frac{s_w}{\hat{\sigma}_\nu}. \quad (1.3.4.108)$$

Estimation by maximum likelihood. The treatment given here is that given by Jenkinson (1969). The likelihood is

$$L(\mathbf{X}/u,\alpha,k) = \prod_{i=1}^{N} f(x_i/u,\alpha,k)$$
$$= \prod_{i=1}^{N} \frac{1}{\alpha}[1-k(x_i-u)/\alpha]^{1/k-1}\, e^{-[1-k(x_i-u)/\alpha]^{1/k}}. \quad (1.3.4.109)$$

Instead of this form for $f(x)$ Jenkinson proceeds from

$$F(x) = \exp(-e^{-y})$$

where $x = u+\alpha(1-e^{-ky})/k$ and writes

$$f(x) = \frac{dF(x)}{dx} = \frac{dF(x)}{dy}\bigg/\frac{dx}{dy} = e^{-y}.\exp(-e^{-y}).\frac{1}{\alpha}e^{ky}$$
$$= \frac{1}{\alpha}\exp(-e^{-y}).e^{-y(1-k)} \quad (1.3.4.110)$$

and inserting this in Equation (1.3.4.109) gives

$$L(\mathbf{X}/u,\alpha,k) = \frac{1}{\alpha^N}\exp(-\Sigma\, e^{-y_i}).\, e^{-\Sigma y_i(1-k)}. \qquad (1.3.4.109a)$$

The forms of $f(x)$ in Equation (1.3.4.109) and in Equation (1.3.4.110) are equivalent as can be seen by inserting the expression

$y = -\frac{1}{k}\ln\left[1 - \frac{x-u}{\alpha}k\right]$ in Equation (1.3.4.110) for y.

Using Equation (1.3.4.109a) the log likelihood is

$$LL(\mathbf{X}/u,\alpha,k) = \ln L(\mathbf{X}/u,\alpha,k)$$

$$= -N\ln\alpha - (1-k)\sum_{i=1}^{N} y_i - \sum_{i=1}^{N} e^{-y_i} \qquad (1.3.4.111)$$

where

$$y_i = -\frac{1}{k}\ln\left(1 - \frac{x_i-u}{\alpha}k\right). \qquad (1.3.4.112)$$

The values of u, α and k for which $LL(\mathbf{X}/u,\alpha,k)$ is a maximum are also the values for which $L(\mathbf{X}/u,\alpha,k)$ is a maximum because the logarithmic function is single valued and monotonic.

Therefore, because the log likelihood function is easier to use than the likelihood function, the maximum likelihood solution will be sought as the values $(\hat{u},\hat{\alpha},\hat{k})$ which maximise $LL(\mathbf{X}/u,\alpha,k)$.

Using Jenkinson's notation the partial derivatives are

$$-\frac{\partial LL}{\partial u} = \frac{Q}{\alpha} \qquad (1.3.4.113)$$

$$-\frac{\partial LL}{\partial \alpha} = \frac{1}{\alpha}.\frac{P+Q}{k} \qquad (1.3.4.114)$$

$$-\frac{\partial LL}{\partial k} = \frac{1}{k}\left(R - \frac{P+Q}{k}\right) \qquad (1.3.4.115)$$

where

$$P = N - \sum_{i=1}^{N} e^{-y_i} \qquad (1.3.4.116)$$

$$Q = \sum_{i=1}^{N} e^{-y_i+ky_i} - (1-k)\sum_{i=1}^{N} e^{ky_i} \qquad (1.3.4.117)$$

$$R = N - \sum_{i=1}^{N} y_i + \sum_{i=1}^{N} y_i\, e^{-y_i} \qquad (1.3.4.118)$$

k	a	b	c	f	g	h
−0.4	1.05	1.29	0.84	0.26	−0.09	0.80
−0.3	0.92	1.29	0.73	0.26	−0.03	0.69
−0.2	0.81	1.28	0.64	0.26	0.04	0.57
−0.1	0.72	1.27	0.55	0.26	0.10	0.46
0.0	0.65	1.25	0.48	0.26	0.15	0.34
0.1	0.61	1.22	0.39	0.24	0.18	0.21
0.2	0.58	1.20	0.33	0.22	0.21	0.09
0.3	0.58	1.17	0.27	0.19	0.23	−0.03
0.4	0.60	1.14	0.21	0.16	0.24	−0.16
0.5	0.63	1.11	0.15	0.13	0.24	−0.30
0.6	0.68	1.08	0.10	0.09	0.22	−0.43
0.8	0.82	1.02	0.03	0.03	0.15	−0.71
1.0	1.00	1.00	0.00	0.00	0.00	−1.00

Table 1.19 Values of a, b, c, f, g and h in the variance–covariance matrix of ML estimates of GEV parameters (after Jenkinson, 1969).

and y_i is as in Equation (1.3.4.112).

The maximum likelihood solutions $(\hat{u},\hat{\alpha},\hat{k})$ must satisfy the conditions

$$-\frac{\partial LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial \hat{u}} = 0, \quad -\frac{\partial LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial \hat{\alpha}} = 0,$$
$$-\frac{\partial LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial \hat{k}} = 0. \quad (1.3.4.119)$$

These equations must be solved iteratively as there is no explicit solution.

Let (u_i,α_i,k_i) be the estimates after the ith step of the iteration. The initial estimates (u_1,α_1,k_1) may be taken as either the moments or sextile estimates. Let δu_i be the difference between the maximum likelihood estimate $\hat{u}$ of u and the current value u_i and define $\delta\alpha_i$ and δk_i similarly. That is

$$\begin{aligned}\hat{u} &= u_i+\delta u_i\\ \hat{\alpha} &= \alpha_i+\delta\alpha_i\\ \hat{k} &= k_i+\delta k_i.\end{aligned} \quad (1.3.4.120)$$

The functions $-\partial LL/\partial u$, $-\partial LL/\partial\alpha$, $-\partial LL/\partial k$ may be expanded in a Taylor series about the maximum likelihood values $(\hat{u},\hat{\alpha},\hat{k})$, for instance

$$\begin{aligned}-\frac{\partial LL(\mathbf{X}/u_i,\alpha_i,k_i)}{\partial u} &= -\frac{\partial LL(\mathbf{X}/\hat{u}-\delta u_i,\hat{\alpha}-\delta\alpha_i,\hat{k}-\delta k_i)}{\partial u}\\ &= -\frac{\partial LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u} + \frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u^2}\delta u_i\\ &\quad + \frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u\,\partial\alpha}\delta\alpha_i + \frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u\,\partial k}\delta k_i\\ &\quad + \text{terms containing higher powers of}\\ &\quad\ \ \delta u_i,\ \delta\alpha_i \text{ and } \delta k_i.\end{aligned} \quad (1.3.4.121)$$

All the terms on the right hand side of Equation (1.3.4.121) are evaluated at $(\hat{u},\hat{\alpha},\hat{k})$ and since $-\partial LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})/\partial u$ is zero by Equation (1.3.4.119), the equation may be rewritten

$$-\frac{\partial LL(\mathbf{X}/u_i,\alpha_i,k_i)}{\partial u} \simeq \frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u^2}\delta u_i + \frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u\,\partial\alpha}\delta\alpha_i + \frac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u\,\partial k}\delta k_i \quad (1.3.4.122)$$

the approximation being due to the omission of terms containing higher powers and cross products of δu_i, $\delta\alpha_i$ and δk_i. Similar expressions may be derived for the other two quantities $-\partial LL(\mathbf{X}/u_i,\alpha_i,k_i)/\partial\alpha$ and $-\partial LL(\mathbf{X}/u_i,\alpha_i,k_i)/\partial k$. The three expressions, after multiplication by -1 may be written:

$$\begin{pmatrix}\dfrac{\partial LL(\mathbf{X}/u_i,\alpha_i,k_i)}{\partial u}\\[2ex] \dfrac{\partial LL(\mathbf{X}/u_i,\alpha_i,k_i)}{\partial\alpha}\\[2ex] \dfrac{\partial LL(\mathbf{X}/u_i,\alpha_i,k_i)}{\partial k}\end{pmatrix} = \begin{pmatrix}-\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u^2} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u\,\partial\alpha} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial u\,\partial k}\\[2ex] -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial\alpha\,\partial u} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial\alpha^2} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial\alpha\,\partial k}\\[2ex] -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial k\,\partial u} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial k\,\partial\alpha} & -\dfrac{\partial^2 LL(\mathbf{X}/\hat{u},\hat{\alpha},\hat{k})}{\partial k^2}\end{pmatrix} \cdot \begin{pmatrix}\delta u_i\\[2ex] \delta\alpha_i\\[2ex] \delta k_i\end{pmatrix}. \quad (1.3.4.123)$$

The elements of the vector on the left hand side may be computed using Equations (1.3.4.112)–(1.3.4.118) with the values (u_i,α_i,k_i) replacing (u,α,k) in these equations. This vector premultiplied by the inverse of the matrix on the right hand side gives the required current increments δu_i, $\delta\alpha_i$ and δk_i. However, all the elements in this matrix are to be evaluated at the maximum likelihood values $(\hat{u},\hat{\alpha},\hat{k})$ which are as yet unknown. Luckily the iteration procedure is not vitiated if an approximation to this inverse matrix is used in its place, though the speed of convergence to the answer may be reduced however. The inverse matrix evaluated at the current values (u_i,α_i,k_i) instead of the unknown $(\hat{u},\hat{\alpha},\hat{k})$ may be used and even the same matrix could be used for two or three iterations before being updated. This would slow the convergence in terms of the number of iterations required but this would be balanced by the reduction of work in the calculation of the matrix elements at each iterative step.

Jenkinson proposed a different variation which reduces the amount of work involved in the calculation of the inverse matrix values. He replaces the elements of the matrix to be inverted by their expected values. The resulting inverted matrix is the large sample maximum likelihood variance–covariance matrix of the estimators $(\hat{u},\hat{\alpha},\hat{k})$. This fact is a well known theorem in statistics (Kendall & Stuart, 1961). In Jenkinson's notation this matrix is

$$\begin{pmatrix} \text{var}\, u & \text{cov}\,(\alpha,u) & \text{cov}\,(u,k) \\ \text{cov}\,(\alpha,u) & \text{var}\,\alpha & \text{cov}\,(\alpha,k) \\ \text{cov}\,(u,k) & \text{cov}\,(\alpha,k) & \text{var}\,k \end{pmatrix} = \frac{1}{N}\begin{pmatrix} \alpha^2 b & \alpha^2 h & \alpha f \\ \alpha^2 h & \alpha^2 a & \alpha g \\ \alpha f & \alpha g & c \end{pmatrix} \qquad (1.3.4.124)$$

and the values of a, b, c, f, g and h depend on k only and have been tabulated by Jenkinson and are reproduced here in Table 1.19. The matrix in Equation (1.3.4.124), computed at each iterative step with α replaced by the current value α_i, premultiplies the vector on the left hand side of Equation (1.3.4.123) to give the current increments δu_i, $\delta\alpha_i$ and δk_i. Providing the values of a, b, c, f, g and h are available, the use of this matrix is the most convenient method of executing the iteration process.

Inserting the expressions for the derivatives $-\partial LL/\partial u$, $-\partial LL/\partial\alpha$ and $-\partial LL/\partial k$ in P, Q and R as in Equations (1.4.3.113), (1.3.4.114) and (1.3.4.115) into Equation (1.3.4.123) and using Equation (1.3.4.124) for the inverse of the matrix on the right of Equation (1.3.4.123) gives

$$\begin{pmatrix} \delta u_i \\ \delta\alpha_i \\ \delta k_i \end{pmatrix} = \frac{1}{N}\begin{pmatrix} \alpha_i^2 b & \alpha_i^2 h & \alpha_i f \\ \alpha_i^2 h & \alpha_i^2 a & \alpha_i g \\ \alpha_i f & \alpha_i g & c \end{pmatrix} \cdot \begin{pmatrix} -\dfrac{Q_i}{\alpha_i} \\ -\dfrac{1}{\alpha_i}\dfrac{P_i+Q_i}{k_i} \\ -\dfrac{1}{k_i}\left(R_i - \dfrac{P_i+Q_i}{k_i}\right) \end{pmatrix} \qquad (1.3.4.125)$$

where the subscript i on P, Q and R indicates that these quantities are evaluated at (u_i,α_i,k_i) the current values of the estimates of (u,α,k). This completes the ith iterative step.

Example. The distribution will be fitted to the 35 annual maxima for 27/1, Nidd at Hunsingore

Sextiles estimate: The 35 values are arranged into order and divided into six groups, the first one having five values and the remainder having six. These values and the sextile means are shown in Table 1.20. The quantities $\bar{w}$, s_w and $\hat{\ell}$ defined in Equations (1.3.4.104)–(1.3.4.106) have the values $\bar{w} = 134.87$, $s_w = 57.45$, $\hat{\ell} = 0.213$.

Table 1.20 Calculation of sextile means.

Sextile no.	Variate values						Sextile mean w_i
1	65.08	65.60	75.06	76.22	78.55	—	72.10
2	81.27	86.73	87.76	88.89	90.28	91.80	87.79
3	91.80	92.82	95.47	100.40	111.54	111.74	100.63
4	115.52	131.82	138.72	148.63	149.30	151.79	139.30
5	153.04	158.01	162.99	172.92	179.12	181.59	167.94
6	189.04	213.70	226.48	251.96	261.82	305.75	241.45

Table 1.18 is entered with $\hat{\ell}$ to give the corresponding estimate of k, namely $k = -0.22$. The quantities $\hat{v}$ and $\hat{\sigma}_v$ are obtained in the same table as 0.854 and 1.606. Then using Equations (1.3.4.107) and (1.3.4.108) $\hat{\alpha} = 57.45/1.606 = 35.77$, $\hat{u} = 134.87-0.854\,\hat{\alpha} = 104.32$, which complete the sextile estimates of u, α and k.

Maximum likelihood estimate: The sextile estimates are used as the initial estimates (u_1,α_1,k_1). The increments $(\delta u_1,\delta\alpha_1,\delta k_1)$ are given by Equation (1.3.4.125) with all quantities in it evaluated at (u_1,α_1,k_1). The quantities a, b, c, f, g, h are obtained by interpolation in Table 1.19 at $k = k_1' = -0.22$ and are, $a = 0.832$, $b = 1.282$, $c = 0.658$, $f = 0.26$, $g = 0.026$, $h = 0.594$.

Next, P_1, Q_1 and R_1 must be calculated and from Equations (1.3.4.116)–(1.3.4.118) it can be seen that these require the values Σy_i, Σe^{ky_i}, Σe^{-y_i}, Σye^{-y_i} and $\Sigma e^{-y_i+ky_i}$ where $y_i = -1/k_1 \ln[1-k_1(x_i-u_1)/\alpha_1]$ for $i = 1, 2, \ldots, N$. These computations are shown in summary form in Table 1.21

Table 1.21 First iteration in estimating GEV parameters by maximum likelihood with starting values $(u_1, \alpha_1, k_1) = (104.32, 35.77, -0.22)$.

i	x	$z = 1-\frac{x-u_1}{\alpha_1}k_1$	$y = -\frac{1}{k_1}\ln z$	e^{k_1y}	e^{-y}	ye^{-y}	e^{-y+k_1y}
STEP 1							
1	189.018	1.5209	1.9060	0.6575	0.1487	0.2834	0.0978
2	91.803	0.9230	−0.3641	1.0834	1.4393	−0.5241	1.5593
3	115.515	1.0689	0.3027	0.9356	0.7388	0.2236	0.6912
4	78.554	0.8415	−0.7843	1.1883	2.1908	−1.7181	2.6033
5	162.987	1.3608	1.4004	0.7348	0.2465	0.3452	0.1811
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
33	138.719	1.2116	0.8723	0.8254	0.4180	0.3646	0.3450
34	305.748	2.2389	3.6635	0.4467	0.0256	0.0939	0.0115
35	226.480	1.7513	2.5472	0.5710	0.0783	0.1995	0.0447
		Sums	22.0683	31.6722	35.1395	−15.5534	39.2204

STEP 2

$P_1 = N-\sum e^{-y} = -0.1395$

$Q_1 = \sum e^{-y+ky}-(1-k)\sum e^{ky} = 0.5803$

$R_1 = N-\sum y+\sum ye^{-y} = -2.6217$

STEP 3

Enter Table 1.19 at $k = -0.22$ and get
$a = 0.832$, $b = 1.282$, $c = 0.658$, $f = 0.26$, $g = 0.026$, $h = 0.594$

STEP 4

$$\delta u_1 = -\frac{\alpha_1}{N}\left[bQ_1+h\frac{P_1+Q_1}{k_1}+\frac{f}{k_1}\left(R_1-\frac{P_1+Q_1}{k_1}\right)\right] = -0.2904$$

$$\delta\alpha_1 = -\frac{\alpha_1}{N}\left[hQ_1+a\frac{P_1+Q_1}{k_1}+\frac{g}{k_1}\left(R_1-\frac{P_1+Q_1}{k_1}\right)\right] = 1.2768$$

$$\delta k_1 = -\frac{1}{N}\left[fQ_1+g\frac{P_1+Q_1}{k_1}+\frac{c}{k_1}\left(R_1-\frac{P_1+Q_1}{k_1}\right)\right] = -0.0557$$

STEP 5

$u_2 = u_1+\delta u_1 = 104.03$

$\alpha_2 = \alpha_1+\delta\alpha_1 = 37.05$

$k_2 = k_1+\delta k_1 = -0.28$

followed by the evaluation of P_1, Q_1 and R_1, Then Equation (1.3.4.125) is written in expanded form and the increments δu_1, $\delta\alpha_1$ and δk_1 are computed followed by the new estimates $u_2 = 104.03$, $\alpha_2 = 37.05$ and $k_2 = -0.28$. Any set of estimates must satisfy Equation (1.3.4.119).

Whether this is so can be checked for the set (u_1,α_1,k_1) when P_1, Q_1 and R_1 have been computed. When u_1, α_1, k_1, P_1, Q_1 and R_1 are inserted into Equations (1.3.4.113)–(1.3.4.115) these yield

$$-\frac{\partial LL(\mathbf{X}/u_1,\alpha_1,k_1)}{\partial u_1} = 0.0162$$

$$-\frac{\partial LL(\mathbf{X}/u_1,\alpha_1,k_1)}{\partial \alpha_1} = -0.0560$$

$$-\frac{\partial LL(\mathbf{X}/u_1,\alpha_1,k_1)}{\partial k_1} = -2.8094.$$

The corresponding quantities based on u_2, α_2 and k_2 must await the values of P_2, Q_2 and R_2 which will emerge when the second iteration is being performed, that is in the calculation of δu_2, $\delta\alpha_2$ and δk_2.

A table similar to Table 1.21 is prepared for this purpose; this gives $P_2 = 0.0176$, $Q_2 = -0.1822$ and $R_2 = 0.3653$. New values of a, b, c, f, g, h are obtained in Table 1.19 at $k = k_2 = -0.28$. These together with P_2, Q_2 and R_2 give as before $\delta u_2 = -0.3848$, $\delta\alpha_2 = -0.4170$ and $\delta k_2 = -0.0146$; and $u_3 = u_2+\delta u_2 = 103.65$, $\alpha_3 = \alpha_2+\delta\alpha_2 = 36.63$, $k_3 = k_2+\delta k_2 = -0.29$.

The derivatives of Equations (1.3.4.113)–(1.3.4.115) may now be calculated with u_2, α_2 k_2, P_2, Q_2 and R_2 and give

$$\frac{-\partial LL}{\partial u_2} = -0.0049, \quad \frac{-\partial LL}{\partial \alpha_2} = +0.0159, \quad \frac{-\partial LL}{\partial k_2} = 0.7948.$$

This is a fourfold reduction in each derivative. After three more iterations the maximum percentage adjustment to the parameters is less than 1% and the function values $-\partial LL/\partial u$, $-\partial LL/\partial \alpha$ and $-\partial LL/\partial k$ are less than 0.007. It goes without saying that this method is better suited to automatic rather than manual computation. A completely self contained program is available but since STEP 1 in Table 1.21 is the most arduous part of each iteration this part alone could be programmed if only a small computer or a programmable calculating machine is available.

The lognormal distributions

As seen earlier there are two parameter and three parameter lognormal distributions. In the latter the third parameter is a lower bound x_0; in such a distribution estimation by maximum likelihood is rarely simple, see for instance the Pearson Type 3 distribution. If ML estimates are required it is suggested that a method analogous to the method outlined for the Pearson Type 3 be used. No formula will be given here for this case.

Graphical estimation. For graphical estimation the form of the variate–reduced variate relationship needs to be considered but in a log distribution two such relationships may be considered, one in the x domain and one in the log domain. In this case the one in the log domain is easier to consider because if x is lognormal then $z = \ln x$ is Normal. Hence, z is related to the standardised or reduced Normal variate by the relation

$$z = \mu_z+\sigma_z y. \qquad (1.3.4.126)$$

Therefore, if the sample values x_1, x_2, ..., x_N are transformed into

$z_1 = \ln x_1$, $z_2 = \ln x_2 \ldots z_N = \ln x_N$ these z values should be plotted as a sample from the Normal distribution (qv), that is at plotting probabilities

$$F_i = \frac{i-3/8}{N+1/4} \qquad (1.3.4.127)$$

or at the corresponding values of y, namely $y_i = \Phi^{-1}(F_i)$ where $\Phi(.)$ is the distribution function of the reduced Normal variate. For sample sizes less than 50 the $E(y_{(i)})$ values in Table 1.13 may be used. The simplest procedure is to plot the $x_1, x_2, \ldots, x_N$ values against the y_i values on semilogarithmic paper, the x's on the logarithmic scale and the y_i's on the linear scale over which a return period scale may be sketched as follows.

T	2	5	10	25	50	100	200
y	0.00	0.84	1.28	1.75	2.05	2.33	2.58

When it is known that the data come from a two parameter lognormal population a straight line guided by eye is drawn through the plot. The value of z (or $\ln x$) at $y = 0$ is the graphical estimate of μ_z while the slope in z units is the graphical estimate of σ_z. If the data come from a three parameter lognormal population the straight line is no longer appropriate and a curve should be drawn but in this case quantiles only and not the individual parameter values can be obtained.

As will be seen later, the maximum likelihood estimates of the parameters in the real domain coincide with both the moments and maximum likelihood estimates in the log domain. This is not necessarily so for other log distributions but in this case its truth would lead to the belief that graphical estimation in the log domain would be satisfactory.

If graphical estimation in the real domain is required a straight line is applicable only if the abscissa is specially graduated in a scale y which is such that $\ln y$ is Normally distributed with mean zero and variance σ_z^2. If σ_z^2 is unknown the sample data $x_1, x_2, \ldots, x_N$ could be plotted at the plotting probabilities of Equation (1.3.4.127) on a Normal, exponential or extreme value base according to whichever gives the least curved plot. Alternatively, if a first estimate of σ_z^2 is available a special abscissa y, such that $\ln y$ is $N(0,\sigma_z^2)$, may be used. If σ_z^2 were known *a priori* the ordinate of a straight line at $y = 0$ on such a plot would estimate x_0 in the three parameter distribution and the slope would equal e^{μ_z} (see Equation (1.2.5.11), Section 1.2).

Estimation by moments—two parameter case. The relations between the first two moments and the parameters μ_z and σ_z^2 are

$$\mu = \exp(\mu_z + \tfrac{1}{2}\sigma_z^2) \qquad (1.3.4.128)$$

$$\sigma = \exp(\mu_z + \tfrac{1}{2}\sigma_z^2)(e^{\sigma_z^2} - 1)^{\frac{1}{2}}$$

or

$$\text{cv} = (e^{\sigma_z^2} - 1)^{\frac{1}{2}}. \qquad (1.3.4.129)$$

Therefore, if $\bar{x}$ and $\widehat{\text{cv}}$ are the sample estimates of the mean and coefficient of variation, $\hat{\sigma}_z^2$ and $\hat{\mu}_z$ are

$$\hat{\sigma}_z^2 = \ln(\widehat{\text{cv}}^2 + 1) \qquad (1.3.4.130)$$

$$\hat{\mu}_z = \ln\bar{x} - \tfrac{1}{2}\hat{\sigma}_z^2. \qquad (1.3.4.131)$$

Estimation by maximum likelihood—two parameter case. The likelihood is

$$L(x_1, x_2, \ldots, x_N/\mu_z, \sigma_z^2) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_z x_i} .\exp\left(-\tfrac{1}{2}\left[\frac{\ln x_i - \mu_z}{\sigma_z}\right]^2\right) \quad (1.3.4.132)$$

and the log likelihood is

$$LL(x_1, x_2, \ldots, x_N/\mu_z, \sigma_z^2) = -\frac{N}{2}\ln(2\pi\sigma_z^2) - \Sigma \ln x_i - \tfrac{1}{2}\Sigma\left(\frac{\ln x_i - \mu_z}{\sigma_z}\right)^2. \quad (1.3.4.133)$$

The maximum likelihood solutions $\hat{\mu}_z$ and $\hat{\sigma}_z^2$ must satisfy

$\partial LL/\partial\hat{\mu}_z = 0, \quad \partial LL/\partial\hat{\sigma}_z^2 = 0.$

These give

$$\frac{1}{\hat{\sigma}_z^2}\Sigma(\ln x_i - \hat{\mu}_z) = 0 \quad \text{or} \quad \hat{\mu}_z = \frac{1}{N}\Sigma\ln x_i \quad (1.3.4.134)$$

and

$$-\frac{N}{2\hat{\sigma}_z^2} + \frac{1}{2(\hat{\sigma}_z^2)^2}\Sigma(\ln x_i - \hat{\mu}_z) = 0, \quad \text{which gives } \hat{\sigma}_z^2 = \frac{\Sigma(\ln x_i - \hat{\mu}_z)^2}{N}. \quad (1.3.4.135)$$

Equation (1.3.4.135) gives a biassed estimate of $\hat{\sigma}_z^2$ but any function of it also has the properties of the maximum likelihood estimate. Hence the quantity

$$\hat{\sigma}_z^2 = \frac{\Sigma(\ln x_i - \hat{\mu}_z)^2}{N-1} \quad (1.3.4.136)$$

which is unbiassed is taken as the maximum likelihood estimator of σ_z^2. Equations (1.3.4.134) and (1.3.4.136) coincide with both the moments and maximum likelihood estimators which would be obtained by fitting a Normal distribution to the sample of logarithms $z_i = \ln x_i$, $i = 1, 2, \ldots, N$.

The three parameter case—estimation by moments. Three relations between moments and parameters are needed. These are

$$\mu = x_0 + \exp(\mu_z + \tfrac{1}{2}\sigma_z^2) \quad (1.3.4.137)$$

$$\sigma = \exp(\mu_z + \tfrac{1}{2}\sigma_z^2).(e^{\sigma_z^2} - 1)^{\frac{1}{2}} \quad (1.3.4.138)$$

and

$$\text{skewness} = g = (e^{\sigma_z^2} - 1)^{\frac{1}{2}}(e^{\sigma_z^2} + 2). \quad (1.3.4.139)$$

The moments μ, σ and g are estimated from the sample $\bar{x}$, $\hat{\mu}$ and $\hat{g}$. First $\hat{\sigma}_z^2$ is obtained from $\hat{g}$ by solving Equation (1.3.4.139). This relation is almost linear (see Equation (1.2.5.9a)), but if this is not considered to be sufficiently accurate the value of $\hat{\sigma}_z^2$ so obtained may be used as starting value in an iterative procedure for solving Equation (1.3.4.139). Given $\hat{\sigma}_z^2$, it is inserted in Equation (1.3.4.138) which is then solved for $\hat{\mu}_z$, σ having been replaced by $\hat{\sigma}$. Finally, Equation (1.3.4.137) is solved for x_0 after μ, μ_z and σ_z^2 have been replaced by $\bar{x}$, $\hat{\mu}_z$ and $\hat{\sigma}_z^2$.

Because $\hat{g}$, the sample estimate of skewness, has a high sampling variance, Sangal & Biswas (1970) proposed a set of estimators which does not involve the sample skewness. On the assumption that σ_z is small the expression

$$x_0 = \zeta - \frac{\sigma^2}{2(\mu - \zeta)} \quad (1.3.4.140)$$

can be shown to hold, where μ, ζ and σ^2 are the mean, median and variance of the x distribution. Sample estimates are inserted for these to give $\hat{x}_0$. Then from Equation (1.3.4.137)

$$\mu - x_0 = \exp(\mu_z + \tfrac{1}{2}\sigma_z^2)$$

so that with Equation (1.3.4.138) this gives

$$\frac{\sigma}{\mu - x_0} = (e\sigma_z^2 - 1)^{\frac{1}{2}}$$

and

$$\sigma_z^2 = \ln\left[1 + \left(\frac{\sigma}{\mu - x_0}\right)^2\right]. \quad (1.3.4.141)$$

The estimate $\hat{\sigma}_z^2$ is obtained by replacing x_0, μ and σ in Equation (1.3.4.141) by $\hat{x}_0$, $\bar{x}$ and $\hat{\sigma}$. Then from Equation (1.3.4.137) using the estimates $\bar{x}$, $\hat{x}_0$ and $\hat{\sigma}_z^2$

$$\hat{\mu}_z = \ln(\bar{x} - \hat{x}_0) - \tfrac{1}{2}\hat{\sigma}_z^2. \quad (1.3.4.142)$$

1.4 Sampling standard errors

1.4.1 Introduction

Methods of estimating parameters of a distribution from a random sample of data were described in Section 1.3. These parameter estimates are then used to obtain quantile estimates as described in Section 1.2. The precision of these estimates is given by their standard errors and the development of formulae for some well known cases is outlined in this section; standard errors are examined again in Section 2.5 where the computation of standard errors for practical purposes is reduced to a very simple operation. The standard error of an estimated quantile can usually be expressed in the form $C\sigma/\sqrt{N}$ where σ is the population standard deviation, N is the sample size and C is a factor depending on the return period and on the form of the distribution. If, as is always the case in practice, an estimated value of σ is used in this expression the expression may differ greatly from its true value. In such cases it may not be worthwhile to split hairs over the precise value of C and this attitude is adopted in Section 2.5. When a quantile is expressed as a linear function of the parameters, Equations (1.1.6.5)–(1.1.6.12) involving variances and covariances of the estimated parameters are used. When estimation is by the methods of moments, the variances of the sample mean and standard deviation and the covariance between these quantities are required. If three parameters have been estimated the sampling variance of the skewness and its covariances with the sample mean and standard deviation are also required. However, because the quantile does not depend linearly on the third moment the treatment is less simple than an application of Equation (1.1.6.6) alone and the standard error of a quantile which depends on three sample moments does not receive widespread attention or use. One such case, the Pearson Type 3, was treated by Kaczmarek (1957) who also considered the Gumbel (EV1) and lognormal cases. The gulf between the two parameter and three parameter distributions in the difficulty of developing and using standard error formulae for quantile estimates is not as apparent in maximum likelihood estimation as in moments estimation.

1.4.2 Standard error of some sample moments

The sample mean $\bar{x} = \Sigma x_i/n$ is an unbiassed, consistent estimator of the population mean μ_1'. The variance of its sampling distribution is

$$\text{var}\ \bar{x} = \frac{\sigma^2}{N} = \frac{\mu_2}{N} \tag{1.4.2.1}$$

regardless of the form of the distribution of x and hence the standard error is

$$\text{se}(\bar{x}) = \frac{\sigma}{\sqrt{N}}. \tag{1.4.2.2}$$

The form of the sampling distribution does of course depend on the form of parent distribution.

The quantity $s^2 = \Sigma(x_i - \bar{x})^2/(N-1)$ is an unbiassed, consistent estimator of the population variance μ_2 or σ^2. The sampling distribution of s^2 depends on the parent distribution of x but its variance is independent of it. This sampling variance is

$$\text{var}(s^2) = \frac{\mu_4 - 3\mu_2{}^2}{N} + \frac{2\mu_2{}^2}{N-1} \tag{1.4.2.3}$$

which, if N is very large so that $N \simeq N-1$, tends to

$$\text{var}(s^2) \simeq \frac{\mu_4 - \mu_2{}^2}{N}. \tag{1.4.2.4}$$

Using the approximate formula

$$\text{var}\ (h(t)) \simeq \left|\frac{\text{d}h}{\text{d}t}\right|^2_{t=E(t)} .\text{var}\ t \tag{1.4.2.5}$$

the variance of s, which estimates the population standard deviation, is

$$\text{var}\ s \simeq \frac{\mu_4 - \mu_2^2}{4N\mu_2}. \tag{1.4.2.6}$$

Note that Equation (1.4.2.6) like both Equations (1.4.2.4) and (1.4.2.5) is correct only for large samples.

The covariance between the mean $\bar{x}$ and the estimated standard deviation s is

$$\text{cov}(\bar{x},s) = \frac{\mu_3}{2N\sigma} \tag{1.4.2.7}$$

which is correct to order $1/N$ only.

The quantity $\hat{g} = \hat{\mu}_3/s^3$ is used to estimate the population skewness and its approximate variance, correct to order $1/N$, is

$$\text{var}\ \hat{g} = \frac{N^3(\mu_6 - \mu_3^2 + 9\mu_2^2 - 6\mu_2\mu_4)}{(N-1)^2(N-2)^2\mu_2^3} + \frac{9\mu_3^2}{4\mu_2^5}\left(\frac{\mu_4 - \mu_2^2}{N} + \frac{2\mu_2^2}{N-1}\right) - \frac{3N^2\mu_3}{(N-1)^2(N-2)\mu_2^4}(\mu_5 - 4\mu_2\mu_3). \tag{1.4.2.8}$$

The Normal distribution. The formulae quoted above are general and do not depend on the form of the distribution. However, they are sometimes confused with the special cases for the Normal distribution and hence to remove the confusion these are listed separately here. The formula for the mean is the same in both cases, Equations (1.4.2.1) and (1.4.2.2). The relation $\mu_4 = 3\mu_2^2$ is introduced into Equation (1.4.2.3) giving

$$\text{var } s^2 = \frac{2\mu_2^2}{N-1} = \frac{2\sigma^4}{N-1} \qquad (1.4.2.9)$$

and using Equation (1.4.2.5)

$$\text{var } s \simeq \frac{\sigma^2}{2(N-1)}. \qquad (1.4.2.10)$$

Since $\mu_3 = 0$ the covariance between $\bar{x}$ and s is zero

$$\text{cov}(\bar{x},s) = 0. \qquad (1.4.2.11)$$

For the skewness the sampling variance is (Fisher, 1930)

$$\text{var } \hat{g} = \frac{6N(N-1)}{(N-2)(N+1)(N+3)}. \qquad (1.4.2.12)$$

For sample sizes of 10, 25 and 50 this gives standard errors of 0.687, 0.464 and 0.337 showing that values of sample skewness much greater than the population value of zero are possible.

1.4.3 Sampling standard errors of quantiles

Quantile values of return period T may be written as

$$Q(T) = A+By(T)$$
$$= A+By \qquad (1.4.3.1)$$

where y is understood to be $y(T)$ for short and where A and B are location and scale parameters of the distribution.

Examples are $Q(T) = \mu+\sigma y$ in the Normal distribution and $Q(T) = u+\alpha y$ in the EV1 distribution, where in both cases y is parameter free and $Q(T) = x_0+\beta y$ in the Pearson Type 3 distribution where y depends on a shape parameter γ. These expressions were encountered in Section 1.2.

Where y does not depend on any parameters the estimate of $Q(T)$ from a sample

$$\widehat{Q(T)} = \hat{A}+\hat{B}y \qquad (1.4.3.2)$$

involves two estimated quantities $\hat{A}$ and $\hat{B}$ and y is a constant, but in the Pearson Type 3 distribution for instance y is also an estimate because it depends on the estimated shape parameter $\hat{\gamma}$; thus in such cases $\widehat{Q(T)}$ must be written

$$\widehat{Q(T)} = \hat{A}+\hat{B}\hat{y}. \qquad (1.4.3.3)$$

The estimated quantities $\hat{A}$ and $\hat{B}$ are sample statistics each with its own sampling distribution and hence the sampling variance of $\widehat{Q(T)}$ in Equation (1.4.3.2) which applies to the Normal and EV1 cases is, following Equation (1.1.6.6),

$$\text{var } \widehat{Q(T)} = \text{var } \hat{A}+2\,\text{cov}(\hat{A},\hat{B}y)+\text{var}(\hat{B}y)$$
$$= \text{var } \hat{A}+2y\,\text{cov}(\hat{A},\hat{B})+y^2\,\text{var } \hat{B}. \qquad (1.4.3.4)$$

Particular cases. Three particular cases will be discussed, the Normal, extreme value Type 1 and general extreme value distributions. The Normal distribution is symmetrical and therefore quantile estimates corresponding to both low and high values of probability experience the same amount of sampling variance in contrast to the other two, applicable to floods, which are asymmetrical and in which quantile estimates in the upper end experience larger sampling variance than those in the lower end.

Normal. In the Normal distribution $A = \mu$ and $B = \sigma$. Both the moments and maximum likelihood estimators take the same form, $\hat{\mu} = \bar{x}$ and $\hat{\sigma} = s$. Then using Equations (1.4.3.4), (1.4.2.1), (1.4.2.10) and (1.4.2.11)

$$\text{var } Q(T) = \frac{\sigma^2}{N} + y^2 \frac{\sigma^2}{2(N-1)}$$

$$= \frac{\sigma^2}{N}\left\{1 + \frac{y^2 N}{2(N-1)}\right\}. \quad (1.4.3.5)$$

Extreme value Type 1. (a) ML estimates. In this distribution $A = u$ and $B = \alpha$. The variance–covariance matrix of the ML estimators is given in Equation (1.3.4.70) which gives var $\hat{u} = 1.11\alpha^2/N$, var $\hat{\alpha} = 0.61\alpha^2/N$ and $\text{cov}(\hat{u},\hat{\alpha}) = 0.26\alpha^2/N$ so that

$$\text{var } \widehat{Q(T)} = 1.11\alpha^2/N + 2(0.26)y\alpha^2/N + y^2 0.61\alpha^2/N$$

$$= \frac{\alpha^2}{N}(1.11 + 0.52y + 0.61y^2). \quad (1.4.3.6)$$

(b) Moments estimates. The population relations $\mu_3 = 1.14\sigma^3$ and $\mu_4 = 5.40\sigma^4$ are used in Equations (1.4.2.7) and (1.4.2.3) to give var s^2 and $\text{cov}(\bar{x},s)$. After some algebraic manipulation (Kaczmarek, 1957; Lowery & Nash, 1970) these lead to

$$\text{var } \widehat{Q(T)} = \frac{\alpha^2}{N}\{1.170 + 0.196y + 1.099y^2\}. \quad (1.4.3.7)$$

If the quantile $Q(T)$ is written in terms of the moments μ and σ

$$Q(T) = \mu + \sigma K \quad (1.4.3.8)$$

where K, Chow's frequency factor, is the EV1 variate with zero mean and unit variance and which is related to y by

$$K = -0.45 + 0.78y \quad (1.4.3.9)$$

the sampling variance is written

$$\text{var } \widehat{Q(T)} = \frac{\sigma^2}{N}\{1 + 1.14K + 1.10K^2\} \quad (1.4.3.10)$$

which is equivalent to Equation (1.4.3.7).

General extreme value. In distributions where $Q(T)$ is given by Equation (1.4.3.3) and $\hat{y}$ is not a constant but depends on an estimated parameter, Equation (1.4.3.4) is incomplete, additional terms being necessary to introduce the components of variance and covariance due to the third parameter. One approximate technique which is widely used is to expand the expression for $Q(T)$ considered as a function of the estimated parameters in a Taylor series about the population parameter values and on squaring to disregard all terms above the second order, and take expectations of all the remaining terms. This gives an expression for the variance which involves the sampling variance of each parameter and all possible sampling covariances between the parameters. This has been done for the general extreme value distribution by Jenkinson (1969), when estimation is by maximum likelihood. The quantile is

$$\widehat{Q(T)} = \hat{u} + \frac{\hat{\alpha}}{\hat{k}}[1-\exp(-\hat{k}y_1)] \quad (1.4.3.11)$$

where y_1 is the EV1 reduced variate of return period T and a constant in this expression. This expression may also be written

$$\widehat{Q(T)} = \hat{u}+\hat{\alpha}W \quad (1.4.3.12)$$

which is of the form of Equation (1.4.3.3) but the Taylor series expansion was carried out on the form in Equation (1.4.3.11). The resulting expression for the ML sampling variance is

$$\text{var}\,\widehat{Q(T)} = \frac{\alpha^2 W^2}{N}\left\{a + \frac{b}{W^2} + \frac{c}{W^2}\left(\frac{\mathrm{d}W}{\mathrm{d}k}\right)^2 + 2\frac{h}{W} + 2\left(\frac{g}{W} + 2\frac{f}{W^2}\right)\frac{\mathrm{d}W}{\mathrm{d}k}\right\}. \quad (1.4.3.13)$$

where the constants a, b, c, f, g, h, depend on k and are tabulated in Table 1.19 and

$$\frac{\mathrm{d}W}{\mathrm{d}k} = (ye^{-ky} - W)/k.$$

This formula is not satisfactory for large positive k values as the expression for var $\widehat{Q(T)}$ gives negative values when the estimated quantile approaches the upper bound of the variate values, despite the fact that the formula is derived in a standard way.

In a theoretical distribution having an upper bound all variate values of return period greater than some arbitrarily chosen value are very close together and it may be that the sampling variance of a quantile is indeed very small. This would suggest that the upper bound itself could be estimated with very small variance. However, if an observed large positive k estimate is found by chance in a sample of flood peaks the small sampling variances given by Equation (1.4.3.13) should not be relied on, as experience with flood peak distributions would suggest that such small sampling variances are spurious.

Other distributions. An expression corresponding to Equation (1.4.3.13) has been developed for the moments estimate of quantiles in another distribution of hydrological interest which has a shape parameter, namely the Pearson Type 3 (Kaczmarek, 1957). Recently, Wallis & Matalas (1973) published the results of some Monte Carlo work on the sampling properties of maximum likelihood estimates of $Q(T)$ from the Pearson Type 3 distribution but no general formula is available for the sampling variances.

In the case of log distributions the usual practice is to calculate the sampling variance or standard error in the log domain and to add some multiple of this quantity to the estimate in the log domain; this is represented in the real domain by the multiplication of the estimate by some factor. This multiplier in the real domain is sometimes referred to as the multiplicative or factorial standard error.

1.5 References

Benson M.A. (1968) Uniform flood-frequency estimating methods for Federal agencies. *Water Resources Research*, **4**, 5, 891–908.

BLOM G. (1958) *Statistical estimates and transformed Beta variables*, pp 68–75 and pp 143–146. John Wiley, New York.

CHOW V.T. (1951) A general formula for hydrologic frequency analysis. *Transactions of American Geophysical Union*, **32** (2), 231–237.

CHOW V.T. (1953) Frequency analysis of hydrologic data with special application to rainfall intensities. *University of Illinois Engineering Experimental Station*, Bulletin series No. 414, 28–29.

CHOW V.T. (1954) The log-probability law and its engineering application. *Proceedings, American Society of Civil Engineers*, **80,** Paper 536, 1–25.

CRAMÉR H. (1946) *Mathematical Methods of Statistics*, pp 424–441. Princeton University Press.

FISHER R.A. (1930) The moments of the distribution for normal samples of measures of departure from normality. *Proceedings of the Royal Society, London*, (*A*), **130,** 16–28.

FISHER R.A. & YATES F. (1963) *Statistical Tables for Biological, Agricultural and Medical Research*, 6th Edn, pp 94–95. Oliver and Boyd, Edinburgh.

FISHER R.A. & TIPPETT L.H.C. (1928) Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Proceedings Cambridge Philosophical Society*, 180–190.

FOSTER H.A. (1924) Theoretical frequency curves and their applications to engineering problems. *Transactions of the American Society of Civil Engineers*, **87,** 142–173.

GRINGORTEN I.I. (1963) A plotting rule for extreme probability paper. *Journal of Geophysical Research*, **68,** 813–814.

GUMBEL E.J. (1937a) *La Durée extrème de la Vie Humaine*. Hermann et Cie, Paris.

GUMBEL E.J. (1937b) Les intervalles extrèmes entre les émissions radioactives. *Journal de Physique* (series VIII) **8,** 321, **11,** 446.

GUMBEL E.J. (1940) Les Crues du Rhône. *Annales de l'Université de Lyon* (series 3), **3,** 39.

HARTER H. L. (1961) Expected values of Normal order statistics, *Biometrika*, **48,** (1) & (2), 151–165.

HARTER H. L. (1969) A new table of percentage points of the Pearson Type 3 distribution. *Technometrics*, **11,** 1, 177–187.

JENKINSON A.F. (1955) The frequency distribution of the annual maximum (or minimum) values of meteorological elements. *Quarterly Journal of the Royal Meteorological Society*, **87,** 158.

JENKINSON A.F. (1969) *Estimation of Maximum Floods*, Chap. 5, pp 183–257. World Meteorological Office Technical Note 98.

KACZMAREK Z. (1957) Efficiency of the estimation of floods with a given return period. *IASH Toronto*, **3,** 145–159.

KENDALL M.G. & STUART A. (1961) *The Advanced Theory of Statistics*, Vol. II, pp 54–56. Griffin and Co., London.

KIMBALL B.F. (1949) An approximation to the sampling variances of an estimated maximum value of given frequency based on the fit of doubly exponential distribution of maximum values. *Annals of Mathematical Statistics*, **20,** 110–113.

KIMBALL B.F. (1960) On a choice of plotting positions on probability paper. *Journal of the American Statistical Association*, **55,** September, 546–560.

LOWERY M.D. & NASH J.E. (1970) A comparison of methods of fitting the double exponential distribution. *Journal of Hydrology*, **10,** 259–275.

MISES R. VON (1936) La distribution de la plus grande de *n* valeurs. *Revue mathématique de l'Union Interbalkanique* (Athens), **7,** 7, 141–160.

PEARSON K. (1934) *Tables of the Incomplete Gamma Function*. Cambridge University Press.

SANGAL B.P. & BISWAS A.K. (1970) The 3-parameter lognormal distribution and its applications in hydrology. *Water Resources Research*, **6**, 2, 505–515.

THOM H.C.S. (1958) A note on the gamma distribution. *Monthly Weather Review*, **86**, 4, 117–122.

UNITED STATES WATER RESOURCES COUNCIL (1967) *A Uniform Technique for Determining Flood Flow Frequencies*. Bulletin No. 15, Water Resources Council, Washington D.C.

WALLIS J.R. & MATALAS N.C. (1973) Eureka! It fits a Pearson Type 3 distribution. *Water Resources Research*, **9**, 2, 281–289.

WILK M.B., GNANADESIKAN R. & HUYETT M.J. (1962) Probability plots for the gamma distribution. *Technometrics*, **4**, 1, 1–20.

2 Statistical flood frequency analysis

2.1 Outline of chapter

2.1.1 Introduction

The statistical tools for the analysis of flood records were presented in Chapter 1. In this chapter these tools are applied to the collected records, especially the long term records, in order to draw empirical conclusions on which to base the methods recommended in the final section, 2.11, of the chapter. These conclusions are described in this introduction.

Three approaches to the analysis of records are presented but the derivation of flood frequencies from the records of a single station requires a long record. Although the choice of distribution for this purpose is discussed in Section 2.4 the recommendation in the usual case with about 5–25 years of record is that the mean annual flood should be estimated from either the annual maximum series or the peaks over a threshold series and that floods of high return periods should be computed from the appropriate region curve which is a general extreme value distribution derived from all the available records in a region.

2.1.2 Summary of chapter contents and results

Section 2.2 Basic elements of flood frequency statistics

In order to estimate the risk that a given flood will be exceeded during the design life of a structure, flood frequency analysis is needed to relate the rarity of a flood to its magnitude. The three main approaches to flood frequency analysis are the time series, the peaks over a threshold and the annual maximum approaches. The first uses the whole flow hydrograph, the second only those peaks which exceed an arbitrary threshold, and the third the highest peak in each year of record. These models are outlined and their relative merits are discussed.

Section 2.3 Elementary properties of annual maximum series

A review of the distributions which have been put forward for the annual maximum series shows that the extreme value distributions alone can be considered to have any theoretical basis, but that the argument is by no means conclusive and moreover the theory does not specify the type. The choice of distribution can therefore only be based on the information contained in observed samples. Average values of coefficient of variation and skewness of annual maximum floods are presented in order to convey an idea of the range of values these quantities display. Distribution free tests for trend and persistence in annual series were carried out on 28 stations with over 30 years of record and it is concluded that the annual maximum records are not entirely free of persistence or trend. The effect of an outlier on the mean annual flood is discussed and a rule is presented for estimating the mean from the median in cases where the maximum flood of record greatly exceeds the mean. A catalogue of the largest recorded floods expressed as a ratio to the mean annual flood is presented.

Section 2.4 Choice of annual maximum distribution by goodness of fit tests

If the correct distribution cannot be chosen from theory, then goodness of fit tests should be useful in choosing distributions for annual maximum series. Seven distributions were tested by calculating goodness of fit indices for a number of long term stations on the hypothesis that each distribution was applicable. Certain tests (χ^2 goodness of fit index and Kolmogorov–Smirnov index) are insufficiently powerful to choose between distributions. Other tests showed that three parameter distributions fitted more closely than two parameter distributions, but the order depended on the choice of index between general extreme value, Pearson Type 3 and log Pearson Type 3. These latter tests are open to criticisms and it is suggested that a general choice of distribution should not be based solely on their results. The Pearson Type 3 and log Pearson Type 3 results are very sensitive to the formulation of the tests and change places in the order of merit when this is changed. The general extreme value distribution is more stable in this respect and for this and other reasons it will be recommended in Section 2.11 as a first choice among distributions of annual maxima.

Section 2.5 Examination of standard error data and practical standard error computations

Theoretical sampling standard errors of quantiles are tabulated as a percentage of quantile value for different distributions, sample sizes and return periods in order to give an idea of the upper limit of the precision available. This is supplemented by an analysis by decades of certain long records. The sampling error of the decade mean is found to be about 15% of the mean which corresponds with the theoretical estimate for the general extreme value distribution. Conclusions about the sampling error of the decade coefficients of variation are more sensitive to outliers; when one station with a high observed value is omitted, the sampling error is found to be about 30% of the mean value of CV.

The sampling standard error of the 25 year flood estimated from each decade is compared with the theoretical sampling error applicable to the extreme value Type 1 distribution but the agreement is not very good. However, because the standard error involves a scale parameter which has to be estimated, the effect of using an approximate standard error formula rather than an exact one is small in comparison to the effect of using an estimated scale parameter instead of the exact value. For this reason a single formula is suggested for all uses.

Section 2.6 Region curves

When a long record is not available to give information about the choice of distribution, knowledge from grouped stations may be used in the form of a region curve, using all records in a region. These records were used to build up the curve by averaging ordinates of individual probability plots in ranges of values of reduced variate. They were extended by splitting the records into groups which were assumed reasonably independent and by plotting the highest floods against the plotting positions corresponding to the number of years of record in the grouped stations. This approach was extended to make use of historical records. A set of region curves is presented in both graphical and numerical forms.

Section 2.7 The peaks over a threshold (POT) model

The peaks over a threshold (POT) model provides an alternative method of estimating floods from the number and magnitude of exceedances of a suitable threshold. Various models have been proposed which differ mainly in the assumptions about the distribution in time of the exceedances, whether this is assumed uniform in time within years or within seasons. These assumptions are less vital than assumptions about the distribution of flood magnitudes, which influence estimates of the flood of high return period. Each of these models is outlined in a standard notation but in this study only the simplest model is used. This assumes a Poisson distribution of numbers of exceedances within a year, and an exponential distribution of the magnitudes of these exceedances. These combine to give a double exponential or EV1 distribution of annual maxima. The estimates of mean annual flood based on this model correspond closely to those based on the annual maximum series. A theoretical comparison of quantile sampling errors between annual maximum and POT models is presented; at the one peak per year level the POT method is more precise only at low return periods while it is more precise at high return periods only if there are at least 1.7 peaks per year included. The assumptions made in the models are examined and an example is presented.

Section 2.8 The treatment of missing peaks or historic floods as censored samples

A method of including historical records or missing peaks in the estimation of parameters of annual maximum and peaks over a threshold distributions is discussed. Historical recorded floods which are known to exceed all intermediate unrecorded ones can be incorporated in the likelihood functions and offer an increase in precision of quantile estimates. Similarly if a series of peaks is truncated by a recording chart limit the number of peaks truncated can supplement the recorded peaks to estimate the *T* year flood. Both these applications use the statistical theory developed for censored samples.

Section 2.9 The time series approach illustrated by the shot noise model

An example of a time series approach to flood frequency analysis is given by the shot noise model, which is described in a number of forms. The parameters of the model can be estimated from the moments of the recorded peak flows. An example of the use of this model is given.

Section 2.10 Choice of design return period

The relationship between design life, risk of failure and design return period is explained and tabulated. The implications of using the largest flood on record as a design flood is discussed. The ratio of the ICE normal maximum flood to the mean annual flood is computed for a number of small upland catchments, and where possible its return period is derived from a region curve. The return period varies very considerably; in many cases it exceeds 1000 years but in some it is less than 50.

Section 2.11 Recommendations and examples

As a result of the research described in the earlier sections recommendations are put forward for the estimation of a flood of given return period and the calculation of the associated standard error. A set of rules is described which depends on the length of record available at the site and the return period for which the estimate is required. However, it is recommended that where possible each problem be examined critically in its own right. The standard error calculations show that the value of extra years of record is greatest when the record is either very short or non-existent; this emphasises the desirability of installing a gauging station as early as possible in any project.

2.2 Basic elements of flood frequency statistics

2.2.1 Introduction

The basic flood frequency requirement is the probability with which a stated flow Q is exceeded during a stated design life. Section 2.2.2 presents a view of how such probabilities fit into the framework of flood frequency, while 2.2.3 explains the probability models which are used to represent the flood aspects of the parent stochastic process and which can be estimated from recorded flood data by statistical methods. In Section 2.2.4 the relative merits of the three models are compared while in 2.2.5 the status of the statistical method is briefly discussed in relation to other methods of specifying a design flood.

2.2.2 Flood magnitude, probability and return period

The streamflow hydrograph contains a random component which implies that the flow hydrograph consists not of a seasonal component alone but contains another component which can only be described in a statistical sense. This stochastic component may also depend on the time of year but it certainly involves a random variable having a distribution such as any one of those described in Section 1.2.

The probabilistic structure of the hydrograph is not simple because there is statistical dependence not only between flow values which occur within a few hours or days of one another but also between values which are separated by months or even years. This property, referred to as memory, is relevant in the time series models but is ignored in the annual maximum and peaks over a threshold model.

Probability in the flood context. The relative frequency probability of any kind of event, denoted A, requires the existence of another event C, the conditioning event, with which A may or may not occur. In flood hydrology the event A may be a flood peak greater than a certain magnitude and C an interval of time. An explicit conditioning event is necessary in the peaks over a threshold model (Section 2.7).

Design life. The time interval of direct interest to the decision maker may be the time to a definite economic horizon, called a design life, L, or it may be of indefinite length. As in many aspects of engineering, the concepts of analysis and design may be separated so that two types of question may be asked:

a What is the probability of a flow Q being exceeded during the design life, L?

b What is the flow Q which has a stated probability of being exceeded during the design life, L?

In these questions, the event that Q is exceeded (event A above) during the design life (conditioning event C above) includes the occurrence of one, two or a greater number of exceedances. The opposite or complementary event is that Q is not exceeded during the design life.

Risk of failure. The exceedance of some value Q by a flood peak is regarded as a failure with respect to that flow value. In this report the probability that at least one failure occurs with respect to some flow value during the design life is called the *risk* of failure, r. It has also been referred to as encounter probability by Borgman (1963).

Risk during design life related to risk during one year. Since L may differ from one application to another it is convenient that r can be expressed simply in terms of the probability, p, that the flow is exceeded during an interval of one year. The probability that Q is not exceeded during L years is $(1-p)^L$. The complement of this event is a failure consisting of one or more exceedances in L years. Therefore

$$\text{risk} = r = 1-(1-p)^L. \qquad (2.2.2.1)$$

Because of this relation it is sufficient to know or find p, the risk of failure during an interval of one year, and this is exploited in all methods of flood frequency analysis.

Number of exceedances during design life. Because of the stochastic component in the flow hydrograph the observation of several L year intervals at the same site would show that the number of peaks exceeding a given flow would vary from interval to interval. The actual number, m, would be a binomially distributed random variable

$$PR(m \text{ peak exceedances}) = \binom{L}{m} p^m (1-p)^{L-m}. \qquad (2.2.2.2)$$

The mean number of exceedances $E(m) = p.L$.

Return period. The time elapsing between successive peak flows exceeding Q is a random variable whose mean value is called the return period T of Q. If T is in years and the probability of Q being exceeded in a year is p then the relation

$$T = \frac{1}{p} \qquad (2.2.2.3)$$

may be assumed provided $T > 10$ (see Section 2.2.4).

The return period in itself is often regarded as a measure of risk. This almost implies that the risk r and the return period T are either synonymous or related in a very simple manner, but as shown above this is not true. The simplest relation arises when L is very small compared with T and then $r \simeq L/T$.

Flood magnitude—return period relationship. The rarity of a flood peak may be conveyed in a number of ways, each expressing the probability of its exceedance or nonexceedance during a time interval, or alternatively each flow value Q may be considered as a function of its associated value of return period T. This unknown relation will be referred to as the POPULATION Q–T RELATION, and all flood frequency research and study is devoted to finding satisfactory approximations to it. It is variously asserted to be one of two forms,

$a \quad Q(T) = a + by$ (2.2.2.4)

$b \quad Q(T) = A + Be^{cy}$ (2.2.2.5)

where in both cases $y \simeq \ln T$ for $T > 10$. Since $e^{c \ln T} = T^c$ these may be better compared when written

$a \quad Q(T) = a + b \ln T$

and

$b \quad Q(T) = A + BT^c$ respectively.

The relation a is equivalent to the statement that the largest value in a year is a Gumbel (EV1) variate while b implies that it is a general extrême value (GEV) variate. In Section 2.6 a value of $c = 0.2$ is found for the averaged distribution for the whole of Britain (c equals $-k$ in the GEV distribution). Approximate relative values for a and b would be $a/b = 1.75$ while $A/B = -1/3$ would be fairly realistic. These relations are sketched in Figure 2.1 although it must be stressed that these are for illustration only.

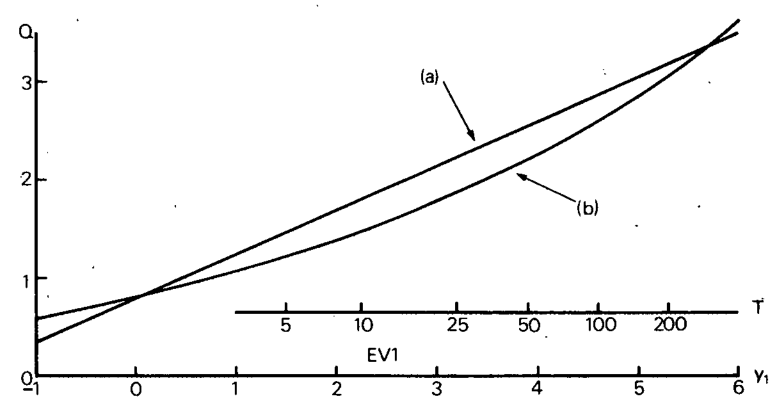


Fig 2.1 Two possible forms of flood magnitude—return period relationship.

2.2.3 *Flood frequency models*

Three types of statistical models are available for estimating the Q–T relation. These are *a* the time series (TS) model,

b the peaks over a threshold (POT) model also known as the partial duration series model,

c the annual maximum (ANNMAX) model.

The time series model approximates all stages of the natural process, both high and low flows, whereas the peaks over a threshold and annual maximum models only represent the flood peak aspects of the parent process.

The time series model. In this model the flow hydrograph is considered to be represented by a series of ordinates at equally spaced intervals of time, called a time series. Ideally if the hydrograph is a stochastic process in continuous time the properties of the time series can be deduced from those of the parent process. The word stochastic is used where the time order of the series is important because of dependence, the word statistical being used otherwise. Time intervals of days, months and years are commonly used in hydrological time series but in flood frequency work only days are of use. The time series of mean daily flows closely represents instantaneous peak flows on large catchments but on small flashy catchments this would not necessarily be so as flood peaks could be smoothed out by daily averaging.

If $Q(t)$ is flow on day t a time series model may be written as the sum of three components, trend, seasonal and stochastic:

$$\text{Flow on day } t = \begin{matrix}\text{Trend}\\ \text{effect}\end{matrix} + \begin{matrix}\text{Seasonal}\\ \text{effect}\end{matrix} + \begin{matrix}\text{Stochastic}\\ \text{effect.}\end{matrix} \qquad (2.2.3.1)$$

This is the form most often used when the parent process is unknown and when both the best model and its parameters have to be inferred from the observed time series.

In such a situation this type of model allows both estimation of parameters and model formation to proceed together through the three components, beginning with trend and finishing with the stochastic effect. Thus, a trend component of a particular form, for instance a linear function of time, could be assumed, its parameters estimated from the sample and then the estimated trend tested to see whether it differs significantly from what would occur by chance from a trend-free model. If found to be statistically significant the trend is subtracted from the observed $Q(t)$ series and the resulting series $Q(t)'$ is then examined for seasonal effects. These seasonal effects may be expressed as the sum of harmonics estimated in pairs, each pair being tested and judged statistically significant before the next harmonic pair is estimated. The total seasonal effect is then subtracted from $Q(t)'$ to give the stochastic component, which is then usually modelled by a Markov process in which the random innovation at each time interval has one of the distributions described in Section 1.2.

Examples of the use of such a model for mean daily flows have been given by Quimpo (1967) on United States data and by Hall & O'Connell (1972) on British data. The latter contains a lucid description of the separate components of the time series; in that application the hypothesis of trend was rejected, six harmonics (13 parameters) were required to describe the seasonal component, while a first order Markov model with a lognormal random element was found to describe the stochastic component.

This step by step method of estimation and model formulation is not ideal but it is relatively economical in effort compared to the estimation of all the parameters in one step.

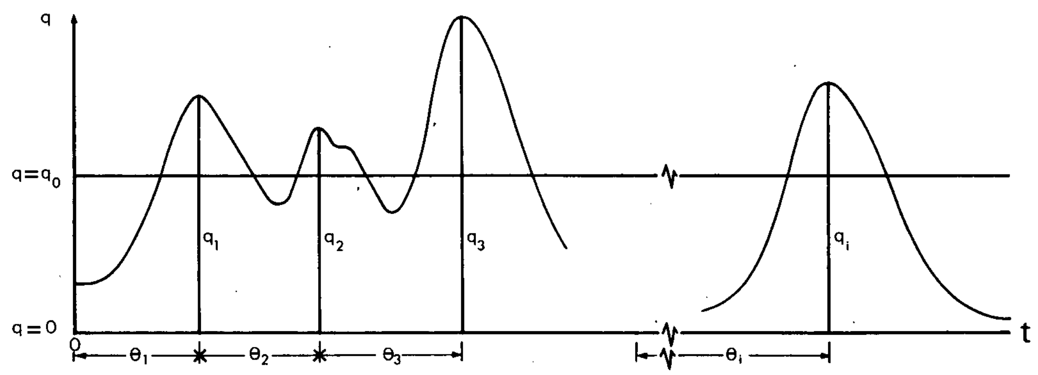
Having fitted the model it should in theory be possible to express the flood of return period T, $Q(T)$, in terms of the model parameters but in practice the statistical mathematics are intractable. Therefore, a long sequence of flows is generated by a Monte Carlo method and the probability distribution of peak flows is obtained from this. This aspect of the model might be considered inelegant in comparison to the other models in which $Q(T)$ may be expressed as explicit and simple functions of their parameters; however in certain circumstances, especially in large catchments, generated flow sequences which may be routed through different hydraulic works may be more useful than flood peaks or a series of isolated hydrographs alone.

This type of time series model was not used extensively in this study because it is considered suitable for specific studies only and the large number of parameters involved could not possibly be realistically related to catchment characteristics. Another time series model, the shot noise model, is discussed in Section 2.9 and some results of its performance as a flood frequency tool are given.

The peaks over a threshold model. In the simplest stochastic processes an expression for $Q(T)$ in terms of the parameters of the parent process can be obtained which involves the rate of occurrence λ of peaks exceeding an arbitrary variate value, q_0, called the threshold, and the conditional distribution of peak magnitudes exceeding q_0, $F(q/q \geqslant q_0)$. The parameters λ and those of $F(q/q \geqslant q_0)$ are functions of the parent process parameters, individual knowledge of which is not needed if $Q(T)$ is required only for large T. Only a portion of the streamflow process is used to derive $Q(T)$, the peak values of the high flows alone being examined and low flows being omitted altogether.

Figure 2.2 shows a sketch of the process which is used instead of the parent process. The time t is measured from t_0 and the occurrences of peaks exceeding the threshold q_0 are marked on the time axis by crosses, their magnitudes being shown by the ordinates $q_1, q_2 \ldots q_i \ldots$. The crosses describe a point stochastic process which is most naturally specified by the joint distribution of the interevent times, $\theta_1, \theta_2 \ldots \theta_i \ldots$. The simplest, known as a renewal process, is that in which the interevent times are identically, independently distributed. Information on the process is also given by the distribution of the number of events in a time interval of stated length, but this is as informative as the distribution of interevent times only in the renewal process.

Fig 2.2 Sketch showing components of the flood hydrograph used in the POT model—interevent time, peak magnitude and threshold level.



Whether the occurrence of flood peaks resembles a renewal process or not has been found to depend on the threshold q_0. If q_0 is so high as to allow only one peak per year on average the interevent times are almost identically independently distributed, but if there are as many as three or four peaks per year there are considerably more peaks in winter than in summer or in other words there is a definite seasonal effect on the interevent time distribution.

The distribution of peak magnitudes $q_1, q_2 \ldots q_i \ldots$ should first be considered as a joint distribution but this requirement can be relaxed as these are found in practice to be independent. This is not always accepted and is discussed later (Section 2.7).

The simplest model consists of an exponential distribution for peak magnitudes

$$F(q/q>q_0) = 1-e^{-(q-q_0)/\beta} \tag{2.2.3.2}$$

and a Poisson distribution for the number of events per year m with mean λ

$$PR(M = m) = e^{-\lambda}\lambda^m/m! \tag{2.2.3.3}$$

which is equivalent to stating that the interevent time θ is exponentially distributed with parameter $1/\lambda$

$$F(\theta) = 1-e^{-\lambda\theta}. \tag{2.2.3.4}$$

As a first approximation, if the number of events per year is taken as nonrandom at a rate of λ per year the effect of neglecting the randomness on the expression for $Q(T)$ is small. This will be discussed in Section 2.7 but meanwhile it can be assumed that if q' has return period T' in sampling units of peaks then q' has return period $T = T'/\lambda$ years.

Models have recently been postulated which take into account the effect of season on the rate of occurrence of peaks such as a time varying Poisson process. This latter implies that the number of events in each season is a Poisson variable whose parameter varies between seasons. However, it is obvious that for floods of high return periods the between season differences are unimportant in comparison to specifying the mean number of peaks correctly.

Historically this model has been called the *partial duration series* method, by analogy with the partial duration curve which is a truncated flow duration curve. When the series contains on average one peak per year (N peaks in N years) the series is traditionally called the *annual exceedance series*. The POT model is discussed in detail in Section 2.7.

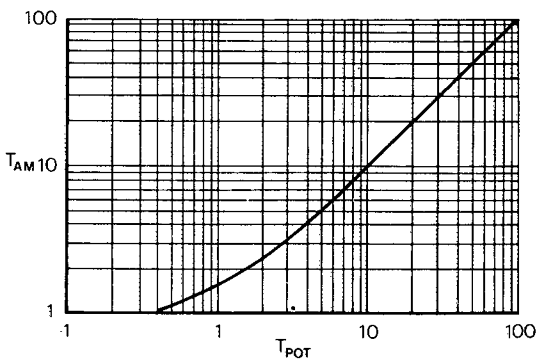
The annual maximum model. This is a special case of a time series approach where the unit of time is one year and the flow representing that time is the highest flow during the year. In practice this time series is statistical rather than stochastic since there is no dependence between successive items which may be considered to be identically, independently distributed.

However, the annual maximum approach did not gain its popularity because it is a special case of a time series, but rather because of the application of the theory of extreme values by Gumbel (1941–1945). If this theory holds, the largest flow in a year has a special statistical property, namely that its distribution must be one of three types. Although the theory is not strictly applicable to annual maxima, Gumbel's active interest in applying the theory to hydrology was so great that by the 1950s the annual maximum method was regarded as supreme among statistical methods of arriving at flood estimates.

All that is required in this model is to specify which of the distributions in Section 1.2 describes the distribution of annual maxima. As an example, Gumbel found empirical support for the hypothesis that annual maxima are distributed according to the extreme value Type 1 distribution and in hydrology this is known as the Gumbel distribution. As a result of recent American work the log Pearson Type 3 has been accepted for the distribution of annual maxima by Federal Agencies there. The case for these distributions in the light of British data is examined in later sections of this chapter.

Relations between TS, POT *and* ANNMAX. A distinction can be drawn between the value of Q for given T in the parent process and the value of the expression for $Q(T)$ evaluted in each of the three models which approximate it in various ways. This distinction may also be viewed by what return period is attributed to it in (a) a time series model, (b) a peaks over a threshold model and (c) an annual maximum model. Each of these values, T_{TS}, T_{POT} and T_{AM}, could be deduced from knowledge of the parent stochastic process and the question is how closely they approximate to T. It is obvious that there is some degree of truncation involved in T_{AM} because each year is condensed into a single time unit. On the other hand in a peaks over a threshold model time is measured on a continuous scale and a value of T can be interpreted in the same way as in the parent process. In the time series model time is also measured in discrete steps but because these are small their use has no practical effect on the interpretation of return period.

For practical purposes it can be assumed that T_{POT} and T_{TS} are equal to T. Langbein (1949) showed that when T is small T_{AM} differs appreciably from T_{POT} and hence from T but differs by only one half year at large values of T. The difference is illustrated in Figure 2.3 where the abscissa is $T_{POT} = T$ and the ordinate is the return period T_{AM} in the annual maximum model of the same variate value. This means that in practical situations there is no need to distinguish between the values $Q(T)$, $Q(T)_{TS}$, $Q(T)_{POT}$ and $Q(T)_{AM}$ when T is large.



T_{POT}	T_{AM}
0·50	1·16
1·00	1·58
1·45	2·00
2·00	2·54
5·00	5·52
10	10·5
50	50·5
100	100·5

Fig 2.3 Relation between return period of a given magnitude in annual maximum and POT models (after Langbein, 1949).

2.2.4 *Relative merits of different models*

In practice, not only the parameter values but even the form of the distribution in these models cannot be stated with certainty and in such conditions each type of model may not be equally effective. At best, a long record of the flow hydrograph is available on the basis of which the form of each type of model is to be inferred and its parameters estimated. The value of $Q(T)$ for some return period T calculated in any of the three models is but an estimate, $\widehat{Q(T)}$, in these circumstances. The choice of model depends on how well $\widehat{Q(T)}$ approximates to $Q(T)$. The mean square deviation of $\widehat{Q(T)}$ from $Q(T)$ depends on sampling effects and the choice of distribution.

Statements about the relative merits of the three types of model have never been totally satisfactory or exhaustive. The statements which have been made can be divided into categories and it is hoped that the comments and facts accompanying these categories below will provide a useful starting point for future discussion and research.

1 Statements which compare different statistical methods of estimating $Q(T)$ in samples from the same population. These are not intermodel comparisons but the properties derived might be used in such exercises.

2 Statements which object to a model by expressing hydrological unease at what might be termed obvious drawbacks in it relative to an alternative model. The two typical statements in this category are (a) 'The annual maximum model omits many high floods and replaces them with smaller ones. In certain cases even the second largest flood on record may be omitted but this is obviously undesirable since it is in the region of large floods and not small ones that estimates of quantiles are required. However, the POT model does not suffer from this drawback'. (b) 'In the POT model the individual floods are considered to be statistically independent. However, since some flood peaks ride on the recession limb of an earlier flood the members of such pairs are obviously not independent and hence the use of the model is vitiated.'

The substance of remarks (a) and (b) are often stated baldly without further comment; it is implied that the POT model would be preferable were it not for the problem of dependence and that the annual maximum model, notwithstanding the weakness mentioned above, is at least valid and should therefore be used. The statement (a) should be viewed in the light of Section 2.7 where it is shown that under one set of assumptions the annual maximum estimate of $Q(T)$ can be more efficient than the POT estimate. With respect to (b) it should be remembered that statistical dependence should be sought in the point stochastic process of peak occurrence rather than between peak magnitudes since plots appearing later in this chapter in Figure 2.20 show that the latter is either nonexistent or else so weak as to be undetectable.

3 Statements which express the loss incurred by assuming the incorrect distribution given that the model form is known. Assume for instance that an annual maximum model is being considered and in the parlance of Matalas & Wallis (1972) suppose the real world distribution of annual maxima is lognormal. The loss incurred in a decision which is based on an estimate of $Q(T)$ got by fitting, say, a Pearson Type 3 distribution to a sample of real world data is studied. This approach is recent and is obviously a within model study but it is the only approach made by hydrologists to tackle the joint problem of both sampling and choice of distribution errors.

4 Statements comparing time series models on the one hand with POT and annual maximum models on the other. The big difference between the two classes is that one uses a very large amount of data including data on low flows while the other models use a summarised version of the data namely all peaks over a threshold or all annual maxima. To date no satisfactory work either theoretical or empirical has been published on this comparison. One advantage which has been put forward in favour of the time series approach is that it can reproduce the very large floods that occur occasionally in natural series, through the joint occurrence of conditions which are not themselves remarkable but are together capable of producing a large flood. However, an annual maximum model can also reproduce unusually large flows and indeed it is remarkable how many very large flows do occur in a sequence generated from a skewed distribution. Because of this the above argument in favour of the time series approach can be considered one sided; in any case it is not the type of argument which allows direct comparison of the two apprcaches.

The comparison of the TS estimates of $Q(T)$ *viz.* $Q(T)_{TS}$ with $Q(T)_{POT}$ or $Q(T)_{AM}$ may indeed be easier than has been thought. In an independent Gaussian process Rice (1954) gave the theoretical expressions for the occurrence of exceedances of a threshold in terms of the parent time series parameters. These expressions have parameters which are functions of the parameters of the original time series model but moreover the expressions are of the same form as if a POT model had been employed originally. The comparison in this, albeit simplified, case could therefore be made between the precision of the parameter and $Q(T)$ estimates obtained (a) directly from a POT model and (b) indirectly by first estimating the parameters of a TS model and deriving from them the POT parameters and $Q(T)$.

Expressions similar to those obtained by Rice (1954) have been suggested for hydrological use by Mejia (1971) and counterparts of them have also been used in work on the distribution of maximum wave heights by Cartwright & Longuet-Higgins (1956) and on the distribution of maximum wind gust speeds by Davenport (1964). The hydrological problem is not, of course, as simple as that arising out of an independent Gaussian process but the principle of this example is worth noting.

2.2.5 Status of statistical method

In hydrology the words 'flood estimation' mean in the broadest sense the specification of a design flood regardless of the criterion used to define what is required or the method used to derive the required estimate from the relevant data. The methods described in this chapter are used when the criterion is a peak magnitude of stated return period and the method to be used is to be one based on the statistical analysis of observed records of peak flows. This is not the only approach used to estimate a flood peak of stated return period; another is to use information on the probabilities of rainfall amount, duration and profile in time in conjunction with a unit hydrograph as described later in Chapter 6. It should be stressed that the unit hydrograph is not a flood frequency tool in itself; its function is to transform specified rainstorms into runoff hydrographs.

2.3 Elementary properties of annual maximum series

2.3.1 Introduction

Basic information about the annual maximum series is given in this section. Theories of annual maximum distributions are reviewed and then averaged values of coefficient of variation and skewness computed from recorded series are presented so as to give a general impression of annual maximum characteristics. Some assumptions such as lack of trend and serial persistence are also examined. The estimation of the mean in a record with an outlier is discussed and a catalogue of the largest recorded floods is presented.

2.3.2 The annual maximum model

Theories of annual maximum frequency distributions

Theories of flood distributions, like other scientific theories, have been formed, tested empirically and replaced if unsatisfactory. In the early part of this century the application of probability theory meant to engineers and hydrologists the theory of errors and the Normal or Gaussian distribution. However, examination of hydrological data, daily flows and annual flood peaks, showed that their distribution is positively skewed and hence that the well known Normal theory of errors is inapplicable. The next step was to assume that the logarithms of the data are Normally distributed. In a sense this was a deductive step since it is known that a logarithmic transformation reduces skewness. There was empirical support for the lognormal distribution so that this could have been accepted as a working hypothesis.

Foster (1924) preferred to deal with untransformed data and hence sought skewed distributions. Pearson's system provided such distributions and he proposed their use in hydrology. The Pearson Type 3 distribution in particular found empirical support from the data, although many hydrologists considered its application too difficult.

The deductive steps described so far were small in comparison with Gumbel's extreme value theory. The statistical theory was developed in the 1920s (Fisher & Tippett, 1928) and during the 1930s was widely promulgated and applied by Gumbel (1935, 1937a, 1937b). His first hydrological applications were to the annual maximum values of mean daily flow. He tested the theory by fitting the Type 1 distribution to long river flow records from many countries and concluded that the evidence was sufficient to support the applicability of the extreme value theory (Gumbel 1941b, 1942, 1945).

Extreme value theory implies that if the random variable Z is the maximum in a sample of size N from some population of x values, then, provided N is sufficiently large, the distribution of Z is one of three limiting types, the choice depending on the distribution of x. Gumbel (1941a) postulated that because the maximum daily flow in a year is the maximum of $N = 365$ values, it should be distributed as an extreme value variate; however the $N = 365$ values are not statistically independent nor are they identically distributed. If therefore the annual maxima have an extreme value distribution it is for other than those reasons stated by Gumbel. Nevertheless, there may be say $M(<N)$ local maxima within the year

which are independent. If this M is large in the context of extreme value theory, then the condition that the annual maxima are the maxima of large samples is satisfied; but even if M is large in that context these M values, although independent, are not identically distributed as can be seen in Quimpo (1967) where the mean and variance of the daily flow are shown to vary with season. Therefore, the theoretical argument for an extreme value distribution of floods is insecure. An even more important problem is the choice of type of extreme value distribution. According to theory this depends on the type of the initial distribution (of the M values mentioned above), but this is not known *a priori*.

In summary therefore the reasoning for the applicability of the extreme value theory is not convincing and does not answer the important question as to which type of extreme value distribution obtains. The empirical evidence presented by Gumbel for the Type 1 distribution would support other distributions also. On the basis of empirical evidence available it is difficult to choose between types and the theory is no help where the need is greatest.

Correct inference about variate values of large return period is more important than correct inference about values of low return period. For instance if the true distribution were Type 1 it would be more serious to use a Type 2 incorrectly than a gamma distribution because for large T the gamma and Type 1 variates both increase with $\ln T$ whereas the Type 2 variate increases with T raised to a power.

The lognormal distribution

Chow (1954) stated that if the annual maximum flood could be considered to be the product of a large number of random effects then it would be lognormally distributed, because the logarithm of the variate could be considered to be the sum of a large number of random effects and would therefore be Normally distributed by the central limit theorem. However, to be valid as a deductive theory these effects would have to be identifiable. Failing this the distribution can only be supported by empirical data.

Other distributions

The Pearson Type 3 distribution was suggested in 1924 by Foster, mainly because, unlike the Normal distribution, it displays skewness. A special case of the Pearson Type 3 distribution, the gamma distribution, was recommended by Moran (1957) on the grounds that it is sufficiently flexible to fit any observed series. Another distribution, the log Pearson Type 3, has been suggested (and also recently recommended for universal usage in the United States) because of its flexibility (U.S. Water Resources Council, 1967). Another distribution, consisting of a mixture of two distributions, was suggested for annual maxima by Singh & Sinclair (1972). If the annual maxima could be classified in some objective manner into two groups, between which there is a noticeable difference in the distribution of variate values, then the concept of a mixture of distributions could be validly and usefully employed. For instance, this is possible with annual maximum wind speed in the North Atlantic where the winds may be classified as being due to either extra-tropical cyclones or tropical cyclones (Thom, 1973). The classification is independent of the magnitude of the wind speeds and hence each item in the record may be attributed to its correct component distribution and the proportion in each may be estimated. In the context of floods the annual maxima might be classified according to whether they arise from thunderstorm rainfall or from other types of precipitation, or alternatively into floods which are influenced by snowmelt and those which are not. In either of these cases a rule for classifying the causative meteorological condition would be necessary and the derivation of such a rule might itself be controversial. This has not yet been done in hydrology.

It could be argued that Singh & Sinclair's (1972) application was simply a device for introducing a five parameter distribution where previously two and three parameter distributions had been used. A mixture of two Normal distributions was proposed:

$$F(q) = p\Phi_1(q)+(1-p)\Phi_2(q)$$

where $\Phi(.)$ is the Normal probability integral. They did not classify the items in the record into two categories and estimate the parameters of each component distribution and the mixture parameter p separately.

In the United Kingdom, the 1947 floods which were affected by snowmelt often prompt the suggestion that snowmelt floods are vastly different from other floods. This is of course a plea for considering the distribution of annual maxima as a mixture of two distributions. However, one snowmelt flood alone is not proof of the necessity because comparison of one such flood, the extreme case, with other floods as a whole does not prove that the distribution of all snowmelt floods is noticeably different from the distribution of other types of floods.

Conclusions

Only the extreme value distributions can be considered as candidates for the annual maximum distribution on the basis of deduction alone. However the deduction as generally understood is based on facts which differ from those required by theory. Furthermore, there is no theoretical way of deducing which type of extreme value variate is appropriate and this is a major shortcoming. Therefore, the distribution to be used must be dictated by the sample distribution of many sets of data considered in some suitable way either singly or jointly.

2.3.3 *Mean, coefficient of variation and skewness of annual maximum distributions*

In this subsection average values of coefficient of variation CV and skewness g are derived. These dimensionless moments give an indication of the kind of distribution involved without singling out any particular algebraic form. Among the distributions described earlier the several parameters do not have a common meaning, as for example in one the location parameter is a lower bound, in another it is the mean and in yet another it is the mode. Similar remarks apply to the scale and shape parameters but the dimensionless moments such as CV and g have the same meaning in all distributions.

The mean of course depends on catchment characteristics and its relation to them is derived in Chapter 4. A typical form of relation is

$$\bar{Q} = C\,\text{AREA}^{0.94}\text{STMFRQ}^{0.27}\text{S1085}^{0.16}\text{SOIL}^{1.23}\text{RSMD}^{1.03}(1+\text{LAKE})^{-0.85} \quad (2.3.3.1)$$

where $\bar{Q}$ = mean annual flood and C is a constant depending on geographical area which ranges from 0.015 to 0.032, while the other quantities are catchment area and measures of channel slope, soil type, average net rainfall and lake fraction which are all defined in Chapter 4.

In Chapter 4, also, attempts are made to relate the coefficient of variation CV to catchment characteristics, and it is shown that the detection of any relation between them is swamped by the sampling variation inherent in the values of CV estimated from short records. The means of CV and g are taken over regions which were drawn up for the preparation of flood frequency curves to be described in Section 2.6. The hydrometric areas defining 10 regions are given in Table 2.1 and the regions can be seen on Figure 2.4.

Table 2.1 Regions of Great Britain defined by constituent hydrometric areas.

Region	Hydrometric area numbers
1	1–16 and 88–97 and 104–108
2	17–21 and 77–87
3	22–27
4	28 and 54
5	29–35
6	36–39
7	40–44 and 101
8	45–53
9	55–67 and 102
10	68–76

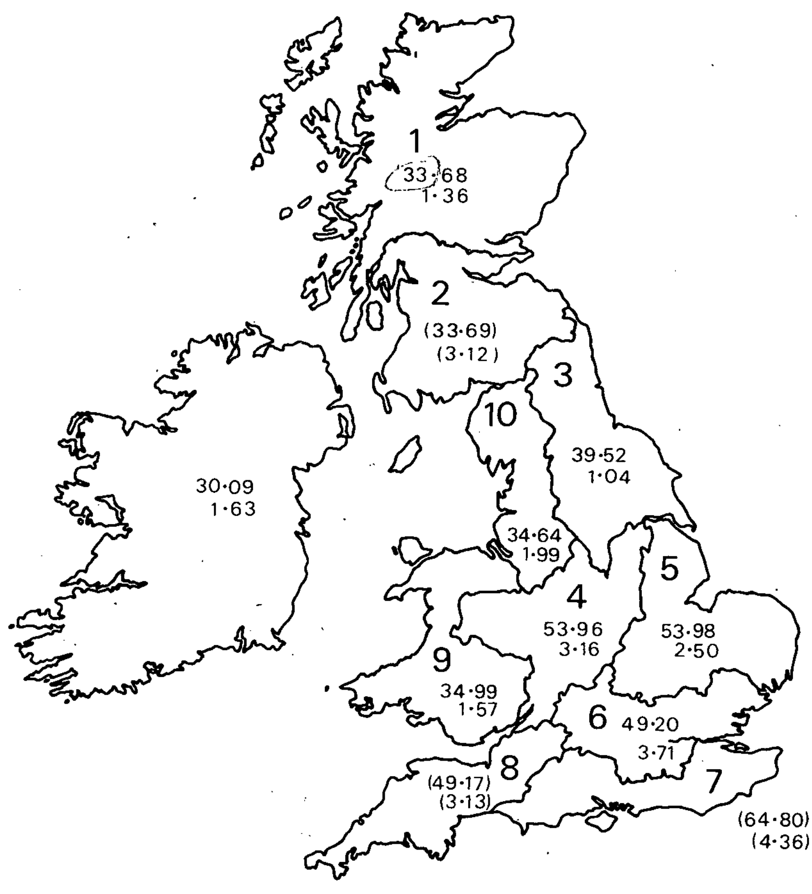


Fig. 2.4 Pooled coefficient of variation and average skewness with weighted correction for each region: taken from Table 2.3 except figures in brackets which are from Table 2.2.

The coefficient of variation

In any region let there be m stations and n_j years of record at the jth station. Let Q_{ij}, $i = 1, 2, \ldots n_j$ be the annual maxima at station j and let $\bar{Q}_{\cdot j}$ be their mean. The coefficient of variation is

$$\text{CV}_j = \left\{ \frac{\sum_{i=1}^{n_j} (Q_{ij} - \bar{Q}_{\cdot j})^2}{n_j - 1} \right\}^{\frac{1}{2}} \Big/ \bar{Q}_{\cdot j} \qquad (2.3.3.2)$$

This can also be considered as the standard deviation of

$$x_{ij} = Q_{ij}/\bar{Q}_{\cdot j} \qquad (2.3.3.3)$$

A mean value of the coefficient of variation of Q is equivalent to a mean value of the standard deviation of x. If the square of this is denoted $\text{var}(x)_j$ a pooled mean value is given by

$$\text{var}\, x = \frac{\text{within records sum of squares}}{\text{total within records degrees of freedom}}$$

$$= \frac{\sum_{j=1}^{m} (n_j - 1)\, \text{var}(x)_j}{\sum_{j=1}^{m} (n_j - 1)}.$$

Since $\text{var}(x)_j = \text{CV}_j^2$ this gives the pooled estimate of CV^2 so that

$$\text{Pooled CV} = \left\{ \frac{\sum_{j=1}^{m} (n_j - 1)\text{CV}_j^2}{\sum_{j=1}^{m} (n_j - 1)} \right\}^{\frac{1}{2}}. \qquad (2.3.3.4)$$

Other average values of CV are the ordinary arithmetic average

$$\overline{\text{CV}} = \frac{1}{m} \sum_{j=1}^{m} \text{CV}_j \qquad (2.3.3.5)$$

and a weighted average, the weighting factor being the degrees of freedom in each record,

$$\text{Weighted } \overline{\text{CV}} = \sum_{j=1}^{m} (n_j - 1)\text{CV}_j \Big/ \sum_{j=1}^{m} (n_j - 1). \qquad (2.3.3.6)$$

These three values of averaged CV are given in Table 2.2 and in Table 2.3 for records of 15 years and over. In the latter table, regions having fewer than 10 records are omitted. The difference between $\overline{\text{CV}}$ and pooled CV is largest in Region 2 (south Scotland) where it is one fifth of the pooled CV value. The pooled CV value is to be preferred. The higher values of the order of 50 are in the south and south east of the country, the lower values of the order 35 covering Wales, the north of England and Scotland. These values based on records of 15 years and more are shown in Figure 2.4.

The skewness. The basic quantity is $(Q_{ij} - \bar{Q}_{\cdot j})^3$ which contributes to the third central moment. The sum $\sum_{j=1}^{n_j} (Q_{ij} - \bar{Q}_{\cdot j})^3$ is associated with $n_j - 1$ degrees of freedom. An estimate of the third central moment μ_3 is

Region	No. of stations	Coefficient of variation %			Skewness			
		Average CV	Weighted average	Pooled CV	No correction	Overall correction	Weighted correction	Average weighted skewness
1	40	37.41	36.17	39.13	2.131	2.402	2.500	0.939
2	60	28.16	29.96	33.69	2.735	3.405	3.121	1.056
3	59	31.64	34.26	36.02	0.868	1.033	0.984	0.695
4	40	39.00	42.90	47.42	2.968	3.477	3.474	1.498
5	50	46.01	48.90	52.60	2.051	2.517	2.361	0.902
6	55	46.40	46.64	50.41	2.684	3.207	3.079	1.234
7	22	52.62	52.97	64.80	3.236	4.245	4.356	1.301
8	39	41.13	42.73	49.17	2.510	3.227	3.125	0.967
9	46	31.50	32.54	34.39	1.183	1.373	1.326	0.724
10	39	29.87	31.91	34.21	1.865	2.160	2.085	0.679
Northern Ireland	1	34.23	34.23	34.23	0.401	0.420	0.420	0.420
Republic of Ireland	112	26.94	27.02	29.08	1.295	1.495	1.485	0.641

Table 2.2 Region averages of CV and g based on all records.

Region	No. of stations	Coefficient of variation %			Skewness			
		Average CV	Weighted average	Pooled CV	No correction	Overall correction	Weighted correction	Average weighted skewness
1	25	35.68	35.17	36.68	1.232	1.351	1.355	0.908
3	13	37.29	38.72	39.52	0.977	1.057	1.041	0.861
4	11	47.63	49.59	53.96	2.951	3.195	3.163	1.500
5	10	49.38	51.66	53.98	2.322	2.527	2.499	1.345
6	14	44.53	46.00	49.20	3.457	3.740	3.714	1.579
9	13	34.03	33.40	34.99	1.443	1.542	1.567	1.020
10	15	33.07	32.95	34.64	1.836	1.994	1.987	0.818
Republic of Ireland	63	27.31	27.71	30.09	1.454	1.643	1.633	0.662

Note: Omitted are Region 2 (seven stations), Region 7 (one station), Region 8 (three stations), Northern Ireland (one station).

Table 2.3 Region averages of CV and g based on records of 15 years and more.

$$\hat{\mu}_3 = \frac{1}{n_j - 1} \sum_{i=1}^{n_j} (Q_{ij} - \bar{Q}_{\cdot j})^3 \tag{2.3.3.7}$$

which is biassed however. An unbiassed estimate is

$$\hat{\mu}_3 = \frac{n_j}{(n_j - 1)(n_j - 2)} \sum_{i=1}^{n_j} (Q_{ij} - \bar{Q}_{\cdot j})^3. \tag{2.3.3.8}$$

The coefficient of skewness is

$$g = \mu_3/\sigma^3 \tag{2.3.3.9}$$

which may also be written

$$g = \frac{\mu_3/(\mu_1)^3}{(\sigma/\mu_1)^3} = \frac{\mu_3/(\mu_1)^3}{\mathrm{CV}^3} \tag{2.3.3.10}$$

in which the numerator and denominator are both dimensionless quantities. To find an average value of g over many catchments of differing sizes the numerator and denominator may be evaluated separately for the region. The CV in the denominator will be taken as the pooled CV value described above. In record j write

$$S3_j = \sum_{i=1}^{n_j} \left(\frac{Q_{ij} - \bar{Q}_{\cdot j}}{\bar{Q}_{\cdot j}} \right)^3 = \sum_{j=1}^{n_j} (x_{ij} - 1)^3. \tag{2.3.3.11}$$

The following three expressions are then candidates for an average numerator;

(i) $$A3 = \sum_{j=1}^{m} S3_j \Big/ \sum_{j=1}^{m} (n_j - 1) \qquad (2.3.3.12)$$

which contains no correction for the bias mentioned in connection with Equation (2.3.3.7) above.

(ii) $$B3 = \frac{\bar{n}}{\bar{n}-2} A3 \qquad (2.3.3.13)$$

where $\bar{n} = \Sigma n_j/m$ is the average length of record. $B3$ can be said to incorporate a lumped correction for bias.

(iii) $$C3 = \sum_{j=1}^{m} \left(\frac{n_j}{n_j-2} \cdot S3_j\right) \Big/ \sum_{j=1}^{m} (n_j - 1) \qquad (2.3.3.14)$$

which incorporates a weighted correction for bias. The pooled estimate of skewness in the region may then be taken as

$$g = \frac{A3}{\text{CV}^3} \text{ or } \frac{B3}{\text{CV}^3} \text{ or } \frac{C3}{\text{CV}^3} \qquad (2.3.3.15)$$

where CV is the regional pooled CV.

The quantity $C3/\text{CV}^3$ is to be preferred. The three quantities are tabulated for each region in Table 2.2 and the corresponding quantities based on 15 years or more of record are given in Table 2.3. The three measures do not differ widely in any region in either table but they differ between tables. In other words there is no great difference between the uses of $A3$, $B3$ and $C3$. The difference due to the omission of short records is not of a constant type as in some regions there is an increase and in others a decrease. Such a comparison is deemed unsuitable in Regions 2, 5 and 8 as each of these had fewer than 10 records of 15 years or more.

The value of $C3/\text{CV}^3$ based on all the records is shown in Figure 2.4.

In Tables 2.2 and 2.3 the weighted arithmetic mean of the individual sample skewness is also shown. This is computed as

$$\bar{g} = \sum_{j=1}^{m} (n_j - 1) g_j \Big/ \sum_{j=1}^{m} (n_j - 1) \qquad (2.3.3.16)$$

where g_j is the sample skewness at station j. These figures are very often considerably smaller than the other averaged values of skewness, Equation (2.3.3.15). However, the g of Equation (2.3.3.15) is the ratio of two averages while the $\bar{g}$ of Equation (2.3.3.16) is the average of sample ratios and it can be shown statistically that under the same set of assumptions for each quantity the latter, Equation (2.3.3.16), would be considerably more biassed than the former, Equation (2.3.3.15). Therefore, the values computed from Equation (2.3.3.15) are to be preferred.

2.3.4 Tests for trend and persistence in annual maximum series

In flood frequency analysis a major assumption is that some set of data is a random sample from a single unknown probability distribution. In this section an attempt is made to check this assumption about the series of annual maximum floods. Randomness cannot be proved but it can be disproved by the presence of some feature of a nonrandom nature, such as trend or serial persistence (positive serial correlation). Distribution free methods were applied to long records of data to test for these conditions.

All United Kingdom records containing over 30 years of continuous annual maximum data were considered for these tests; in all, 28 records which are listed in Table 2.4 with the number of years of continuous record available, were accepted. Some records were rejected because they were of low grade (D or E), reflecting poor quality of observation or of rating (see Volume IV, Chapter 1).

Table 2.4 Stations used in distribution free tests for randomness.

Station no.	Station name	Grade	Catchment area (km²)	Years record
6/901	Ness at Ness Castle Farm	B	1790	33
10/1	Ythan at Ardlethan	A	448	31
12/1	Dee at Cairnton	B	1370	40
15/4	Inzion at Lintrathen	B	24.7	43
15/5	Melgam at Lintrathen	B	40.9	41
27/1	Nidd at Hunsingore	A2	484	35
27/2	Wharfe at Flint Mill Weir	A1	759	33
27/10	Hodge Beck at Bransdale Weir	A2	18.9	32
27/21	Don at Doncaster	B	1260	34
28/10	Derwent at Longbridge	B	1120	34
28/801	Burbage Brook	A2	9.1	43
28/804	Trent at Trent Bridge	B	7490	82
39/1	Thames at Teddington	A2	9870	88
54/1	Severn at Bewdley	A	4330	46
54/2	Avon at Evesham	A2	2210	32
55/1	Wye at Cadora	A1	4040	32
55/2	Wye at Belmont	B	1900	32
55/4	Irfon at Abernant	C	72.8	31
55/5	Wye at Rhayader	C	167	31
55/7	Wye at Erwood	B	1280	31
67/2	Dee at Erbistock	B	1040	31
67/803	Dee at Chester Weir	B	1830	71
68/1	Weaver at Ashbrook	A2	609	32
68/5	Weaver at Audlem	B	262	33
69/1	Mersey at Irlam Weir	A2	679	35
69/2	Irwell at Adelphi Weir	C	559	33
69/6	Bollin at Dunham Massey	B	256	33
83/802	Irvine at Kilmarnock	B	218	55

Distribution free tests. Any statistical test includes (i) naming a null hypothesis, (ii) naming a test statistic and its distribution under the null hypothesis, (iii) naming a critical region for the test statistic in which, under the null hypothesis, the value of the test statistic falls with probability α, (iv) computing the test statistic from a sample, and (v) rejecting the null hypothesis or not according to whether the observed test statistic value falls in the critical region.

If step (ii) does not depend on the form of the parent distribution from which the sample is drawn, then the test is distribution free as are all the tests reported here.

The test statistics computed on each sample of data were

a Rank order serial correlation coefficient s_1

b Arithmetic serial correlation coefficient r_1

c Rank order correlation coefficient of trend r_2

d Split sample tests

- *i* Mann–Whitney test (for location difference) U
- *ii* Wald–Wolfowitz runs test (any difference) R

e Number of turning points p

Tests *a* and *b* test for persistence, *e* is a general randomness test while *c* and *d(i)* test for a progressive change in the mean value with time and *d(ii)* tests for a change in any measure of the distribution (mean, variance . . .) with time. Each of these statistics was computed for each station listed in Table 2.4. Siegel (1956) provides a good account of tests of statistical hypotheses including those used here. In each case the test statistics are defined below. The annual maxima are denoted Q_1, Q_2 . . . Q_i . . . Q_N in chronological order of occurrence.

(a) Spearman rank order serial correlation coefficient

The definition of serial correlation adopted is

$$s_1 = 1 - \frac{6\sum_{i=1}^{m}(d_i^2)}{m^3 - m} \tag{2.3.4.1}$$

where d_i is the difference in ranks between Q_i and Q_{i+1}, m is the original sample size minus one $= N-1$.

If two or more values of Q are equal they are each given the mean rank and the formula is adjusted to take this into account. For each tie the quantity

$$T = (t^3 - t)/12 \tag{2.3.4.2}$$

is computed where t is the number of observations tied at a given rank. Then with x referring to the series Q_1, Q_2 . . . Q_{N-1} and y to the series

$$s_1 = \frac{\sum_{i=1}^{m} x^2 + \sum_{i=1}^{m} y^2 - \sum_{i=1}^{m} d_i^2}{2\sqrt{\Sigma x^2 \Sigma y^2}} \tag{2.3.4.3}$$

where $\Sigma x^2 = (m^3 - m)/12 - \Sigma T_x$ and $\Sigma y^2 = (m^3 - m)/12 - \Sigma T_y$ and ΣT_x and ΣT_y are the sum of all T values arising in the x and y series respectively. According to Siegel (1956) the quantity

$$t = s_1 \left(\frac{m-2}{1-s_1^2}\right)^{\frac{1}{2}} \tag{2.3.4.4}$$

is distributed as Student's t with df $= m-2$ when $m > 10$.

(b) Arithmetic serial correlation coefficient

Like the rank order serial correlation coefficient, the value of this statistic reveals the presence or absence of serial correlation. Denote the statistic by r_1. Let u_i be the departure of Q_i from the overall sample mean $\bar{Q}$. Then define

$$r_1 = \frac{\sum_{i=1}^{m}(u_i\, u_{i+1})/m}{\sum_{i=1}^{N} u_i^2/N} \tag{2.3.4.5}$$

where N is the length of the entire series and $m = N-1$.

For large N the variance of r_1 in a random series is (Yule & Kendall, 1950)

$$\text{var } r_1 = \frac{1}{N-1}. \qquad (2.3.4.6)$$

It will be assumed here that the quantity analogous to t in Equation (2.3.4.4) with s_1 replaced by r_1 is distributed as Student's t with $m-2$ degrees of freedom.

(*c*) *Rank order correlation test for trend*

Trend may be viewed as a correlation between the series and time. If Q_t represents the series and t represents time then Q_t and t are correlated if Q_t contains trend. Therefore, a test for correlation between Q_t and t constitutes a test for trend. The Spearman rank order correlation coefficient, r_2, between Q_t and t is used as the basis for the test. One set of ranks gives R_t the rank of Q_t in the series, the other set, T_t, being 1, 2, 3 . . . N.

The coefficient is computed as

$$r_2 = 1 - \frac{6\sum_{t=1}^{N} d_t^2}{N^3 - N} \qquad (2.3.4.7)$$

where N is the length of the series and $d_t = R_t - T_t$.

As with the rank order serial correlation test, corrections should be made for ties in the ranking of the data. The corrections are of exactly the same form in both cases, and the test statistic of Equation (2.3.4.4) is used with r_2 replacing s_1 with $N-2$ degrees of freedom.

(*d*) *Split sample tests*

The sample is split into two subsamples nearly equal in size, and the hypothesis that the two subsamples come from the same population is tested by either of two methods. Table 2.5 is used to illustrate the calculation of both the Mann–Whitney and Wald–Wolfowitz statistics.

Mann–Whitney U test. This tests for a location difference between the two subsamples; if they are not well mixed and the entire sample is ranked, the elements of one subsample display relatively low rank numbers while those of the other display relatively high rank numbers. The test statistic, U, reflects these differences with a low value arising when the subsamples are not well mixed. Therefore, if the observed U value is less than a certain critical value U_{cr} the hypothesis that there is no location difference between subsamples is rejected. The quantity U_{cr} depends on the confidence level and the subsample sizes and is tabulated by Siegel (1956); when both subsamples are larger than 20, U is approximately Normally distributed with mean $n_1n_2/2$ and standard deviation $[n_1n_2(n_1+n_2+1)]^{\frac{1}{2}}/12$.

The test statistic is calculated as the smaller of V_1 and V_2 where

$$V_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1 \qquad (2.3.4.8)$$

and

$$V_2 = n_1n_2 - V_1 \qquad (2.3.4.9)$$

n_1 = size of first subsample, n_2 = size of second subsample, R_1 = sum of ranks attributed to members of the first subsample in the ranked total sample.

Q_i	Rank	Ranked data	Subsample	Runs
First subsample A				
12.62	8	31.03	A	1
7.57	24	19.46	A	—
11.62	9	17.36	B	2
19.46	2	14.73	A	3
8.73	21	14.25	B	4
10.99	12	13.73	B	—
6.94	26	12.73	A	5
10.52	14	12.62	A	—
10.26	15	11.62	A	—
31.03	1	11.31	B	6
10.26	15	11.20	B	—
14.73	4	10.99	A	7
10.68	13	10.68	A	—
12.73	7	10.52	A	—
8.94	20	10.26	A	—
9.73	18	10.26	A	—
	209			
Second subsample B				
11.20	11	10.26	B	8
13.73	6	9.73	A	9
8.63	23	9.21	B	10
11.31	10	8.94	A	11
6.42	27	8.73	A	—
4.63	32	8.73	B	12
5.52	30	8.73	B	—
10.26	15	7.57	A	13
14.25	5	7.42	B	14
7.42	25	6.94	A	15
9.21	19	6.43	B	16
6.10	28	6.10	B	—
6.10	28	6.10	B	—
8.73	21	5.52	B	—
5.52	30	5.52	B	—
17.36	3	4.63	B	—

Table 2.5 Calculation of Mann–Whitney and Wald–Wolfowitz test statistics.

Data are tabulated in columns 1 and 2 of Table 2.5 in the required manner; column 1 lists the data in chronological order while the first and second subsamples, each of size 16, are also indicated. The ranked position of Q_i in the entire series is shown for each i in column 2 (columns 3, 4 and 5 refer to the next test). R_1, the sum of these ranks of members of the first subseries, is 209.

Therefore, using $n = 32, n_1 = n_2 = 16$ and $R_1 = 209$ gives $V_1 = 16(16) + 16(17)/2 - 209 = 183$ and $V_2 = 73$ while U being the smaller of these two is $U = 73$. The critical value at the 5% significance level is 75 so that the hypothesis of no location difference between the subsamples is rejected at that level.

Wald–Wolfowitz runs test. This tests for any difference (location, dispersion, skewness . . .) between the subsamples. Columns 3, 4 and 5 of Table 2.5 refer to this test. Column 3 shows the entire series ranked in order. In column 4 an A or B is shown opposite each ranked value; an A if it is from the first subsample and a B if it is from the second subsample. If the two subsamples are alike the As and Bs are well mixed. The number of runs of like characters in column 4 measures how well the As and Bs are mixed and therefore measures the similarity between the subsamples. The cumulative number of runs is given in column 5 at the beginning of each new run. The total number of runs R, is $R = 16$.

Too few runs indicate too little mixing of the two subsamples and hence some difference between them. Too many runs indicate a nonrandom

Table 2.6 Results of distribution free tests for trend and persistence.

Station	No. of years	(a) Rank order serial correlation test for persistence (one tailed)		(b) Arithmetic serial correlation test for persistence (one tailed)		(c) Rank order correlation test for trend (two tailed)		d(i) Mann–Whitney 'U' test for split sample difference (one tailed) ($\alpha = 5\%$)		d(ii) Wald–Wolfowitz runs test for split sample difference (two tailed) ($\alpha_L = \alpha_U = 5\%$)			(e) Number of turning points test for randomness (two tailed)		
	N	s_1	t on $N-3$ df	r_1	t on $N-3$ df	r_2	t on $N-2$ df	U	U_{cr} or $t = N(0,1)$	R	R_L or $t = N(0,1)$	R_U	p	$E(p)$	se (p)
6/901	33	0.11	0.60	0.21	1.19	0.39	2.35**	95.0	89	21	11	24	21	20.67	2.35
10/1	31	−0.08	−0.41	0.20	1.11	−0.22	−1.22	96.0	77	17	10	23	19	19.33	2.28
12/1	40	0.14	0.84	0.02	0.15	0.50	3.59***	106.5	138***	18	14	28	26	25.33	2.61
15/4	43	−0.18	−1.23	0.02	0.99	−0.02	−0.15	222.5	$t =$ 0.21	22	$t =$ 0.18		26	27.33	2.71
15/5	41	0.11	0.71	−0.02	−0.14	0.02	0.12	196.5	$t =$ −0.35	20	$t =$ 0.47		25	26.00	2.59
27/1	35	−0.07	−0.41	−0.02	−0.10	−0.13	−0.75	138.0	102	16	12	25	26	22.00	2.43
27/10	32	0.07	0.39	−0.02	−0.12	0.36	2.09**	75.5	83	18	11	23	23	20.00	2.30
27/2	33	−0.04	−0.23	−0.22	−0.12	0.001	−0.01	108.5	89	22	11	24	22	20.67	2.35
27/21	34	0.03	0.16	0.12	0.69	−0.04	−0.21	144.0	96	21	11	25	24	21.33	2.39
28/10	34	0.06	0.32	−0.03	−0.14	0.18	1.03	121.0	96	20	11	25	20	21.33	2.32
28/801	43	0.12	0.77	−0.03	−0.99	−0.22	−1.43	199.0	$t =$ −0.78	23	$t =$ 0.47		24	27.33	2.71
28/804	71	−0.07	−0.54	0.09	0.71	−0.08	−0.66	567.5	$t =$ −0.72	37	$t =$ 0.21		40	46.00	3.51
39/1	88	−0.07	−0.64	−0.08	−0.73	−0.26	−2.50**	747.0	$t =$ −1.48**	40	$t =$ 1.07		57	57.33	3.92
54/1	46	0.19	1.25	0.26	1.82**	−0.18	−1.22	217.5	$t =$ −1.03	28	$t =$ 1.19		30	30.00	2.74
54/2	32	0.11	0.62	0.02	0.09	−0.04	−0.19	119.0	83	20	11	23	21	20.00	2.80
55/1	32	0.09	0.50	0.02	0.12	0.16	0.87	110.5	83	16	11	23	21	20.00	2.32
55/2	61	0.36	2.91***	0.07	0.05	−0.49	−4.36***	146.0	$t =$ −4.6***	22	$t =$ 2.43**		39	39.33	3.24
55/4	32	0.19	1.03	0.08	0.42	−0.14	−0.79	111.5	83	20	11	23	19	20.00	2.32
55/5	31	0.10	0.55	0.08	0.43	−0.44	−2.65**	61.0	77***	14	10	23	19	19.33	2.32
55/7	31	0.30	1.68	0.10	0.53	0.12	0.65	112.5	77	13	10	23	21	19.33	2.28
67/2	31	0.07	0.36	0.10	0.52	0.11	0.60	110.0	77	14	10	23	21	19.33	2.28
67/803	72	0.33	2.90***	0.34	3.04***	−0.15	1.20	509.0	$t =$ −1.57	26	$t =$ 2.61***		44	46.00	3.52
68/1	35	0.35	2.01**	0.11	0.64	−0.24	−0.13	117.0	102	14	11	23	16	20.00	2.32
68/5	33	0.28	1.62*	0.26	1.52*	−0.41	−0.23	123.5	89	18	11	24	18	20.67	2.35
69/1	35	0.05	0.31	0.24	1.44*	−0.10	−0.57	151.5	102	15	12	25	26	22.00	2.43
69/2	33	−0.34	−1.95	0.12	0.69	−0.18	−1.01	109.5	89	21	11	24	26	20.67	2.35
69/6	33	0.03	0.19	0.12	0.69	0.32	1.90*	99.0	89	19	11	24	24	20.67	2.35
83/802	55	0.18	1.30	0.12	0.91	−0.38	−2.95***	216.5	$t =$ −2.72***	18	$t =$ 2.86***		34	35.33	3.08

In (a) (b) and (c) t is Student's t. In d(i) U is in region of rejection if less than U_{cr}. In d(ii) the region of rejection is $R < R_L$ and $R > R_U$.
Statistic values significant at 10%, 5% and 1% are marked *, ** and *** respectively.

arrangement of the As and Bs and hence a nonrandom allocation of the entire series between the two subsamples. Under the null hypothesis that the subsamples are from the same population R has mean $1+2n_1n_2/(n_1+n_2)$ and variance $2n_1n_2(2n_1n_2-N)/N^2(N-1)$. If n_1 and n_2 are less than 20, critical values of B, both upper and lower values, are given by Siegel (1956). For larger values of n_1 and n_2 it may be assumed that R is Normally distributed with the above mean and variance and critical values may be obtained from Normal tables.

(e) Test for randomness based on number of turning points

A turning point is either a peak or a trough in the graph of Q_t against t (i.e. against a year of occurrence). In a purely random series of N values the expected number of peaks and troughs is

$$E(p) = 2(N-2)/3 \qquad (2.3.4.10)$$

with large sample variance

$$\text{var}(p) = (16N-29)/90. \qquad (2.3.4.11)$$

For large N, p is approximately Normally distributed (Yule & Kendall, 1950). Too few or too many turning points indicate nonrandomness.

Results

The test statistic values for each test on each station listed in Table 2.4 are shown in Table 2.6. In addition to the statistic values information is given on which their significance may be judged. The form of this information depends on the availability of tables of critical points for each particular test statistic and is indicated in the table footnotes. The number of times the null hypothesis is rejected in the 28 records taking each test in turn is summarised in Table 2.7.

Table 2.7 Number of cases in which the null hypothesis is rejected at stated level out of 28 cases.

Test significance level	(a) Persistence	(b) Persistence	(c) Trend	(d) Split sample Mann–Whitney	(d) Split sample Wald–Wolfowitz	(e) Randomness
1%	2	1	3	3	1	0
5%	2	2	7	4	3	1
10%	4	4	8	4	3	3

Out of 28 cases there is less than a 5% chance of getting two or more significant values at the 1% level (i.e. with $p = 0.01$, PR (two successes or more in 28 trials) <0.05). Therefore, taking the 28 records as a whole it must be concluded that they are not free of persistence (test *a*) or trend (tests *c* and *d(i)*). At the 10% level, four occurrences are not unusual but eight would have less than a 1% chance of occurring under the null hypothesis. Therefore, there is strong evidence of trend when all the records are viewed as a whole. Looking at individual records the following display the most marked departures from the null hypothesis of zero persistence (tests *a* and *b*) and trend (tests *c* and *d*).

12/1 Dee at Cairnton — Trend and split sample difference at 1% level

39/1	Thames at Teddington	Trend and split sample difference at 5% level
55/2	Wye at Belmont	Persistence, trend and split sample difference at 1% level
55/5	Wye at Rhayader	Trend at 5% level and split sample difference at 1%
67/803	Dee at Chester Weir	Persistence at 1% and split sample difference at 5%
83/802	Irvine at Kilmarnock	Trend and split sample difference at 1%

Of these six the Thames at Teddington is the only one which does not show some statistical significance at the 1% level. In particular, the Dee at Chester shows strong persistence while the Dee at Cairnton (a different river), the Wye at Belmont and the Irvine at Kilmarnock show strong trend, while persistence at Belmont is also significant at the 1% level.

Evidence of persistence in 2 out of 28 records is not conclusive especially since it is not supported by other records on these same rivers (Wye and Dee). On the other hand the evidence for trend is stronger, being shown at 6 out of 28 stations. Unfortunately, the older and longer stations are not the most reliable but the conclusion must be that the possibility of trend cannot be dismissed. Besides hydrometric reasons such as rating curve uncertainties and weir changes there are of course possible hydrological reasons for trend based on land use changes such as hydroelectric development, channel improvements and land drainage.

2.3.5 Some properties of largest recorded floods

Catalogue of very large outliers

In Table 2.8 the values of $Q_{max}/Q_{med} \geqslant 2.50$ in each region are given, while in Table 2.9 the 30 largest values of Q_{max}/Q_{med} in the country are given. The use of Q_{med} as divisor gives figures which on the average are 7% larger than the use of $\bar{Q}$ (see below) but further it allows the truly large floods in short records to stand out as they should, in records where $\bar{Q}$ might be exaggerated and $Q_{max}/\bar{Q}$ decreased. The 25th largest value is just over 3.50 and since these figures are based on the data of 449 stations it can be stated that approximately 5% of Q_{max}/Q_{med} values are greater than 3.50. If the count were extended further it would be seen that approximately 9% of all values are greater than 3.00.

Relation of mean to median annual flood

It is useful to know the average of the ratio $\bar{Q}/Q_{med}$ because when a record contains an outlier the value of $\bar{Q}$ is inflated whereas Q_{med} is not necessarily affected. Table 2.10 gives the mean value of $\bar{Q}/Q_{med}$ in each of the 10 regions defined in Table 2.1. In records with large values of Q_{max}, particularly where the record is short, the $\bar{Q}/Q_{med}$ value is large, but there are a sufficient number of ordinary stations to ensure that these large values do not overinfluence the mean. It can be seen that the countrywide average value of $\bar{Q}/Q_{med}$ is just over 1.07, the major anomalous region being Region 7 where the mean is 1.32. However, in this region the longest record is only 15 years and the 1968 flood was very large in

Table 2.8 Values of Q_{max}/Q_{med} exceeding 2.50.

Station No.	Q_{max}/Q_{med}
Region 1	
7/2	6.66
8/7	3.39
8/8	2.81
95/801	2.79
8/6	2.76
8/1	2.76
12/1	2.76
8/2	2.61
Region 2	
21/3	5.40
83/802	3.26
21/2	3.00
79/3	2.78
19/8	2.63
Region 3	
27/10	3.02
25/5	2.73
27/1	2.65
Region 4	
28/801	6.44
54/6	5.62
28/10	3.60
54/3	2.76
54/19	2.69
28/11	2.69
54/2	2.62
54/14	2.52

Station No.	Q_{max}/Q_{med}
Region 5	
29/3	30.53
34/2	5.93
32/6	4.93
32/10	4.07
33/11	3.39
33/13	3.31
32/8	3.28
33/14	3.10
34/6	3.05
32/2	3.05
32/3	2.76
35/1	2.73
Region 6	
39/15	10.79
39/3	7.62
36/8	5.89
39/813	5.89
39/1	3.51
36/1	3.32
38/4	3.29
36/6	3.09
38/13	2.93
38/2	2.83
39/830	2.75
36/7	2.63
38/7	2.59
36/2	2.56
39/820	2.55

Station No.	Q_{max}/Q_{med}
Region 7	
40/12	16.48
40/6	9.52
40/19	7.75
41/7	3.67
40/7	2.62
41/5	2.51
Region 8	
West Lyn	18.41
53/4	8.89
45/5	4.91
45/3	3.09
45/1	2.76
53/3	2.75
50/1	2.62
Region 9	
55/8	4.25
55/15	2.80
67/7	2.74
67/6	2.60
55/10	2.56
67/2	2.51
Region 10	
71/804	9.50
68/1	4.18
72/807	3.82
69/802	3.19

many cases. Hence, the figure 1.32 must be considered high even for Region 7. If the average value of $\bar{Q}/Q_{med}$ is calculated from those stations for which $Q_{max}/Q_{med} < 3$, that is stations without outliers by the previous definition, it is found to be 1.05.

Estimate of mean when the record contains an outlier

The sample mean $\bar{Q}$ is an unbiassed estimate of the population mean μ; it is the best estimate when nothing is known *a priori* about the population from which the sample was drawn. In particular, an estimator such as $0.80\bar{Q}$ is biassed because its expected value over all possible samples is 0.80μ. If, however, the mean $\bar{Q}$ of a sample is known to be from portion C of the sampling distribution as in Figure 2.5 then it is also known that the average value of $\bar{Q}$ taken over all *such samples* exceeds μ. Consequently, if all samples could be divided correctly into categories, the estimator to be applied to samples in C should be of the form $\lambda\bar{Q}$, $\lambda < 1$, rather than $\bar{Q}$.

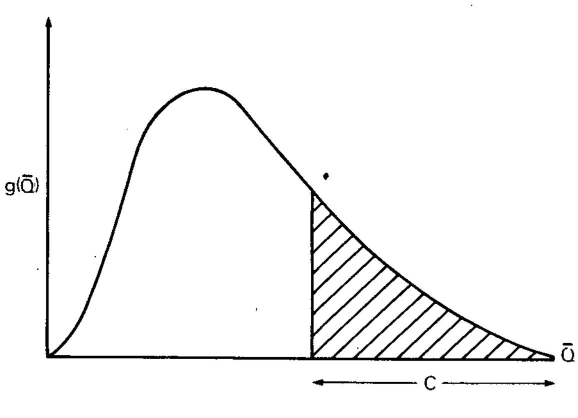


Fig 2.5 Sketch of sampling distribution of the mean, $\bar{Q}$.

Rank		Station		Q_{max}/Q_{med}	Water year
1	29/3	Lud at Louth		30.53	1920
2	—	West Lyn at Lynmouth		18.41†	1952
3	40/12	Darent at Hawley	A	16.48	1967
4	39/15	Whitewater at Lodge Farm	C	10.79	1946
5	40/6	Bourne at Hadlow	A	9.52	1967
6	71/804	Dunsop at Footholme		9.50†	1966
7	53/4	Chew at Compton Dando	B	8.89	1967
8	40/19	Eden at Vexour Bridge	A	7.75	1967
9	39/3	Wandle at Connolly's Mill	A	7.62	1967
10	7/2	Findhorn at Forres		6.66	1969
11	28/801	Burbage Brook		6.44	1957
12	34/2	Tas at Shotesham	A	5.93	1967
13	36/8	Stour at West Mill	A	5.89	1967
14	39/813	Mole at Ifield Weir	A	5.89	1967
15	54/6	Stour at Kidderminster		5.62	1954
16	21/3	Tweed at Peebles		5.40	1948
17	32/6	Kislingbury Branch at Upton	C	4.93	1946
18	45/5	Otter at Dotton	B	4.91	1967
19	55/8	Wye at Cefn Brwyn		4.25	1973
20	68/1	Weaver below Ashbrook		4.18	1945
21	32/10	Nene at Wansford		4.07	1946
22	72/807	Wenning at Hornby		3.82	1966
23	41/7	Arun at Park Mound	A	3.67	1967
24	28/10	Derwent at Longbridge		3.60	1965
25	39/1	Thames at Teddington		3.51	1894
26	33/11	Little Ouse at Euston		3.39	1967
27	8/7	Spey at Invertruim		3.39	1966
28	36/1	Stour at Stratford St Mary	A	3.32	1967
29	33/13	Sapiston at Euston (Rectory)		3.31	1967
30	38/4	Rib at Wadesmill		3.29	1967

†Q_{med} taken as MAF/1.07 where MAF obtained from catchment characteristics (Equation 2.3.3.1).

29/3 Q_{max} = 138 (4870 cusecs) as in recent reworking,
West Lyn Q_{max} = 252 (indirect method),
39/15 Q_{max} = 9.565 (338 cusecs) basis of estimate unknown,
71/804 Q_{max} = 272 (indirect method),
55/8 Q_{max} = 67 (indirect method).

A is September 1968 event, B is July 1968 event, C is March 1947 event.

Table 2.9 30 largest available values of Q_{max}/Q_{med}.

An annual maximum sample whose $\bar{Q}$ value lies in category C can be recognised, because in about 90% of available records $Q_{max}/Q_{med} < 3.0$. In the remaining 10%, Q_{max} is very large in comparison with the remainder of the sample values, a fact which is seen very easily on a probability plot. Such samples can be considered to have $\bar{Q}$ in category C and therefore an estimate of the form $\lambda\bar{Q}$, $\lambda < 1$, is closer than $\bar{Q}$ to μ. When Q_{max} is large the difference between $\bar{Q}$ and μ is greatest when the sample size is small. Therefore, the best value of λ is dependent on the sample size and its exact determination requires an assumption about the form of the distribution. A crude short cut is to avoid the determination of a λ value as a function of sample size and distribution and to adopt the following rule

When $Q_{max}/Q_{med} > 3.0$ let $\hat{\mu} = 1.07 Q_{med}$ and otherwise let $\hat{\mu} = \bar{Q}$.

It must be remembered that it is crude to use a single value of

Region	Average $\bar{Q}/Q_{med}$	No. of stations	Average $\bar{Q}/Q_{med}$ excluding values >3
1	1.089	40	1.069
2	1.048	59	1.042
3	1.057	59	1.057
4	1.036	40	1.010
5	1.086	47	1.060
6	1.073	57	1.042
7	1.315	22	1.110
8	1.093	39	1.055
9	1.034	45	1.034
10	1.033	39	1.022
Overall	1.073	447	1.047

Table 2.10 Average value of $\bar{Q}/Q_{med}$ in each region.

Q_{max}/Q_{med} (=3.0) for dividing samples into two categories, one of which is to receive unconventional treatment for the calculation of μ. The dividing value should obviously depend on sample size but the value 3.0 does capture the most extreme outliers.

2.4 Choice of annual maximum distribution by goodness of fit indices

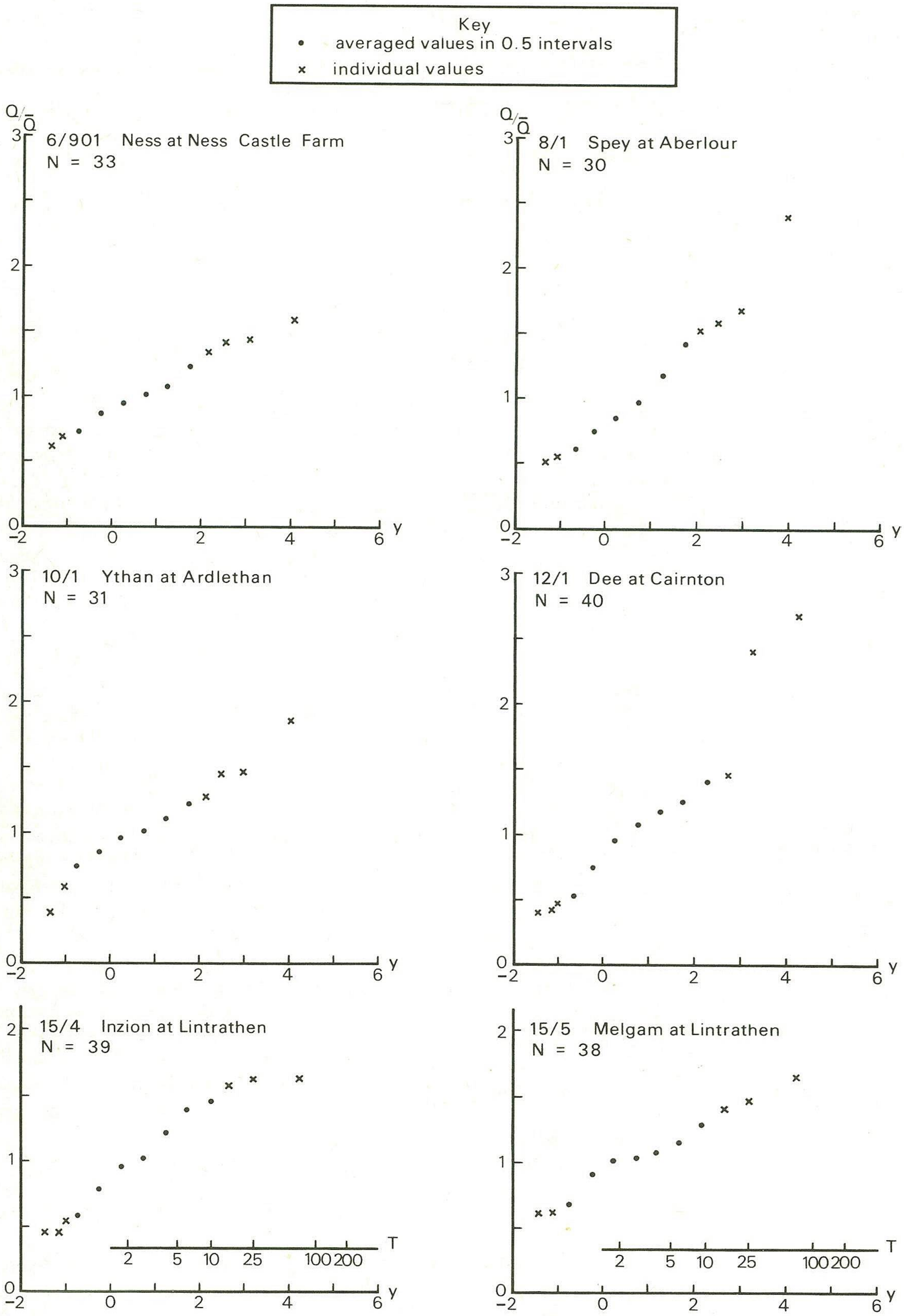
2.4.1 Introduction

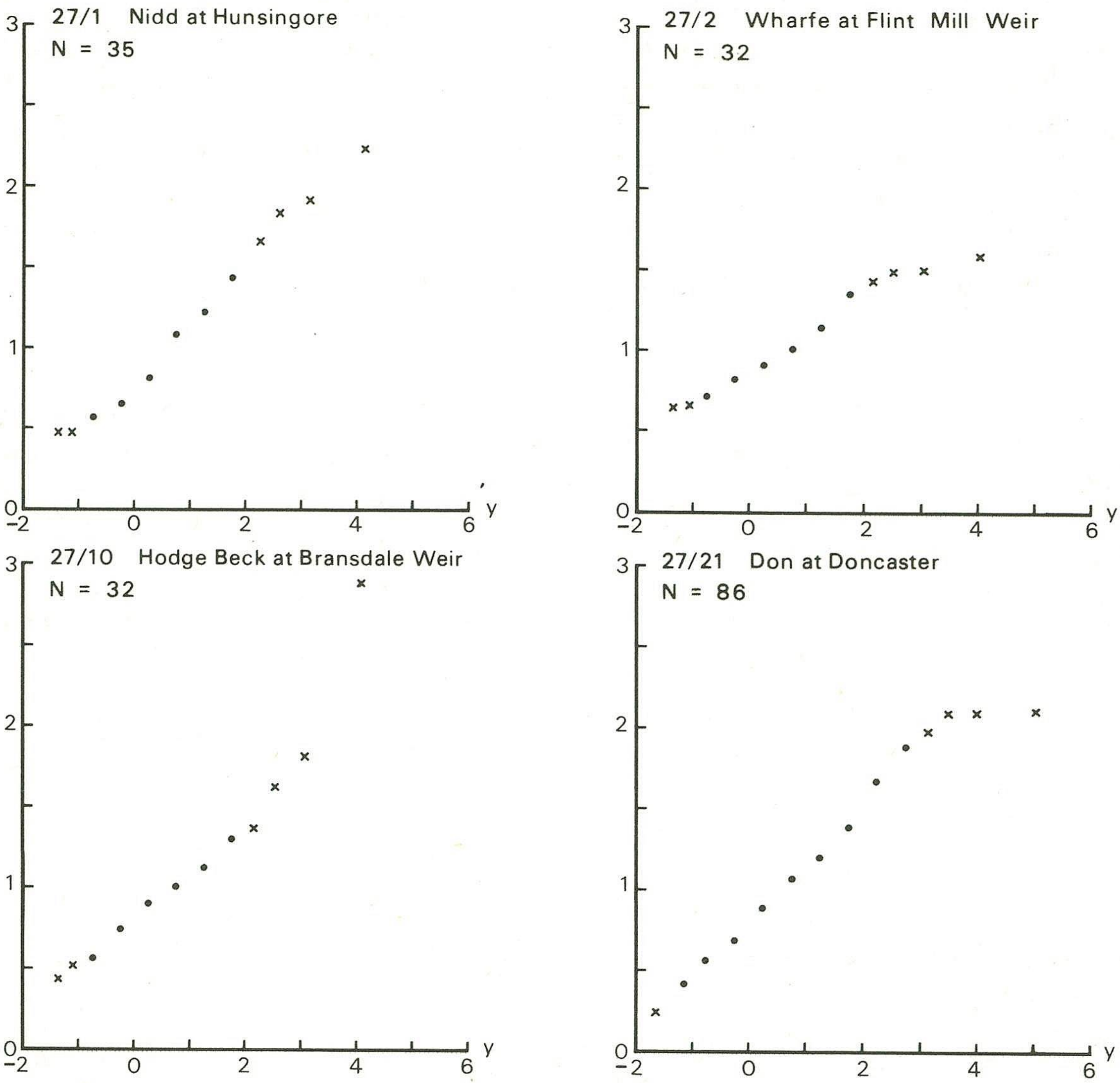
In theory, a goodness of fit index should be useful in discriminating between different distributions for the same application. If a single sample is available the goodness of fit index corresponding to each candidate distribution may be calculated and the distribution giving rise to the optimum value of the index chosen. In practice, a single inference may be sought from the joint study of many samples but a drawback is that the several alternative indices do not always point to the same distribution choice.

In this section the results are presented of calculating goodness of fit indices for records of annual maxima of 30 years and over on the hypothesis of different distributions. Out of 31 stations the χ^2 goodness of fit index rejected the hypothesis of the extreme value Type 1 (Gumbel), lognormal and general extreme value distributions about an equal number of times and hence supported no single choice. Out of the same 31 stations the Kolmogorov–Smirnov goodness of fit index rejected the extreme value Type 1 and the lognormal distributions once and the two parameter gamma distribution twice at the 10% level, but did not reject the general extreme value distribution at any station. It appears that with such sample sizes this test is insensitive to any departures from these distributions and is of little help in choosing one. This conclusion was borne out by the results from seven Irish stations having records of between 23 and 44 years.

Other goodness of fit indices were also computed. These are based on the deviations of the plotted sample points from each of the distribution curves on probability or *x–y* plots. Typically the index is the mean absolute value or the mean square value of these deviations but the sampling distributions of these indices are unknown and the procedure of rejecting or accepting hypotheses cannot be followed. Instead these indices were calculated for seven distributions for 28 stations in Great Britain with more than 30 annual maxima and seven stations in Ireland with more than 23 annual maxima in order to pick the distribution with the best performance. The seven distributions are extreme value Type 1 (Gumbel), gamma, lognormal, general extreme value, Pearson Type 3, log Pearson Type 3 and log gamma. This approach was unsatisfactory because the outcome is sensitive to whether the mean absolute deviation or the mean squared deviation is used as index. With the former the log Pearson Type 3 is indicated while with the latter the Pearson Type 3 is indicated, but these distributions are very different as variate values increase with T, for large T, much more rapidly in the former than in the latter.

The type of index used by the work group of the United States Water Resources Council Hydrology Committee is also based on differences between plotted points and theoretical distribution curves on probability or *x–y* plots (Benson, 1968). This American study was repeated on six stations' data but the result is found to be sensitive to plotting position


Key
averaged values in 0.5 intervals
individual values
Q/Q̄
6/901 Ness at Ness Castle Farm
N = 33
8/1 Spey at Aberlour
N = 30
10/1 Ythan at Ardlethan
N = 31
12/1 Dee at Cairnton
N = 40
15/4 Inzion at Lintrathen
N = 39
15/5 Melgam at Lintrathen
N = 38
T
2 5 10 25 100 200
y
−2 0 2 4 6
0 1 2 3



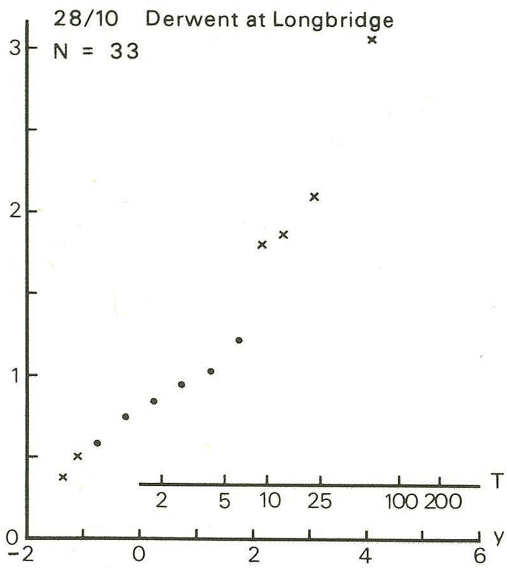
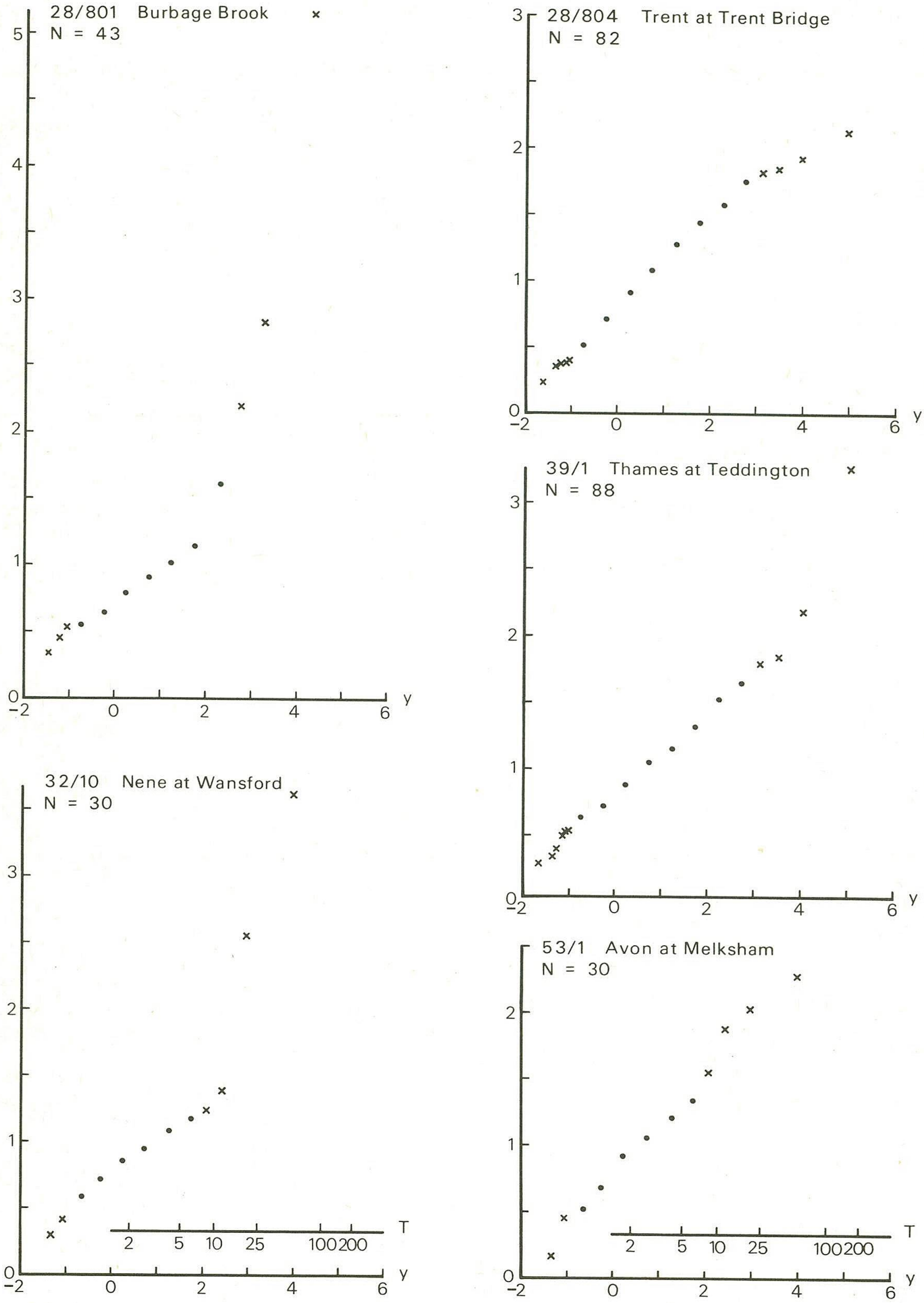


Fig 2.6 Annual maximum data from long term stations plotted as $Q/\bar{Q}$ against extreme value Type 1 reduced variate. To avoid overcrowding average values are plotted except at the extremes.


28/801 Burbage Brook
N = 43
28/804 Trent at Trent Bridge
N = 82
39/1 Thames at Teddington
N = 88
32/10 Nene at Wansford
N = 30
53/1 Avon at Melksham
N = 30
T
2 5 10 25 100 200
y
-2 0 2 4 6

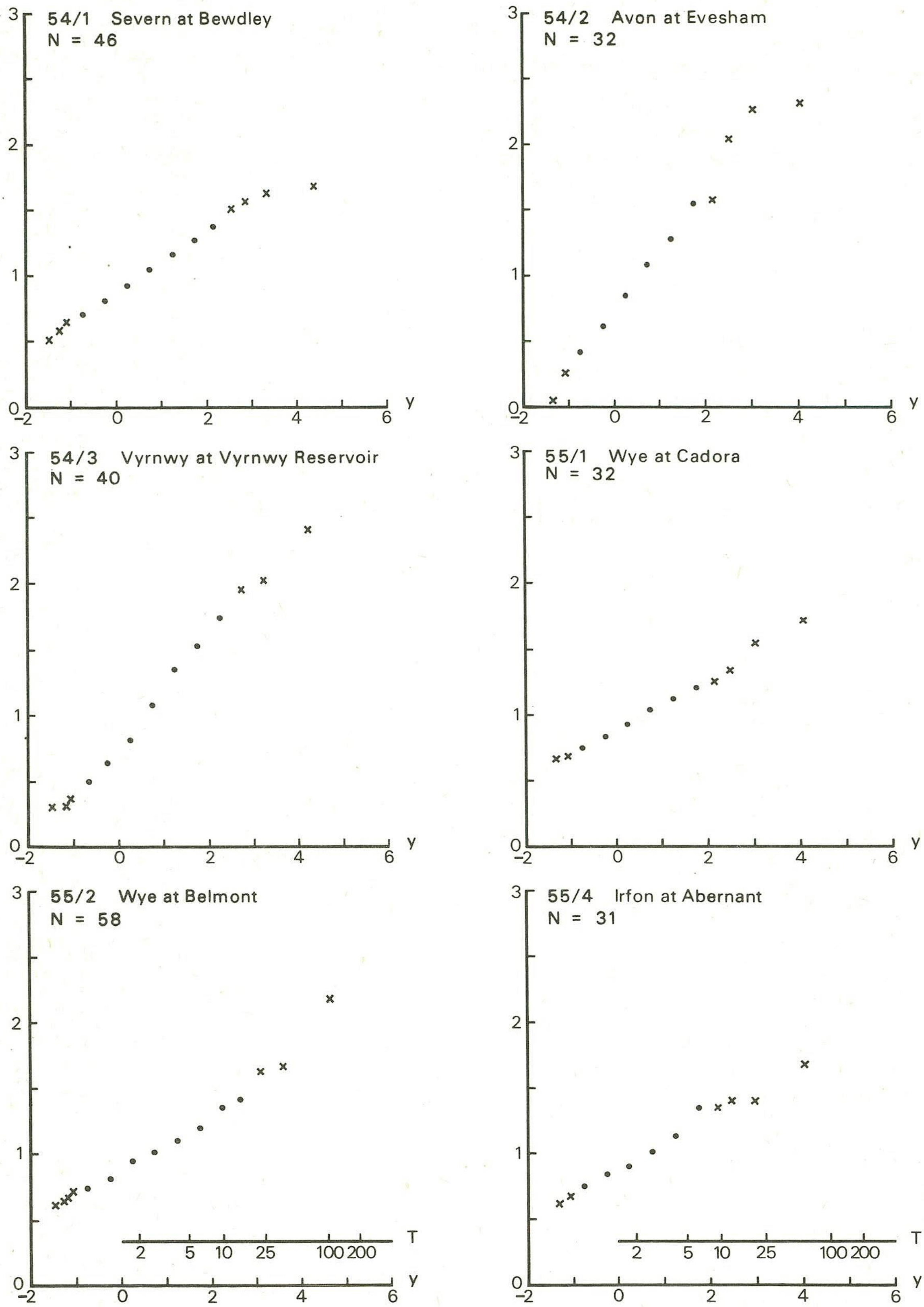
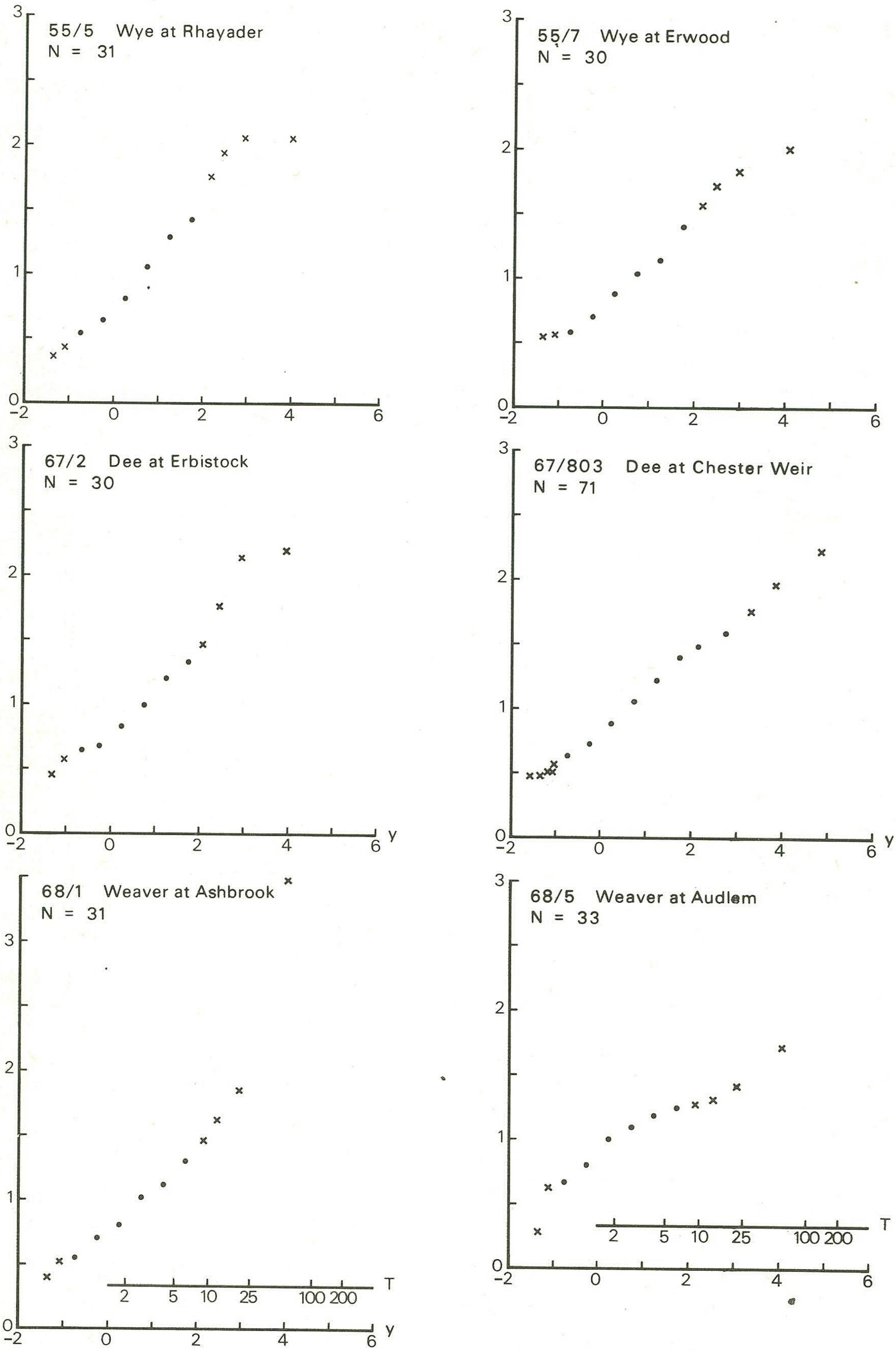


Fig. 2.6 continued


55/5 Wye at Rhayader
N = 31
55/7 Wye at Erwood
N = 30
67/2 Dee at Erbistock
N = 30
67/803 Dee at Chester Weir
N = 71
68/1 Weaver at Ashbrook
N = 31
68/5 Weaver at Audlem
N = 33
y
T
2 5 10 25 100 200
-2 0 2 4 6
0 1 2 3

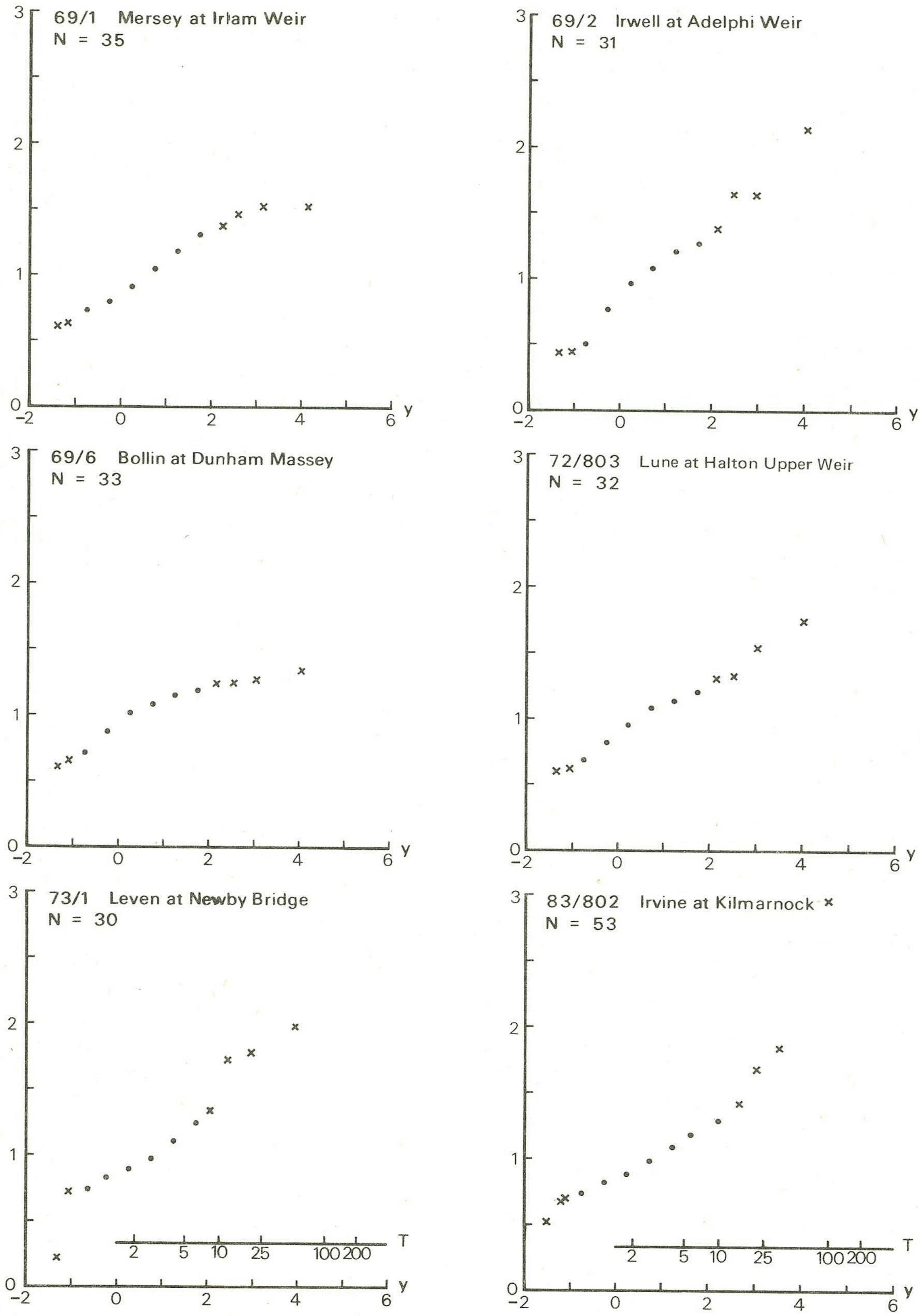
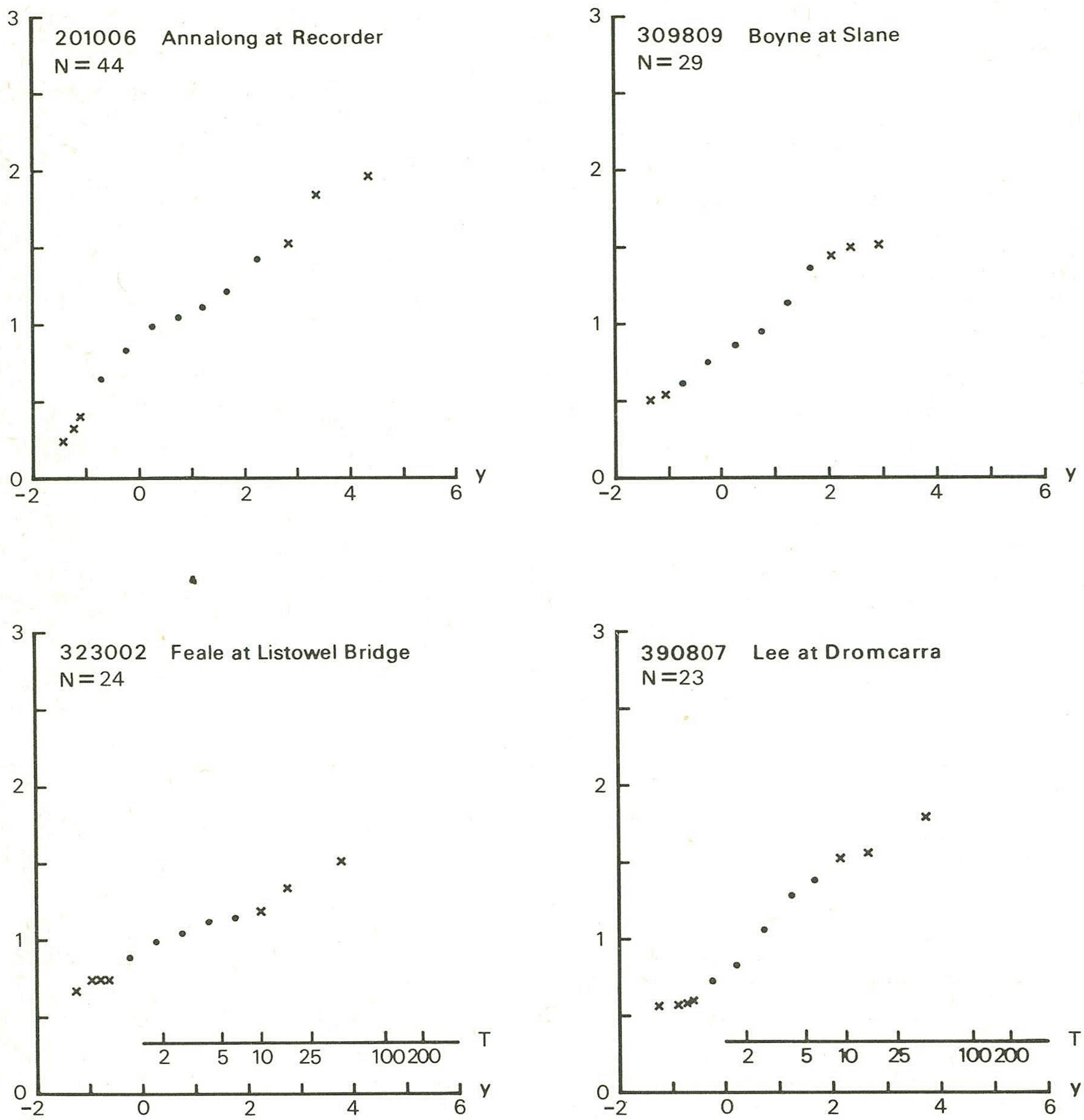


Fig. 2.6 continued



formula. When the Weibull formula $F_i = i/(N+1)$ is used the results agree with the results given by the Work Group and the log Pearson Type 3 comes out best but when the plotting positions which are regarded as the correct ones (see Section 1.3) are used the result is different, the Pearson Type 3 distribution being the best fit. Regardless of plotting position formula the general extreme value distribution was the most stable being a close second in each case. This distribution was found to describe satisfactorily the empirically derived region flood frequency curves in Section 2.6.

In all, 31 stations in Great Britain and seven stations in Ireland were used in these tests and probability plots of these are shown in Figure 2.6.

Some of the details and numerical results of these tests are given in the following pages, which may be omitted on first reading. The conclusion to be drawn from the foregoing summary is that groups of best fitting

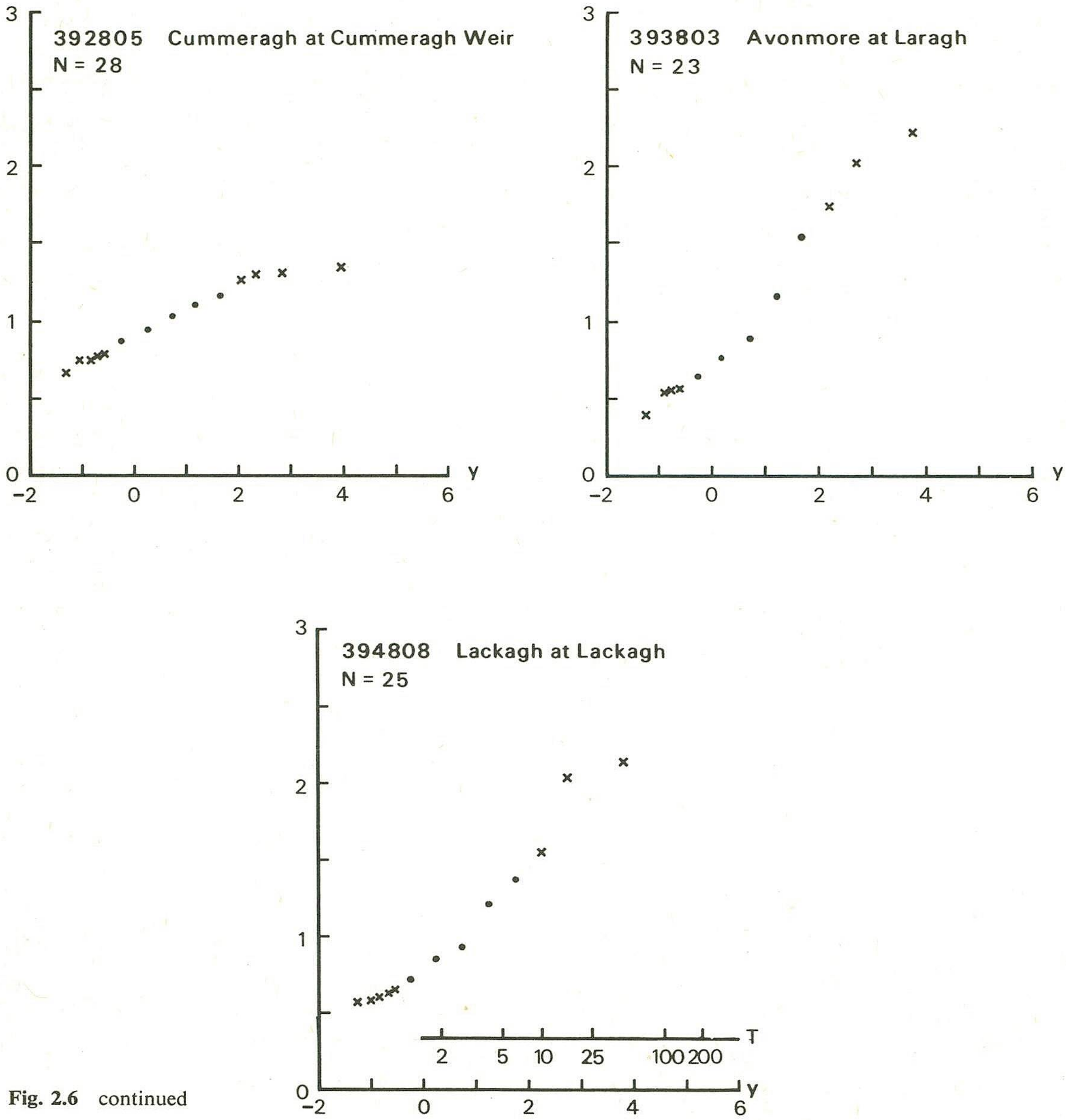


Fig. 2.6 continued

distributions can be picked out but the choice of a single distribution may depend on which index is arbitrarily chosen from a group of indices and that the general extreme value was least sensitive to change of index.

2.4.2 χ^2 and Kolmogorov–Smirnov goodness of fit indices

A goodness of fit index expresses the agreement between an observed sample of annual maxima and some theoretically specified population. The latter may be specified without any reference to the sample data or it may result from fitting a distribution to the sample. The index is a sample statistic having a distribution. If the observed index value lies in the tail of its sampling distribution it throws doubt on the hypothesis that the sample came from the specified distribution.

The χ^2 index

Figure 2.7(a) shows a histogram of the annual maximum data for the Nidd at Hunsingore, 27/1, with $M = 12$ class intervals having observed frequencies $O_1, O_2 \ldots O_{12}$, while Figure 2.7(b) shows an EV1 distribution

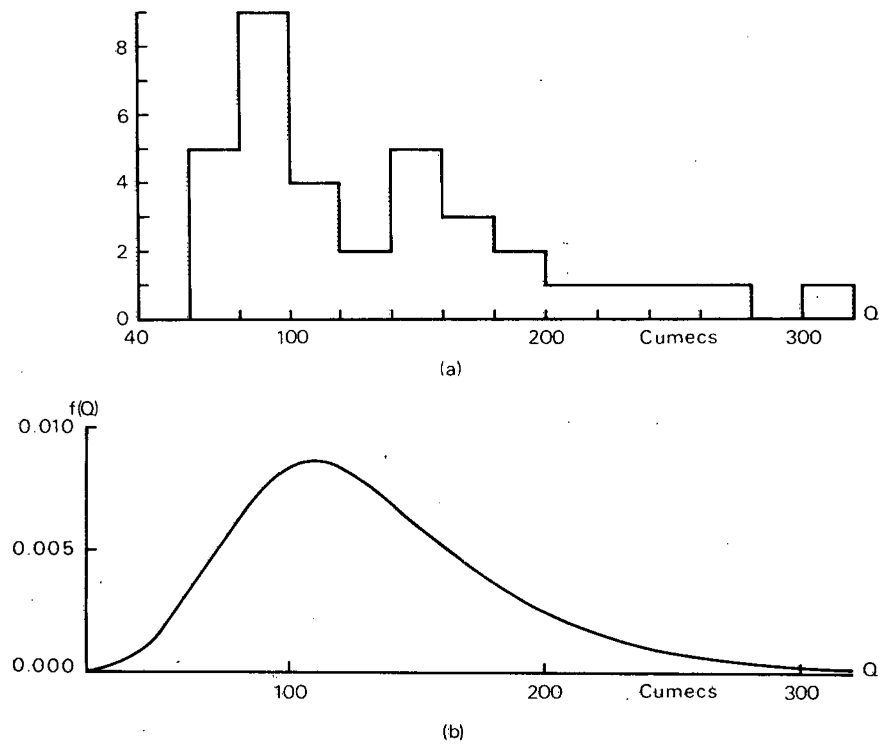


Fig 2.7 (a) Histogram of annual maximum data at 27/1.
(b) Hypothetical extreme value Type I distribution fitted to the data.

Table 2.11 Histogram data for Figure 2.7.

Class interval	No. of values in each class	Class interval	No. of values in each class
60–80	5	200–220	1
80–100	9	220–240	1
100–120	4	240–260	1
120–140	2	260–280	1
140–160	5	280–300	0
160–180	3	300–320	1
180–200	2		

whose goodness of fit to the sample is to be expressed. The expected number of sample members falling in the j^{th} class interval is $E_j = NP_j$ where P_j is the cumulative probability in that group. The quantity $D_j = O_j - E_j$ is an obvious quantity upon which to base a goodness of fit index as large values of D indicate poor agreement between sample and distribution. The χ^2 goodness of fit index is defined as

$$\chi^2 = \sum_{j=1}^{M} \frac{D_j^2}{E_j} = \sum_{j=1}^{M} \frac{(O_j - E_j)^2}{E_j}. \qquad (2.4.2.1)$$

If $E_j > 5$ for $j = 1, 2, \ldots M$ this quantity is distributed as χ^2 with $M-1$ degrees of freedom and this is approximately true even if one or two E values are as small as 2. When the distribution being tested has been fitted to the sample the degrees of freedom are reduced by the number of parameters estimated.

There are no theoretical rules to guide the choice of class intervals but for the sake of consistency and some degree of objectivity they are taken to correspond to preassigned probability values as shown in Table 2.12. For sample sizes less than $N = 20$ the groups would be too few in number or too small so no rule is given for such cases.

Table 2.12 Scheme for computing group boundaries in χ^2 goodness of fit test.

Sample size N	No. of class intervals M	Group boundaries on probability scale expressed as percentages, P
20–29	5	0, 12.5, 37.5, 62.5, 87.5, 100
30–39	7	0, 14, 28, 43, 58, 72, 86, 100
40–49	9	0, 11, 22, 34, 45, 56, 67, 78, 89, 100
50–100	10	0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100

The above example is repeated using this scheme. Since $N = 35$, M is taken to be 7. Each probability value is converted into a reduced variate value $y = -\ln -\ln(P/100)$ and then into the variate of the distribution under test by $Q = u + \alpha y$. With $u = 109.93$ and $\alpha = 42.95$ this gives the following set of group boundaries.

P	0	14	28	43	58	72	86	100
y	$-\infty$	−0.6761	−0.2413	0.1696	0.6075	1.1132	1.8916	∞
Q	$-\infty$	80.89	99.57	117.21	136.02	157.74	191.17	∞

The sample histogram is then formed with these group boundaries and the following values of O, E and $O-E$ for each group obtained.

O	5	9	4	1	5	6	5
E	4.9	4.9	5.25	5.25	4.9	4.9	4.9
$O-E$	0.1	4.1	−1.25	−4.25	0.1	1.1	0.1

The goodness of fit index $\Sigma(O-E)^2/E$ is 7.42; under the hypothesis that the sample came from an EV1 distribution with $u = 109.93$ and $\alpha = 42.95$ the index is distributed as χ^2 where df $= M-3 = 4$ and not taken as $M-1 = 6$ because the parameter values u and α have been estimated from the sample data.

The results of calculating a χ^2 goodness of fit index as described on three distributions for 31 stations with 30 years or more of record in Great Britain and for seven stations of between 23 and 44 years in Ireland are shown in Table 2.13. The distributions are EV1 (Gumbel), lognormal and general extreme value; these were fitted to the data by maximum likelihood except in two cases. These are stations 28/801 and 73/1 where for the general extreme value distribution the iterative approach to the ML estimates did not converge and the sextile estimates were used to define the hypothesised distribution. The results are summarised in Table 2.14. Since each distribution was rejected as often as the others, the choice of one distribution on the basis of the χ^2 test alone is inconclusive. However another interpretation of these results is that each distribution is rejected, since at the 10% level the expected number of rejections would be 4, and more than 8 would be most unlikely under the null hypothesis.

Table 2.14 Number of times out of 38 that distributions were rejected by χ^2 test.

Significance level	Extreme value Type 1 (Gumbel)	Lognormal	General extreme value
0.10	17	15	14
0.05	7	7	11
0.01	2	3	2

Station no.	Name	No. of years	Grade	Extreme value type			Lognormal			General extreme value		
				χ^2	df	$P(\chi^2)$	χ^2	df	$P(\chi^2)$	χ^2	df	$P(\chi^2)$
Great Britain												
6/901	Ness at Ness Castle Farm	33	B	3.39	4	0.50	1.65	4	0.80	1.65	3	0.50
8/1	Spey at Aberlour	30	B	2.51	4	0.50	4.92	4	0.20	4.27	3	0.20
10/1	Ythan at Ardlethan	31	A1	5.76	4	0.20	5.76	4	0.20	4.04	3	0.20
12/1	Dee at Cairnton	40	B	12.65	6	0.02**	12.56	6	0.05**	12.65	5	0.02**
15/5	Melgam at Loch Lintrathen	38	B	29.03	4	<0.001***	22.89	4	<0.001***	27.59	3	<0.001***
27/1	Nidd at Hunsingore	35	A2	7.42	4	0.10*	7.42	4	0.01***	7.07	3	0.05**
27/2	Wharfe at Flint Mill Weir	32	A1	2.75	4	0.50	3.19	4	0.50	1.60	3	0.50
27/10	Hodge Beck at Bransdale	32	A2	1.91	4	0.70	1.85	4	0.70	2.21	3	0.50
27/21	Don at Doncaster	86	B	13.53	7	0.05**	11.21	7	0.10*	13.3	6	0.02**
28/10	Derwent at Longbridge	33	B	6.94	4	0.10*	4.46	4	0.30	5.11	3	0.10*
28/801	Burbage Brook	43	A2	11.47	6	0.05**	8.36	6	0.20	5.26	5	0.30†
28/804	Trent at Trent Bridge	82	B	3.61	7	0.80	4.34	7	0.70	6.78	6	0.30
32/10	Nene at Wansford	30	A2	2.37	4	0.50	5.56	4	0.20	3.11	3	0.10*
39/1	Thames at Teddington	88	A2	8.14	7	0.30	7.00	7	0.30	8.82	6	0.20
54/1	Severn at Bewdley	46	B	0.84	6	0.99	2.02	6	0.90	1.63	5	0.90
54/2	Avon at Evesham	32	A2	3.82	4	0.30	4.24	4	0.30	3.82	3	0.20
55/1	Wye at Cadora	32	A1	3.12	4	0.50	3.12	4	0.50	0.87	3	0.80
55/2	Wye at Belmont	58	C up to 1935, B after.	6.83	7	0.30	8.90	7	0.20	11.66	6	0.10*
55/4	Irfon at Abernant	31	C	7.10	4	0.10*	6.91	4	0.10*	4.51	3	0.20
55/5	Wye at Rhayader	31	C	6.91	4	0.10*	4.19	4	0.30	4.19	3	0.20
55/7	Wye at Erwood	31	A	6.50	4	0.10*	5.41	4	0.20	8.63	3	0.02**
67/2	Dee at Erbistock Rectory	30	B	7.22	4	0.10*	7.11	4	0.10*	7.81	3	0.05**
67/803	Dee at Chester Weir	71	B	7.45	7	0.30	13.08	7	0.05**	13.37	6	0.02**
68/1	Weaver at Ashbrook	31	A2	7.56	4	0.10*	10.04	4	0.02**	3.36	3	0.30
68/5	Weaver at Audlem	33	B	5.53	4	0.20	6.99	4	0.10*	4.56	3	0.20
69/1	Mersey at Irlam Weir	35	A2	5.99	4	0.20	6.78	4	0.10*	6.40	3	0.05**
69/2	Irwell at Adelphi Weir	31	C	5.47	4	0.20	5.47	4	0.20	4.41	3	0.20
69/6	Bollin at Dunham Massey	33	B	14.98	4	0.001***	11.00	4	0.02**	1.49	3	0.50
72/803	Lune at Halton Upper Weir	32	B	7.43	4	0.10*	2.79	4	0.50	4.58	3	0.20
73/1	Leven at Newby Bridge	30	B	6.06	4	0.10*	6.51	4	0.10*	6.51	3	0.05**†
83/802	Irvine at Kilmarnock	53	B	6.06	7	0.50	11.72	7	0.10*	17.38	6	0.001***
Ireland												
201/6	Annalong at Recorder	44	A2	11.35	6	0.05**	18.65	6	0.001***	10.12	5	0.05**
309/809	Boyne at Slane	29	B	1.21	2	0.50	1.21	2	0.50	0.93	1	0.30
323/2	Feale at Listowel Bridge	24	A1	1.67	2	0.30	1.00	2	0.50	1.00	1	0.30
390/807	Lee at Dromcarra	23	A	2.04	2	0.30	2.04	2	0.30	0.65	1	0.70
392/805	Cummeragh at Cummeragh Weir	28	B	3.57	2	0.10*	3.57	2	0.10*	1.29	1	0.20
393/803	Avonmore at Laragh	23	B	7.78	2	0.02**	3.09	2	0.20	1.52	1	0.20
394/808	Lackagh at Lackagh Bridge	25	B	1.40	2	0.30	0.44	2	0.80	0.60	1	0.30

The Kolmogorov–Smirnov index

This is based on the difference between the empirical distribution function and the distribution function under test. Figure 2.8 shows these functions plotted for the annual maximum data of 27/1 and the extreme value Type 1 distribution used in the χ^2 example above with u = 109.93 and α = 42.95. The empirical distribution function $S_N(Q)$ is defined by

$$S_N(Q) = \begin{cases} 0 & Q < Q_{(1)} \\ \frac{i}{N} & Q_{(i)} \leqslant Q < Q_{(i+1)} \\ 1 & Q_{(N)} \leqslant Q \end{cases} \qquad (2.4.2.2)$$

where $Q_{(1)}$ and $Q_{(N)}$ are the smallest and largest sample members and $Q_{(i)}$ is the ith smallest.

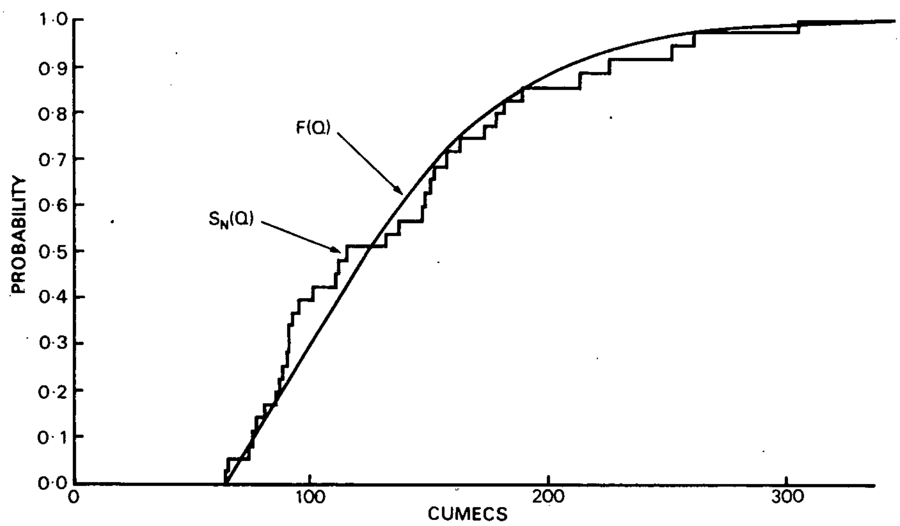


Fig 2.8 Empirical distribution function, $S_N(Q)$, and fitted EV1 distribution function $F(Q)$ for 27/1.

At each observed Q value, $Q_{(i)}$, i = 1, 2, . . . N the difference between $F(Q)$ and $S_N(Q)$ has two values as $S_N(Q)$ changes at each such value of Q. Denote these two values, which are illustrated in Figure 2.9, by $\partial+$ and $\partial-$

$$\partial_i^+ = i/N - F(Q) \qquad (2.4.2.3)$$

$$\partial_i^- = F(Q) - (i-1)/N$$

and let

$$d_i = \max(\partial_i^+, \partial_i^-) \qquad (2.4.2.4)$$

The maximum of all the d_i values is the Kolmogorov–Smirnov goodness of fit index, D_N

$$D_N = \max(d_1, d_2, \ldots d_N). \qquad (2.4.2.5)$$

The sampling distribution of D_N is known for all values of N when the sample actually comes from the distribution under test. It depends only on N and not on the form or parameters of the hypothetical distribution. For large N, $N \geqslant 35$, the upper 10, 5 and 1 % points of the distribution are 1.22/$\sqrt{N}$, 1.36/$\sqrt{N}$ and 1.63/$\sqrt{N}$ respectively. If the sample is not drawn from the hypothetical distribution these values are expected to be exceeded more frequently than 10, 5 or 1 % of the time as the case may be. In practice,

Table 2.13 χ^2 goodness of fit values. (One, two and three asterisks indicate rejection of hypothesis at significance levels of 0.10, 0.05 and 0.01 respectively.)

†KS and χ^2 calculated using sextile estimates u, α and k.

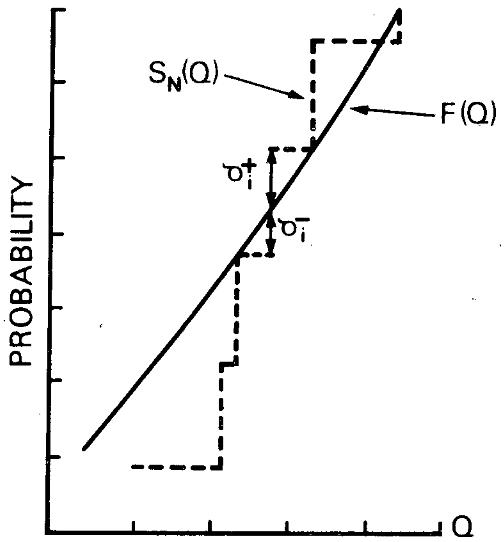


Fig 2.9 Illustration of the two distinct differences, ∂^+ and ∂^- between $F(Q)$ and $S_N(Q)$ at each data point as used in Kolmogorov–Smirnov test.

if an observed value of D_N exceeds one of these critical values the distribution under test is rejected at that level of confidence.

Table 2.15 shows in abridged form how D_N is calculated for the annual maximum data of 27/1 and the fitted extreme value Type 1 distribution (u = 109.93, α = 42.95). The value of D_N is 0.1535. The 10, 5 and 1% of the distribution of D_N under this EV1 hypothesis are 0.21, 0.23 and 0.27 based on N = 35. Therefore, the observed D_N is not significantly large. By analogy with the χ^2 test procedure the percentage points were also computed on a value of $N-2$ to allow for the estimated parameters but the percentage points were the same to two decimal places.

Table 2.15 Example calculation of Kolmogorov–Smirnov D_N.

Order i	Sample value $Q_{(i)}$	$F(Q)$	$S_N(Q)$	$\partial_i^- = F(Q)-\left(\frac{i-1}{N}\right)$	$\partial_i^+ = \frac{i}{N}-F(Q)$	$d_i = \max(\partial_i^+, \partial_i^-)$
1	65.08	0.0583	0.0286	0.0583	−0.0297	0.0583
2	65.60	0.0604	0.0571	0.0318	−0.0033	0.0318
3	75.06	0.1052	0.0857	0.0481	−0.0195	0.0481
4	76.22	0.1117	0.1143	0.0260	+0.0026	0.0260
.	.	.	.	.	.	.
13	92.82	0.2255	0.3714	−0.1174	+0.1459	0.1459
14	95.47	0.2465	0.4000	−0.1249	+0.1535	0.1535
15	100.40	0.2870	0.4286	−0.1130	+0.1416	0.1416
16	111.54	0.3817	0.4571	−0.0469	+0.0754	0.0754
.	.	.	.	.	.	.
33	251.96	0.9640	0.9429	0.0497	−0.0211	0.0497
34	261.82	0.9713	0.9714	0.0284	+0.0001	0.0284
35	305.75	0.9896	1.0000	0.0182	+0.0104	0.0182
				D_N^- = 0.0948	D_N^+ = 0.1535	D_N = 0.1535

The D_N statistic was calculated for the stations listed in Table 2.13 for four distributions—the extreme value Type 1, two parameter gamma, lognormal and general extreme value—and is tabulated in Table 2.16 together with the 0.10, 0.05 and 0.01 points of the D_N distribution in each case. The numbers of times out of 38 that these significance levels were exceeded are tabulated in Table 2.17. Only four rejections of the null

Station no.	Sample size	Distribution: Extreme value Type 1	Gamma	Lognormal	General extreme value	Kolmogorov–Smirnov critical points: 10% = $1.22/\sqrt{N}$	5% = $1.36/\sqrt{N}$	1% = $1.63/\sqrt{N}$
Great Britain								
6/901	33	0.097	0.127	0.116	0.105	0.212	0.237	0.284
8/1	30	0.095	0.122	0.098	0.084	0.223	0.248	0.298
10/1	31	0.117	0.113	0.108	0.119	0.219	0.244	0.293
12/1	40	0.125	0.113	0.141	0.140	0.193	0.215	0.258
15/5	38	0.223**	0.193	0.209*	0.186	0.198	0.221	0.264
27/1	35	0.153	0.151	0.141	0.135	0.206	0.230	0.276
27/2	32	0.099	0.114	0.099	0.088	0.216	0.240	0.288
27/10	32	0.084	0.118	0.091	0.105	0.216	0.240	0.288
27/21	86	0.066	0.065	0.064	0.065	0.132	0.147	0.176
28/10	33	0.161	0.204	0.171	0.134	0.212	0.237	0.284
28/801	43	0.146	0.206*	0.154	0.116	0.186	0.207	0.249
28/804	82	0.055	0.049	0.069	0.048	0.135	0.150	0.180
32/10	30	0.117	0.160	0.125	0.129	0.223	0.248	0.298
39/1	88	0.071	0.092	0.075	0.068	0.130	0.145	0.174
54/1	46	0.055	0.068	0.050	0.054	0.180	0.201	0.240
54/2	32	0.084	0.095	0.142	0.079	0.216	0.240	0.288
55/1	32	0.079	0.114	0.101	0.076	0.216	0.240	0.288
55/2	58	0.069	0.098	0.081	0.090	0.160	0.179	0.214
55/4	32	0.106	0.132	0.117	0.098	0.216	0.240	0.288
55/5	31	0.123	0.134	0.104	0.105	0.219	0.244	0.293
55/7	31	0.126	0.113	0.116	0.127	0.219	0.244	0.293
67/2	30	0.127	0.126	0.113	0.112	0.223	0.248	0.298
67/803	71	0.117	0.118	0.111	0.106	0.145	0.161	0.193
68/1	31	0.112	0.155	0.122	0.098	0.219	0.244	0.293
68/5	33	0.129	0.106	0.125	0.089	0.212	0.237	0.284
69/1	35	0.113	0.121	0.115	0.110	0.206	0.230	0.276
69/2	31	0.127	0.115	0.139	0.109	0.219	0.244	0.292
69/6	33	0.170	0.154	0.164	0.095	0.212	0.237	0.284
72/803	32	0.119	0.105	0.103	0.107	0.216	0.240	0.288
73/1	30	0.187	0.186	0.206	0.158	0.223	0.248	0.298
83/802	53	0.132	0.176*	0.155	0.124	0.168	0.187	0.224
Ireland								
201/006	44	0.146	0.139	0.167	0.138	0.184	0.205	0.246
309/809	29	0.126	0.155	0.130	0.076	0.227	0.253	0.303
323/002	24	0.116	0.092	0.092	0.093	0.249	0.278	0.333
390/807	23	0.161	0.170	0.148	0.170	0.254	0.284	0.340
392/805	28	0.126	0.096	0.098	0.100	0.231	0.257	0.308
393/803	23	0.194	0.213	0.184	0.129	0.254	0.284	0.340
394/808	25	0.159	0.190	0.165	0.092	0.244	0.272	0.326

Table 2.16 Kolmogorov–Smirnov test statistic D_N.

hypothesis are registered so that in contrast to the results of the χ^2 test this test allows each of these hypotheses to be entertained. On the other hand it is unable to discriminate between the distributions.

	Extreme value Type 1 (Gumbel)	Two parameter gamma	Lognormal	General extreme value
0.10	1	2	1	0
0.05	1	0	0	0
0.01	0	0	0	0

Table 2.17 Number of times out of 38 that distributions were rejected by Kolmogorov–Smirnov test.

Comment on χ^2 and Kolmogorov–Smirnov tests

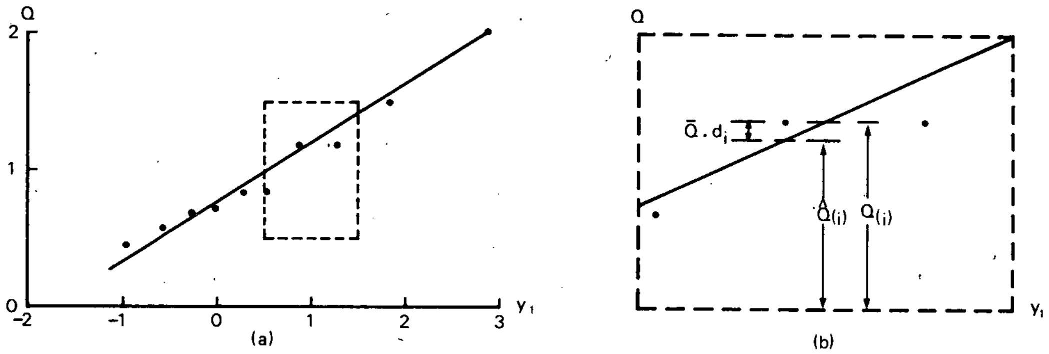
These goodness of fit indices and tests are the main statistical indices which are available from theoretical statistics, perhaps because the derivation of their sampling distributions was found to be mathematically tractable. Other, possibly more appropriate, goodness of fit indices have not been developed to the point of common usage. Some goodness of fit indices which have been devised by hydrologists (see 2.4.3) lack a complete statistical treatment. The dependence of the sampling distribution of these indices on the parent distribution under test was seldom considered relevant, though they have been used as a basis of choosing a best fitting distribution.

2.4.3 Goodness of fit indices based on probability or x–y plot

The distribution and the sample to be compared are represented by a continuous line or curve and a set of plotted points on the same diagram, as shown on Figure 2.10. Denote the variate value on the curve at the *i*th plotting position by $\hat{Q}_{(i)}$, the observed *i*th value by $Q_{(i)}$ and the mean of the sample data by $\bar{Q}$. The quantity

$$d_i = \frac{Q_{(i)} - \hat{Q}_{(i)}}{\bar{Q}} \qquad (2.4.3.1)$$

Fig 2.10 (a) A probability plot. (b) Illustration of d_i used in some goodness of fit tests.



is a standardised measure of the difference between the observed sample and the distribution at the *i*th plotting position. Division by $\bar{Q}$ is meant to remove the effect of catchment size and other catchment characteristics so that any statistic computed from $d_1, d_2, \ldots d_i \ldots d_N$ would express goodness of fit alone and allow comparable figures for different catchments to be obtained, thus allowing the choice of best distribution as the one giving the lowest average index value.

The index itself may take one of a number of forms, for instance the mean absolute value of $d_1, d_2, \ldots d_N$, or their standard deviation; but in hydrological studies (Singh & Sinclair, 1972; Prasad, 1970) only the mean absolute value

$$\overline{|d|} = \frac{1}{N} \sum_{i=1}^{N} |d_i| \qquad (2.4.3.2)$$

has been used. However, the choice of a single distribution from among a set of competing distributions can depend on which of these two indices is used as criterion. Another source of profusion of these types of index arises from the manner in which $\hat{Q}_{(i)}$ and hence d_i is computed: $\hat{Q}_{(i)}$ depends on the plotting probability which is traditionally taken as the Weibull formula $F_i = i/(N+1)$ which is not recommended because of its bias. Three types of index A, B and C are distinguished here.

A Index based on variate differences, d

The differences d_i are taken as

$$d_i = \frac{Q_{(i)} - E(Q_{(i)})}{\bar{Q}}. \tag{2.4.3.3}$$

That is, $\hat{Q}_{(i)} = E(Q_{(i)})$, which depends both on the parameters and form of the distribution and implies that the plotting position $E(y_{(i)})$ or its equivalent on the probability scale is used.

B Index based on probability differences, d'

The differences d_i' are taken as

$$d_i' = F(Q_{(i)}) - E(F_i) \tag{2.4.3.4}$$

where $F(Q_{(i)})$ is the hypothesised distribution function evaluated at $Q_{(i)}$ and $E(F_i) = i/(N+1)$ is the mean value of $F(Q_{(i)})$ over all possible samples. This is independent of the form of $F(Q)$. $F(Q_{(i)})$ is itself a random variable having a Beta distribution with parameters i and $N+1-i$.

C Hybrid index (d″)

The differences d_i'' are taken as

$$d'' = \frac{Q_{(i)} - Q(i/(N+1))}{\bar{Q}} \tag{2.4.3.5}$$

where $Q(i/(N+1))$ is the value of Q corresponding to $F(Q) = i/(N+1)$. This difference is of Type A if $Q(i/(N+1))$ is interpreted as an approximation to $E(Q_{(i)})$. This interpretation is exact only if the distribution is uniform but is quite inaccurate with most annual maximum distributions. However, because of the general misconception that the Weibull plotting probability $F_i = i/(N+1)$ is best it has been widely used recently in hydrology. For instance Singh & Sinclair (1972) used the mean absolute value of d'' to decide between the log Pearson Type 3 and a mixture of two Normal distributions and Prasad (1970) used it to choose the best fitting of eight distributions fitted to each of 329 sets of data. If $Q(i/(N+1))$ were replaced by $Q((i-\frac{1}{2})/N)$ the index would be of the same type but since $Q((i-\frac{1}{2})/N)$ is not an unreasonable approximation to $E(Q_{(i)})$ in any distribution the resulting index might be called an index of Type A.

Statistics based on deviations of any of the three types A, B and C when the sample sizes are not equal should be treated with caution. For a given sample size a given root mean square value or mean absolute value of any one of these deviations might be statistically small for one distribution but statistically large for another distribution. To what extent this affects the conclusions can only be provided by the (unknown) sampling properties of these statistics.

Computations

Goodness of fit indices have been computed for seven distributions on the data of 35 long term stations. They are extreme value Type 1 (Gumbel), gamma, lognormal, general extreme value (Jenkinson), Pearson Type 3, log gamma and log Pearson Type 3. The parameter estimates used were obtained by maximum likelihood except for the Pearson Type 3 and log Pearson Type 3 distributions where moments estimates were used.

The index values computed were all of Type A or C in terms of the earlier classification, being comparisons of variate values rather than comparisons of probability values. Because of its widespread use the hybrid index $\{Q_{(i)} - Q(i/(N+1))\}/\bar{Q}$ was computed for each distribution. The index $\{Q_{(i)} - Q((i-\frac{1}{2})/N)\}/\bar{Q}$ corresponding to the Hazen formula (Hazen, 1914, 1930) was also computed for each distribution. In addition the index $\{Q_{(i)} - E(Q_{(i)})\}/\bar{Q}$ was computed for both the Gumbel and lognormal distributions. In these cases the Gringorten (1963) and Blom (1958) formulae were used to provide good approximations to $E(Q_{(i)})$. Computations were carried out for the general extreme value distribution using the Blom and Gringorten formulae as well as the Hazen and Weibull formulae. The values of $E(Q_{(i)})$ cannot be computed exactly so the following scheme was used:

for $k < -0.15$	base $E(Q_{(i)})$ approximation on the Hazen formula
k between -0.15 and 0.15	base it on the Gringorten formula which is almost exact when $k = 0$
$k > 0.15$	base it on the Blom formula.

This involves some compromise but it recognises the fact that $E(Q_{(i)})$ depends on the value of k. When computing $E(Q_{(i)})$ for the gamma, log gamma, Pearson Type 3 and log Pearson Type 3 the approximation $Q((i-\frac{1}{2})/N)$ was used although $Q\{(i-2/5)/(N+1/5)\}$ might have been an equally acceptable compromise. The exact formulae for these distributions are unknown.

Results

The main results are shown in Tables 2.18–2.21. In each table there is an index value for each of the seven distributions for each of 35 stations. These are

Table 2.18: Mean absolute value of $\{Q_{(i)} - E(Q_{(i)})\}/\bar{Q}$

Table 2.19: Mean absolute value of $\left\{Q_{(i)} - Q\left(\frac{i-\frac{1}{2}}{N}\right)\right\}\Big/\bar{Q}$

Table 2.20: Mean absolute value of $\left\{Q_{(i)} - Q\left(\frac{i}{N+1}\right)\right\}\Big/\bar{Q}$

Table 2.21: Root mean square value of $\{Q_{(i)} - E(Q_{(i)})\}/\bar{Q}$

For each station the seven distributions are ranked by their index values, from smallest to largest. At the bottom of each column the sum of the ranks attributed to each distribution is shown.

The distributions may be ranked on the basis of these sums. The entries in Table 2.22(a) are the subtotals in Tables 2.18–2.21 for the stations in Great Britain while the totals including Irish stations are given in Table 2.22(b).

	Gumbel	Gamma	Lognormal	GEV	Pearson 3	Log Pearson 3	Log gamma
Formula	Gringorten	Hazen	Blom	Blom, Gringorten Hazen	Hazen	Blom	Blom
Great Britain							
6/901	2.46^{6}	2.79^{7}	2.45^{5}	2.32^{2}	2.33^{3}	2.28^{1}	2.41^{4}
10/1	5.06^{7}	4.17^{1}	4.49^{4}	4.26^{2}	4.27^{3}	5.02^{6}	4.60^{5}
12/1	8.49^{2}	8.87^{6}	8.68^{3}	8.47^{1}	10.15^{7}	8.69^{4}	8.74^{5}
15/4	4.33^{6}	3.42^{2}	4.17^{5}	3.69^{3}	3.73^{4}	3.18^{1}	5.89^{7}
15/5	5.93^{7}	5.14^{2}	5.06^{1}	5.30^{5}	5.18^{3}	5.27^{4}	5.47^{6}
27/1	6.90^{6}	6.19^{5}	5.18^{2}	7.81^{7}	4.96^{1}	5.20^{3}	5.50^{4}
27/2	3.75^{3}	4.71^{7}	4.12^{5}	3.61^{1}	4.11^{4}	3.63^{2}	4.36^{6}
27/10	6.27^{5}	7.52^{6}	6.17^{4}	5.04^{1}	8.30^{7}	5.58^{3}	5.43^{2}
27/21	11.32^{7}	4.63^{4}	4.32^{1}	4.34^{2}	4.77^{5}	4.36^{3}	4.78^{6}
28/10	11.05^{6}	13.59^{7}	10.99^{5}	7.86^{2}	8.91^{3}	7.72^{1}	10.31^{4}
28/801	18.68^{6}	21.29^{7}	17.00^{5}	9.61^{1}	13.18^{3}	9.82^{2}	13.52^{4}
28/804	4.00^{5}	3.35^{4}	5.49^{6}	3.18^{3}	2.79^{1}	2.82^{2}	7.23^{7}
39/1	3.75^{2}	4.88^{6}	3.88^{4}	3.62^{1}	5.81^{7}	3.84^{3}	4.06^{5}
54/1	1.92^{2}	2.59^{7}	1.99^{3}	1.99^{3}	2.12^{5}	1.89^{1}	2.58^{6}
54/2	5.34^{2}	6.47^{4}	16.63^{6}	5.12^{1}	5.44^{3}	10.66^{5}	30.54^{7}
55/1	2.22^{4}	2.96^{6}	2.43^{5}	1.90^{2}	1.77^{1}	1.94^{3}	4.70^{7}
55/2	2.58^{3}	4.43^{7}	3.56^{6}	1.83^{1}	2.68^{4}	1.96^{2}	3.33^{5}
55/4	3.10^{3}	4.54^{7}	3.90^{6}	3.07^{2}	3.54^{4}	3.03^{1}	3.74^{5}
55/5	7.33^{7}	6.95^{5}	6.12^{2}	6.72^{4}	7.06^{6}	6.05^{1}	6.21^{3}
55/7	4.91^{5}	5.45^{7}	4.29^{3}	5.42^{6}	4.26^{1}	4.27^{2}	4.40^{4}
67/2	8.04^{7}	7.55^{6}	6.41^{4}	6.57^{5}	4.16^{1}	5.33^{2}	5.35^{3}
67/803	3.37^{5}	3.89^{7}	3.07^{2}	3.79^{6}	3.05^{1}	3.10^{3}	3.12^{4}
68/1	9.40^{5}	11.53^{7}	8.55^{4}	6.52^{1}	10.44^{6}	6.90^{2}	7.89^{3}
68/5	7.90^{6}	4.81^{4}	7.10^{5}	3.73^{2}	3.38^{1}	3.79^{3}	9.25^{7}
69/1	3.85^{5}	4.13^{7}	3.75^{3}	3.83^{4}	3.92^{6}	3.58^{1}	3.71^{2}
69/2	6.09^{5}	5.85^{3}	6.62^{6}	5.83^{2}	5.81^{1}	6.03^{4}	7.55^{7}
69/6	5.78^{7}	3.57^{4}	4.19^{5}	1.89^{1}	2.26^{2}	2.29^{3}	4.44^{6}
83/802	5.29^{3}	8.76^{7}	6.99^{6}	4.29^{2}	6.98^{5}	4.01^{1}	6.29^{4}
Subtotals	137	152	116	73	98	69	138
	5	7	4	2	3	1	6
Ireland							
201/6	9.46^{5}	7.50^{4}	10.11^{6}	6.65^{2}	6.31^{1}	7.49^{3}	14.00^{7}
309/809	6.55^{4}	8.24^{7}	6.61^{5}	4.84^{1}	6.97^{6}	5.33^{2}	6.17^{3}
323/2	3.66^{6}	3.32^{1}	3.47^{3}	3.52^{5}	3.40^{2}	3.50^{4}	4.55^{7}
390/807	6.71^{6}	6.58^{5}	6.41^{1}	7.80^{7}	6.56^{4}	6.52^{3}	6.48^{2}
392/805	2.58^{7}	1.89^{3}	1.93^{5}	1.83^{1}	1.91^{4}	1.85^{2}	2.15^{6}
393/803	10.85^{6}	11.00^{7}	8.73^{4}	8.64^{3}	9.42^{5}	7.56^{1}	8.43^{2}
394/808	8.41^{6}	8.43^{7}	7.17^{5}	6.84^{3}	4.95^{2}	4.49^{1}	6.99^{4}
Subtotals	40	34	29	22	24	16	31
Totals	177	186	145	95	122	85	169

Table 2.18 Approximation to mean absolute value of $\frac{Q_{(i)}-E(Q_{(i)})}{\bar{Q}}$ as a percentage.

These show that

a three parameter distributions fit more closely than two parameter distributions,

b the lognormal distribution fits more closely than any other two parameter distribution,

c (i) on the basis of the mean absolute deviations the order is log Pearson Type 3 better than GEV better than Pearson Type 3 and this is independent of the plotting position formula used,

c (ii) on the basis of the root mean square deviation the Pearson Type 3 is better than both the log Pearson Type 3 and the GEV.

Examination of the ranks in the original Tables 2.18–2.21 shows that the ranks attributed to the log Pearson Type 3 distribution vary less between stations than those attributed to the GEV distribution.

The main practical disadvantage of the results is that if the mean absolute deviation is used as criterion then the log Pearson Type 3 distribution is indicated whereas if the root mean square deviation is used as criterion the Pearson Type 3 is indicated. But these distributions are quite

	Gumbel	Gamma	Lognormal	GEV	Pearson 3	Log Pearson 3	Log gamma
Great Britain							
6/901	2.51^{6}	2.79^{7}	2.36^{4}	2.29^{1}	2.33^{3}	2.31^{2}	2.39^{5}
10/1	5.25^{7}	4.17^{1}	4.45^{4}	4.27^{2}	4.27^{2}	5.01^{6}	4.61^{5}
12/1	8.45^{2}	8.87^{6}	8.52^{4}	8.37^{1}	10.15^{7}	8.50^{3}	8.53^{5}
15/4	4.44^{6}	3.42^{2}	4.39^{5}	3.70^{3}	3.73^{4}	3.27^{1}	6.14^{7}
15/5	6.03^{7}	5.14^{1}	5.15^{2}	5.23^{5}	5.18^{3}	5.19^{4}	5.64^{6}
27/1	6.71^{6}	6.19^{5}	5.13^{2}	7.81^{7}	4.96^{1}	5.49^{3}	5.54^{4}
27/2	3.78^{2}	4.71^{7}	4.15^{5}	3.69^{1}	4.11^{4}	3.78^{2}	4.43^{6}
27/10	6.18^{5}	7.52^{6}	6.01^{4}	5.04^{1}	8.30^{7}	5.48^{3}	5.34^{2}
27/21	11.42^{7}	4.63^{4}	4.46^{3}	4.37^{1}	4.77^{5}	4.42^{2}	4.98^{6}
28/10	10.96^{6}	13.59^{7}	10.81^{5}	7.86^{2}	8.91^{3}	7.15^{1}	10.01^{4}
28/801	18.61^{6}	21.29^{7}	16.78^{5}	9.61^{2}	13.18^{4}	8.83^{1}	13.09^{3}
28/804	4.03^{5}	3.35^{4}	5.67^{6}	3.18^{3}	2.79^{1}	2.84^{2}	7.52^{7}
39/1	3.71^{2}	4.88^{7}	3.83^{5}	3.58^{1}	5.81^{4}	3.79^{3}	4.08^{6}
54/1	1.94^{2}	2.59^{6}	2.01^{4}	1.97^{3}	2.12^{5}	1.87^{1}	2.66^{7}
54/2	5.42^{2}	6.47^{4}	18.06^{6}	5.16^{1}	5.44^{3}	10.72^{5}	33.18^{7}
55/1	2.14^{4}	2.96^{6}	2.39^{5}	1.91^{3}	1.77^{1}	1.78^{2}	4.62^{7}
55/2	2.54^{3}	4.43^{7}	3.51^{6}	1.80^{1}	2.68^{4}	1.94^{2}	3.27^{5}
55/4	3.10^{1}	4.54^{7}	3.87^{6}	3.13^{2}	3.54^{4}	3.22^{3}	3.67^{5}
55/5	7.37^{7}	6.95^{5}	6.30^{3}	6.72^{4}	7.06^{6}	6.24^{1}	6.28^{2}
55/7	4.75^{5}	5.45^{7}	4.37^{2}	5.42^{6}	4.26^{1}	4.38^{3}	4.49^{4}
67/2	7.85^{7}	7.55^{6}	6.01^{4}	6.57^{5}	4.16^{1}	5.42^{3}	5.31^{2}
67/803	3.33^{5}	3.89^{6}	3.06^{2}	3.89^{6}	3.05^{1}	3.28^{4}	3.21^{3}
68/1	9.28^{5}	11.53^{7}	8.58^{4}	6.52^{1}	10.44^{6}	6.57^{2}	7.82^{3}
68/5	8.10^{6}	4.81^{4}	7.47^{5}	3.73^{2}	3.38^{1}	3.79^{3}	9.71^{7}
69/1	3.85^{4}	4.13^{7}	3.78^{3}	3.87^{5}	3.92^{6}	3.61^{1}	3.71^{2}
69/2	6.30^{5}	5.85^{3}	7.08^{6}	5.80^{1}	5.81^{2}	5.98^{4}	8.07^{7}
69/6	5.87^{7}	3.57^{4}	4.28^{5}	1.89^{1}	2.26^{2}	2.34^{3}	4.54^{6}
83/802	5.25^{3}	8.76^{7}	7.00^{6}	4.22^{2}	6.98^{5}	3.91^{1}	6.26^{4}
Subtotals	133	150	121	73	96	71	137
Ireland							
201/6	9.62^{5}	7.50^{4}	10.48^{6}	6.61^{2}	6.31^{1}	7.45^{3}	14.54^{7}
309/809	6.46^{4}	8.24^{7}	6.59^{5}	4.84^{1}	6.97^{6}	5.00^{2}	6.09^{3}
323/2	3.74^{6}	3.32^{2}	3.30^{1}	3.43^{5}	3.40^{4}	3.32^{2}	4.81^{7}
390/807	6.83^{6}	6.58^{2}	6.65^{3}	7.80^{7}	6.56^{1}	6.80^{5}	6.73^{4}
392/805	2.62^{7}	1.89^{3}	1.95^{5}	1.87^{1}	1.91^{4}	1.87^{1}	2.16^{6}
393/803	10.67^{6}	11.00^{7}	8.74^{4}	8.64^{3}	9.42^{5}	7.83^{1}	8.13^{2}
394/808	8.22^{6}	8.43^{7}	6.82^{4}	6.84^{5}	4.95^{2}	4.75^{1}	6.53^{3}
Subtotals	40	32	28	24	23	15	32
Totals	173	182	149	97	119	86	169

Table 2.19 Mean absolute value of $\dfrac{Q_{(i)} - Q\left(\frac{i-\frac{1}{2}}{N}\right)}{\bar{Q}}$ as a percentage.

different. Since there is no logical basis for choosing one criterion rather than the other this result is unsatisfactory. The popular American practice has been to use the mean absolute deviation as the criterion in this type of work. On this basis the log Pearson Type 3 and general extreme value distributions are best. These distributions are not incompatible, because within a certain range of parameter values $Q(T)$ increases with T in the same manner in both.

In the work described the three parameter Pearson Type 3 and log Pearson Type 3 distributions both fared well; the fact that these were both fitted by moments might have given them an unfair advantage over distributions fitted by maximum likelihood, because in positively skewed distributions the largest sample values are most variable and the moments are affected most by the largest values. On the other hand the maximum likelihood estimators give relatively more weight to the smaller values. However, repetition of the computations for the EV1, gamma and log-normal distributions with moments estimates showed that the inference drawn could not be attributed to the method of estimation.

	Gumbel	Gamma	Lognormal	GEV	Pearson 3	Log Pearson 3	Log gamma
Great Britain							
6/901	2.66^{1}	3.19^{7}	2.90^{4}	2.93^{6}	2.85^{3}	2.79^{2}	2.91^{5}
10/1	4.79^{6}	4.29^{1}	4.48^{4}	4.32^{2}	4.32^{2}	4.98^{7}	4.62^{5}
12/1	8.81^{1}	8.92^{2}	9.01^{4}	8.95^{3}	10.73^{7}	9.05^{5}	9.21^{6}
15/4	3.75^{4}	3.37^{2}	3.78^{5}	3.84^{6}	3.74^{3}	3.13^{1}	5.39^{7}
15/5	5.37^{3}	5.57^{7}	5.33^{2}	5.56^{6}	5.55^{5}	5.52^{4}	5.22^{1}
27/1	8.01^{7}	6.94^{6}	6.21^{3}	6.50^{4}	5.76^{2}	5.58^{1}	6.65^{5}
27/2	3.79^{3}	4.92^{7}	4.16^{5}	3.38^{1}	3.94^{4}	3.53^{2}	4.38^{6}
27/10	6.79^{5}	7.78^{6}	6.62^{4}	5.95^{1}	8.30^{7}	5.97^{2}	6.21^{3}
27/21	11.46^{7}	4.65^{5}	4.19^{1}	4.21^{2}	4.67^{6}	4.24^{3}	4.39^{4}
28/10	11.73^{6}	14.70^{7}	11.48^{5}	9.37^{2}	10.13^{3}	8.89^{1}	10.93^{4}
28/801	18.99^{6}	21.56^{7}	17.41^{5}	12.17^{2}	14.02^{3}	11.71^{1}	14.37^{4}
28/804	3.91^{5}	3.38^{3}	5.20^{6}	3.54^{4}	3.09^{2}	2.88^{1}	6.58^{7}
39/1	4.02^{4}	5.05^{6}	3.96^{3}	3.79^{1}	5.68^{7}	3.92^{2}	4.09^{5}
54/1	2.10^{2}	2.83^{7}	2.18^{3}	2.21^{4}	2.23^{5}	2.06^{1}	2.64^{6}
54/2	5.38^{1}	5.78^{3}	13.38^{6}	5.92^{4}	5.64^{2}	10.48^{5}	24.88^{7}
55/1	2.76^{5}	3.08^{6}	2.67^{4}	2.51^{3}	2.18^{1}	2.33^{2}	4.89^{7}
55/2	2.85^{4}	4.49^{7}	3.65^{6}	2.10^{1}	2.71^{3}	2.14^{2}	3.49^{5}
55/4	3.65^{3}	4.53^{7}	3.98^{5}	3.35^{1}	3.65^{3}	3.35^{1}	4.01^{6}
55/5	7.70^{7}	7.03^{6}	6.27^{1}	6.99^{5}	6.56^{3}	6.27^{1}	6.57^{4}
55/7	5.92^{6}	6.25^{7}	5.10^{4}	4.98^{3}	4.91^{2}	4.60^{1}	5.31^{5}
67/2	9.21^{7}	8.45^{6}	7.46^{5}	6.59^{4}	5.16^{1}	6.03^{2}	6.52^{3}
67/803	3.92^{6}	4.26^{7}	3.31^{3}	3.35^{4}	3.21^{2}	3.12^{1}	3.40^{5}
68/1	10.10^{5}	11.42^{7}	8.79^{4}	7.45^{2}	10.83^{6}	7.40^{1}	8.22^{3}
68/5	6.69^{6}	4.54^{4}	6.18^{5}	3.75^{2}	3.51^{1}	3.77^{3}	8.14^{7}
69/1	4.02^{6}	4.10^{7}	3.79^{2}	3.97^{5}	3.84^{3}	3.70^{1}	3.84^{3}
69/2	6.22^{5}	6.01^{3}	6.50^{6}	5.94^{1}	5.94^{1}	6.09^{4}	6.76^{7}
69/6	5.32^{7}	3.53^{4}	4.00^{5}	2.05^{1}	2.20^{2}	2.33^{3}	4.23^{6}
83/802	5.58^{3}	8.56^{7}	6.93^{5}	4.59^{2}	7.08^{6}	4.38^{1}	6.31^{4}
Subtotals	131	154	115	82	95	61	140
Ireland	5	7	4	2	3	1	6
201/6	8.57^{5}	7.43^{3}	9.26^{6}	6.75^{2}	6.58^{1}	7.58^{4}	12.69^{7}
309/809	7.26^{5}	7.96^{7}	6.65^{4}	5.90^{1}	7.54^{6}	5.95^{2}	6.54^{3}
323/2	4.10^{7}	3.89^{1}	3.89^{1}	4.02^{6}	3.98^{5}	3.92^{3}	3.95^{4}
390/807	7.63^{6}	7.70^{7}	6.63^{2}	6.65^{3}	7.09^{5}	6.16^{1}	6.92^{4}
392/805	2.46^{7}	2.02^{3}	2.06^{4}	2.11^{5}	1.90^{1}	1.99^{2}	2.32^{6}
393/803	12.08^{6}	12.15^{7}	9.97^{3}	7.92^{1}	9.99^{4}	7.94^{2}	10.11^{5}
394/808	9.58^{7}	9.29^{6}	8.17^{5}	5.47^{1}	5.82^{3}	5.74^{2}	8.09^{4}
Subtotals	43	34	25	19	25	16	33
Totals	174	188	140	101	120	77	173

Table 2.20 Mean absolute deviation

$$\text{of } \frac{Q_{(i)} - Q\left(\frac{i}{N+1}\right)}{\bar{Q}} \text{ as a percentage.}$$

2.4.4 Goodness of fit indices used in the United States Water Resources Council study

In 1966 the Hydrology Committee of the United States Water Resources Council was directed to set up a work group on flow frequency methods. Its brief was to 'present a set of techniques of frequency analysis that are based on the best known hydrological and statistical procedures' and that 'its report should describe those procedures among the suitable methods which, in its judgement, should be standardised in Federal practice'. The Committee's findings were published by Benson (1968).

The work group tested the goodness of fit of six distributions to each of 10 records, the shortest of which was 40 years. At each station the value found by interpolation between adjacent observations on a probability plot at return periods 2, 5, 10, 25 and 50 years is compared with the fitted value obtained from each distribution and the difference expressed as a percentage of the data value. These percentage differences and in particular the average values at each return period for each distribution form the basis of the study. The lognormal, log Pearson Type 3 and Hazen

	Gumbel	Gamma	Lognormal	GEV	Pearson 3	Log Pearson 3	Log gamma
Formula	Gringorten	Hazen	Blom	Blom, Gringorten Hazen	Hazen	Blom	Blom
Great Britain							
6/901	3.00 [1]	3.61 [7]	3.16 [5]	3.00 [1]	3.00 [1]	3.00 [1]	3.16 [5]
10/1	6.32 [6]	5.66 [3]	5.57 [2]	5.83 [5]	5.48 [1]	7.28 [7]	5.66 [3]
12/1	13.64 [6]	14.39 [7]	12.96 [4]	12.25 [1]	12.96 [4]	12.73 [3]	12.65 [2]
15/4	8.31 [6]	5.66 [4]	7.48 [5]	4.69 [1]	5.20 [3]	4.90 [2]	11.31 [7]
15/5	7.00 [7]	6.00 [1]	6.08 [2]	6.16 [5]	6.08 [2]	6.08 [2]	6.56 [6]
27/1	8.49 [6]	8.00 [5]	6.48 [3]	16.76 [7]	6.24 [1]	6.40 [2]	6.71 [4]
27/2	5.20 [4]	5.66 [6]	5.10 [2]	6.08 [7]	5.10 [2]	5.00 [1]	5.20 [4]
27/10	15.97 [6]	16.22 [7]	14.80 [5]	9.70 [1]	10.58 [2]	10.91 [3]	12.69 [4]
27/21	13.30 [7]	6.63 [2]	8.12 [5]	7.21 [4]	6.63 [2]	6.48 [1]	10.58 [6]
28/10	21.17 [7]	20.86 [6]	18.55 [5]	12.17 [3]	11.22 [1]	11.36 [2]	18.14 [4]
28/801	48.75 [7]	44.09 [6]	43.81 [5]	25.73 [3]	19.87 [1]	23.58 [2]	39.09 [4]
28/804	6.40 [5]	4.69 [4]	9.22 [6]	4.00 [3]	3.74 [2]	3.61 [1]	12.92 [7]
39/1	10.25 [6]	11.75 [7]	9.64 [5]	8.49 [1]	9.06 [3]	9.27 [4]	8.60 [2]
54/1	3.32 [5]	3.32 [5]	2.83 [1]	3.00 [4]	2.83 [1]	2.83 [1]	3.32 [5]
54/2	7.68 [3]	9.80 [4]	31.45 [6]	7.55 [2]	7.48 [1]	16.06 [5]	65.38 [7]
55/1	3.16 [4]	4.69 [6]	4.12 [5]	2.45 [1]	2.45 [1]	2.65 [3]	5.10 [7]
55/2	5.74 [4]	7.62 [7]	6.86 [5]	2.65 [1]	3.74 [3]	3.46 [2]	6.86 [5]
55/4	4.36 [2]	5.29 [7]	4.80 [6]	4.36 [2]	4.36 [2]	4.24 [1]	4.69 [5]
55/5	9.85 [5]	9.00 [2]	9.17 [3]	13.38 [7]	8.89 [1]	9.64 [4]	10.10 [6]
55/7	6.78 [5]	7.00 [6]	5.74 [3]	9.27 [7]	5.48 [2]	5.39 [1]	6.00 [4]
67/2	12.45 [7]	10.91 [5]	9.85 [4]	10.91 [5]	6.71 [1]	8.06 [2]	8.19 [3]
67/803	4.47 [5]	5.10 [6]	3.87 [2]	5.29 [7]	3.74 [1]	4.00 [3]	4.12 [4]
68/1	25.59 [7]	24.27 [6]	22.78 [5]	13.86 [1]	14.70 [2]	15.23 [3]	20.88 [4]
68/5	11.49 [6]	6.56 [3]	9.38 [5]	5.29 [2]	4.69 [1]	6.71 [4]	12.45 [7]
69/1	5.20 [6]	4.80 [3]	4.58 [1]	5.74 [7]	4.69 [2]	4.80 [3]	4.80 [3]
69/2	7.68 [5]	7.21 [1]	8.19 [6]	7.42 [3]	7.28 [2]	7.48 [4]	9.22 [7]
69/6	8.72 [7]	4.69 [4]	5.48 [5]	2.24 [1]	2.83 [2]	2.83 [2]	6.00 [6]
83/802	16.37 [4]	16.94 [7]	16.55 [6]	12.33 [3]	9.33 [2]	9.17 [1]	16.37 [4]
Subtotals	149	137	117	95	49	70	135
	7	6	5	3	1	2	4
Ireland							
201/6	10.91 [5]	8.19 [3]	11.40 [6]	7.81 [2]	7.35 [1]	10.49 [4]	17.78 [7]
309/809	17.12 [7]	16.91 [6]	16.00 [5]	9.22 [1]	10.54 [2]	10.82 [3]	15.75 [4]
323/2	5.92 [6]	5.20 [1]	5.57 [3]	5.57 [3]	5.39 [2]	5.57 [3]	6.32 [7]
390/807	8.06 [6]	7.75 [2]	7.48 [1]	12.41 [7]	7.81 [3]	7.87 [4]	7.94 [5]
392/805	4.12 [7]	2.45 [1]	2.65 [5]	2.45 [1]	2.45 [1]	2.45 [1]	2.83 [6]
393/803	15.13 [7]	13.34 [4]	12.00 [3]	15.07 [6]	11.18 [1]	11.31 [2]	13.45 [5]
394/808	12.67 [7]	11.92 [5]	10.68 [3]	16.58 [6]	7.42 [1]	7.48 [2]	10.86 [4]
Subtotals	45	22	26	26	11	19	38
Totals	194	159	143	121	60	89	173

Table 2.21 Approximations to root mean square deviation of $\frac{Q_{(i)}-E(Q_{(i)})}{\bar{Q}}$ as a percentage.

distributions showed smaller average deviations than the gamma, Gumbel and log Gumbel distributions. Here the Hazen distribution is not a theoretically specified distribution but can be considered as a lognormal distribution modified by empirically derived factors. It is also close to a log Pearson Type 3 distribution. It has not been used in this Floods Study. The log Gumbel distribution is equivalent to the extreme value Type 2 distribution (general extreme value distribution with negative k value) as discussed in Section 1.2.5.

In the American study, because the lognormal is a special case of the log Pearson Type 3 and the Hazen distribution depends on factors derived by empirical and graphical methods, the work group accepted the log Pearson Type 3 distribution as being the most general and most objective of the best three distributions and hence recommended it for general use.

The same type of analysis as reported by Benson (1968) was repeated on the data of six stations in the United Kingdom, which are listed in Table 2.23. The distributions tested were two parameter gamma, extreme

Criterion	Mean abs. dev. of $[Q_{(i)}-E(Q_{(i)})]/\bar{Q}$		Mean abs. dev. of $[Q_{(i)}-Q\{(i-\frac{1}{2})/N\}]/\bar{Q}$		Mean abs. dev. of $[Q_{(i)}-Q\{i/(N+1)\}]/\bar{Q}$		RMSD of $[Q_{(i)}-E(Q_{(i)})]/\bar{Q}$	
					(a)			
Rank	From Table 2.18		From Table 2.19		From Table 2.20		From Table 2.21	
1	Log Pearson 3	69	Log Pearson 3	71	Log Pearson 3	61	Pearson 3	49
2	GEV	73	GEV	73	GEV	82	Log Pearson 3	70
3	Pearson 3	98	Pearson 3	96	Pearson 3	95	GEV	95
4	Lognormal	116	Lognormal	121	Lognormal	115	Lognormal	117
5	Gumbel	137	Gumbel	133	Gumbel	131	Log gamma	135
6	Log gamma	138	Log gamma	137	Log gamma	140	Gamma	137
7	Gamma	152	Gamma	150	Gamma	154	Gumbel	149
					(b)			
1	Log Pearson 3	85	Log Pearson 3	86	Log Pearson 3	77	Pearson 3	60
2	GEV	95	GEV	97	GEV	101	Log Pearson 3	89
3	Pearson 3	122	Pearson 3	119	Pearson 3	120	GEV	121
4	Lognormal	145	Lognormal	149	Lognormal	140	Lognormal	143
5	Log gamma	169	Log gamma	169	Log gamma	173	Gamma	159
6	Gumbel	177	Gumbel	173	Gumbel	174	Log gamma	173
7	Gamma	186	Gamma	182	Gamma	188	Gumbel	194

Table 2.22 Summary of sum of ranks attributed to different distributions in Tables 2.18–2.21, (a) stations in Great Britain only and (b) stations in Great Britain and Ireland.

value Type 1 (Gumbel), lognormal, general extreme value (Jenkinson), Pearson Type 3, log two parameter gamma and log Pearson Type 3. The first four were fitted by maximum likelihood, the Pearson Type 3 was fitted by moments, the log gamma was fitted by maximum likelihood to the logarithms of the data and the log Pearson Type 3 was fitted by moments to the logarithms of the data. For each station quantiles corresponding to T = 2, 5, 10, 25 and 50 years are prepared from the data by interpolation between adjacent points on a probability plot. As in Benson's

Station no.	Location	Grade	Drainage area (km²)	Years of record
6/901	Ness at Ness Castle Farm	B	1790	33
27/1	Nidd at Hunsingore Weir	A2	484	35
27/21	Don at Doncaster	B	1260	86
54/1	Severn at Bewdley	A	4330	46
68/1	Weaver below Ashbrook	A2	609	32
83/802	Irvine at Kilmarnock	B	218	55

Table 2.23 Stations subjected to U.S. Water Resources Council type of study.

Station no.	Return period				
	2	5	10	25	50
6/901	−0.7148	−0.8881	−6.3929	− 6.7695	—
27/1	11.5504	−0.2982	−10.0369	−11.7464	—
27/21	1.3816	3.0136	− 6.1942	− 8.4173	−1.6250
54/1	2.0548	0.0600	− 4.3069	− 6.1627	—
68/1	14.0337	16.7798	2.9105	−31.8702	—
83/802	6.8550	6.9424	5.2803	−12.3229	−38.8155
Total	35.2956	25.6095	−18.7401	−77.2890	−40.4405
Average	5.8826	4.2683	− 3.1234	−12.8815	−20.2203

Table 2.24 Values of $(Q(T)-Q_D(T))/Q_D(T)$ computed for gamma distribution.

paper the Weibull plotting positions were used; therefore only stations having records of 50 years or more have a point plotted to the right of $T = 50$. The interpolated values are denoted $Q_D(T)$, the subscript D indicating data. For each return period and for each distribution the quantity $100(Q(T)-Q_D(T))/Q_D(T)$ is calculated where $Q(T)$ is the variate value in the fitted distribution. This gives a table with a section for each distribution; the section corresponding to the two parameter gamma distribution is reproduced here as an example, Table 2.24.

The average value of this deviation over all stations is shown for each return period in the last line of the table and is shown for each return period and distribution for ease of comparison in Table 2.25(a). For return periods $T = 2$ and $T = 5$ the average values are positive in each case showing that the estimated values exceed the data values while the opposite is true for $T = 10$, 25 and 50.

At low return periods ($T = 2, 5$) the general extreme value and Pearson Type 3 distributions have the smallest average deviations while at large return periods ($T \geqslant 10$) the log Pearson Type 3 has the smallest average deviations. It should be noted that these are three parameter distributions while the other four less well fitting distributions have only two parameters. Notice also that two of the three closest fitting distributions have been fitted by moments.

+, When fitted values exceed data values, over estimation.
—, When fitted values less than data values, under estimation.

Table 2.25(a) Summary of average deviations such as those given in Table 2.24.

Distribution	Return period				
	2	5	10	25	50 based on two stations
Gamma	5.88	4.27	−3.12	−12.88	−20.22
Gumbel	3.45	1.17	−5.05	−12.44	−19.31
Lognormal	3.47	2.81	−2.96	−10.46	−15.16
GEV	0.99	0.26	−2.33	−7.00	−15.49
Pearson 3	0.90	9.86	−1.25	−6.75	−11.09
Log gamma	2.49	2.07	−3.06	−9.60	−12.60
Log Pearson 3	2.21	2.70	−0.78	−5.89	−9.05

The effect of plotting position

The tables and results referred to thus far are based on the use of the Weibull plotting formula $i/(N+1)$ in the preparation of Tables 2.24 and 2.25(a). This was done so that the results would be comparable with Benson's. Benson investigated the effect of using the Hazen plotting formula $F_i = (i-\frac{1}{2})/N$ by repeating the analyses of Table 2.24 and 2.25(a) for 25 and 50 year return periods using that formula. He found that

a The results for the two parameter gamma and Gumbel were little affected but the bias at the higher return period was slightly reduced. (Here, bias is the quantity tabulated in Table 2.25(a).)

b The bias was increased at the higher return period in the log Gumbel case; the magnitude of the average departures was not changed but the sign of these departures was changed.

This *b* was not commented on by Benson probably because it did not cause the findings based on the Weibull plotting formula to be modified.

The present study was also repeated using a different plotting position. The Gringorten formula $(i-0.44)/(N+0.12)$ which is the correct formula for the Gumbel distribution was used. This differs from Benson's alternative which was the Hazen formula $(i-\frac{1}{2})/N$. However, the difference between these two is small in comparison with the difference between either of them and the Weibull formula $i/(N+1)$. Table 2.25(b) corresponds to Table 2.25(a) and shows that the Pearson Type 3 displays the smallest average deviations when taken as a whole over all return periods and the general extreme value (Jenkinson) comes a close second while the log Pearson 3 is distinctly relegated to third place. Of the remaining distributions the log gamma has the smallest average deviations up to $T = 25$.

It should be noted that at $T = 50$, and there only, the log Pearson 3 has a smaller average deviation than the GEV but is still not as good as the Pearson Type 3.

+, When fitted values exceed data values, over estimation.
−, When fitted values less than data values, under estimation.

Table 2.25(b) Summary of average deviations as in Table 2.25(a) but with different plotting positions.

Distribution	Return period				
	2	5	10	25	50
Gamma	5.86	4.92	−1.14	−6.75	−9.44
Gumbel	3.45	1.80	−3.10	−6.49	−5.98
Lognormal	3.47	3.46	−0.97	−4.20	−4.06
GEV	0.99	1.38	−0.96	−0.29	−2.55
Pearson 3	−0.20	2.28	0.79	0.22	−0.41
Log gamma	2.49	2.72	−1.08	−1.70	11.40
Log Pearson 3	2.21	3.35	0.97	0.91	2.41

Therefore, on changing the plotting position the same group of three distributions are picked out as best but there is now a definite ordering among the three. Further this ordering is different from that found by the United States Water Resources Council Committee (Benson, 1968).

2.4.5 Criticism of indices based on x–y plot

A major drawback in these methods is that no allowance is made for the fact that not all members of an ordered sample are subject to the same amount of sampling variation. The observations at the upper end of a sample, in particular, have large sampling variances. As a result the line or curve fitted to the true distribution may pass further from the largest value than that fitted to some more flexible distribution, in which case the latter would have a smaller goodness of fit index. Such tests really pick out the most flexible distributions; as an example if the true population were EV1 it is to be expected that the GEV distribution with its third parameter k could show better agreement with many samples of data than the parent EV1. A similar remark could be made about the lognormal and the log Pearson Type 3 distributions. Further research is required to examine these hypotheses but in the meantime it has to be noted that the superiority of the three parameter distributions might be seen to be less if different kinds of tests were used.

It might be asked whether the two parameter distributions should be discarded completely but it is suggested that they should be retained. There are two main reasons for this:

a it has not been shown conclusively that they are inapplicable and
b it has been observed that their use when small samples only are available leads to results which are often more sensible than results obtained by fitting a three parameter distribution. A three parameter distribution fitted to a small sample does in some cases imply from the combination of estimated parameter values that there is an upper bound to the variate values of little more than twice the mean annual flood. There may be an upper limit to flood magnitude but it is certainly higher than twice the mean annual flood (see Tables 2.8 and 2.9 for instance).

A further reason is that:
c when a small sample only is available a fitted distribution will usually not be extrapolated to high return periods. Figure 2.1 shows that a straight line representing a two parameter distribution is a reasonable approxima-

tion to the curve representing a three parameter distribution over periods up to $T = 100$; therefore a two parameter distribution would always be a reasonable approximation and in small samples gives more sensible results than one involving the estimation of a third parameter.

2.5 Examination of standard error data and practical standard error computations

2.5.1 Introduction

In Section 1.4 formulae for sampling variances and standard errors of moments and quantile estimates in samples from some theoretical distributions are given. In 2.5.2 the quantile standard errors are tabulated as a percentage of the true quantile values for distributions having coefficients of variation of 0.40 which is typical of annual maximum series. These are lower limits as they take sampling variation alone into account. The true standard error is greater than the computed standard error if there is secular, even though undetectable, as well as statistical variation in the population from which the data come. In 2.5.3 and following subsections the results of analysing long records into decades and examining some of the sampling properties of these ten year records are described. For this purpose three records of 7 decades, a further five of 4 decades and a further five of 3 decades were used. The three 7 decade records suggest that the decade mean has a distribution which is approximately Normal but which nevertheless could be as skewed as a Gumbel or EV1 distribution. A pooled estimate of the variance of the decade mean corrected for station differences by the analysis of variance technique is obtained. This includes the effect of both random sampling variation and genuine between decade differences, if any. It shows that the standard error of the decade mean is about 15% of the mean itself. Since 10 years is about the average length of record this figure should be a guide to the amount of mean annual flood sampling variation which could remain as residual variance in the regression analyses described in Chapter 4 even if the regression model were perfect. However, it should be borne in mind that the value of 15% given here is based on a small number of stations.

The sampling variation in the decade coefficient of variation CV is also examined but the results obtained are not as conclusive as those obtained for the mean because of the greater variance inherent in CV. The results obtained depend on whether or not the four early decades from the three longest stations are included in the analysis but the overall indication is that the standard error of the decade CV is about 30% of its mean value.

The sampling standard error of the 25 year flood estimated from each decade is also examined and compared with the theoretical sampling variance applicable to the EV1 distribution but the agreement is not good in all cases. However, because the value of scale parameter α used in the formula has to be estimated, the use of any standard error formula gives results which vary enormously from decade to decade. Therefore, even if the correct formula were known the standard error value could be given only to within about 25% of its true value; for this reason a single formula is suggested to cover all practical cases.

2.5.2 *Standard error of quantile estimate as a fraction of the quantile*

It is of interest to relate the standard error of a quantile estimate to the population value of the quantile being estimated. Illustrations are given for three cases here, Normal, EV1 and GEV with $k = -0.2$. The first case is presented as an example of a symmetrical distribution where the sampling variation is present to the same degree for quantiles at both ends of the variate but the other two are presented in the belief that they enclose between them the majority of the candidate distributions of annual maxima. The quantity standard error (se) $(\widehat{Q(T)})/Q(T)$ as a percentage is given in Table 2.26 for sample sizes of $N = 10, 25, 50$ and 100 and return periods $T = 2, 10, 25, 100$ and 1000.

Table 2.26 Standard error $(\widehat{Q(T)})/Q(T)$ as a percentage for different distributions, sample sizes and return periods.

Sample size	Return period 2	10	25	100	1000	Distribution
10	12.65	11.57	12.24	13.13	14.21	Normal
25	8.00	7.20	7.58	8.10	8.74	$\sigma = 0.40\mu$
50	5.66	5.07	5.33	5.69	6.13	cv = 0.40
100	4.00	3.58	3.75	4.00	4.32	
10	12.39	15.01	16.29	17.72	19.28	Extreme value Type I
25	7.84	9.50	10.30	11.20	12.20	$\alpha = 0.38u$
50	5.54	6.71	7.29	7.92	8.62	cv = 0.40
100	3.92	4.75	5.15	5.60	6.10	
10	14.17	23.15	33.29	52.77	92.55	General extreme value
25	8.96	14.64	21.05	33.38	58.54	$k = -0.2$
50	6.33	10.35	14.89	23.60	41.39	$\alpha = 0.38u$
100	4.48	7.32	10.53	16.69	29.27	

In the fitting of a distribution to any annual maximum series the figure in Table 2.26 for the EV1 distribution should be considered as a lower bound for the sampling variation of quantile estimates from the sample of stated size. These are, of course, smaller than the corresponding quantities for the EV2 distribution with parameter $k = -0.2$. The figures in this table are those which are attributable only to sampling error when the real population has the stated distribution. Since there is considerable room for doubting that any particular distribution is the correct one its fitting to a sample of annual maxima involves a bias owing to error of distribution choice in addition to sampling errors. Therefore the mean square deviation of all estimates of $Q(T)$ over all possible samples of given size is larger than the theoretical sampling standard error given in Table 2.26.

2.5.3 *Sampling properties of 10 year records*

The stations and the number of decades of data available at them are listed in Table 2.27. The main objective is to study the between decade variation of sample statistics to see if there are any obvious departures from the behaviour expected of random samples from the same population and in addition where possible to get some information on the form of the sampling distributions of statistics.

The decade mean and coefficient of variation for each station are given in Table 2.28 for each decade. Some decades' estimates at the beginning of a record are based on 9 years only; some records contain gaps which are not considered to have contained an annual maximum while in a few cases a missing year has been filled in by selecting randomly an annual maximum from a few spare years at the beginning or end of the record. The eighth decade of data at station 27/21 is not from 1890–99 as in the other two long records but 1880–89. Therefore, the eighth decade is rejected when stations are being pooled together for analysis but when decades from a single station are being studied in isolation all available decades are used.

Table 2.27 Stations used in decade analysis of $\bar{Q}$, cv and $Q(25)$.

Station no.	Station name	No. of decades available
12/1	Dee at Cairnton	4
15/4	Inzion at Lintrathen	4
15/5	Melgam at Loch Lintrathen	4
27/21	Don at Doncaster	8
28/801	Burbage Brook	4
28/804	Trent at Trent Bridge	8
39/1	Thames at Teddington	8
54/1	Severn at Bewdley	4
55/1	Wye at Cadora	3
55/2	Wye at Belmont	3
55/7	Wye at Erwood	3
69/1	Mersey at Irlam Weir	3
69/6	Bollin at Dunham Massey	3

When stations are pooled together the variance due to sampling fluctuations and to between decade differences, if any, is estimated by using the technique known as analysis of variance, ANOVA. Let x_{ij} be the quantity being considered, measured at the ith station and the jth decade, and suppose there are eight stations and four decades. Then the variance is

$$\operatorname{var} x = \frac{1}{31}\sum_{i=1}^{8}\sum_{j=1}^{4}(x_{ij}-\bar{x})^2 \qquad (2.5.3.1)$$

and this variation is due to three causes or in statistical terms this variance has three components. These are due to

i random sampling from a statistical population

ii the population mean value differing from station to station (station effect)

iii the population mean value differing from decade to decade (decade effect),

but either *ii* or *iii* or both may be zero. If $\bar{x}_{i\cdot}$ is the mean value at station i and $\bar{x}_{\cdot j}$ is the mean value in decade j the sum of squares in Equation (2.5.3.1) may be shown to be

$$\sum_{i=1}^{8}\sum_{j=1}^{4}(x_{ij}-\bar{x})^2 = 8\sum_{j=1}^{4}(\bar{x}_{\cdot j}-\bar{x})^2 + 4\sum_{i=1}^{8}(\bar{x}_{i\cdot}-\bar{x})^2 + \sum_{i=1}^{8}\sum_{j=1}^{4}(x_{ij}-\bar{x}_{i\cdot}-\bar{x}_{\cdot j}+\bar{x})^2 \qquad (2.5.3.2)$$

that is

$$\mathrm{SS_T} = \mathrm{SS_D}+\mathrm{SS_S}+\mathrm{SS_R}. \qquad (2.5.3.3)$$

The first quantity on the right hand side is associated with the between decade differences and is associated with 3 degrees of freedom, the second represents the between stations component of variance and has 7 degrees of freedom while the last term represents what is called the residual effect which in this case is the random sampling effect (i) above. It is based on 21 degrees of freedom this being found from the requirement that the sum of the degrees of freedom on the right hand side of Equation (2.5.3.2) should equal those on the left, namely 31. The sum of ss_D and ss_R divided by 24 gives an estimate of the variance due to both random sampling and decade differences if any of the latter exist.

The result of the decomposition of the sum of squares is usually summarised in an analysis of variance table of which there are some examples below. The algebraic treatment can be found in any text book on statistics. It should be pointed out that there are no distributional assumptions involved in the preparation of an ANOVA table. In many applications an ANOVA table is prepared in the course of testing certain statistical hypotheses and the applicability of the tests requires that the data be Normally distributed. In what follows variances alone are calculated; in the few cases where tests were made the assumptions were checked and were found to hold.

The decade mean

The eight decade means for 27/21, 28/804 and 39/1 are shown ranked in order of magnitude in Table 2.29 with their plotting positions y in both the Normal and EV1 distributions. Figure 2.11 shows the three sets of data plotted against both reduced variates and surprisingly both distributions describe the data well. When judged against the synthetic samples from an EV1 population (Figure 1.18) the scatter in these plots seems unnaturally small. However, the distribution of the mean cannot be exactly Normal although it converges to Normal as the sample gets

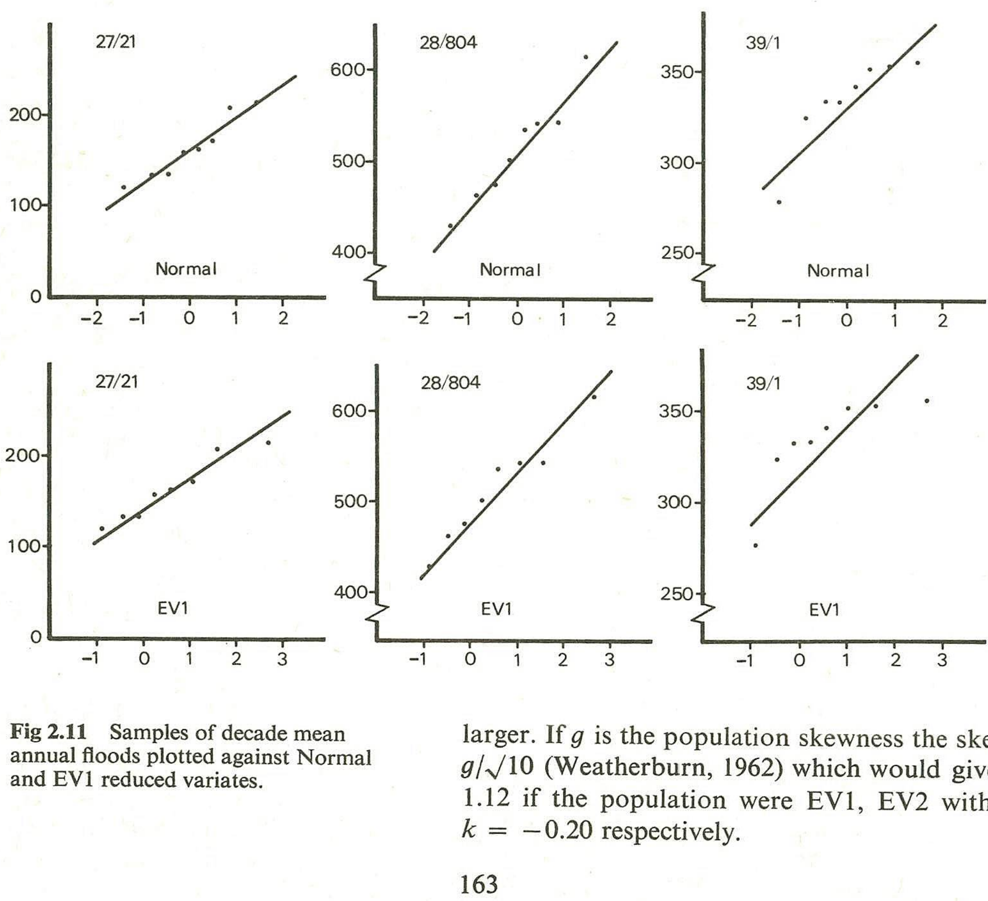


Fig 2.11 Samples of decade mean annual floods plotted against Normal and EV1 reduced variates.

larger. If g is the population skewness the skewness of the decade mean is $g/\sqrt{10}$ (Weatherburn, 1962) which would give skewness of 0.36, 0.60 and 1.12 if the population were EV1, EV2 with $k = -0.10$ and EV2 with $k = -0.20$ respectively.

Station	1960–69	1950–59	1940–49	1930–39	1920–29	1910–19	1900–09	1890–99
12/1								
$\bar{Q}$	337.284	416.368	416.917	529.230	—	—	—	—
cv	0.464	0.556	0.298	0.435	—	—	—	—
15/4								
$\bar{Q}$	6.264	6.233	6.369	6.188	—	—	—	—
cv	0.455	0.385	0.277	0.222	—	—	—	—
15/5								
$\bar{Q}$	14.552	14.234	15.737	15.825	—	—	—	—
cv	0.270	0.286	0.239	0.185	—	—	—	—
27/21								(1880–89)
$\bar{Q}$	162.900	118.190	171.989	207.508	157.317	135.286	133.545	214.455
cv	0.404	0.241	0.622	0.464	0.366	0.399	0.309	0.307
28/801								
$\bar{Q}$	4.976	7.092	4.050	5.945	—	—	—	—
cv	0.285	1.070	0.306	0.699	—	—	—	—
28/804								
$\bar{Q}$	429.233	542.682	616.834	541.474	501.642	536.232	475.789	461.848
cv	0.464	0.373	0.404	0.469	0.249	0.379	0.517	0.480
39/1								
$\bar{Q}$	342.510	333.747	333.642	324.556	354.611	352.737	277.370	356.778
cv	0.366	0.193	0.485	0.397	0.385	0.284	0.402	0.766
54/1								
$\bar{Q}$	410.093	379.603	455.558	341.460	—	—	—	—
cv	0.198	0.206	0.317	0.270	—	—	—	—
55/1								
$\bar{Q}$	523.390	491.500	602.109	—	—	—	—	—
cv	0.286	0.206	0.237	—	—	—	—	—
55/2								
$\bar{Q}$	517.148	457.359	528.868	—	—	—	—	—
cv	0.380	0.140	0.215	—	—	—	—	—
55/7								
$\bar{Q}$	684.919	451.122	661.722	—	—	—	—	—
cv	0.509	0.247	0.246	—	—	—	—	—
69/1								
$\bar{Q}$	179.895	165.614	196.243	—	—	—	—	—
cv	0.243	0.205	0.287	—	—	—	—	—
69/6								
$\bar{Q}$	37.774	39.453	46.216	—	—	—	—	—
cv	0.264	0.203	0.091	—	—	—	—	—

Table 2.28 Decade values of mean annual flood, $\bar{Q}$ and cv.

27/21	28/804	39/1	Plotting positions	
			EV1	Normal
118.189	429.233	277.370	−0.90	−1.42
133.545	461.848	324.556	−0.45	−0.85
135.286	475.789	333.642	−0.10	−0.47
157.317	501.642	333.747	0.23	−0.15
162.900	536.232	342.510	0.59	0.15
171.989	541.474	352.737	1.01	0.47
207.508	542.682	354.611	1.59	0.85
214.455	616.834	356.778	2.66	1.42

Table 2.29 Decade means ranked in order of magnitude.

The variance of the decade mean as a fraction of the mean

Before this analysis each decade mean was divided by the mean of all available decades at the station except at 27/21, 28/804 and 39/1 where the eighth decade was discarded. This division has the effect of standardising the data so that there is little or no difference in the resulting figures between stations. The ANOVA table for the four decades 1930–69 at the eight stations 12/1, 15/4, 15/5, 27/21, 28/801, 28/804, 39/1, and 54/1 is given in Table 2.30.

Table 2.30 ANOVA table of decade means.

Source	SS	df	MS	RMS
Stations	0.0143,	7	0.0020	
Decades	0.0636	3	0.0212	
Residual	0.4963	21	0.0236	
Total	0.5742	31	0.0185	
Decades + residuals	0.5599	24	0.0233	0.1527

The last line in the table estimates the true variation in the decade mean; it includes any secular variation as well as pure sampling effects. This shows that the standard error of the decade mean is about 15% of the mean itself. If all 13 stations are taken together and all decades except the eighth are used, the sums of squares expressions in Equation (2.5.3.2) need alteration because the number of decades is not the same at all stations; when this is done it is found that a figure of 13.8% results instead of the 15% above. A similar result is obtained if the three stations of seven decades each are analysed as a group (13.7%) or if the three decades 1940–69 common to all 13 stations are analysed as a group (13.9%).

The decade coefficient of variation, CV

Apart from the question of the magnitude of the sampling variation of CV as a percentage of CV it is also of great interest to investigate whether the variation in CV between stations is significantly larger than that due to random sampling and secular effects within stations. If there is a difference in CV between stations it would be necessary to seek an explanation of it; a relation should be sought between CV and catchment characteristics.

In decade 1 the CV values range from 0.198 to 0.509 and in decade 2 from 0.140 to 1.070. These ranges are very large and it is tempting to say that these 13 stations have populations of annual maxima which are quite heterogeneous in CV. However, before this can be asserted the difference between stations must be compared to the inherent variation in CV from decade to decade at the same station. Thus, for 12/1 the CV values vary from 0.298 to 0.556 which is almost as much variation as that displayed between the stations in decade 1. Excluding the eighth decade in each of the three long term stations there are 56 decades available and the mean value of CV in these is 0.345.

The analysis of variance on (a) four decades 1930–69 from eight stations and (b) on seven decades 1900–69 from three stations is shown in ANOVA tables in Table 2.31. The results of analysing three decades 1940–69 from all 13 stations are very similar to the results from (a). The decades + residual root mean square is the standard error of CV due to sampling effects. This quantity differs greatly from (a) to (b) showing that the introduction of four extra decades has an effect. Further, it is seen that the decade mean square is predominant in (b) whereas in (a) it is the station mean square which predominates. These two great differences between (a) and (b) could be said to be due to the very high inherent sampling variability in the CV values and that the results being so dependent on which stations are included in the analysis are therefore so undependable as to be valueless.

There is, however, an alternative and acceptable explanation. This is that the CV value of 1.070 in decade 2 at station 28/801 is sufficiently abnormal to affect any analysis in which it is included, for instance (a) above.

Source	SS	df	MS	Ratio	RMS
(a) 4 decades (1930–69) from eight stations					
Stations	0.3506	7	0.0501	$F = 1.681$ (not significant)	—
Decades	0.0015	3	0.0005	—	—
Residuals	0.6260	21	0.0298	—	—
Total	0.9781	31	0.0316	—	—
Decades + residuals	0.6275	24	0.0261	—	0.162
(b) 7 decades (1900–69) from three stations					
Stations	0.0105	2	0.0053	$F = 0.80$	—
Decades	0.1079	6	0.0180	$F = 2.727$ (significant at 1%)	—
Residuals	0.0744	12	0.0066	—	—
Total	0.1978	20	0.0099	—	—
Decades + residuals	0.1823	18	0.0101	—	0.1005

Table 2.31 ANOVA tables of CV values.

In 420 records of annual maxima only nine, say 2%, have CV values exceeding 1.00. Therefore, this observed value of 1.070 at 28/801 is indeed exceptional and an analysis which ignores it would be representative of the great majority of CV values which actually occur.

Therefore, analyses were performed with station 28/801 omitted. These are (c) analysis of four decades from seven stations and (d) analysis of all decades (52) from the 12 stations. The ANOVA tables corresponding to these are given in Table 2.32. From the point of view of the decades + residual root mean square (c) and (d) give results which are practically identical to that given by (b) in Table 2.31 justifying the suspicion that the outlier in 28/801 was a cause of the discrepancy between analyses (a) and (b). The decade + residual root mean square may be taken as 0.10 which is about 30% of the mean value of CV.

Analyses (c) and (d) show that the station mean square predominates so that the dominance of the decade mean square in (b) is anomalous and seems to be due to the three decades 1900–29. This anomaly cannot be resolved with the amount of data available.

Source	SS	df	MS	Ratio	RMS
(c) 4 decades (1930–69) from seven stations					
Stations	0.1691	6	0.0283	$F = 2.63$ (significant at 5%)	—
Decades	0.0150	3	0.0050	$F < 1$	—
Residual	0.1936	18	0.0108	—	—
Total	0.3777	27	0.0140	—	—
Decades + residuals	0.2086	21	0.0099	—	0.0997
(d) All available decades from 12 stations					
Stations	0.3192	11	0.0290	$F = 3.466$ (significant at 0.5%)	—
Decades	0.0814	6	0.0136	$F = 1.62$	—
Residuals	0.2847	34	0.0084	—	—
Total	0.6853	51	0.0134	—	—
Decades + residuals	0.3661	40	0.0092	—	0.096

Table 2.32 ANOVA tables of CV values, station 28/801 excluded.

2.5.4 Sampling distribution of the 25 year and 100 year floods

Eight decades from the long term stations 27/21, 28/804 and 39/1 were used to study this distribution. The quantiles are estimated by the graphical method. Although it is recognised that this method allows a certain amount of subjectivity into the estimates it is felt that the conclusions should not be dependent on any distributional assumptions. Further it is felt that on occasions it is better to obtain some basic facts on the distribution of hydrologically relevant statistics and determine whether these accord with theoretical formulae than to split hairs over methods of estimation. The agreement need not necessarily be exact in order to be satisfactory. The estimated $Q(25)$ values are shown ranked in order of magnitude in Table 2.33 and the corresponding EV1 and Normal reduced variate plotting positions are also given in that table. The estimates are shown plotted on Figure 2.12, and eye guided straight lines are superimposed on them. At each station the agreement with the EV1 distribution is very close by comparison to what is usually found in random samples from that distribution, see for instance Figure 1.18, but the plotted points agree even better with the Normal distribution. However, the sampling distributions of quantile estimates from skewed distributions by the Monte Carlo

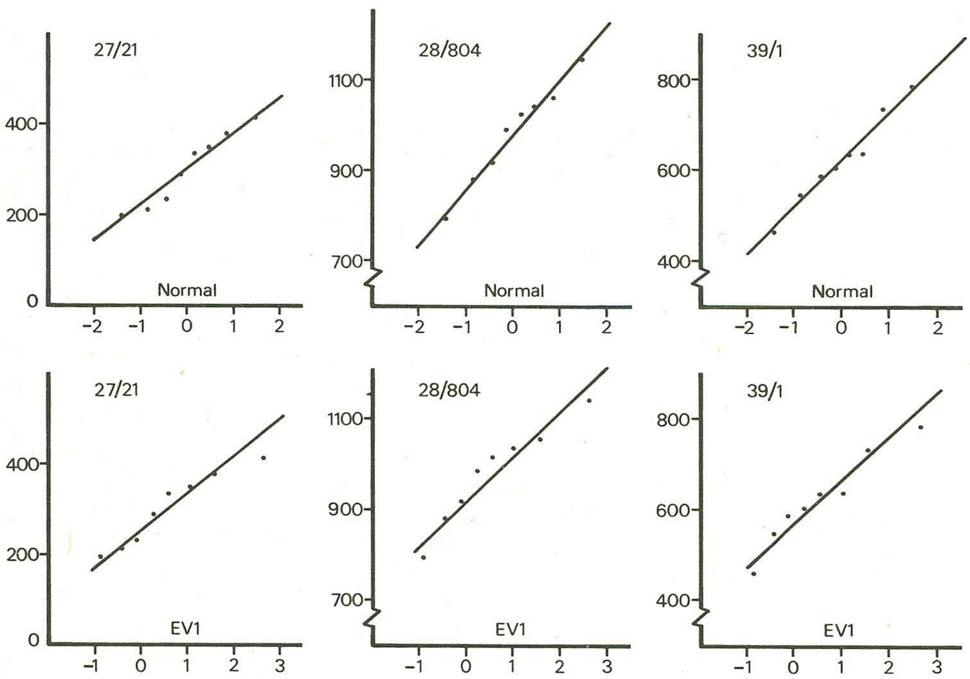


Fig 2.12 Samples of decade estimates of $\hat{Q}(25)$ plotted against Normal and EV1 reduced variates.

Table 2.33 Ranked $\hat{Q}(25)$ values estimated from decades.

	Station			Plotting positions	
	27/21	28/804	39/1	EV1	Normal
	195.01	792.33	460.57	−0.90	−1.42
	213.67	879.93	548.50	−0.45	−0.85
	231.34	917.13	585.25	−0.10	−0.47
	289.96	988.35	602.84	0.23	−0.15
	335.08	1018.19	634.93	0.59	0.15
	349.56	1039.63	637.07	1.01	0.47
	379.73	1056.38	734.96	1.59	0.85
	412.77	1141.14	784.06	2.66	1.42
Standard deviation	81.5	110.8	101.8		

method show departures from Normality. Table 2.34 shows the graphical estimates of the 100 year flood, $\hat{Q}(100)$, and these are plotted on Figure 2.13 with results similar to those of Figure 2.12.

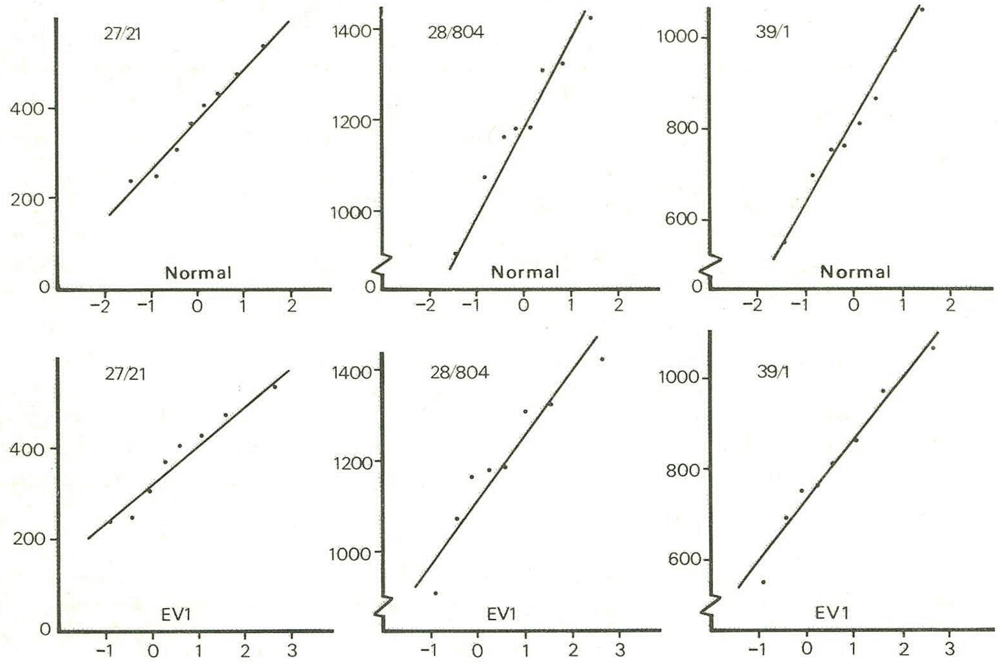


Fig 2.13 Samples of decade estimates of $\hat{Q}(100)$ plotted against Normal and EV1 reduced variates.

Table 2.34 Ranked $\hat{Q}(100)$ values estimated from decades.

Station			Plotting positions	
27/21	28/804	39/1	EV1	Normal
238.74	907.97	550.68	−0.90	−1.42
248.39	1073.08	697.80	−0.45	−0.85
307.10	1166.77	754.45	−0.10	−0.47
366.52	1179.71	762.41	0.23	−0.15
404.31	1186.95	811.30	0.59	0.15
428.91	1310.37	863.13	1.01	0.47
471.04	1322.69	970.44	1.59	0.85
533.17	1424.89	1067.65	2.66	1.42

2.5.5 Sampling variance of the 25 year flood

The standard deviations included in the bottom row of Table 2.33 are computed from the $Q(25)$ estimates above them. This is the observed standard error of $\hat{Q}(25)$ based on eight values, which admittedly is a very small sample. It is of interest to compare these standard errors with those computed on the assumption that the basic data, the annual maxima, are from an EV1 distribution. The moments estimate of $Q(25)$ in such circumstances has sampling standard error given by Equation (1.4.3.7) evaluated at $y = 3.20$. This gives

$$\text{se } Q(25) = \frac{3.61\alpha}{\sqrt{10}} = \frac{2.82\sigma}{\sqrt{10}} \qquad (2.5.5.1)$$

since $\alpha = \sigma\sqrt{6}/\pi = 0.78\sigma$ by Equation (1.2.4.4). This expression is tabulated in the third row of Table 2.35. The differences between these and the observed values are shown in the fourth row; at station 28/804 the difference is very large while at the other two stations the differences would be regarded as tolerable for practical purposes.

Table 2.35 Observed and computed standard errors of $\widehat{Q}(25)$.

	Station		
	27/21	28/804	39/1
Estimated parent population standard deviation, σ	72.5	215.6	140.7
Observed standard error of $\widehat{Q}(25)$	81.5	110.8	101.8
Standard error of $\widehat{Q}(25)$ under EV1 assumption	64.6	192.3	125.5
Difference between observed and EV1 standard errors %	−20.7	+73.5	+18.8

Therefore, on this evidence the use of the standard error formula appropriate to the Gumbel distribution cannot always be said to give a satisfactory approximation to the real sampling standard error. This is regrettable because when a small sample only is available it is not possible to choose between alternative candidate distributions such as Gumbel and Pearson Type 3 distributions. Such different distributions yield different expressions for the sampling standard errors of estimated quantiles $\widehat{Q}(T)$. Since a choice cannot be made between distributions on the basis of a small sample, it would be convenient if the expression for sampling error appropriate to one of them could be used, with the assurance that it would give a number which is of the right order of magnitude when compared with actual observed standard errors such as those in Table 2.35.

In Table 2.35 the formula (2.5.5.1) is tested by using a value of σ estimated from 80 years of record. In practice such a long record is seldom available and the value of σ used in a standard error formula is estimated from a short record. To assess the usefulness of the standard error based on a value of σ estimated from a short record it is only necessary to look at the variability of that estimate of σ. Table 2.36 shows σ estimated from each decade. For a given parent distribution and a given return period, T, the standard error of $\widehat{Q}(T)$ is but a constant multiple of σ. Assuming $T = 25$ and and EV1 distribution the standard error is given by Equation

Table 2.36 Decade standard deviation and theoretical standard error of $\widehat{Q}(25)$.

Decade	27/21		28/804		39/1	
	$\hat{\sigma}$	$\frac{2.82\hat{\sigma}}{\sqrt{10}}$	$\hat{\sigma}$	$\frac{2.82\hat{\sigma}}{\sqrt{10}}$	$\hat{\sigma}$	$\frac{2.82\hat{\sigma}}{\sqrt{10}}$
1	65.81	58.68	199.35	177.78	125.48	111.90
2	28.48	25.40	202.48	180.57	64.54	57.56
3	106.98	94.50	249.21	222.24	161.70	144.20
4	96.28	85.86	253.90	226.43	128.87	114.92
5	57.58	51.36	124.86	111.35	136.42	121.65
6	53.98	48.14	203.04	181.07	100.26	89.41
7	41.27	36.80	246.01	219.38	111.48	99.42
8	65.84	58.71	221.66	197.67	273.16	243.60
Mean		57.54		189.56		122.83
Minimum		25.40		111.35		57.56
Maximum		95.40		222.24		243.70

(2.5.5.1) and this multiplied by $\hat{\sigma}$ for each decade is also given in Table 2.36. This shows that if an estimated value, $\hat{\sigma}$, is used to calculate a standard error the resulting quantity may diverge widely from the true value.

In practical situations the use of a slightly different multiplier of σ in the standard error formula would have a small effect in comparison to the effect produced by using a highly variable estimate $\hat{\sigma}$ instead of σ. It is therefore suggested that in hydrological situations a single standard error can be used regardless of the distribution being fitted or of the method of fitting. It must then be understood that the resulting standard error figure serves only to convey an order of magnitude rather than an exact value. A further step may also be warranted and this is to assume a constant value of population CV, say 40%, and to use $\sigma = (\text{CV})\ \bar{Q}$ in the standard error formula, $\bar{Q}$ being the sample mean. It is likely that this σ is closer to the true value than that estimated from a short record since $\bar{Q}$ is subject to lesser sampling variance than $\hat{\sigma}$.

2.5.6 *Practical standard errors*

The standard error formula takes the form

$$\text{se}\,(\widehat{Q(T)}) = \frac{\text{constant}.\sigma}{\sqrt{N}} = \frac{\text{constant}.\text{CV}.\mu}{\sqrt{N}} \qquad (2.5.6.1)$$

where the constant depends on the form of the distribution, the method of fitting and on T. Assuming an EV1 distribution fitted by moments the standard error constant, C, is obtained from Equation (1.4.3.7) with α replaced by 0.78 σ as described earlier and y being the EV1 reduced variate value of return period T.

To get the standard error, C in row 3 of Table 2.37 is multiplied by $\sigma/\sqrt{N}$ or if an average value of CV = 0.40 is assumed the quantity D of row 4 is multiplied by $\mu/\sqrt{N}$ or $\bar{Q}/\sqrt{N}$. If C is plotted against y the relation is seen to be practically linear and is adequately described by

$$C = 0.35 + 0.80\,y$$
$$\simeq 0.35 + 0.80\,\ln(T - \tfrac{1}{2}). \qquad (2.5.6.2)$$

T	5	10	20	25	50	100	200	500	1000
y	1.50	2.25	2.97	3.20	3.90	4.60	5.30	6.21	6.91
Standard error constant, C	1.55	2.09	2.64	2.82	3.37	3.93	4.48	5.22	5.78
$D = 0.40\,C$	0.62	0.84	1.06	1.13	1.35	1.57	1.79	2.09	2.31

Table 2.37 Standard error constants.

2.6 Region curves

2.6.1 *Introduction*

Flood frequency analysis for a station with a long record can be based almost exclusively on that record alone; distributions may be fitted to the annual maximum series and a choice made on the basis of either numerical goodness of fit indices or visual agreement of fitted curve and plotted points on a probability plot. When only a short record is available at the site the choice of distribution cannot be based on the sample alone and prior knowledge about the form of the distribution must be used. This prior knowledge can take a number of forms; for instance the results of Section 2.4, in which a group of distributions consisting of log Pearson Type 3, general extreme value and Pearson Type 3 distributions is singled out as best, provide one form of prior information. Another form is a region curve which is the mean distribution of all recorded floods scaled by a single parameter in a region. When there is no record whatever available at the site the region curve may be used together with an estimate of the mean annual flood obtained from catchment characteristics (see Chapter 4). The three distributions mentioned above cannot however be so used until three independent relations have been established between the distribution of floods and the characteristics of catchments on which they occur.

A region curve is derived for each of 10 regions in Great Britain and one for Ireland. Each of the curves derived makes use of any historical data which can be regarded as reliably expressed in terms of discharge (see Volume IV). A curve is also prepared for the whole of Great Britain and an extremely tentative extension of this based on the very largest individual floods tabulated earlier in Table 2.9 is also presented.

2.6.2 *Definition and outline of derivation of region curve*

A region curve is essentially a frequency distribution of $Q/\bar{Q}$. It associates a return period T with $Q/\bar{Q}$ and this relation is assumed to be valid for all catchments in one region, or alternatively to represent the mean of the different relationships for the different catchments in the region.

The distribution of $Q/\bar{Q}$ is to be described by considering the variate to be a function, as yet unnamed, of the reduced Gumbel variate y. If the data conform to a Gumbel distribution they will be represented by a linear relation but departures from linearity can be allowed for in the graphical derivation of the region curve. In practice with up to 50 stations each contributing an average of 10 years to such a plot the lower part of the curve is well defined up to $y = 3$ ($T \simeq 20$). At higher return periods the shape and position of the curve is less well defined by the data but some guidance as to the appropriate shape can be obtained by pooling the data of several regions to obtain a single curve.

For each station in the region the annual maximum flows were assembled in the form of $Q/\bar{Q}$ together with the plotting positions appropriate to the Gumbel distribution expressed as reduced variate values y. The ith smallest in the record of N years is associated with $y = E(y_{i;N})$, the expected value of the ith smallest order statistic in a sample of size N from the reduced Gumbel variate (Table 1.16). If each record were of the same length the ith smallest in each record would have the same plotting position. The mean value of all these ith smallest values could therefore be plotted at this plotting position. However, all records are not of the same length and the plotted points do not fall into such neat categories.

Instead the y axis was divided into categories of width 0.5 units, for instance -2.0 to -1.5, -1.5 to -1.0. All the points which would be plotted in a given category were replaced by the average value of $Q/\bar{Q}$ plotted at the average value of y. This reduced the mass of plotted points and effected a smoothness at the expense of great local detail which was not required. The resulting plot usually covers the range $y = -1.5$ to $y = 3.5$ or 4.0 and is usually quite smooth up to $y = 3.0$. Above $y = 3.0$ the plotted points are averages of just a few points. In a typical 50 station region the averages in divisions between $y = -1.0$ and $y = 2.0$ are based on between 50 and 100 values while those in divisions to the right of $y = 3.0$ ($T \simeq 20$) are based on between 5 and 10 values and to the right of $y = 4.0$ ($T \simeq 50$) perhaps just single points are available.

The plot was extended out to $y = 5.5$ or 6.0 ($T = 200$–400) in the following way. The stations in a region were divided into four or five groups. A major requirement was that stations which are close neighbours do not appear in the same group. If this requirement is met it was assumed that the samples in a group are statistically independent. The four highest values of $Q/\bar{Q}$ in the group—regardless of station—were noted and were considered to be the four highest in a sample size M, where M is the total number of station years in the group. They were associated with the expected order statistics $E(y_{(1;M)})$, $E(y_{(2;M)})$. . . . Four such values are taken from each group. The number M usually varied from group to group but an attempt was made to keep it approximately constant. These new values from all groups were also averaged over intervals of y such as 4.0–4.5, 4.5–5.0 and the averages plotted.

In most cases these additional points form a fairly continuous plot with the lower points. There is some statistical dependence between the two types of plotted points but it is hoped that this is small if M in each group is large. Because the stations in each group are not close neighbours it is assumed that the highest four may be considered as the largest in a random sample of M and this enables correct plotting positions to be assigned to the largest values. However, independence between groups is not assumed but since each group affords an unbiassed estimate of the population curve between $y = 3.0$ and $y = 5.5$ or 6.0, the mean of the groups is unbiassed.

When these points had been plotted a smooth eye guided curve was drawn through the lower points and extended up to about $y = 5.5$ ($T \simeq 200$) with the guidance of the additional points obtained from the group maxima. This curve is taken as the estimate of the population curve.

When a number of such hand drawn curves had become available it was decided to try to summarise each one in parametric form. Equations of the form

$$\frac{Q}{\bar{Q}} = a + be^{cy} \qquad (2.6.2.1)$$

were considered and found to be satisfactory. However, instead of writing the equation as in (2.6.2.1) it was written in the more usual form of the general extreme value variate,

$$\frac{Q}{\bar{Q}} = u + \alpha\left(\frac{1 - e^{-ky}}{k}\right) \qquad (2.6.2.2)$$

where u is the value of $Q/\bar{Q}$ when $y = 0$, α is the gradient of the curve when $y = 0$ and k is a curvature parameter. In each case the hand drawn curve was regarded as the definitive estimate of the population curve to begin with and then by a series of trials a curve of the form (2.6.2.2) was chosen. The usual procedure was to set u = ordinate of hand drawn curve at $y = 0$, α = slope of hand drawn curve at $y = 0$, and then sketch in the curves corresponding to a few trial values of k. The curve that provided the best description of the hand drawn curve was chosen.

In some instances where the upper points were well scattered the chosen theoretical curve might be some distance from the original hand drawn curve but because of the scatter it looked as good a description of the data as the latter. This theoretical curve was then taken as the final estimate.

Equations of the form (2.6.2.2) have proved to be so suitable that they are used to summarise the hand drawn curves; they are to be regarded purely as descriptors of the curves. Too much reliance should not be placed on the α and k values *per se* because where α_1 and k_1 are quoted it often happens that a slightly different pair, α_2 and k_2, would have been equally satisfactory. Therefore, any countrywide pattern in these values should be treated with some caution although it is true that k values for Wales and the north of England are closer to zero than those for the south east.

Region	u	α	k	CV
1	0.82	0.22	−0.20	0.40
2	0.84	0.18	−0.30	0.43
3	0.84	0.27	0.00	0.35
4	0.80	0.25	−0.175	0.43
5	0.79	0.26	−0.325	0.65
6/7	0.77	0.28	−0.25	0.57
8	0.78	0.28	−0.10	0.43
9	0.84	0.23	−0.10	0.34
10	0.85	0.21	−0.10	0.31
Great Britain	0.80	0.24	−0.20	0.44
Ireland	0.87	0.21	−0.05	0.29

Table 2.38 Values of u, α, k for each region curve $Q/\bar{Q} = u+\alpha(1-e^{-ky})/k$ and the corresponding theoretical coefficient of variation.

Region	Return period						
	2	5	10	25	50	100	200
1	0.90	1.20	1.45	1.81	2.12	2.48	2.89
2	0.91	1.11	1.42	1.81	2.17	2.63	3.18
3	0.94	1.25	1.45	1.70	1.90	2.08	2.27
4	0.89	1.23	1.49	1.87	2.20	2.57	2.98
5	0.89	1.29	1.65	2.25	2.83	3.56	4.46
6/7	0.88	1.28	1.62	2.14	2.62	3.19	3.86
8	0.88	1.23	1.49	1.84	2.12	2.42	2.74
9	0.93	1.21	1.42	1.71	1.94	2.18	2.45
10	0.93	1.19	1.38	1.64	1.85	2.08	2.32
Great Britain	0.89	1.22	1.48	1.88	2.22	2.61	3.06
Ireland	0.95	1.20	1.37	1.60	1.77	1.96	2.14

Table 2.39 Region curve ordinates.

Ten geographical regions in Great Britain were picked out at an early stage in the study and they are defined by hydrometric area numbers in Table 2.1 in Section 2.3. Because of the lack of long records Region 7, the South Coast, was pooled with Region 6, Essex River Authority, the Thames and Lee Conservancies. Ireland was treated as one region. The values of u, α, and k which summarise the curves for these regions are given in Table 2.38 as well as the values of u, α, k obtained by pooling the data for Great Britain and treating it as one region. In all these cases historical data have been taken into account as well as recently recorded data. The curves corresponding to these are shown on Figures 2.14 and 2.16 and a selection of the main quantiles from these are given in Table 2.39. The differences between some of these are small but discussion of this fact is deferred until later in this section.

2.6.3 Example of region curve derivation

This subsection is a detailed illustration of the principles of region curve derivation already outlined and the general reader may like to turn to Section 2.6.4.

Region 5 which consists of hydrometric areas 29–35 is used to illustrate the procedure used in the region curve derivation. The 47 stations used, with a total of 532 years of record, are listed in Table 2.40. Further historical information will be listed below. Table 2.41 shows the results of grouping the data into intervals of width 0.5 along the y axis and averaging them. The mean $Q/\bar{Q}$ values are plotted against the mean y values in Figure

2.15(a). The extension of the plot by the method of grouping described earlier proceeds as shown in Table 2.42.

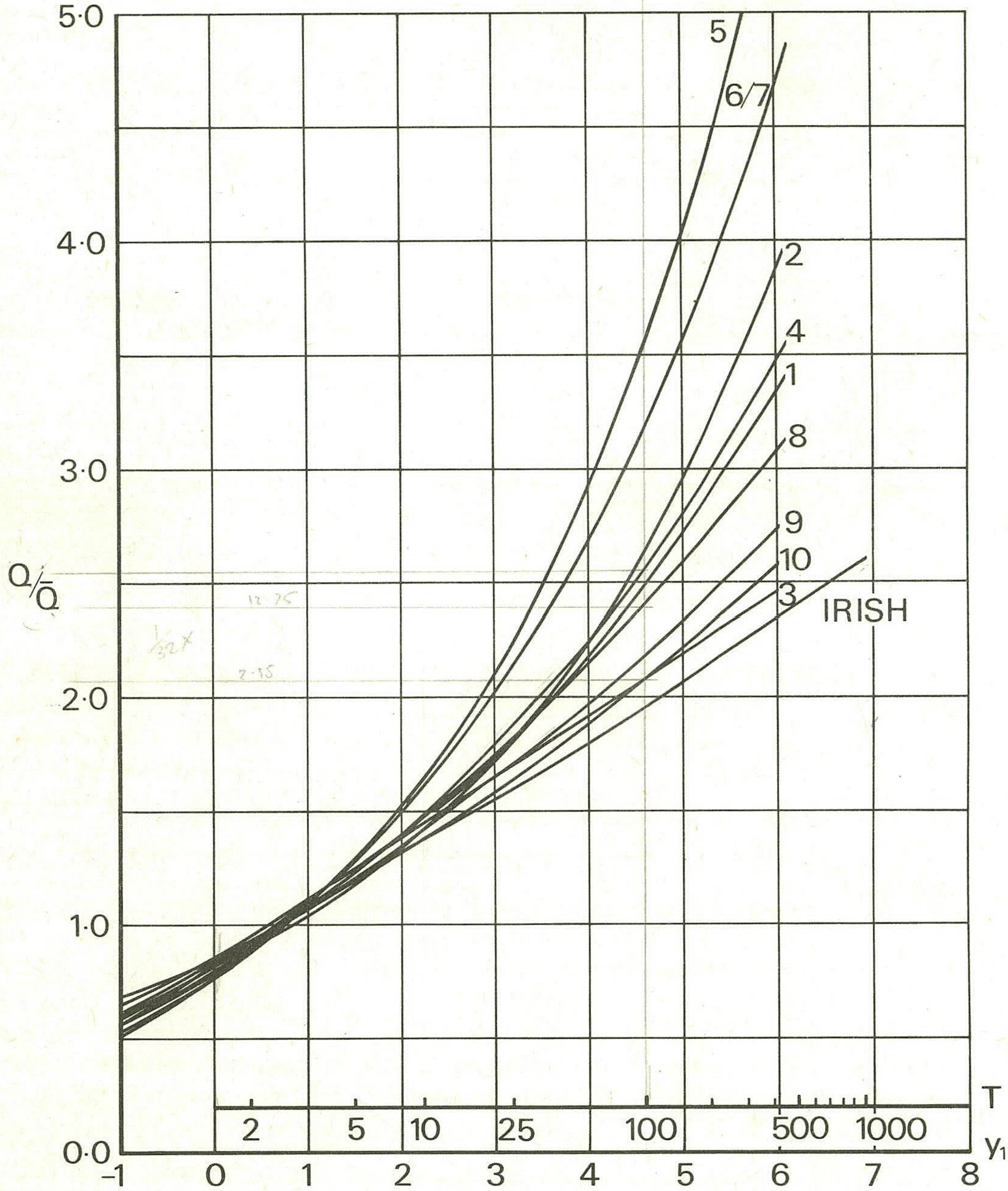


Fig 2.14 Region curves showing average distribution of $Q/\bar{Q}$ in each region.

The four highest values of $Q/\bar{Q}$ in each group are given in Table 2.43, together with their plotting positions y as if these were the largest in a sample of size M. Thus, in group 1, 3.63 is the largest value of $Q/\bar{Q}$ and $y = 5.33$ is the plotting position of the largest in a sample of size $M = 116$ from the standardised Gumbel distribution. These 20 values were grouped into intervals according to their y values. Thus, in the y interval 4.0–4.5 the $Q/\bar{Q}$ values of 2.56, 2.97, 2.41, 2.66 and 2.01 appear with an average of 2.52, while the corresponding y values are 4.30, 4.40, 4.15, 4.00 and 4.17 with an average of 4.21. The other similarly derived pairs of averages are given in Table 2.44.

	Stations used	No. of years
32/801	Flore experimental catchment	5
33/6	Wissey at Northwold	13
32/8	Whilton Branch at Dodford	22
33/11	Little Ouse at Euston County Bridge	8
35/4	Ore at Beversham Bridge	4
35/8	Gipping at ICI Stowmarket	5
30/2	Barlings Eau at Langworth Bridge	9
35/3	Alde at Farnham	8
34/3	Bure at Ingworth	10
30/4	Partney Lymn at Partney Mill	6
32/3	Harpers Brook at Old Mill Bridge	28
34/1	Yare at Colney	11
33/17	Great Ouse at St Ives Staunch	14
33/29	Stringside at White Bridge	4
29/1	Waithe Beck at Brigsley	9
29/3	Lud at Louth (both stations)	4
33/13	Sapiston at Euston (Rectory Bridge)	9
33/809	Bury Brook at Bury Bridge	6
33/21	Rhee at Burnt Mill Weir	7
33/813	Mel at Meldreth	5
33/805	Beechamwell Brook at Beechamwell	5
33/14	Lark at Temple Weir	9
33/18	Tove at Cappenham	7
33/23	Kennett (Lea Brook) at Beck Bridge	6
33/24	Cam at Dernford	6
34/6	Waveney at Needham Mill	6
33/12	Kym at Meagre Farm	9
33/15	Ouzel at Willen Weir	7
34/5	Tud at Costessey Park	8
33/2	Great Ouse at Bedford	10
33/5	Great Ouse at Thornborough	19
33/20	Alconbury Branch at Brampton	6
34/2	Tas at Shotesham	11
35/801	Bucklesham Mill River at Newbourne	17
30/1	Witham at Claypole Mill	10
30/3	Bain at Fulsby Lock	7
31/2	Glen at Kates Bridge and King St Bridge	10
32/4	Ise Brook at Harrowden Old Mill	25
32/7	Brampton Branch at St Andrews Mill	29
33/9	Great Ouse at Harrold Mill	18
34/4	Wensum at Costessey Mill	9
32/2	Willow Brook at Fotheringhay	29
33/19	Thet at Melford Bridge	8
35/1	Gipping at Constantine Weir	8
32/10	Nene at Wansford	30
32/6	Kislingbury Branch at Upton	29
31/5	Welland at Tixover	7
	47 stations	532 years total

Table 2.40 The 47 stations contributing to region curve 5.

y interval	No. in interval	Mean y	Mean $Q/\bar{Q}$
−1.5–−1.0	19	−1.17	0.34
−1.0–−0.5	77	−0.75	0.49
−0.5– 0.0	94	−0.24	0.70
0.0– 0.5	97	0.25	0.87
0.5– 1.0	76	0.74	1.05
1.0– 1.5	61	1.22	1.24
1.5– 2.0	40	1.73	1.39
2.0– 2.5	27	2.27	1.54
2.5– 3.0	29	2.76	1.94
3.0– 3.5	4	3.31	1.52
3.5– 4.0	8	3.84	2.72
	532		

Table 2.41 Averaged values of $Q/\bar{Q}$ and plotting positions y from 47 stations in Region 5.

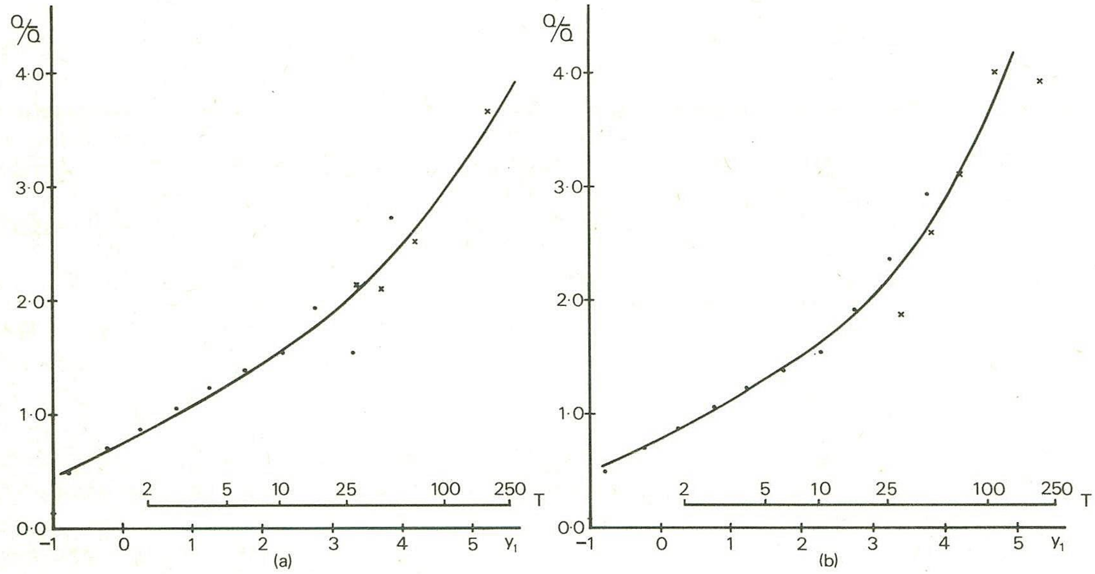


Fig 2.15 Region curve for Region 5. (a) Without historical data and (b) after inclusion of historical data.

Table 2.42 Grouping of stations in Region 5. M is the number of station years in a group. Stations with historical information are marked H.

	Group 1	Group 2	Group 3	Group 4	Group 5
	32/4	32/7	31/5 H	31/2 H	32/801
	32/10 H	32/2	32/6	32/8	32/3
	33/18	33/9	33/2	33/15 H	33/5
	33/12	33/17 H	33/809	33/20	33/813
	33/14	33/805	33/24	33/21	33/23
	33/11	34/3	33/6	33/13	33/19
	34/1	34/2	34/6	33/29	34/5
	35/3	35/8	35/801	34/4	35/4
	30/2	30/3	30/4	29/3 H	29/1
				35/1 H	30/1
M	116	128	100	86	102

Table 2.43 Maximum values and their plotting positions in each group where M is the group size.

M	116		128		100		86		102	
	$Q/\bar{Q}$	y	$Q/\bar{Q}$	y	$Q/\bar{Q}$	y	$Q/\bar{Q}$	y	$Q/\bar{Q}$	y
H1	3.63	5.33	4.55	5.43	4.58	5.18	2.78	5.03	2.74	5.20
H2	2.56	4.30	2.97	4.40	2.41	4.15	2.66	4.00	2.01	4.17
H3	2.52	3.80	1.91	3.90	1.92	3.65	2.51	3.50	1.89	3.67
H4	2.42	3.47	1.87	3.57	1.84	3.32	2.46	3.16	1.85	3.34

Table 2.44 Average values of $Q/\bar{Q}$ and plotting position y among the four largest in the five groups.

y interval	No. of values	Mean y	Mean $Q/\bar{Q}$
3.0–3.5	4	3.32	2.14
3.5–4.0	6	3.68	2.10
4.0–4.5	5	4.21	2.52
4.5–5.0	—	—	—
5.0–5.5	5	5.23	3.65

The $Q/\bar{Q}$ and y values of Table 2.41 are shown on Figure 2.15 as dots and the values in Table 2.44 are shown as crosses. The point plotted at $y = 3.31$ and $Q/\bar{Q} = 1.52$ falls considerably below the trend of the rest of the points largely because of one exceptional record.

Inclusion of historical data

At some stations there is some information on one or more historical floods in addition to the flood information in the recent recorded series (see Volume IV). In Region 5 there are seven stations in which historical information may be put to use in the construction of a region curve. The way in which it is used is similar to the way in which historical information is used when a theoretical distribution is being fitted to a record at a single site (see Section 2.8). The historical information may be of two types:

a a large flood which occurred many years before recording began may be regarded as the largest value in a period of n_1+n_2 years where n_1 is the length of record and the flood occurred n_2 years before the beginning of the record. This means that the historical flood can be plotted as the largest in a sample size n_1+n_2 and the recorded data are unaltered

b both a large flood during the n_1 years of record and one some n_2 years before the beginning of record where it is known that the historic flood had not been exceeded until the large flood in the record occurred.

An example of the first type occurs at station 33/15, Ouzel at Willen Bridge Weir. There are 7 years of record at the site and in addition it is known that the flood of 14 March 1947 was not surpassed in the period 1946–69 making it the largest in 23 years. The 1947 flood has been estimated and its ratio to the mean $\bar{Q}$ of the 7 years of record is 4.06. Therefore, in addition to the seven items from the continuous recent record plotted as a sample of size 7 this value of 4.06 is plotted as the largest in a sample of 23 that is at $y = 3.71$.

An example of the second type occurs at 31/2, Glen at Kates Bridge, where there are 10 years of recent record. These 10 annual maxima contributed to the preparation of Table 2.41. In addition, the flood of 17 March 1947 is known not to have been exceeded between 1946 and 1968 so that its ratio to the mean $\bar{Q}$ which is 4.09 can be plotted as the largest in a sample of size 22 that is at $y = 3.67$. It is also known that the largest flood in the recent record which occurred in 1968 was not exceeded between 1947 and the beginning of the recent record making it the second largest in 22 years. Therefore, the entries in Tables 2.41, 2.43 and 2.44 were changed to take into account that this station's data are to be used as follows: the nine lowest of the recent record to be treated as the nine lowest in a sample of size 10, the highest in the recent record to be treated as the second highest in a sample of size 22, a y value of 2.60 instead of 2.88, and the 1947 flood as the largest in a sample of size 22, a y value of 3.62. This meant that $Q/\bar{Q} = 2.08$ was moved from $y = 2.88$ to $y = 2.60$ in preparing a revised version of the tables and that $Q/\bar{Q} = 4.09$ is introduced at $y = 3.62$.

The seven stations with usable historical information of these types are listed in Table 2.45 and the manner in which they affected the entries in Table 2.41 was arranged for systematic calculations as in Table 2.46; the calculations can be understood from the footnote in that table.

A new table corresponding to Table 2.41 which includes the adjustments due to historical information is given in Table 2.47. These entries are plotted as dots on Figure 2.15(b).

The group maxima had to be altered to include the effects of historical floods. In Table 2.42 the stations with historical information are marked by an H. Table 2.48 is the counterpart of Table 2.43 which gives the number of years of record M in each group and the group maxima.

In each group which contains an H the value of M is increased as

Station	Recent record		Recent record				Historical floods and/or recent floods with revised plotting position				
	No. of years	Record mean	Floods with plotting positions affected by historical information								
	(n_1)	($\bar{Q}$)	Date	$Q/\bar{Q}$	Order	y	Date	$Q/\bar{Q}$	Order	Period	y
29/3 Lud at Louth	4	4.72	—	—	—	—	29.5.1920	29.2	1	50	4.49
33/15 Ouzel at Willen Bridge Weir	7	16.73	—	—	—	—	14.3.1947	4.06	1	23	3.71
33/17 Great Ouse at St Ives	14	102.51	—	—	—	—	16.3.1947	3.02	1	23	3.71
35/1 Gipping at							1938/39	5.45	1	36	4.16
Constantine Weir	8	20.72	—	—	—	—	March 1947	5.45	2	36	3.12
31/2 Glen at							17.3.1947	4.09	1	21	3.62
Kates Bridge	10	16.61	2.11.1968	2.08	1	2.88→	2.11.1968	2.08	2	21	2.60
31/5 Welland at							14.3.1947	2.38	1	22	3.67
Tixover	7	35.14	3.11.1968	1.60	1	2.52→	3.11.1968	1.60	2	22	2.64
32/10 Nene at			18.3.1947	3.62	1	3.98→	18.3.1947	3.63	2	64	3.70
Wansford	30	70.53	9.2.1940	2.56	2	2.96→	9.2.1940	2.57	3	64	3.20
							1905	unknown but >3.63	1	64	

Table 2.45 Historical flood information in Region 5. In the first three stations it is of type *a* above and of type *b* in the last four.

indicated in Table 2.48. The length of a recent record is replaced by the corresponding historical length so that in effect their difference is added

Interval	From Table 2.41			From LHS Table 2.45 Subtract			From RHS Table 2.45 Add			Amended entries corrected by historical data		
	N	y	$Q/\bar{Q}$	N	y	$Q/\bar{Q}$	N	y	$Q/\bar{Q}$	N	y	$Q/\bar{Q}$
2.5–3.0	29	2.76	1.94	1	2.88	2.08	1	2.60	2.08	28	2.747	1.918†
				1	2.52	1.60	1	2.64	1.60			
				1	2.96	2.56						
							2	5.24	3.68			
				3	8.36	6.24						
3.0–3.5	4	3.31	1.52	—	—	—	1	3.12	5.45	6	3.26	2.35
							1	3.20	2.57			
							2	6.32	8.02			
3.5–4.0	8	3.84	2.72	1	3.98	3.62	1	3.71	4.06	12	3.763	2.943
							1	3.71	3.02			
							1	3.62	4.09			
							1	3.67	2.38			
							1	3.70	3.63			
							5	18.41	17.18			
4.0–4.5	—	—	—		—	—	1	4.49	29.20	2	4.32	17.32
							1	4.16	5.45			
							2	8.65	34.65			

$\dagger N = 29-3+2 = 28$,
$y = (29(2.76)-8.36+5.24)/28$
$= 2.747$,
$Q/\bar{Q} = (29(1.94)-6.24+3.68)/28$
$= 1.918$.

Table 2.46 Computation of amendments to Table 2.41 due to introduction of historical information.

to M. The values of y corresponding to the four highest values of $Q/\bar{Q}$ are based on this value of M. Thus, in Table 2.48 in Group 1 $y = 5.59$ is the plotting position of the largest in a sample of size $M = 150$. The $Q/\bar{Q}$ values in Table 2.43 are compared with $Q/\bar{Q}$ values of stations in the same group listed in Table 2.45 and the four largest from these two sources listed in Table 2.48.

The averages of these over y intervals are shown in Table 2.49 which corresponds to Table 2.44. Entries in Table 2.49 are plotted as crosses on Figure 2.15(b) which is the counterpart of Figure 2.15(a) and contains the effects of all usable historical information in the region.

Table 2.47 Counterpart of Table 2.41 after inclusion of historical data.

y interval	No. in interval	Mean y	Mean $Q/\bar{Q}$
−1.5−−1.0	19	−1.17	0.34
−1.0−−0.5	77	−0.75	0.49
−0.5− 0.0	94	−0.24	0.70
0.0− 0.5	97	0.25	0.87
0.5− 1.0	76	0.74	1.05
1.0− 1.5	61	1.22	1.24
1.5− 2.0	40	1.73	1.39
2.0− 2.5	27	2.27	1.54
2.5− 3.0	28	2.75	1.92
3.0− 3.5	6	3.26	2.35
3.5− 4.0	12	3.76	2.94
4.0− 4.5	2	4.32	17.32

Table 2.48 Maximum values and their plotting positions in each group after inclusion of historical data.

$M = 150$		$M = 137$		$M = 115$		$M = 187$		$M = 102$ (no change)	
$Q/\bar{Q}$	y	$Q/\bar{Q}$	y	$Q/\bar{Q}$	y	$Q/\bar{Q}$	y	$Q/\bar{Q}$	y
3.63	5.59	4.55	5.50	4.58	5.32	29.2	5.81	2.74	5.20
2.56	4.56	3.02	4.47	2.41	4.29	5.45	4.78	2.01	4.17
2.52	4.06	2.97	3.97	2.38	3.79	5.45	4.28	1.89	3.67
2.42	3.73	1.87	3.64	1.92	3.46	4.09	3.95	1.85	3.34

Table 2.49 Average values of $Q/\bar{Q}$ and plotting positions y among the four largest in the five groups after inclusion of historical data.

y interval	y	$Q/\bar{Q}$
3.0–3.5	3.400	1.885
3.5–4.0	3.792	2.603
4.0–4.5	4.254	3.082
4.5–5.0	4.670	4.005
5.0–5.5	5.340	3.957
5.5–6.0	5.700	16.410

2.6.4 *Averaged countrywide distribution of $Q/\bar{Q}$*

A curve based on the whole country as a region is derived from the data which had been assembled for the derivation of the region curves. For instance these data in the case of Region 5 are given in Tables 2.47 and 2.49 which contain the effect of all usable historical information. A new table, Table 2.50, was computed from the 10 tables like Table 2.47 and the

Table 2.50 Averaged values of $Q/\bar{Q}$ and plotting positions y from 449 stations in Great Britain.

y interval	No. in interval	Mean y	Mean $Q/\bar{Q}$
−1.0−−0.5	775	−0.74	0.61
−0.5− 0.0	1001	−0.24	0.76
0.0− 0.5	1020	0.25	0.90
0.5− 1.0	811	0.73	1.03
1.0− 1.5	669	1.23	1.17
1.5− 2.0	402	1.73	1.31
2.0− 2.5	299	2.24	1.49
2.5− 3.0	243	2.75	1.72
3.0− 3.5	122	3.22	1.81
3.5− 4.0	72	3.72	2.27
4.0− 4.5	37	4.18	3.14
4.5− 5.0	11	4.75	2.45
5.0− 5.5	8	5.24	2.21
5.5− 6.0	2	5.63	3.50
6.0− 6.5	1	6.18	2.71
	5473		

entries in it have the same meaning as the corresponding entries in that table. As described earlier in the worked examples for Region 5 groups of stations were picked in each region such that the stations in each group could be considered independent and the four highest values of $Q/\bar{Q}$ in each group were given the plotting positions y of the four highest in a random sample of size M, the number of station years in the group. For the entire country groups were also picked which for convenience were composed from the corresponding groups in each of the regions. Thus, the stations in group 1 consisted of the stations in group 1 in each region and so on. The four largest values of $Q/\bar{Q}$ in each are obtained by listing the corresponding values in that group in each region. Table 2.51 gives such a list and its compilation can be understood by noting that the fifth row referring to Region 5 is taken from the first column of Table 2.48. The four largest values in group 1 (5.79, 5.03, 4.98 and 4.68) are in italics in Table 2.51 and are considered to be the four largest values in a random sample of size 1301. Their plotting probabilities are

$$F_1 = \frac{1301-0.44}{1301+0.12},\ F_2 = \frac{1300-0.44}{1301+0.12},\ F_3 = \frac{1299-0.44}{1301+0.12},\ F_4 = \frac{1298-0.44}{1301+0.12}$$

and the corresponding y values, $y = -\ln-\ln F$, are $y_1 = 7.75$, $y_2 = 6.73$, $y_3 = 6.23$ and $y_4 = 5.90$. For each of the other five groups four pairs of

Region	Four largest values of $Q/\bar{Q}$ in group 1				No. of station years
1	*4.68*	2.36	2.07	1.93	154
2	2.51	2.42	1.91	1.64	101
3	1.85	1.70	1.58	1.54	72
4	2.33	1.93	1.82	1.75	104
5	3.62	2.56	2.52	2.42	150
6	*5.79*	3.61	3.38	2.51	120
7	*5.03*	3.07	2.17	1.50	45
8	*4.98*	2.08	1.99	1.89	92
9	3.44	2.04	2.03	2.01	267
10	1.83	1.73	1.63	1.62	196
					1301

Table 2.51 Four largest values of $Q/\bar{Q}$ and number of station years in group 1 in each region.

$Q/\bar{Q}$ and y values were found in the same way, and averaged over intervals of y to give the quantities in Table 2.52. These $Q/\bar{Q}$ and y values together with the values in Table 2.50 are plotted in Figure 2.16, the latter as dots, the former as crosses. The highest point of 29.2 at $y = 8.09$ is based on a single extreme event (29/3, Lud at Louth) and plots so far away from the

y interval	y	$Q/\bar{Q}$
4.5–5.0	4.75	3.23
5.0–5.5	5.41	2.50
5.5–6.0	5.79	3.79
6.0–6.5	6.26	3.74
6.5–7.0	6.78	4.67
7.0–7.5	7.07	5.45
7.5–8.0	7.78	5.11
8.0–8.5	8.09	29.2

Table 2.52 Average values of $Q/\bar{Q}$ and y taken over Great Britain group maxima.

rest of the points that it is not allowed to influence the drawing of an eye guided curve unduly (see Section 1.3). Such a curve is shown in Figure 2.16 and it has been found that the equation

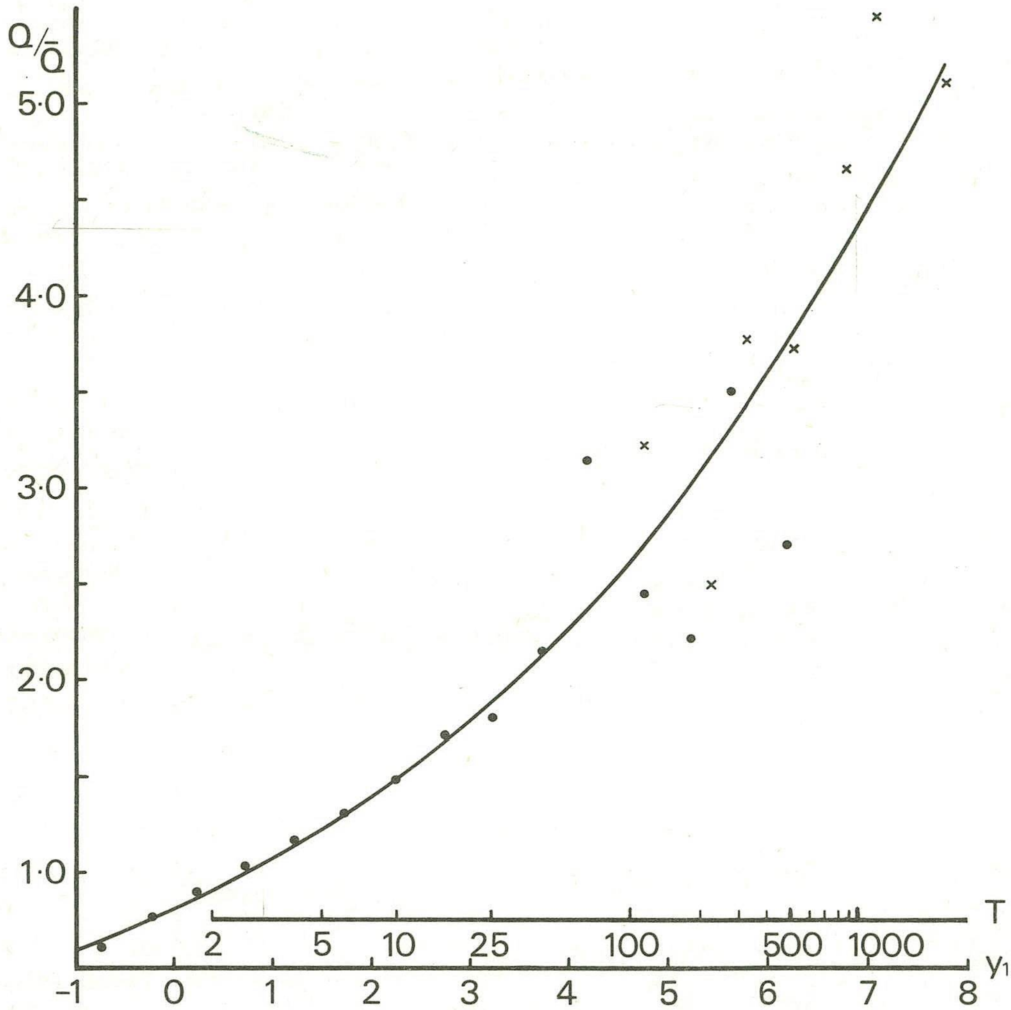


Fig 2.16 Region curve based on all records in Great Britain: 420 stations which span 7760 station years.

$$\frac{Q}{\bar{Q}} = u + \alpha\left(\frac{1-e^{-ky}}{k}\right) \quad (2.6.4.1)$$

with $u = 0.80$, $\alpha = 0.24$, $k = -0.20$, describes the eye guided curve very closely. These values of u, α and k are those given in Table 2.38 under the Great Britain entry. The number of individual $Q/\bar{Q}$ values represented in Figure 2.16 is 5473, the sum of column 2 in Table 2.50, which includes the historic floods. The inclusion of historic floods has the effect of increasing the number of station years over which the plotted points span in time to 7760 years. This last figure is not the number of station years of data; it is the number of station years spanned by the data.

2.6.5 Differences between region curves

It is seen from Figure 2.14 that there are quite large differences between some pairs of region curves especially at high return periods. However, experience with records of annual maxima suggests that very large sampling fluctuations occur with data from such distributions. It is not unreasonable therefore to suggest that the data on which each region curve is based might well have come from the distribution, Figure 2.16, for the whole of Great Britain. This hypothesis has not been tested but is to be the subject of future research. Pending the result it is suggested that the separate region curves be adopted. Notwithstanding any statistical tests there is a hydrological reason for accepting different curves. This is that in a region where the percentage runoff of rainfall causing floods varies widely between floods of mean annual flood magnitude and 100 year flood magnitude, the $Q/\bar{Q}$ curve must be steeper than in a region where percentage runoff varies less between small and large floods (see also Chapter 6). Furthermore, there is a difference in the frequency of rainfalls of equal amount in different parts of the country and these complement rather than conflict with the argument based on runoff percentage.

2.6.6 *The effect of outliers on $\bar{Q}$*

In Section 2.3 the effect of outliers on the estimate of $\bar{Q}$ was discussed and an alternative method of estimating $\bar{Q}$ from a sample of data when the ratio Q_{max}/Q_{med} exceeds 3.0 was given. Some of the records contributing to the region curves described above should have been so treated before inclusion but this would have required changes in the data processing system which would have slowed the progress of the work. However, because such large amounts of data were used it is felt that the effects would not be sufficient to alter the results substantially.

2.6.7 *The effect of catchment area*

In each region separate curves were derived from groups of catchments selected by catchment size. For instance curves were derived from the data of catchments with (a) areas less than 100 km^2, (b) areas between 100 and 500 km^2, and (c) areas greater than 500 km^2 although other divisions by area were also tried in some specific regions. Because the number of catchments in each group was not large the resulting curves were not as well defined as the curve based on all the stations' data in the region and it was impossible to detect any consistent effect of catchment area on the curves.

It might be thought *a priori* that small catchments would have steeper frequency curves than large ones but it is not possible to establish the truth of this even on plots averaged over several stations' data. The following specific example illustrates the converse, showing that catchment geology and climate can play a larger part than area alone. The Allt Uaine (87/801) drains a small catchment area of 3.13 km^2 and has a high annual rainfall fairly evenly distributed throughout the year. The catchment provides little soil storage with the result that the runoff coefficient throughout the year is large and not as affected by storm size as in a chalk covered catchment. The runoff per unit area of the largest flood of 11.32 cumecs in 18 years' record is very large by countrywide standards when considered as runoff per unit area but the same is true for the mean annual flood so that their ratio is small giving an extremely flat plot of $Q/\bar{Q}$ *versus* y as in Figure 2.17. This is so flat in comparison to the region curves of Figure 2.14 that it is sufficient to refute the hypothesis that small catchments necessarily give steeper frequency curves. In this case it appears that the lack of variation in percentage runoff between small and large storms is the dominant factor.

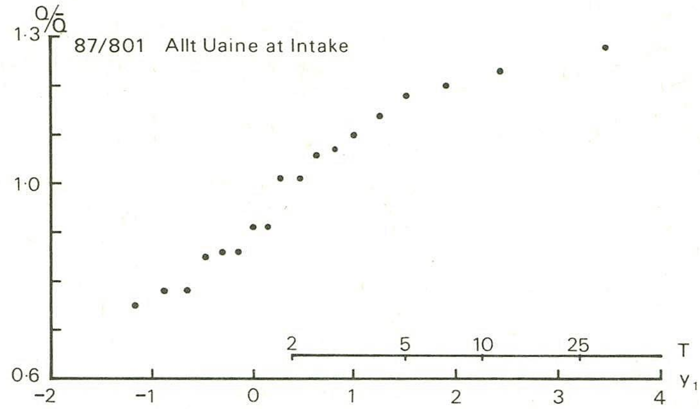


Fig 2.17 Example of a record from a small catchment where the largest floods are but small multiples of the mean annual flood.

2.6.8 Standard errors

In this context it is necessary to distinguish between two kinds of statistical variation (a) the variation of the region curve as estimated from different sets of data and (b) the variation, if any, from station to station in the region of the population distribution of $Q/\bar{Q}$. To illustrate consider a fixed return period T and let $Q(T)/\bar{Q}$ be the region curve ordinate estimated from the records currently available. If a different set of records of equal length were available the resulting estimate of $Q(T)/\bar{Q}$ would also be different and this is the variation referred to in (a) above. If the true value of $Q(T)/\bar{Q}$ differs from station to station the region curve is an estimate of the mean of $Q(T)/\bar{Q}$ taken over all stations; the variation from station to station is that mentioned in (b) above and is inherent in the second of the two interpretations of the region curve concept described in the beginning of this section.

Because of the manner in which the region curve is estimated a completely satisfactory theoretical derivation of an expression for the standard deviation of these two kinds of variation is necessarily complex and is not available. However, an estimate of the order of magnitude of these quantities was obtained from the results of fitting distributions to the records at each site in a region and examining the variation in the estimated T year floods after division by $\bar{Q}$ in each case. This was done with the data of 40 stations in Region 4. Quantile estimates of return period up to $T = 1000$, were obtained at each station and while this entailed gross extrapolation it must be remembered that the region curve derivation also entails it. The standard deviation of the 40 estimates of $Q(T)/\bar{Q}$ at each return period expressed as a percentage of the region curve ordinate at that return period was found to increase approximately linearly with $\ln T$ in the range $T = 10$ to $T = 1000$ being 14% at $T = 10$, 32% at $T = 100$ and 50% at $T = 1000$. This approximate relation can be summarised by

$$S_b = -3.5 + 7.7 \ln T, \qquad (2.6.8.1)$$

for $10 < T < 1000$

where

$$S_b = \frac{\text{standard deviation of individual station estimates of } Q(T)/\bar{Q}}{\text{region curve value of } Q(T)/\bar{Q}} \cdot 100$$

and is a measure of the variation (b) mentioned above.

The variation (a) above is necessarily smaller than this because the region curve ordinate is essentially an average of the ordinates at each station and therefore the corresponding standard error must be given by

$$S_a = S_b/\sqrt{L} \qquad (2.6.8.2)$$

where S_a = standard deviation between data sets of region curve $Q(T)/\bar{Q}$ as a percentage of the region curve ordinate itself, and L = number of statistically independent stations contributing to the region curve.

Here $L = 40$ but it would be disputed that all these stations are statistically independent. If L is taken as 25, $\sqrt{L} = 5$ and this would give standard error values S_a of approximately 3, 6 and 10% at $T = 10$, 100 and 1000 respectively. These figures apply only to the region curve ordinates; when the region curve is interpreted as the mean of many distributions and it estimates $Q(T)/\bar{Q}$ at a particular station, the standard error S_b must be associated with the estimate.

The quantity S_b at best gives the order of magnitude of the standard error required and even then only for Region 4 but these figures for orders of magnitude give at least a basis on which to proceed until better standard error estimates are available.

The sampling variance of a $Q(T)$ estimate obtained via a region curve uses the fact that

$$\text{var}(x.z) \simeq E(z)^2\, \text{var}x + E(x)^2\, \text{var}z \qquad (2.6.8.3)$$

provided x and z are statistically independent. The region curve ordinate $Q(T)/\bar{Q}$ can be considered statistically independent of a $\bar{Q}$ estimate. Then

$$\text{var}\, Q(T) = \text{var}\,(\bar{Q}\,.\,Q(T)/\bar{Q}) = E(\bar{Q})^2\, \text{var}\,(Q(T)/\bar{Q}) + E\left(\frac{Q(T)}{\bar{Q}}\right)^2 \text{var}\,\bar{Q}. \qquad (2.6.8.4)$$

The quantity $\text{var}(Q(T)/\bar{Q})$ is obtained from Equation (2.6.8.1) as

$$\text{var}(Q(T)/\bar{Q}) = \left(\frac{S_b}{100}\,\frac{Q(T)}{\bar{Q}}\right)^2 \qquad (2.6.8.5)$$

while $\text{var}\,\bar{Q}$ depends on the manner in which $\bar{Q}$ is estimated. In practice of course $\bar{Q}$ is inserted for $E(\bar{Q})$. If $\bar{Q}$ is obtained from catchment characteristics its variance is about equal to the variance of a $\bar{Q}$ estimated from just one or two annual maxima so that if σ is the population standard deviation $\text{var}\,\bar{Q}$ lies between σ^2 and $\sigma^2/2$. Since σ is unknown an average value of CV may be taken and σ replaced by $\text{CV}\,.\,E(\bar{Q})$ in theory and $\text{CV}\,.\,\bar{Q}$ in practice. Therefore if CV is taken as 0.40, for instance, $\text{var}\,\bar{Q}$ lies between $0.16\bar{Q}^2$ and $0.08\bar{Q}^2$. If $\bar{Q}$ is obtained from a record of annual maxima $\text{var}\,\bar{Q} = \sigma^2/N$ or $(\text{CV}\,.\,\bar{Q})^2/N$. The average value of CV may be taken from Figure 2.4 or as the CV of the region curve given in Table 2.38.

As an example if $T = 25$ and the site in question is in Region 1, Table 2.38 gives $Q(T)/\bar{Q} = 1.81$ and Equation (2.6.8.1) gives $S_b = -3.5 + 7.7 \ln 25 = 21.3$ so that $\text{var}(Q(T)/\bar{Q}) = [0.01\ \ S_b\,.\,Q(T)/\bar{Q}]^2 = [(0.213)(1.81)]^2 = 0.149$. If $\bar{Q} = 200$ is estimated from a 10 year record and CV is assumed to be 0.40, $\text{var}\,\bar{Q} = (0.40\bar{Q})^2/10 = (80)^2/10 = 640$. Then

$$\widehat{Q(25)} = (1.81)(200) = 362$$

$$\text{var}\,\widehat{Q(25)} = (200)^2(0.149) + (1.81)^2\, 640 = 5960 + 2097 = 8057$$

and se $\widehat{Q(25)} \doteq 90$, which is 25% of $\widehat{Q(25)}$.

2.6.9 *$Q(T)/\bar{Q}$ of 10 000 year return period—a conjecture*

So far in this section stations have been grouped in a manner which allows the records at each to be assumed statistically independent of each other thus allowing the largest values to be considered as the largest in a sample of size M, where M is the number of station years in the group. By this method the countrywide curve is defined for return periods of up to approximately 1000 years, the variate $Q/\bar{Q}$ extending almost to 5.0. There are however 16 instances of $Q/\bar{Q}$ exceeding 5.0 as can be seen from Table 2.10 by using the relation $Q/\bar{Q} = Q/(1.07\ Q_{med})$.

These data were plotted as a truncated portion of a sample of size 7500 on extreme value paper through which a line was drawn giving little weight to the three highest floods (29/3, Lud at Louth, West Lyn at Lynmouth and 40/19, Eden at Vexour Bridge). This line intersected the countrywide region curve at the $T = 1000$ mark but diverged upwards from it at higher return periods giving $Q(T)/\bar{Q} \simeq 18$ at $T = 10\,000$ whereas the extrapolated GB curve gives $Q(T)/\bar{Q} \simeq 7.5$. It must be stressed that discussion of the 10 000 year event involves gross extrapolation but a value in between the above two, say $Q(T)/\bar{Q} = 12$, might be taken as the basis of conjecture.

2.7 The peaks over a threshold (POT) model

2.7.1 *Introduction*

The POT approach corresponds to the very simplest statistical treatment of flood peaks but in recent years it has been studied by increasingly sophisticated methods. The simplest treatment consists of enumerating all flood peaks exceeding a series of different levels. The number of peaks exceeding a given level, divided by the number of years of record, gives the rate of exceedance, λ, which can then be used to find the expected number of exceedances in a period of specified length of time; for instance $N\lambda$ is the expected number of exceedances in N years. This counting of peaks is less useful for high flows because the flows in large ranges are assigned the same probability; it is then necessary to use continuously varying probability values rather than discrete values. The probability distribution of magnitudes which depends on the level over which peaks are counted must be combined in some way with rate of occurrence of peaks.

An outline of particular POT models is given, ranging from the simplest up to one where the rate of occurrence of peaks and the distribution of peak magnitudes vary with season. The estimation of parameters and their sampling variance in a simple model is then presented; the assumptions on which the model is based are examined and it is concluded that uncritical use of the model is dangerous except for very low return periods. The mean annual flood can be estimated from POT data and it is shown that the resulting estimate agrees well with the value estimated from the annual maximum series. Under certain assumptions estimates of the T year flood from both annual maximum and POT data can be compared from the point of view of efficiency and it is shown that the dictum 'more data, better estimates' is not necessarily true. Successive peak magnitudes have been examined for dependence but it is concluded that they are independent. This section ends with examples of use of the POT model for estimating $Q(T)$.

2.7.2 *Principles of POT models*

The distribution of all peak magnitudes exceeding some threshold q_0 is a conditional distribution and statements must be of the type 'the probability that a peak value q is >180, given that it is $>120(=q_0)$, is 0.15'. Such a statement means that 85% of all peaks which exceed 120 are less than 180 but it does not mean that 85% of *all* peaks are less than 180. This latter statement is the kind of statement which is required. It is a statement which is unconditional on the value of q_0 which, being arbitrary, has no place in the statement finally required. To pass from the conditional to the unconditional statement requires knowledge of the probability of the condition. This transition is usually expressed as

$$PR(A.B) = PR(A/B)\,PR(B). \qquad (2.7.2.1)$$

In the above example B is the event that a peak which exceeds 120 does occur, A/B is the event that a peak picked from those which exceed 120 also exceeds 180 while $A.B$ is the event that any peak picked from among all peaks exceeds 180 (as well as 120).

The quantity $PR(A/B)$ is obtained from the conditional distribution of magnitudes which exceed q_0, 120 in this example. To date in this approach the alternative distributional forms used for this have been exponential, gamma or Erlangian. The last is a special case of the gamma distribution with the shape parameter γ constrained to integer values. Of course the exponential distribution is a member of the Erlangian family.

The quantity $PR(B)$, the probability of an occurrence of a peak above a threshold, cannot be specified without reference to a time interval because obviously for a given threshold q_0 the probability of B during an interval of 1 year is less than its probability during 10 years. If for a given q_0 the number of peaks exceeding q_0 in successive years is counted it will be seen that the same number does not occur in each year; if the year is broken up into seasons then the same number does not occur in each season. In the various treatments proposed for $PR(B)$ these facts are either taken into account or ignored. The basic principle is extremely simple. Suppose for example that 120 cumecs in the above example is exceeded twice every year on average. It is known that of those which do occur 15% exceed 180 which means that there is a probability of 0.30 that 180 is exceeded in a year or in other words that $Q = 180$ has return period $T = 1/0.3 = 3.33$. Repeating this for several values of Q gives the magnitude–return period relationship. In the next subsection the various methods devised for expressing this concept more formally are presented; each one introduces a new feature to make the model accord more closely with reality. However, it is claimed here that procedures introduced to represent the distribution of peak occurrences within the year do not contribute an improvement to the specification of floods of large return period.

2.7.3 *Outline of particular models*

There are three models, to be called models 1, 2 and 3, which differ from each other in the way the number of peaks over the threshold each year is treated. A fourth model is also outlined in which the distribution of flood peak magnitudes is allowed to vary from season to season. Such a model has only been proposed in the literature and the need for it has not been investigated for United Kingdom use. In each model described the conditional distribution of magnitudes will be taken to be exponential

$$F(Q \leqslant q/q \geqslant q_0) = 1 - e^{-(q-q_0)/\beta} \qquad (2.7.3.1)$$

or in the notation given above

$$PR(A/B) = PR(Q \geqslant q/q \geqslant q_0) = e^{-(q-q_0)/\beta} \qquad (2.7.3.2)$$

but this assumption can be accepted only if supported by data. In most of the literature on POT models this distribution is used in the examples. It will be seen that this assumption implies an EV1 distribution of annual maxima which from Sections 2.4 and 2.6 seems not to be entirely the case in practice. Therefore, such models should be used with care.

After each model is described the expression for the flood of return period T is given and this is followed by the derivation of the corresponding distribution of annual maxima.

Model 1

The variation between years and between seasons in the number of peaks exceeding the threshold q_0 ($=120$ for example) is ignored and a constant number of exceedances, λ, is assumed to occur each year.

Return period in years and in sampling units. There are λ peaks greater than q_0 each year so that there are λT such peaks in T years. Therefore, the T year flood occurs once on average among every λT peaks which exceed q_0 which is equivalent to saying that it has a return period $T' = \lambda T$ sampling units where a sampling unit is a peak over the threshold. Therefore, the T year flood $Q(T)$ is given implicitly by

$$F(Q \leqslant Q(T)/Q(T) \geqslant q_0) = 1 - \frac{1}{\lambda T}. \qquad (2.7.3.3)$$

When the conditional distribution of magnitudes is given by Equation (2.7.3.2) this gives

$$Q(T) = q_0 + \beta \ln \lambda + \beta \ln T. \qquad (2.7.3.4)$$

Distribution of annual maxima. During a time interval of one year the probability that r of the λ threshold exceedances also exceed a higher value q, such as 180, is given by the binomial distribution as

$$PR(r/\lambda) = \binom{\lambda}{r} [PR(A/B)]^r [1 - PR(A/B)]^{\lambda - r} \qquad (2.7.3.5)$$

where $PR(A/B)$ is the probability of a single threshold exceedance being greater than 180 as defined in Equation (2.7.3.2). If $r = 0$ the annual maximum Q_{max} is less than q. Therefore

$$PR(Q_{max} \leqslant q) = PR(0/\lambda) = [1 - PR(A/B)]^{\lambda} \qquad (2.7.3.6)$$

and when Equation (2.7.3.1) gives the distribution of magnitudes

$$PR(Q_{max} \leqslant q) = [1 - e^{-(q-q_0)/\beta}]^{\lambda}. \qquad (2.7.3.7)$$

Model 2

This model in which the variation between years in the number of peaks exceeding the threshold is formally acknowledged, but the variation between seasons within the year is ignored, has been described by many authors (Borgman, 1963; Shane & Lynn, 1964; Bernier, 1967; Todorovic & Zelenhasic, 1970). The number of peaks in a year is considered to be a random variable with mean λ. Let $p_0, p_1, \ldots p_i$ be the probability of 0, 1 . . . i peaks over the threshold in a year. If i peaks exceed the threshold in a year the probability that r of them exceed $q \geqslant q_0$ is

$$PR(r \text{ peaks} > q/i) = \binom{i}{r}(PR(A/B))^r(1-PR(A/B))^{i-r}. \tag{2.7.3.8}$$

This is a conditional probability, conditional on i peaks $>q_0$ occurring. Obviously r peaks $>q$ may occur with any value of i providing $r \leqslant i$. The unconditional probability that r peaks exceed q in a year is therefore

$$PR(r \text{ peaks} > q) = \sum_{i=r}^{\infty} PR(r \text{ peaks} > q/i)p_i$$

$$= \sum_{i=r}^{\infty}\binom{i}{r}(PR(A/B))^r\,(1-PR(A/B))^{i-r}p_i$$

$$= \sum_{j=0}^{\infty}\binom{j+r}{r}(PR(A/B))^r\,(1-PR(A/B))^j p_{j+r}. \tag{2.7.3.9}$$

The number of events as a Poisson variate. The number of events exceeding the threshold q_0 in a year has sometimes been taken as a Poisson variate, that is

$$p_i = \frac{e^{-\lambda}\lambda^i}{i!}. \tag{2.7.3.10}$$

Adoption of this distribution is consistent with the assumption that peaks which exceed the threshold are scattered in a completely random manner throughout the year although this is not a necessary condition. Inserting for p_{j+r} in Equation (2.7.3.9) gives

$$PR(r \text{ peaks} > q) = \sum_{j=0}^{\infty}\binom{j+r}{r}PR(A/B)^r\,(1-PR(A/B))^j\frac{e^{-\lambda}\lambda^{j+r}}{(j+r)!}$$

$$= \frac{e^{-\lambda}\lambda^r}{r!}\,(PR(A/B))^r\sum_{j=0}^{\infty}\frac{\lambda^j(1-PR(A/B))^j}{j!}$$

$$= \frac{e^{-\lambda}\lambda^r PR(A/B)^r}{r!}\,e^{\lambda\,(1-PR(A/B))}$$

$$= \frac{e^{-\lambda PR(A/B)}\,.\,[\lambda PR(A/B)]^r}{r!}. \tag{2.7.3.11}$$

This shows that the distribution of the number of peaks exceeding the higher level q is also a Poisson distribution with parameter $\lambda PR(A/B)$. Here, λ is the rate per year at which the threshold q_0 is exceeded and $PR(A/B)$ is the proportion of those floods which exceed q_0 which also exceed the higher level q, as in Equation (2.7.3.2).

The T year flood. This is found by treating the flow $q > q_0$ as a variable and deriving the value of q which is exceeded on average once in T years. From Equation (2.7.3.11) it is seen that the number of peaks exceeding q

each year is a Poisson variate with mean $\lambda PR(A/B)$ and because of the additive property of Poisson variates the number of peaks exceeding q during intervals of length λT years is a Poisson variate with mean $\lambda T\,PR(A/B)$. If q is such that this last quantity is one then q is the T year flood. That is

$$\lambda T\,PR(A/B) = 1 \tag{2.7.3.12}$$

or inserting for $PR(A/B)$ from Equation (2.7.3.2)

$$F(Q \leqslant Q(T)/Q(T) \geqslant q_0) = 1 - \frac{1}{\lambda T} \tag{2.7.3.13}$$

and when $F(.)$ is exponential this reduces to

$$Q(T) = q_0 + \beta \ln \lambda + \beta \ln T. \tag{2.7.3.14}$$

Equations (2.7.3.13) and (2.7.3.14) correspond to Equations (2.7.3.3) and (2.7.3.4) in model 1 and it is seen that the expressions are identical for both models. Thus, the adoption of a random Poisson model instead of a constant for the number of peaks exceeding the threshold q_0 has added nothing new in the expression for $Q(T)$. This is not surprising since any stipulation which does not affect the mean number in a year could not be expected to alter $Q(T)$ if T is a large number of years.

The distribution of annual maxima in model 2. According to Equation (2.7.3.10) there is a probability $p_0 = e^{-\lambda}$ that no flood exceeds the threshold q_0. This means that in the proportion $e^{-\lambda}$ of years the annual maximum flow is less than q_0. Since the distribution of magnitudes is specified for peaks exceeding q_0 there is no possibility of describing that portion of the distribution of annual maxima in the range $q<q_0$. The distribution of annual maxima which exceed q_0 can however be derived. From Equation (2.7.3.11) the unconditional probability that no peaks, $r = 0$, exceed the level q in a year can be obtained and this is the probability that the annual maximum is less than q.

$$PR(Q_{\max} \leqslant q) = PR(\text{No peaks} > q) = e^{-\lambda PR(A/B)} \tag{2.7.3.15}$$

or rewriting $PR(A/B)$

$$PR(Q_{\max} \leqslant q) = e^{-\lambda[1 - F(Q \leqslant q/q \geqslant q_0)]}. \tag{2.7.3.16}$$

In the particular case where $F(.)$ is exponential as in Equation (2.7.3.2) Equation (2.7.3.16) gives

$$PR(Q_{\max} \leqslant q) = e^{-\lambda e^{-(q-q_0)/\beta}}$$

and noting that $\lambda = e^{\ln \lambda}$ and letting $u' = q_0 + \beta \ln \lambda$ this may be written

$$PR(Q_{\max} \leqslant q) = e^{-e^{-(q-u')/\beta}} \tag{2.7.3.17}$$

which is recognised as the form of the EV1 or Gumbel distribution. Therefore, in the annual maximum distribution corresponding to model 2 the T year flood is

$$Q(T) = u' + \beta y, \tag{2.7.3.18}$$

if $Q(T) \geqslant q_0$, where y is the Gumbel standardised or reduced variate. Since the approximation

$$y \simeq \ln(T - \tfrac{1}{2}) \tag{2.7.3.19}$$

is correct to 3 significant digits for $T \geqslant 5$, Equation (2.7.3.18) can be rewritten

$$\begin{aligned} Q(T) &\simeq u' + \beta \ln (T - \tfrac{1}{2}) \\ &= q_0 + \lambda \ln \lambda + \beta \ln(T - \tfrac{1}{2}), \end{aligned} \qquad (2.7.3.20)$$

provided $Q(T) \geqslant q_0$ and $T \geqslant 5$.

In this model 2 with an exponential distribution of magnitudes the true Q–T relation is given by Equation (2.7.3.14) and Equation (2.7.3.20) shows that the Q–T relation in the corresponding annual maximum series differs from it by very little when $T \geqslant 5$. In fact the right hand side of Equation (2.7.3.20) is such that it would correspond to $Q(T - \frac{1}{2})$ in Equation (2.7.3.14). In other words the $(T - \frac{1}{2})$ year flood in the original model has a return period T in the deduced annual maximum model. This result was derived by Langbein (1949) without invoking either the exponential distribution of magnitudes or the Poisson distribution of the number of events per year. In the same paper the result of Equation (2.7.3.15) was derived without the Poisson assumption. His derivation was the equivalent of the following. Let there be n peaks per year and let ε peaks per year exceed q. Then $(1-\varepsilon/n)^n$ is the probability that all floods in a year are less than q and if $\varepsilon \ll n$ this may be written $e^{-\varepsilon}$. But ε has the same meaning as $\lambda PR(A/B) = \lambda[1 - F(Q \leqslant q/q \geqslant q_0)]$ in Equation (2.7.3.15). However, his assumption $\varepsilon \ll n$ is not inconsistent with a Poisson assumption, for the latter is the limiting form of the binomial distribution on which the expression $(1-\varepsilon/n)^n$ is based.

Model 3

This model takes seasonal variation in the number of peaks exceeding the threshold into account. As an example let the year be divided into two seasons and let the number of peaks occurring in the first season have Poisson distribution with mean λ_1 and let the number occurring in the second season have Poisson distribution with mean λ_2. Under these assumptions the total number of events per year is also a Poisson variate with mean $\lambda = \lambda_1 + \lambda_2$ because the sum of any number of Poisson variates is a Poisson variate. Therefore, the results appropriate to model 2 are also appropriate to this model and in particular the T year flood is

$$Q(T) = q_0 + \beta \ln(\lambda_1 + \lambda_2) + \beta \ln T. \qquad (2.7.3.21)$$

An extension of this is the general time dependent Poisson process where the number of peaks exceeding the threshold in any time interval follows a Poisson law the parameter of which depends on not only the length of the interval but also the time of year in which it begins. If $t = 0$ indicates the beginning of the year let $\Lambda(t_1)$ be the average number of peaks exceeding the threshold between time $t = 0$ and $t = t_1$. Then in any year the probability of r events between $t = 0$ and $t = t_1$ is

$$PR[r \text{ in } (t_0, t_1)] = \frac{e^{-\Lambda(t_1)}[\Lambda(t_1)]^r}{r!}. \qquad (2.7.3.22)$$

If the interval extending from time t_1 to t_2 is considered, the number of peaks exceeding the threshold is a Poisson variate with mean $\Lambda(t_2) - \Lambda(t_1)$. Such a time dependent Poisson process was suggested by Borgman (1963) and used explicitly by Todorovic & Zelenhasic (1970). For the purpose of defining $\Lambda(t)$ in a practical way Todorovic & Zelenhasic divide the water year 1 October to 30 September into seventeen 20 day and one 25 day periods. The average number of events over all years in the intervals 0–20 days, 0–40 days . . . 0–365 days is determined from observed data and the form of the function $\Lambda(t)$ is determined. In the example cited by Todorovic & Zelenhasic the following form of expression was used

$$\Lambda(t) = a+bt+c_1 \cos\left(\frac{\pi t}{9}+d_1\pi\right)+c_2 \cos\left(\frac{2\pi t}{9}+d_2\pi\right)$$
$$+c_3 \cos\left(\frac{2\pi t}{6}+d_3\pi\right)+c_4 \cos\left(\frac{2\pi t}{3}+d_4\pi\right) \quad (2.7.3.23)$$

the constants a, b, $c_1 \ldots c_4$, $d_1, \ldots d_4$ having to be estimated from the data of a particular station. When $t = 365$, $\Lambda(365)$ is the mean number of peaks per year.

Distribution of annual maxima. For the distribution of annual maxima Equations (2.7.3.15)–(2.7.3.20) still hold, with λ replaced by $\Lambda(t = 365)$. That is the introduction of the function $\Lambda(t)$ does not alter the distribution of annual maxima between model 2 and model 3. Since the number of peaks over the threshold in the season between t_1 and t_2 is a Poisson variate with mean $\Lambda(t_2)-\Lambda(t_1)$ and the peak magnitudes have a common distribution, the theory of annual maxima of model 2 is directly applicable to seasonal maxima here. Thus, if Q^s_{max} is the maximum in the season its distribution function is from Equation (2.7.3.16)

$$PR(Q^s_{max} \leqslant q) = e^{-[\Lambda(t_2)-\Lambda(t_1)].[1-F(Q\leqslant q/q\geqslant q_0)]} \quad (2.7.3.24)$$

If the distribution of magnitudes is exponential with parameters q_0 and β then by Equation (2.7.3.17) Q^s_{max} values which exceed q_0 have the Gumbel distribution

$$PR(Q^s_{max} \leqslant q) = e^{-e^{-(q-u'')/\beta}} \quad (2.7.3.25)$$

where

$$u'' = q_0+\beta \ln[\Lambda(t_2)-\Lambda(t_1)]. \quad (2.7.3.26)$$

Model 4

There is yet a further generalisation introduced by Todorovic & Rousselle (1971) where in addition to accommodating varying rates of occurrence of events within the year this model accommodates different distributions of peak magnitudes during different seasons of the year.

Let the year be divided into M seasons and let the occurrence of peaks in any season be a time dependent process exactly as in model 3. Let the ith season begin at time T_{i-1} and end at time T_i. The function $\Lambda(t)$ is as described for model 3. Let λ_i be the number of peaks over the threshold q_0 in season i,

$$\lambda_i = \Lambda(T_i)-\Lambda(T_{i-1}) \quad (2.7.3.27)$$

and let the distribution of magnitudes have distribution function

$$PR(Q\leqslant q \text{ in season } i) = F_i(Q\leqslant q/q\geqslant q_0). \quad (2.7.3.28)$$

Distribution of seasonal maxima. Let Q^i_{max} be the maximum in season i. The theory of model 2 may be applied to this or any individual season and therefore by Equation (2.7.3.16)

$$PR(Q^i_{max} \leqslant q) = e^{-\lambda_i[1-F_i(Q \leqslant q/q \geqslant q_0)]}. \tag{2.7.3.29}$$

Distribution of annual maxima. Let Q_{max} be the maximum value in the year,

$$Q_{max} = \max(Q^1_{max}, Q^2_{max} \ldots Q^M_{max}). \tag{2.7.3.30}$$

Then by the multiplication theorem of probability

$$\begin{aligned} PR(Q_{max} \leqslant q) &= \prod_{i=1}^{M} PR(Q^i_{max} \leqslant q) \\ &= \prod_{i=1}^{M} e^{-\lambda_i[1-F_i(Q \leqslant q/q \geqslant q_0)]} \\ &= \exp - \sum_{i=1}^{M} \lambda_i[1-F_i(Q \leqslant q/q \geqslant q_0)]. \end{aligned} \tag{2.7.3.31}$$

If the distribution of magnitudes in each season is exponential with common threshold q_0 and scale parameter β_i in the ith season this gives

$$\begin{aligned} PR(Q_{max} \leqslant q) &= e^{-\Sigma\lambda_i e^{-(q-q_0)/\beta_i}} \\ &= e^{-\Sigma e^{-(q-u_i')/\beta_i}} \end{aligned} \tag{2.7.3.32}$$

where

$$u_i' = q_0 + \beta_i \ln \lambda_i. \tag{2.7.3.33}$$

An obvious particular case is the distribution of the maximum in seasons $j, j+1, j+2, \ldots j+k$. This is obtained by replacing the limits of summation in Equation (2.7.3.31) from $i = 1, \ldots M$ to $i = j, j+1, \ldots j+k$. The account given here under the heading model 4 substantially describes the aspects of the paper by Todorovic & Rousselle (1971) which are of immediate interest. The early part of that paper presented the theoretical results on which the model is based in a manner more formal than used here.

2.7.4 *Suitability and applicability of* POT *models 1–4*

In the presentation of these models assumptions about distributions were made, (Poisson distribution of number of events in a season or year and exponential distribution of magnitudes), which helped to make them more meaningful than if presented in an abstract manner. It must be understood that these distributions have no special prior entitlement except that they have also been used by earlier expositors; their use must be justified by the support they receive from the data. The exponential and Poisson assumptions are discussed under separate headings in subsections 2.7.6 and 2.7.7. Since the expression for $Q(T)$ in model 1 (Equation (2.7.3.3)) is of the same form as that for $Q(T)$ in model 2 (Equation 2.7.3.13)) the use of the Poisson distribution does not affect the result. When T is small the random element in the number of peaks per year is relatively important. In model 1 no information about the way peaks occur in time within the year is given. This would be an important omission if the flood which is expected to occur on average twice in three years needs to be specified but quite unimportant if the once in 100 years event is required. Model 2 makes good this omission but in an unrealistic way because it is to be assumed that they are randomly distributed in time throughout the year. This would also give unrealistic results for very small return periods if there was a marked seasonal effect in the occurrence of peaks over a threshold.

Therefore, when very low return periods are of interest model 3 would be necessary to take the seasonal effect into account.

On the other hand, for large return periods the use of the correct distribution of magnitudes is all important in comparison to the distribution of occurrence of peaks within the year. Provided the average number of events per year is correct, the within year distribution of occurrence is irrelevant for large T but a mistake in the distribution of peak magnitudes is crucial. In this study the major use made of the POT model is in the provision of an alternative mean annual flood estimate which might be advantageous when only a short record is available. For this purpose model 2 is used as described above where the resulting estimates of mean annual flood are compared with those obtained from the annual maxima.

2.7.5 Estimation of parameters in model 2

This is the model in which the number of peaks exceeding the threshold each year is considered to be a random variable having a Poisson distribution with parameter λ and the distribution of the peak magnitudes is exponential with parameters q_0 and β. The estimation of parameters in models 3 and 4 follow exactly similar principles the only difference being that parameters have to be estimated separately for each season. There are two distinct methods of abstracting data from the discharge record and the method of estimating the parameters and hence the T year flood depends on the method of abstraction.

Abstraction method 1

In the first method the threshold q_0 is fixed before the data are abstracted while the number of peaks over the threshold M and their magnitudes $q_1, q_2 \ldots q_M$ are unknown. These latter are abstracted from the record whose length is taken as N years.

If the number of events per annum is regarded as a Poisson variate then both the moments and maximum likelihood estimates of λ are given by the mean

$$\hat{\lambda} = M/N \qquad (2.7.5.1)$$

but it should be noted that this is the estimate of λ that would be used even if the form of the distribution were unspecified. Therefore, the Poisson assumption does not play a large part here.

The magnitudes $q_1, q_2, \ldots q_M$ are treated as a random sample from the exponential distribution with q_0 known and β unknown. Graphical and numerical estimates are discussed in Section 1.3. The maximum likelihood and moments estimates of β coincide and are

$$\hat{\beta} = \bar{q} - q_0 \qquad (2.7.5.2)$$

where $\bar{q} = \sum_{i=1}^{M} q_i/M$.

The T year flood is then written

$$Q(T) = q_0 + \hat{\beta} \ln \hat{\lambda} + \hat{\beta} \ln T \qquad (2.7.5.3)$$

Graphical estimation. If the distribution of magnitudes is estimated graphically by a curve rather than a straight line on an exponential base the estimated distribution is not exponential. In that case the value of y (abscissa) corresponding to a return period $T' = \lambda T$ sampling units is computed as $y = \ln \lambda T$ and the ordinate at that value of y gives the graphical estimate of $Q(T)$.

Abstraction method 2

The second and distinct method of estimating the parameters is to extract a fixed number of floods M. This means that the average number of floods per year is fixed before extracting the data from a list containing more than M items. In this case λ is known but q_0 and β are initially unknown and must be estimated from the sample $q_1, q_2 \ldots q_M$.

If POT data are abstracted by using an arbitrary threshold value q_0 which corresponds if possible to a convenient stage height or discharge reading on the record chart then an analysis performed on these data would regard q_0 as known and fixed. If subsequently a specified number of peaks such as the highest $3N$ values are picked from this list the threshold which would give the $3N$ peaks exactly cannot be found without reference to the data. In this case any value between the $3N$th and $3N+1$th largest values would serve for the purposes of extraction but the exact value must be treated as an unknown less than the $3N$th largest value. This unknown value is estimated statistically.

In this method therefore the location and scale parameters q_0 and β of an exponential distribution need to be estimated jointly from a sample of fixed size M. The method of estimation in such a case is described in Section 1.3 both for graphical and maximum likelihood methods. The maximum likelihood estimates of q_0 and β, after correction for bias, are

$$\hat{q}_0 = \frac{Mq_{\min} - \bar{q}}{M-1} = q_{\min} - \frac{\hat{\beta}}{M} \tag{2.7.5.4}$$

and

$$\hat{\beta} = \frac{M(\bar{q} - q_{\min})}{M-1} \tag{2.7.5.5}$$

and the estimated T year flood is

$$\widehat{Q(T)} = \hat{q}_0 + \hat{\beta}(\ln \lambda + \ln T). \tag{2.7.5.6}$$

Equation (2.7.5.6) resembles Equation (2.7.5.3); in Equation (2.7.5.6) however q_0 and β are estimated, but in Equation (2.7.5.3) λ and β are estimated.

In graphical estimation if a curve is drawn through the plotted points the same remarks apply as were made for abstraction method 1, and $\widehat{Q(T)}$ is obtained as the ordinate at $y = \ln(\lambda T)$.

Sampling variance of $\widehat{Q(T)}$. The variance of $\widehat{Q(T)}$ of Equation (2.7.5.3) is, noting that q_0 is constant, and using Equation (1.1.6.6)

$$\text{var } \widehat{Q(T)} = \text{var}(\hat{\beta} \ln \hat{\lambda}) + 2 \ln T \,\text{cov}(\hat{\beta} \ln \hat{\lambda}, \hat{\beta}) + (\ln T)^2 \text{ var } \hat{\beta}. \tag{2.7.5.7}$$

The distribution of peak magnitudes is independent of the distribution of number of events per year so that $\text{cov}(\hat{\beta}, \hat{\lambda}) = 0$. Under the Poisson assumption $\text{var } \hat{\lambda} = \lambda/N$ and $\text{var } \hat{\beta} = \text{var } q/M = \beta^2/M$ since $\hat{\beta}$ is the estimate of the mean of the exponential distribution. Using large sample formulae for the variance of a function of random variables and for the covariance of two functions of random variables (Kendall & Stuart, 1961, Vol. I, Equations 10.12 and 10.13) it can be shown that var($\hat{\beta}$ ln $\hat{\lambda}$) = (ln λ)^2 var $\hat{\beta}$+(β/λ)^2 var $\hat{\lambda}$ and cov($\hat{\beta}$ ln $\hat{\lambda}$, $\hat{\beta}$) = (ln λ) var $\hat{\beta}$. Substituting these expressions in Equation (2.7.5.7) gives

$$\text{var } \widehat{Q(T)} = \frac{\beta^2}{N}\left[1 + \frac{(\ln T + \ln \lambda)^2}{\lambda}\right] \quad (2.7.5.8)$$

which will also be written for convenience as

$$\text{var } \widehat{Q(T)} = \frac{\beta^2}{N} PD1(T, \lambda) \quad (2.7.5.9)$$

where $PD1(T, \lambda)$ depends only on T and λ and is defined by the expression in square brackets in Equation (2.7.5.8).

In abstraction method 2 a different expression for var $\widehat{Q(T)}$ is required because the estimated parameters appearing in Equation (2.7.5.6) differ from those in Equation (2.7.5.3). For convenience rewrite (2.7.5.6) as

$$\widehat{Q(T)} = \hat{q}_0 + \hat{\beta}G \quad (2.7.5.10)$$

where

$$G = \ln \lambda + \ln T \quad (2.7.5.11)$$

is not a random variable. Then by Equation (1.1.6.6)

$$\text{var } \widehat{Q(T)} = \text{var } \hat{q}_0 + 2G \text{ cov}(\hat{q}_0, \hat{\beta}) + G^2 \text{ var } \hat{\beta}. \quad (2.7.5.12)$$

Sukhatme (1937) shows that $2(M-1)\,\hat{\beta}/\beta$ is distributed as χ^2 on $2(M-1)$ degrees of freedom and therefore var $\beta = \beta^2/(M-1)$. Further, he shows that $q_{\min}$ and $\hat{\beta}$ are statistically independent so that cov($q_{\min}$, $\hat{\beta}$) = 0. It was shown in Section 1.3 that $q_{\min}$ is exponentially distributed with parameter β/M so that var $q_{\min} = \beta^2/M^2$. These last two facts applied in the variance of $\hat{q}_0$ in Equation (2.7.5.4) lead to the expressions var $\hat{q}_0$ = $\beta^2/[M(M-1)]$ and cov($\hat{q}_0$, $\hat{\beta}$) = $-\beta^2/[M(M-1)]$. Insertion of these expressions in Equation (2.7.5.12) gives

$$\text{var } Q(T) = \frac{\beta^2}{M}\left[\frac{(1-G)^2}{M-1} + G^2\right]$$

which on inserting for G from Equation (2.7.5.11) gives

$$\text{var } \widehat{Q(T)} = \frac{\beta^2}{N}\left[\frac{(1-\ln \lambda - \ln T)^2}{N\lambda - 1} + (\ln \lambda + \ln T)^2\right]\Big/\lambda. \quad (2.7.5.13)$$

This will also be referred to as

$$\text{var } \widehat{Q(T)} = \frac{\beta^2}{N} PD2(T, \lambda, N) \quad (2.7.5.14)$$

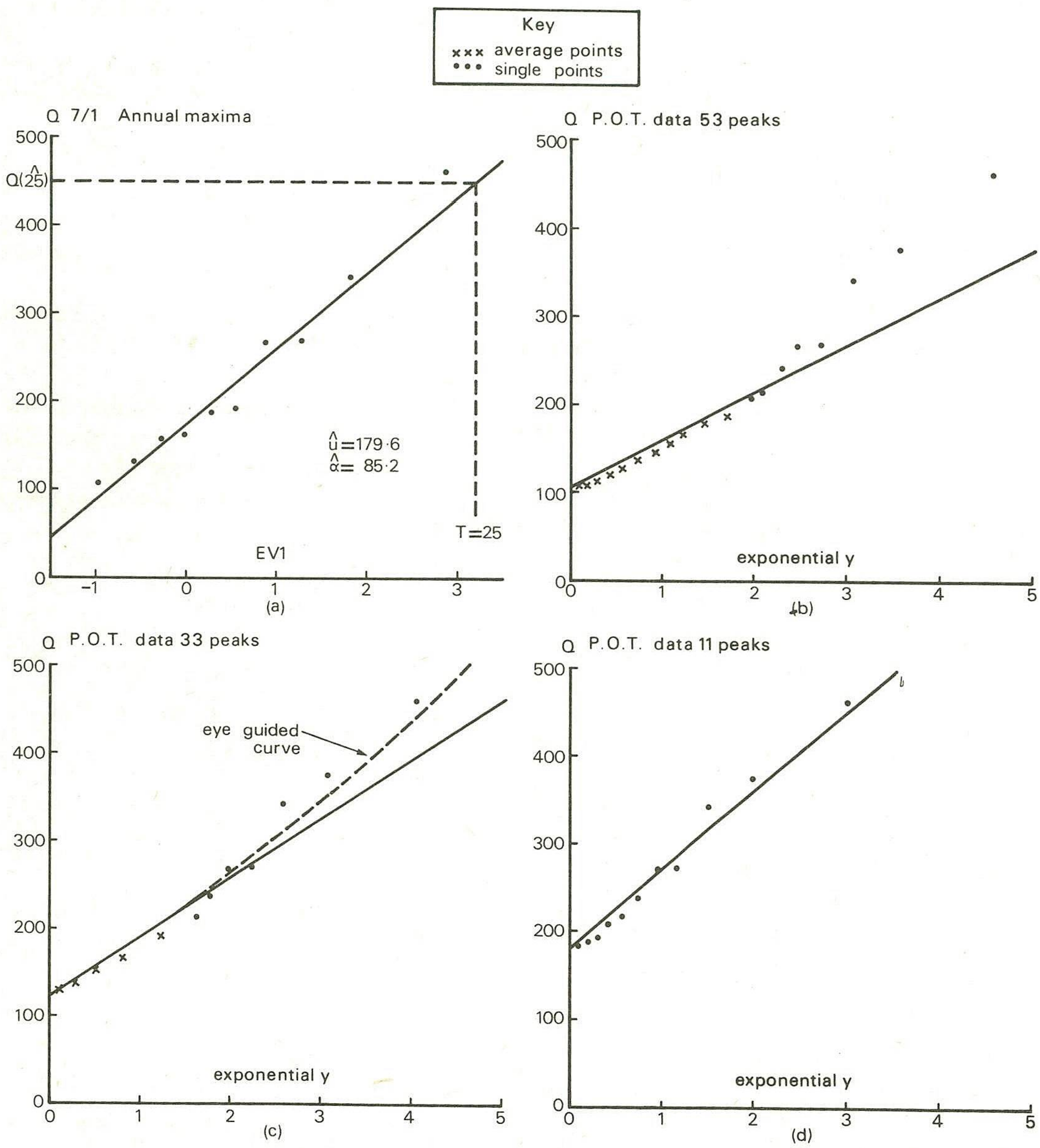
which resembles Equation (2.7.5.9) but note that the multiplier of β^2/N in (2.7.5.9) depends on T and λ only whereas in (2.7.5.14) it depends on N as well.

2.7.6 The exponential assumption

An exhaustive study of the suitability of the exponential distribution for all stations was not carried out but there is sufficient evidence to show that it is not universally applicable. The most striking form of departure from the assumption is a distinct change of slope in the probability plot of the POT magnitudes. The data of 7/1, Findhorn at Shenachie give such an example. Figure 2.18 shows the annual maxima and the POT data corresponding to one, three and five peaks per year. Close to the threshold the POT data are packed together very closely and for this reason the data are grouped at the lower end and the group averages only are plotted, a procedure which does not invalidate the plot in any way. The annual maxima appear to fit very well about a straight line representing an EV1 distribution fitted by moments, with parameters $\hat{u} = 179.6$ and $\hat{\alpha} = 85.2$. The 25 year flood is estimated by

$$\widehat{Q(25)} = \hat{u} + 3.20\,\hat{\alpha} = 451.7.$$

Fig 2.18 (a) Annual maxima at 7/1, Findhorn at Shenachie, and the peaks over a threshold data corresponding to (b) five peaks per year, (c) three peaks per year, (d) one peak per year.



The 25 year flood estimated from the several POT series are given by Equation (2.7.3.4)

$$\widehat{Q(25)} = q_0 + \beta \ln \lambda + \beta \ln 25.$$

The values of q_0 and β corresponding to λ values of approximately 1, 2, 3, 4 and 5 are given in Table 2.53 and $\widehat{Q(25)}$ is shown in the bottom row. These figures are based on 10 years' data but there are an extra 2 months at the beginning of the record which contribute two peaks over the first four thresholds and one over the highest threshold. All these peaks are included in the determination of the distribution of peaks but they are not included in determining the rate of occurrence of peaks per year. Thus, there are 32 peaks over the third threshold but only 30 of these occurred during the 10 complete water years and so $\hat{\lambda} = 3.0$. For low thresholds the estimate of $Q(25)$ is low in comparison to the estimate from annual maxima

Table 2.53 Estimates of $Q(25)$ from POT data with low and high thesholds.

λ	5.1	4.0	3.0	2.0	1.0
q_0	107.0	111.5	125.2	141.5	177.0
β	53.2	62.1	65.1	74.7	91.5
$\widehat{Q(25)}$	364.9	397.5	408.0	433.7	471.5

and a glance at Figure 2.18 (b) and (c) can show why. The fitted distribution is dominated by the values near the threshold rather than by the largest values because the slope of the fitted line β is the mean amount by which the plotted points exceed q_0. The ordinate at $y = \ln 25 = 3.22$ is $\widehat{Q(25)}$; at this point the plotted points differ consistently from the fitted line. The POT estimates at the high threshold Figure 2.18(c) is in agreement with the annual maximum estimate.

If a distribution other than exponential giving a curve lying close to the plotted points were fitted to the data the estimates of $\widehat{Q(25)}$ so obtained would agree more closely with the annual maximum estimate. For example an eye guided curve drawn through the data of Figure 2.18(c) represents a distribution in which the 25 year flood has return period $T' = \lambda T = (3.2)$ $25 = 80$. The value of abscissa with this return period is $y = \ln 80 = 4.44$ and the ordinate at this point is 475 which is an improvement on the earlier figure of 408. A Pearson Type 3 distribution was fitted to these data and while it provided a closer fit to the plotted data for high thresholds the agreement was not as good as that given by the exponential straight line at the lower thresholds.

The plots of POT magnitudes corresponding to four thresholds at 22 long term stations were examined; 11 of them were similar to 7/1 of the above example and the others could be said to agree well with the exponential assumption. Although the example, 7/1, is not typical it shows what can result from the uncritical use of any distribution.

The fact that many records considered jointly in Section 2.6 indicated a general extreme value distribution with negative k for annual maxima is in conflict with an exponential distribution for POT magnitudes although small samples from a GEV population with negative k could give the appearance of being almost exponential.

2.7.7 The Poisson assumption

The Poisson assumption is introduced into models 2, 3 and 4 as one method of expressing the random manner in which peaks over a threshold occur. For large return periods the mean number of peaks per year needs to be known but specification of the correct distribution is less important. In this subsection the results of examining this assumption on a selection of records show that when the series contains two peaks per year or more the assumption is unreasonable. The Poisson assumption is consistent with a completely random occurrence within the year but it does not imply it; it is also consistent with a varying rate of occurrence of peaks within the year with the number occurring in any time interval being a Poisson variate.

Let m be the random variable denoting the number of events in a year with mean value λ. Then under the Poisson hypothesis the variance of the distribution is also λ and hence the ratio observed variance/observed mean should be approximately 1, the departure from 1 being due to sampling fluctuations. The Fisher dispersion test for a Poisson distribution uses this fact. A statistic d is computed from the observed data which is distributed as χ^2 if the Poisson hypothesis is true. If it is false d will either be exceedingly small or exceedingly large.

If the mean λ is sufficiently large, $\lambda > 5$, the distribution of m is almost symmetrical. As λ gets larger, m becomes Normally distributed. Hence

$$m \sim N(m, \sigma) = N(\lambda, \sqrt{\lambda})$$

where $\sim$ means 'is distributed as', and

$$\frac{m-\lambda}{\sqrt{\lambda}} \sim N(0, 1)$$

therefore

$$\sum_{i=1}^{N} \left(\frac{m_i - \lambda}{\sqrt{\lambda}}\right)^2 \sim \chi^2_N$$

where N = number of years.

If λ is estimated by $\bar{m}$ a degree of freedom is lost and then

$$d = \sum_{i=1}^{N} \frac{(m_i - \bar{m})^2}{\bar{m}} \sim \chi^2_{N-1}. \quad (2.7.7.1)$$

Thus, if $\bar{m} > 5$ (about), d is distributed as χ^2_{N-1}. However, even if $\bar{m}$ is less than five Sukhatme (1938) has shown that assuming d to be χ^2_{N-1} is adequate for testing the difference between Poisson mean and variance provided $N > 5$ if $\bar{m} > 1$ and that $N > 15$ if $\bar{m} < 1$.

Threshold levels

When the data were extracted the original threshold level was chosen so as to include between five and six peaks per year on average. Analysis is carried out at this threshold level and a new threshold level is then found above which there are exactly four events per year and an analysis is carried out on this subset of events. This process is repeated for thresholds which give exactly three, two and one events per year on average. Thus, for each station the behaviour of the Poisson assumption at each of five threshold levels can be observed and further it is believed that because of the manner in which the thresholds are chosen comparison of findings can be made between catchments. In some cases it is not possible to define a threshold exceeded by exactly one, two, three or four events per year because of several equal flow values at the threshold.

Table 2.54 shows for each of 26 stations the ratio of observed variance to the mean of the number of peaks per year, R, and the statistic d defined by Equation (2.7.7.1). The relation between R and d is

$$R = (N-1)d \quad (2.7.7.2)$$

where N is the number of years of record. The probability that a χ^2 variate with $N-1$ degrees of freedom should exceed the observed d value is also given. The exact figures are not given but the tabular values from Fisher & Yates (1963, Table IV) are used so that the true probability is usually a little greater than the quoted value but less than the next tabular value. At the upper end, the tabular values are 0.5, 0.3, 0.2, 0.1, 0.05, 0.02, 0.01, 0.001 so that a quoted value of 0.3 means 'a value somewhere in (0.5, 0.3)', whereas one quoted as 0.01 means 'a value somewhere in (0.02, 0.01)'.

The 26 stations represent 20 distinct catchments and therefore under the Poisson hypothesis the average number of values of d significant at the 5% level is 1 while 3 or 4 would be highly unusual. As can be seen at the bottom of Table 2.54 there are 7 significant values at the one per year threshold level and at least 12 at the other thresholds which firmly rejects the Poisson hypothesis for these 20 catchments taken as a whole.

The nature of the departure from Poisson behaviour

Inspection of records of POT data shows that there are some years which have a very large number of peaks and that peaks over the threshold sometimes occur in bunches. The magnitudes of successive peaks do not appear to be statistically dependent on one another (see 2.7.8) but the bunching reflects statistically in the point stochastic process which gives rise to the peaks. The interevent times are not identically distributed random variables nor are they always independent. They may be so in summer but in winter there is some dependence.

The negative binomial distribution is an alternative to the Poisson distribution for discrete valued variates and departs from it in the same way as the data in that its variance is greater than its mean. It has two parameters and its pdf is

$$p_m = \frac{\gamma(\gamma+1)\ldots(\gamma+m-1)}{m!}\alpha^m(1-\alpha)^\gamma$$

and has mean and variance

$$E(m) = \frac{\alpha}{1-\alpha}\gamma$$

$$\text{var}(m) = \frac{\alpha}{(1-\alpha)^2}\gamma.$$

This distribution is consistent with the assumption that the number of peaks in a year comes from a Poisson distribution with mean r which itself is drawn randomly from a family of Poisson distributions in which r is a random variable with a gamma distribution having scale and shape parameters $\alpha/(1-\alpha)$ and γ. It is also consistent with the assumption that a Poisson number of events occurs each year and that prescribed proportions of these have one peak, two peaks, three peaks . . . and so on which would account for bunching during wet years.

This distribution was fitted to several sets of data and since there are

Station		Threshold at One/year	Two/year	Three/year	Four/year	Five/year	Years of record, N
7/1	R	1.33	1.45	1.11	1.27	1.76	10
	d	11.97	13.05	9.99	11.43	15.84	
	$PR(\chi^2>d)$	0.20	0.10	0.30	0.20	0.05	
7/3	R	1.17	1.63	1.40	1.03	0.99	11
	d	11.7	16.3	14.0	10.3	9.9	
	$PR(\chi^2>d)$	0.30	0.05	0.10	0.30	0.30	
8/2	R	1.04	1.60	1.29	2.17	1.73	18
	d	17.68	27.20	21.93	36.89	29.41	
	$PR(\chi^2>d)$	0.30	0.05	0.10	0.001	0.02	
8/3	R	1.30	1.86	1.76	1.76	1.59	18
	d	22.10	31.62	29.92	29.92	27.03	
	$PR(\chi^2>d)$	0.10	0.01	0.02	0.02	0.05	
8/4	R	1.06	1.12	1.39	1.20	1.08	18
	d	18.02	19.04	23.63	20.40	18.36	
	$PR(\chi^2>d)$	0.30	0.30	0.10	0.20	0.30	
8/5	R	1.30	2.01	1.72	1.98	1.98	18
	d	22.10	34.17	29.24	33.66	33.66	
	$PR(\chi^2>d)$	0.10	0.001	0.02	0.001	0.001	
8/6	R	1.06	0.82	1.14	1.38	1.84	18
	d	18.02	13.94	19.38	23.46	31.28	
	$PR(\chi^2>d)$	0.30	0.50	0.20	0.10	0.01	
8/7	R	2.26	1.71	1.56	2.75	3.42	17
	d	36.16	27.36	24.96	44.00	54.72	
	$PR(\chi^2>d)$	0.001	0.02	0.05	<0.001	<0.001	
8/8	R	1.75	1.68	2.04	2.69	1.89	17
	d	28.00	26.88	32.64	43.04	30.24	
	$PR(\chi^2>d)$	0.02	0.02	0.001	<0.001	0.01	
8/10	R	1.56	1.72	1.91	2.24	2.09	17
	d	24.96	27.52	30.56	35.84	33.44	
	$PR(\chi^2>d)$	0.05	0.02	0.01	0.001	0.001	
24/4	R	0.89	1.85	1.33	0.67	0.63	10
	d	8.01	16.65	11.97	6.03	5.67	
	$PR(\chi^2>d)$	0.50	0.05	0.20	0.70	0.70	
27/1	R	1.55	1.56	1.63	2.17	—	35
	d	52.70	53.04	55.42	73.78	—	
	$PR(\chi^2>d)$	0.02	0.01	0.01	<0.001	—	
28/3	R	1.07	1.15	1.23	1.72	1.72	14
	d	13.91	14.95	15.99	22.36	22.36	
	$PR(\chi^2>d)$	0.30	0.30	0.20	0.05	0.05	
28/4	R	1.50	1.91	1.27	1.79	2.20	13
	d	18.00	22.92	15.24	21.48	28.60	
	$PR(\chi^2>d)$	0.10	0.02	0.20	0.02	0.001	
28/5	R	1.00	1.41	2.05	1.75	2.28	13
	d	12.00	16.92	24.60	21.00	27.36	
	$PR(\chi^2>d)$	0.30	0.10	0.01	0.05	0.001	
28/7	R	1.85	1.69	1.44	1.66	1.54	14
	d	24.05	21.97	18.72	21.58	20.02	
	$PR(\chi^2>d)$	0.02	0.05	0.10	0.05	0.05	
34/1	R	1.60	2.70	2.57	—	—	11
	d	16.0	27.0	25.7	—	—	
	$PR(\chi^2>d)$	0.05	0.001	0.001	—	—	
45/1	R	1.83	2.41	2.88	2.56	2.23	13
	d	21.96	28.92	34.68	30.72	26.76	
	$PR(\chi^2>d)$	0.02	0.001	<0.001	0.001	0.001	
46/3	R	0.60	0.50	1.13	0.85	1.20	11
	d	6.0	5.0	11.3	8.5	12.0	
	$PR(\chi^2>d)$	0.80	0.80	0.30	0.50	0.30	
53/1	R	1.26	1.29	1.77	1.98	2.16	31
	d	37.8	38.7	53.1	59.4	64.8	
	$PR(\chi^2>d)$	0.10	0.10	0.001	0.001	<0.001	

Station		Threshold at One/year	Two/year	Three/year	Four/year	Five/year	Years of record, N
53/4	R	1.40	2.50	3.00	3.15	3.24	
	d	14.0	25.0	30.0	31.5	32.4	11
	$PR(\chi^2>d)$	0.10	0.001	<0.001	<0.001	<0.001	
54/6	R	2.0	2.37	2.04	3.22	5.86	
	d	32.00	37.92	32.64	51.52	93.76	17
	$PR(\chi^2>d)$	0.01	0.001	0.001	<0.001	<0.001	
55/8	R	1.33	1.96	1.84	1.86	1.66	
	d	23.94	35.28	33.12	33.48	29.88	19
	$PR(\chi^2>d)$	0.10	0.001	0.01	0.01	0.02	
59/1	R	2.00	2.09	1.39	1.41	1.53	
	d	22.00	22.99	15.29	15.51	16.83	12
	$PR(\chi^2>d)$	0.02	0.02	0.10	0.10	0.10	
60/1	R	1.20	1.48	1.40	1.65	1.85	
	d	12.0	14.8	14.0	16.5	18.5	11
	$PR(\chi^2>d)$	0.20	0.10	0.10	0.05	0.02	
62/1	R	1.11	0.68	0.54	1.53	—	
	d	9.99	6.12	4.86	13.77	—	10
	$PR(\chi^1>d)$	0.30	0.70	0.80	0.10	—	
Number of values significantly large at 5% level out of 26 stations on 20 catchments		7	13	12	13 out of 25	14 out of 23	—

Table 2.54 Ratio of observed variance to observed mean of number of peaks over a threshold per year and dispersion test statistic.

two parameters it must be expected to provide better agreement with the data than that given by a simple Poisson distribution. However, it did not appear to offer a vast improvement and although no formal statistical tests of a negative binomial hypothesis were carried out the results do not suggest that it is ideal. An example of the fit in comparison to the Poisson distribution is given in Figure 2.19. The data are from the two per year threshold at 8/2, Spey at Kinrara, with 18 years of record. The negative binomial gives a closer fit to the data on the cumulative histogram but its superiority is not so marked on the histogram (lower part of Figure 2.19).

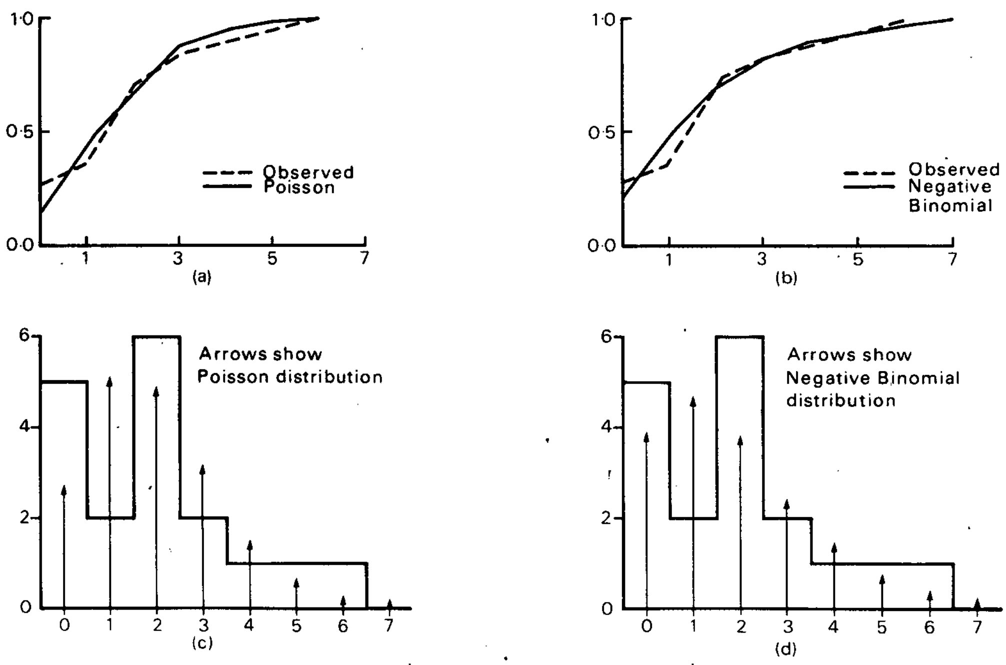


Fig 2.19 The distribution of the number of peaks per year at the two per year level at 8/2, Spey at Kinrara. (a) and (b) Distribution functions fitted to Poisson and negative binomial distributions. (c) and (d) Corresponding discrete probability functions.

2.7.8 *Statistical independence of successive peaks*

It has often been stated that one of the drawbacks of the POT approach is that successive items in the series are not independent, and this is sometimes taken to mean that the successive peak magnitudes are correlated. Successive peaks were examined for correlation and no evidence of this could be found so that the dependence exists as some form of persistence in the point stochastic process producing the peaks.

The correlation coefficient between successive members of the entire series of POT data is a poor measure of dependence since the time interval between peaks varies and because inspection of records shows that the length of time between successive peaks in summer and autumn is so long that these peaks are due to different meteorological events. Therefore, dependence between peaks should be sought in the first instance among peaks which are separated by short time intervals. Scatter diagrams of peaks q_1 against q_2 which occur within 10 days of one another are shown for four stations in Figure 2.20 where different symbols show those peaks which occur within 5 days of one another and those which occur within 6–10 days. Even among the plotted points in the first category there is no trend on which a prediction of peak magnitude q_2, which is known to occur within 5 days of an earlier one q_1, may be based on knowledge of q_1. Therefore, it is concluded that successive peak magnitudes in the POT series are independent and that dependence in the POT series is in the interevent times only.

2.7.9 *Use of POT series for estimation of mean annual flood*

The previous two subsections examined the suitability of the Poisson and exponential distributions in Model 2; it was stated there that the Poisson assumption is not vital unless return periods of less than a year are to be considered but that the exponential distribution could not be used blindly for magnitudes. However even when the exponential distribution is an inadequate description of the data as a whole it does in nearly all cases represent the flow values near to the threshold fairly closely and the discrepancy when it does exist is due to the failure of the distribution to represent the upper end of the data. There is therefore the possibility that model 2 could be used in all circumstances to estimate floods of low return period. One such flood is the mean annual flood. This is computed as described below and compared with the value computed from the annual maximum series. The agreement between the two sets of figures is considered to be good enough to suggest that the POT series affords a valuable method of estimating the mean annual flood.

It was seen that the distribution of those annual maxima which exceed the threshold q_0 can be deduced from the POT model and its parameters expressed in terms of the POT model parameters. When the conditional distribution of magnitudes exceeding the threshold is exponential these annual maxima have the EV1 distribution with parameters β and $u' = q_0 + \beta \ln \lambda$. On the assumption that the entire annual maxima have the EV1 distribution the mean is

$$\begin{aligned} \mu &= u' + 0.5772\,\beta \\ &= q_0 + \beta \ln \lambda + 0.5772\,\beta. \end{aligned} \qquad (2.7.9.1)$$

If q_0, β and λ are available from the POT model Equation (2.7.9.1) can be

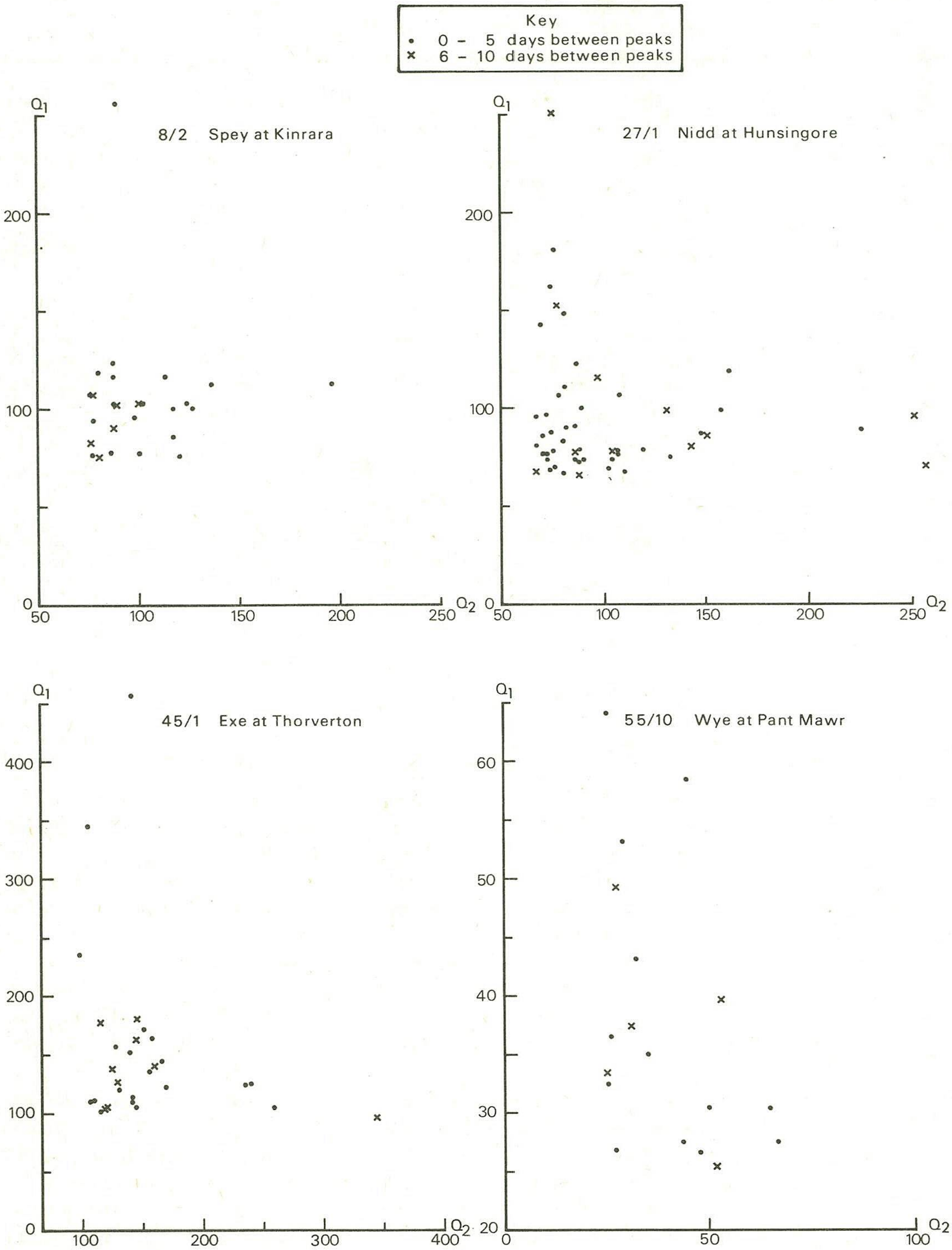


Fig 2.20 Magnitudes of successive peaks over a threshold occurring within 10 days of each other plotted one against another and showing no trace of statistical dependence.

evaluated to give the mean of the annual maxima. This depends of course on the assumption that those annual maxima less than q_0 conform to the lower end of an EV1 distribution even though no distributional information about values less than q_0 is contained in the POT model specification. In the

EV1 distribution a proportion $e^{-e^{-0.5772}}$ of the variate values are less than the mean while in the POT model a proportion $e^{-\lambda}$ of annual maximum values are less than the threshold q_0. Therefore, q_0 is less than the mean if and only if

$$e^{-\lambda} < e^{-e^{-0.5772}}$$

that is

$$\lambda > e^{-0.5772} = 0.5615. \qquad (2.7.9.2)$$

Therefore, if q_0 is so high that fewer than an average of 0.56 peaks per year over the threshold occur the mean is less than q_0, the quantity $\beta \ln \lambda + 0.5772\,\beta$ in Equation (2.7.9.1) being negative. In such a case when the parameters of the POT model have to be estimated from data the resulting estimate of the mean is an extrapolation outside the range of definition of the POT model. Common sense requires that this should be avoided but this is not difficult since $\lambda = 0.56$ corresponds to a very high threshold and usually lower thresholds would be considered desirable. When there are an average of one, two or three peaks over the threshold per year Equation (2.7.9.1) gives

$$\mu = q_0 + 0.5772\,\beta \qquad \text{when } \lambda = 1$$

$$\mu = q_0 + 1.2704\,\beta \qquad \text{when } \lambda = 2$$

$$\mu = q_0 + 1.6758\,\beta \qquad \text{when } \lambda = 3.$$

These quantities estimated from data are given for each station in Volume IV and are called PT1MAF, PT2MAF, and PT3MAF.

A number of simple comparisons were made between mean annual floods derived from annual maxima (AMAF) and from Equation (2.7.9.1) with $\lambda = 1$, 2, and 3. Figure 2.21 shows a plot of AMAF versus PT3MAF for 70 of the longest records available. Log-log paper is used only for the purposes of fitting all points on a single graph. The plotted points lie sufficiently close to the line of equality to support the conclusion that PT3MAF is an acceptable alternative to AMAF. Plots of PT2MAF and PT1MAF against AMAF look very similar to Figure 2.21. This result is for long records and the behaviour for short records based on 10 year subsets of longer records is shown next. In Table 2.55 the deviation of PT1MAF, PT2MAF and PT3MAF from AMAF is tabulated as a percentage for individual decades; the decade AMAF is tabulated also. In some decades the figures corresponding to PT3MAF or PT2MAF are blank as POT data are available only for high thresholds. The very large negative deviations for the 1950–59 decade at 28/801 are due to the fact that in 1957 a large flood of about six times the median flood occurred and this affects AMAF much more than the POT estimates. In this case the POT estimates are far more realistic than the AMAF value although they are still higher than the mean of the entire record. The large positive deviations at 12/1 for the decade 1960–69 provide one instance of a genuine difference between the two sets of estimates. The decade mean annual flood is low because the five smallest annual maxima in the decade are widely separated from the upper four (only 9 years data were available); in the corresponding POT series only some of these low flow values are included and their effect is diminished by the higher flows. Apart from these two exceptional cases the deviations in the other decades are less than 12%, one third of them being less than 5%. The decade PT1MAF estimates are shown plotted against decade AMAF estimates on Figure 2.22.

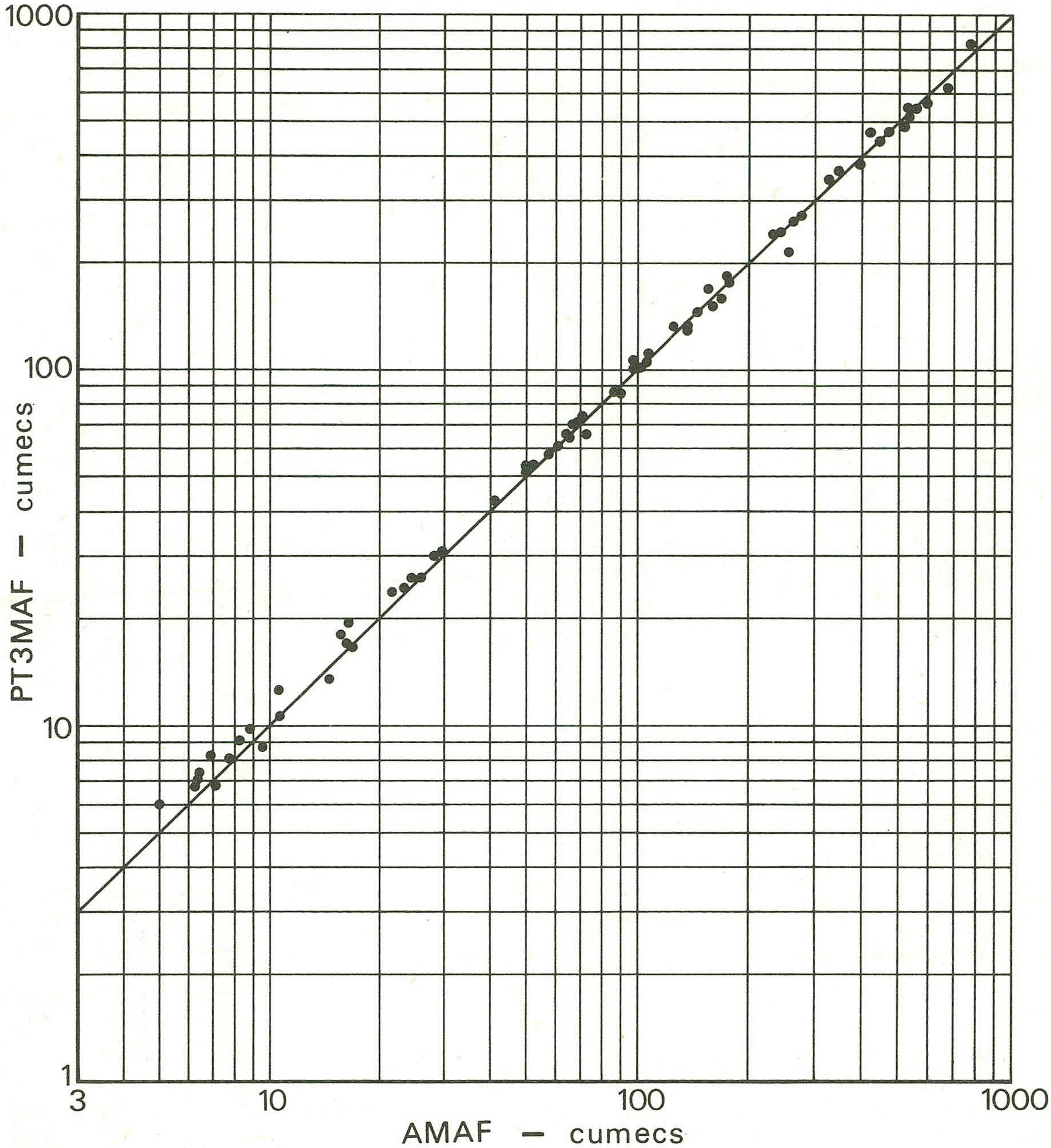


Fig 2.21 Mean annual flood estimated from POT data at three per year level compared with AMAF estimated from annual maxima at 70 long term stations. The log scales are used for convenience of plotting.

As a further measure of agreement between the two kinds of estimates of mean annual flood the correlation coefficient between them may be quoted. Based on all stations available the simple correlation coefficient between log (AMAF) and log (PT1MAF) or log (PT2MAF) or log (PT3MAF) is 0.999.

On the basis of these comparisons the conclusions are that despite the doubts over the exponential assumption it has little effect on floods of low return period and that the mean annual flood estimated from POT data is a satisfactory approximation to the estimate derived from the annual maximum data.

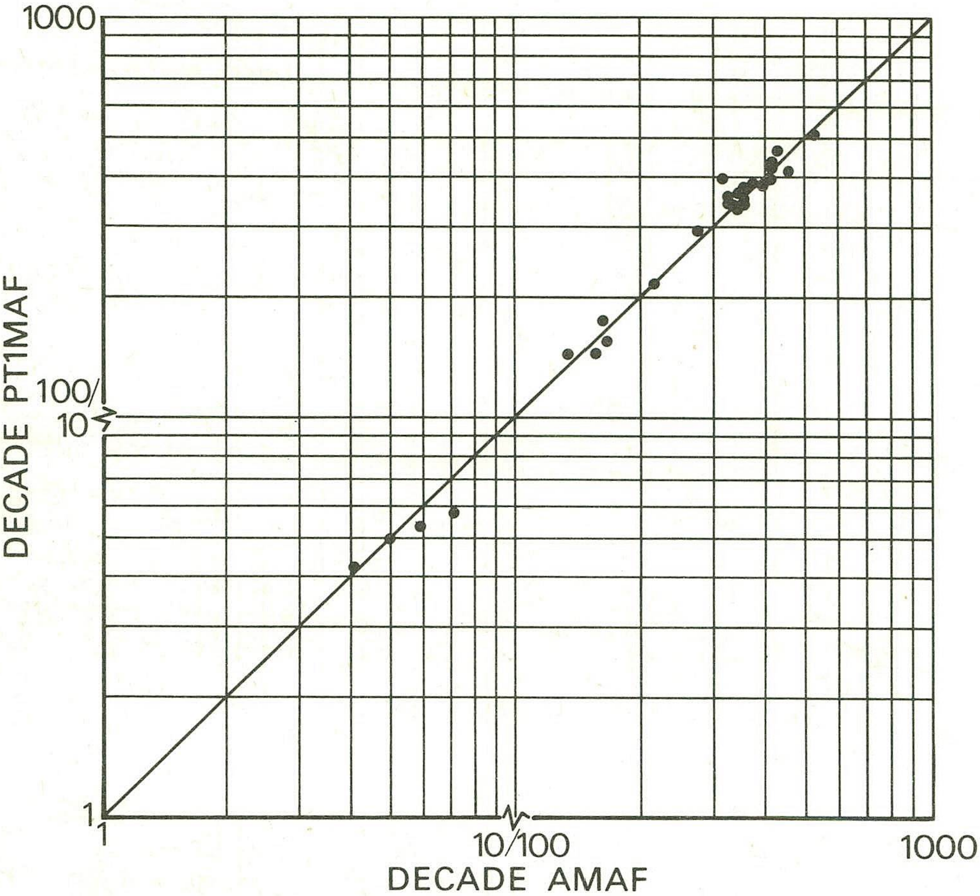


Fig 2.22 Decade mean annual flood from POT data at one per year level compared with that obtained from decade annual maxima at five stations.

Station	% deviation from AMAF	1880–89	1890–99	1900–09	1910–19	1920–29	1930–39	1940–49	1950–59	1960–69	Total record
	PT3MAF	—	—	—	—	—	2.2882	3.2412	7.2032	—	11.5166
12/1	PT2MAF	—	—	—	—	—	−1.6609	2.3249	5.5460	21.0305	10.0388
	PT1MAF	—	—	—	—	—	−2.3903	2.3777	4.0522	26.1668	9.3540
	AMAF	—	—	—	—	—	529	417	416	337	424
	PT3MAF	7.0900	—	—	12.1329	—	—	—	—	—	—
27/21	PT2MAF	−0.4034	—	—	6.9069	−8.3631	—	—	—	4.5678	−5.9125
	PT1MAF	1.6483	—	—	7.6239	−7.8228	—	—	—	9.4297	−5.7797
	AMAF	214	—	—	135	157	—	—	—	163	166
	PT3MAF	—	—	—	—	—	−8.8310	9.6296	−19.3457	−0.5225	−7.9828
28/801	PT2MAF	—	—	—	—	—	−5.9714	4.6914	−15.3976	−0.7235	−6.4987
	PT1MAF	—	—	—	—	—	−10.0084	2.7160	−18.2177	−0.5525	−7.2408
	AMAF	—	—	—	—	—	5.95	4.05	7.09	4.98	5.39
	PT3MAF	—	—	—	−3.8150	5.1885	9.1060	3.6800	4.7949	9.3866	6.1685
39/1	PT2MAF	—	−0.7982	6.2083	−2.8199	5.6764	9.4449	1.7018	4.0069	7.4888	5.6759
	PT1MAF	—	−3.8113	5.2926	−0.5888	6.8579	10.8252	2.0795	2.9732	6.0932	6.0737
	AMAF	—	357	277	353	355	325	334	334	343	327
	PT3MAF	—	—	—	—	—	−1.2827	−8.4200	1.4692	−3.3804	−3.6792
54/1	PT2MAF	—	—	—	—	—	0.0791	−8.3849	0.3944	−0.9176	−2.9892
	PT1MAF	—	—	—	—	—	−1.5873	−9.6098	1.2927	−3.4438	−2.4855
	AMAF	—	—	—	—	—	341	456	380	410	397

Table 2.55 Deviation of POT estimates of mean annual flood from annual maximum estimate.

2.7.10 *Comparison of POT estimate of $Q(T)$ with annual maximum estimate*

It was shown earlier, in Section 2.7.3, that when the distribution of POT magnitudes is exponential the annual maxima which exceed the threshold have a Gumbel distribution. This is true regardless of whether the POT model is model 1, 2, or 3. Because $Q(T)$ can be estimated in both models and the sampling variances by both methods are also available a valid comparison can be made of the estimates from each model from the point of view of statistical efficiency.

Assume that the peaks over the threshold q_0 behave as postulated in model 2; that is the number per year have a Poisson distribution and the peaks are exponentially distributed. A sample of POT data from this model would provide an estimate of the T year flood $Q(T)$, Equations (2.7.5.3) or (2.7.5.6), with sampling variance given by Equation (2.7.5.9) or (2.7.5.14). A sample of annual maxima from the same flow record can be considered as a sample from the Gumbel distribution the parameters of which are related to those of the POT model. An estimate, $Q(T)_{AM}$, together with its sampling variance can be obtained from the sample. Since the same population value is being estimated from both the POT sample and the annual maxima the sampling variances of each method afford a valid method of comparing the different models.

The equations for the sampling variances are summarised here for this comparison

Annual maxima: $\text{var } Q(T)_{AM} = \dfrac{\beta^2}{N} AM(T)$

where $AM(T) = (1.11+0.52\,y+0.61\,y^2)$ from Equation (1.4.3.6), and

y is Gumbel reduced variate of return period T,

$$y = -\ln - \ln \frac{T-1}{T}$$

Peaks over a threshold: Method (1) $\text{var } Q(T)_{POT} = \dfrac{\beta^2}{N} PD1\,(T, \lambda)$ from Equation (2.7.5.9) above.

Method (2) $\text{var } Q(T)_{POT} = \dfrac{\beta^2}{N} PD2\,(T, \lambda, N)$ from Equation (2.7.5.14) above.

Either of these last two equations must have a value of λ specified in order to be evaluated and compared with $(\beta^2/N)\,AM(T)$.

An approximate and interesting comparison arises when $\lambda = 1$ in which case the POT series is known as the annual exceedance series. In method 2 the length of record in years must be specified. In Figure 2.23 the three expressions for variance summarised above, after the division by the common factor β^2/N, are shown plotted for $\lambda = 1$ in the POT method, and a record of 20 years in POT method 2. This figure shows that the POT method gives smaller sampling variance for return periods less than $T = 5$ or $T = 10$ depending on the method in which the POT sample is extracted. For higher return periods the annual maximum estimate has smaller variance than the POT estimates.

It may be asked what value of λ is necessary to make the POT sampling

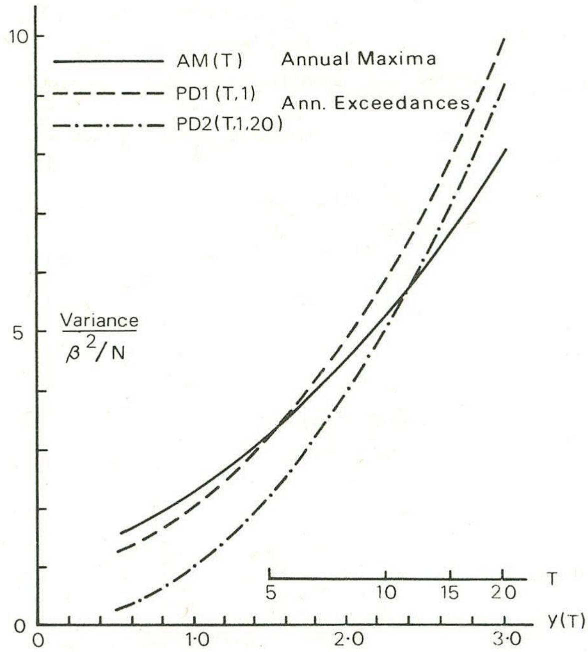


Fig 2.23 Sampling variance of $Q(T)$ divided by β^2/N estimated by annual maximum and POT methods.

variance as small as the annual maximum sampling variance for larger values of T. This is answered by setting the efficiency to unity and solving for λ. In method 1 this is

$$E_1 = \text{var}(Q(T)_{AM})/\text{var}\ Q(T)_{PD1} = AM(T)/PD1(T, \lambda) \qquad (2.7.10.1)$$

and in method 2 it is

$$E_2 = \text{var}(Q(T)_{AM})/\text{var}\ Q(T)_{PD2} = AM(T)/PD2(T, \lambda, N). \qquad (2.7.10.2)$$

Equating E_1 to unity and solving for λ at a series of values of T shows that $\lambda = 1$ at $T = 5$ and that λ increases with T until at $T = 50$ it has almost attained an asymptotic value of 1.65. This would mean for instance that with $\lambda = 1.50$ (i.e. $\lambda < 1.65$) the annual maximum estimate of $Q(T)$, $T > 5$, from a 20 year record has smaller sampling variance than the POT estimate by method 1 based on 30 peaks from the 20 year record.

Equating E_2 to unity, when $N = 20$, and solving for λ shows that $\lambda = 1$ at $T \simeq 10$ and that λ increases with T until it attains an asymptotic value of 1.68 at about $T = 50$. The chain curve on Figure 2.23 is based on $N = 20$ but it has been found that the curve does not depend greatly on N and hence the values of λ required to make E_2 equal to unity do not depend greatly on N.

It must be remembered that these findings are based on very strict assumptions which are not necessarily true in reality. However, they serve to warn that it is dangerous to assume that the use of a larger number of peaks in the POT method necessarily leads to a more efficient estimate than the use of possibly fewer peaks in the annual maximum method. In other words the dictum 'more data, better estimates' is not universally true. One possible explanation why the annual maximum estimates can be more efficient than annual exceedance estimates is that each annual maximum conveys a magnitude as well as the information that it is the largest in a given time interval whereas a POT peak conveys only a magnitude. Thus perhaps one extra annual maximum value of medium size might convey more information than one extra POT peak value which is large.

If as in reality the form of the distribution is not known exactly then an extra POT peak value which is large might be *more useful* than one extra annual maximum value of medium size for the purpose of helping to identify the correct form of distribution in that range of the variate which is most important. The words 'more useful' are used here instead of more efficient because an increase in efficiency is of use only after the correct model is known.

2.7.11 Estimation of T year flood by POT method

The use of the POT method will be demonstrated by using the data in Table 2.56 which lists the 60 highest peaks during 18 years for 8/7, Spey at Invertruim.

	Q	Date		Q	Date
1	276.92	17.12.66	31	64.61	12.12.61
2	237.50	11. 2.62	32	63.36	18.10.54
3	136.35	27. 3.68	33	63.36	30.10.68
4	131.35	5. 3.67	34	62.95	14. 8.56
5	124.04	28.12.55	35	62.95	13.10.68
6	118.12	4.12.54	36	62.54	5. 3.54
7	116.38	15. 2.62	37	61.31	2. 9.53
8	114.07	31. 1.62	38	60.90	25.11.54
9	109.54	15.12.56	39	60.90	21. 1.60
10	97.61	3. 3.67	40	60.90	1.11.65
11	96.04	14. 1.68	41	59.30	2.12.53
12	93.58	17. 3.70	42	59.30	26.12.54
13	91.91	2.11.69	43	58.90	15.11.64
14	90.91	28. 9.61	44	57.32	22.11.54
15	90.91	2.12.54	45	56.15	7.12.59
16	86.92	24.10.61	46	55.00	20. 1.57
17	85.94	15. 1.62	47	55.00	25. 3.67
18	84.96	29. 9.62	48	54.23	11.10.68
19	82.07	20.12.57	49	54.23	14. 4.60
20	81.12	11. 1.65	50	54.23	7. 1.65
21	81.12	7.11.53	51	52.35	19. 1.54
22	80.17	1. 3.56	52	51.61	21.12.54
23	80.17	15.12 62	53	51 61	10. 2.61
24	78.30	17.10.59	54	51.24	29. 7.56
25	71.94	18.12.54	55	50.87	10.12.61
26	70.61	8. 8.61	56	50.14	22.10.61
27	67.57	25. 9.65	57	49.78	28.12.54
28	67.14	27. 1.61	58	49.06	7.11.61
29	67.14	12.12.64	59	49.06	14.12.56
30	65.45	21.12.57	60	47.99	1.12.54

Table 2.56 8/7 Spey at Invertruim. Record 1 October 1952 to 30 September 1970. $N = 18$ years.

Example 1 Analyse the data corresponding to the three peaks per year threshold and estimate the 10 and 50 year floods using model 2.

There are $N = 18$ years and the rate of occurrence is fixed at $\lambda = 3$ per year; therefore the highest $M = \lambda N = 54$ peaks are taken from Table 2.56. The mean and minimum of these values are: $\bar{q} = 82.78$, $q_{\min} = 51.24$.

The estimated β and q_0 of the exponential distribution are from Equations (2.7.5.5) and (2.7.5.4)

$$\hat{\beta} = \frac{M(\bar{q}-q_{\min})}{M-1} = 32.13$$

$$\hat{q} = q_{\min} - \frac{\hat{\beta}}{M} = 50.65.$$

These estimates are inserted in Equation (2.7.3.14) to give the estimated T year flood.

$$\widehat{Q(T)} = \hat{q}_0 + \hat{\beta} \ln \lambda + \hat{\beta} \ln T$$

$$\widehat{Q(10)} = 50.65 + 32.13 (\ln 3 + \ln 10) = 133.06$$

$$\widehat{Q(50)} = 50.65 + 32.13 (\ln 3 + \ln 50) = 211.64.$$

The sampling variance of $\widehat{Q(T)}$ in this model is by Equation (2.7.5.14)

$$\text{var } \widehat{Q(T)} = \frac{\beta^2}{N} PD2(T, \lambda, N)$$

where $PD2(T, \lambda, N)$ is defined by Equation (2.7.5.13); for $\lambda = 3$ and $N = 18$ it is related to T as follows

T	2	5	10	25	50	100
$PD2(T,3,18)$	1.074	2.463	3.892	6.283	8.504	10.984

Therefore,

$$\text{var } \widehat{Q(10)} = \frac{(32.13)^2}{18} 3.892 = 223.20; \text{ se } \widehat{Q(10)} = 14.94$$

$$\text{var } \widehat{Q(50)} = \frac{(32.13)^2}{18} 8.504 = 487.72; \text{ se } \widehat{Q(50)} = 22.08.$$

The Poisson assumption is of little importance but major errors may result from misuse of the exponential assumption and this should be checked. A probability plot is useful because it shows whether the fitted distribution is reasonable or not. The judgement is subjective but it is the best that is available because as seen in Section 2.4 the χ^2 and Kolmogorov–Smirnov goodness of fit indices are not sufficiently sensitive. The main point to remember when judging a probability plot is that the largest values in a sample are the most variable so that a single large value might not be sufficient to cause alarm. The probability of obtaining such a large value from the fitted distribution can be easily obtained by using the relation

$$PR(Q_{max} \leqslant q) = [F(q)]^N$$

where $F(q) = PR(Q \leqslant q)$ in the fitted distribution and N is the sample size.

The plotting positions for the exponential distribution, in terms of the reduced variate y, are

$$y_i = \sum_{j=1}^{i} \frac{1}{N+1-j}$$

so that when $N = 54$ these are

$$y_1 = \frac{1}{54} = 0.0185$$

$$y_2 = \frac{1}{54} + \frac{1}{53} = 0.0374$$

$$y_{54} = \frac{1}{54}+\frac{1}{53}+\frac{1}{52}+\ldots+\frac{1}{3}+\frac{1}{2}+1 = 4.58.$$

These differ a little from the y values obtained by computing the plotting probabilities $F_i = (i-0.44)/N+0.12)$ and converting to y values by $y_i = -\ln(1-F_i)$. Regardless of which set of y values is used it will be

Table 2.57 8/7 Spey at Invertruim. POT data at three/year level. Average plotting position y and variate value Q in ordered groups of stated size.

Group size	Plotting position y	Variate Q
10	0.11	53.57
10	0.34	60.43
5	0.55	63.94
5	0.72	68.88
3	0.89	79.55
3	1.03	81.44
3	1.20	85.94
3	1.40	91.24
3	1.65	95.74
3	1.99	113.33
Six highest individuals		
1	2.29	118.12
1	2.49	124.04
1	2.74	131.35
1	3.08	126.35
1	3.58	237.50
1	4.58	276.92

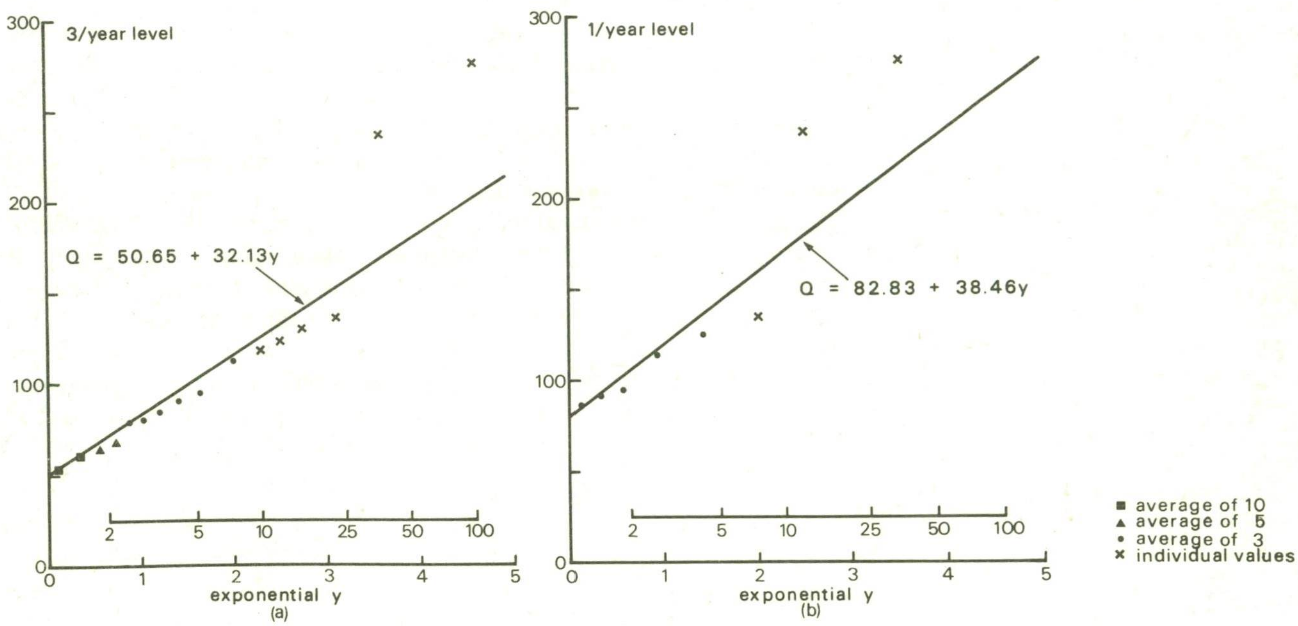


Fig 2.24 8/7, Spey at Invertruim. Peaks over a threshold magnitudes at (a) three per year and (b) one per year level plotted against exponential reduced variate. At the lower end averages of many points only are plotted to avoid congestion but without loss of important detail.

noticed that they are very close together at the lower end of the sample. For this reason the lowest and second lowest 10 points are represented by their average values as are further groups of the ordered data as indicated in Table 2.57. These values are plotted in Figure 2.24(a) where the six very highest points are plotted individually. Whether grouping is employed is a matter of choice but it is convenient for plotting and the detail which is omitted makes little difference to the conclusions drawn from the plot.

All but the two highest points lie close to the fitted exponential distribution shown by the straight line on Figure 2.24(a). As mentioned throughout this chapter the highest values are most variable and it is worth investigating the probability that the maximum Q_{max} in a sample of $N = 54$ should be as large as 276.9. The df of Q_{max} is

$$PR(Q_{max} \leqslant q) = [F(q)]^N = [1 - e^{-(q-q_0)/\beta}]^N$$

and inserting $q = 276.9$, $N = 54$, $\hat{q}_0 = 50.65$ and $\hat{\beta} = 32.13$ this gives

$$PR(Q_{max} \leqslant 276.9) = 0.9538$$

which means that only about one in 20 samples of 54 from the fitted distribution would have a maximum value of 276.9 or greater. This fact may be used in the same manner as the result of a goodness of fit test which rejects the hypothesis that the fitted exponential distribution is appropriate. This raises a difficult practical problem because the fitted distribution agrees well with all but the two highest flows. If flows of less than 10 year return period only were being considered the fitted distribution would be accepted as adequate but for larger return periods it is necessary to consider the effects of the two largest floods more fully.

Two possible steps are (i) to use a more flexible distribution such as the Pearson Type 3 and (ii) to move the threshold to a higher level and see whether the two highest points are in agreement with the distribution of values remaining after the exclusion of the very lowest ones. The use of the Pearson Type 3 distribution offers no real solution because the fitted distribution has $\gamma = 0.91$ which is very close to the exponential distribution in which $\gamma = 1$. When the curve (not shown) corresponding to it is superimposed on Figure 2.24(a), it is barely distinguishable from the straight line.

When the threshold is raised to the one/year level there are 18 values shown plotted on Figure 2.24(b). The estimated parameters are $\hat{q}_0 = 82.83$ and $\hat{\beta} = 38.46$. The probability that the largest value in a sample of 18 should be less than or equal to 276.9 is 0.89 showing that these large values are more in accord with this distribution than with the one including more lower flows. This is to be preferred to the fitting of a more flexible distribution to the larger sample provided at the three/year level where the low flows still have more effect than the large ones which are really important. The use of such an increased sample is only valuable if the correct form of the distribution is known with certainty. In the fitted exponential distribution Equation (2.7.3.14) with $\lambda \triangleq 1$ gives

$$\widehat{Q(T)} = 82.83 + 38.46 \ln T$$

and therefore

$$\widehat{Q(10)} = 82.83 + 38.46\,(2.30) = 171.29$$

$$\widehat{Q(50)} = 82.83 + 38.46\,(3.91) = 233.21.$$

To compute the standard errors the function $PD2(T, \lambda, N)$ of Equation (2.7.5.13) must be evaluated with $\lambda = 1$ and $N = 18$. For $T = 10$ it is 5.402 and for $T = 50$ it is 15.803. Therefore,

$$\text{var } \widehat{Q(10)} = \frac{(38.46)^2}{18} \cdot 5.402 = 443.92; \text{ se} = 21.07$$

$$\text{var } \widehat{Q(50)} = \frac{(38.46)^2}{18} \cdot 15.803 = 1298.63; \text{ se} = 36.04.$$

2.8 The treatment of missing peaks or historic floods as censored samples

2.8.1 Introduction

Those concerned with maximum flood estimation will be familiar with at least two types of nonstandard data found in connection with flood series: missing peaks in continuous chart records and historic flood marks.

Missing peaks occur when the level is so high that the recording pen runs off the top of the chart. It may be possible to estimate a missing peak from the shape of the available truncated hydrograph but the approach used here is to assume only that a flood which has exceeded a chart limit has a peak discharge greater than the flow corresponding to the chart limit.

Historic flood marks are usually to be found on walls, bridges or on specially constructed flood stones. They indicate the levels of flood which have risen above a fixed point during some historic period. In certain circumstances, it may be assumed that all such floods have been marked, and that floods in the intervening years for which no marks exist have failed to reach the fixed point.

These two types of data have this in common; values are only specified if they lie on one side of a given threshold. Samples which exhibit this property are known as censored samples, the threshold being called the censoring point. If the threshold is fixed, as it is in these cases, the proportion of censored events is a random variable, and censoring is type I, as defined by Kendall & Stuart (1961). Such data may form part of a series either of annual maxima or of peaks over a threshold. The incorporation of either a missing peak or a 'historic record' into a standard annual maximum sample is treated in Section 2.8.2. An example of 'missing peaks' in a series of peaks over a threshold is discussed in Section 2.8.3. The 'historic flood' record is treated in Section 2.8.3 as a special case of a record of peaks over a threshold.

2.8.2 Censored annual maximum series

The method of maximum likelihood may conveniently be used to estimate distribution parameters from a censored sample; for example, the general form of the likelihood function L_c for a sample of $N+M$ values, of which N are below the censoring point x_c and are specified as $x_1, x_2, \ldots x_N$, and M are above x_c and are unknown, is

$$L_c = \prod_{i=1}^{N} f(x_i) \left[\int_{x_c}^{\infty} f(x)\mathrm{d}x \right]^M, \qquad (2.8.2.1)$$

where $f(x)$ is the appropriate pdf. A similar expression may be obtained for censoring below a censoring point, and maximum likelihood equations may be obtained from either expression by differentiation.

The Gumbel distribution (EV1) is used to illustrate the method (1.2.4).

The distribution function is

$$F(x) = \exp\left[-e^{-\left(\frac{x-u}{\alpha}\right)}\right] \tag{2.8.2.2}$$

where $F(x)$ is the probability of obtaining a value not exceeding x, and α and u are the parameters to be estimated. In this case the pdf is

$$f(x) = \frac{\mathrm{d}F(x)}{\mathrm{d}x} = \frac{1}{\alpha}\exp(-y-e^{-y}) \tag{2.8.2.3}$$

where the reduced variate y is given by

$$y = \left(\frac{x-u}{\alpha}\right). \tag{2.8.2.4}$$

Censoring above a threshold (missing peaks)

Suppose an annual maximum sample consists of N known maxima, x_1, $x_2, \ldots x_N$ and M missing peaks which are known to be above the chart limit x_c. The likelihood function L_c and maximum likelihood equations for the Gumbel distribution are then

$$L_c = \prod_{i=1}^{N} \frac{1}{\alpha}\exp(-y_i - e^{-y_i}).(1-\omega)^M, \tag{2.8.2.5}$$

and

$$-\frac{1}{\alpha}\left[N - \sum_{i=1}^{N} y_i + \sum_{i=1}^{N} y_i e^{-y_i} - \frac{M\omega}{1-\omega} e^{-y_c} y_c\right] = 0$$

$$-\frac{1}{\alpha}\left[-N + \sum_{i=1}^{N} e^{-y_i} - \frac{M\omega}{1-\omega} e^{-y_c}\right] = 0, \tag{2.8.2.6}$$

where

$$y_i = \frac{x_i - u}{\alpha}; \qquad y_c = \frac{x_c - u}{\alpha}; \qquad \omega = e^{-e^{-y_c}}.$$

Censoring below a threshold (historic flood marks)

Suppose that during a historic flood period of length J years, r annual maximum floods rose above a threshold x_h and were marked, leaving $L = J - r$ floods unmarked and assumed to be less than x_h. Suppose that in addition there are N years of continuous record from recent times providing $N+r$ completely specified values. The likelihood function L_h and maximum likelihood equations are then

$$L_h = \prod_{i=1}^{N+r} \frac{1}{\alpha}\exp(-y_i - e^{-y_i}).\omega^L \tag{2.8.2.7}$$

and

$$-\frac{1}{\alpha}\left[(N+r) - \sum_{i=1}^{N+r} y_i + \sum_{i=1}^{N+r} y_i e^{-y_i} + L e^{-y_h} y_h\right] = 0$$

$$-\frac{1}{\alpha}\left[-(N+r) + \sum_{i=1}^{N+r} e^{-y_i} + L e^{-y_h}\right] = 0 \tag{2.8.2.8}$$

where

$$y_i = \frac{x_i - u}{\alpha}; \qquad y_h = \frac{x_h - u}{\alpha}; \qquad \omega = e^{-e^{-y_h}}.$$

Similar expressions may be obtained for distributions other than the Gumbel distribution by substituting the appropriate pdf in Equation (2.8.2.1) or its equivalent for censoring below a threshold.

An iterative technique for solving the maximum likelihood for a standard sample is given in Jenkinson (1969) and is described in Section 1.3.4 with a worked example. This technique may be used with slight adaptation for the solution of Equations (2.8.2.6) or (2.8.2.8); satisfactory results are obtained if the iteration matrix is left unchanged, i.e. given values appropriate to an uncensored sample. However, care should be taken in the choice of initial values if the proportion of censored values is high. Formulae for the large sample standard errors of the parameter estimates are given by Leese (1973).

Examples

The following examples use some of the records given in Volume IV. They indicate the changes in parameter estimates, and the reduction in their standard errors, which result from the addition of historical data to a standard sample.

The first application is from 53/3, Avon at Bath, where a fairly long continuous record is preceded by historical records dating back to 1850. In this example the parameters remain virtually unchanged, and a reliable comparison is obtained between the standard errors of the estimates based on all the data and those based only on the recent record. The basic data and the results are shown in Tables 2.58 and 2.59 respectively.

The second example is taken from 54/2, Avon at Evesham. The parameter u remains unchanged but α is increased, resulting in a 10% change in the estimate of the 100 year flood. This change (from 437 to 478 cumecs) is, however, only of the order of magnitude of the standard errors of the estimates, as may be seen in Table 2.60 and 2.61.

Reliable historic flood records from 69/2, Irwell at Adelphi Weir, date back to 1896; joint use of the recent record and the 10 historical flood levels between 1896 and 1936 (the beginning of the recent record) results in a 20% increase in the estimate of the 100 year flood. The change in this case (from 537 to 648 cumecs) is quite considerable in relation to the standard errors of the estimates, which are about 50 cumecs. The results are shown in Tables 2.62 and 2.63.

Additional nonstandard data are of course not always consistent with the assumptions of the censoring model considered above. For example, historic data often appear in the form of a statement that a given flood was the 'highest within living memory'. This is an important case, but unfortunately its treatment is statistically more complex than those described above because the censoring point (the highest flood, below which all other floods are said to have fallen, is a random variable). Nevertheless, it is probably reasonable to use the maximum likelihood equations derived for the simpler case, always assuming that the number of annual maxima that the highest flood is said to have exceeded is known with some degree of confidence.

Water year	Flood	Water year	Flood	Water year	Flood
1865	206	1941	84	1955	128
1866	228	1942	149	1956	107
1874‡	121	1943	73	1957	138
1875	218	1944	118	1958	169
1879†	264	1945	128	1959	169
1882	362	1946	282	1960	352
1888	204	1947	98	1961	121
1894	375	1948	104	1962	103
1896‡	154	1949	113	1963	277
1899	239	1950	229	1964	110
1900	302	1951	136	1965	178
1902‡	186	1952	116	1966	172
1924	255	1953	96	1967	311
1940	148	1954	296	1968	125

†Date of this flood since revised.
‡These floods were included in the recent sample (see Leese, 1973).
Hence $N = 32$; $J = 58$; $L = 48$; $r = 10$; $x_h = 200$.

Table 2.58 Annual maximum floods: 53/3, Avon at Bath (cumecs).

	Parameter				Sample size
	α	u	$\bar{Q}$	*Q100*	
Historic data + recent data					
Estimate	47	128	155	344	29 recent values + 13 historic values
Large sample standard error	±5	±7	±7	±19	+48 censored values
Recent data alone					
Estimate	48	128	156	348	29 recent values
Large sample standard error	±7	±9	±11	±31	

Table 2.59 Parameter estimates: 53/3, Avon at Bath.

Water year	Flood	Water year	Flood	Water year	Flood
1848	430	1914	263	1949	149
1852	396	1917	232	1950	182
1867	283	1919	246	1951	130
1872	275	1925	227	1953	86
1874	329	1927	210	1954	191
1875	354	1931	351	1955	94
1877	229	1932	303	1956	138
1878	297	1935	309	1957	138
1879	190	1937	47	1958	244
1880	263	1938	240	1959	246
1881	181	1939	316	1960	215
1882	371	1940	187	1961	92
1885	293	1941	184	1962	68
1888	348	1942	201	1963	117
1894	138	1943	8	1964	41
1895	289	1944	103	1965	148
1899	462	1945	86	1966	131
1907	241	1946	356	1967	362
1910	229	1947	67	1968	199
1911	261	1948	91		

$N = 32$; $J = 88$; $L = 58$; $r = 30$; $x_h = 180$.

Table 2.60 Annual maximum floods: 54/2, Avon at Evesham (cumecs).

	Parameter				Sample size
	α	u	$\bar{Q}$	*Q100*	
Historic data + recent data					
Estimate	78	117	162	478	32 recent values + 30 historic values
Large sample standard error	±7	±10	±10	±33	+ 58 censored values
Recent data alone					
Estimate	70	117	156	437	32 recent values
Large sample standard error	±10	±13	±16	±50	

Table 2.61 Parameter estimates: 54/2, Avon at Evesham.

Water year	Flood	Water year	Flood	Water year	Flood
1896	311	1941	205	1955	295
1901	396	1942	186	1956	152
1908	255	1943	246	1957	278
1911	287	1944	249	1958	114
1919	348	1945	496	1959	176
1921	311	1946	101	1961	252
1923	289	1947	272	1962	257
1924	289	1948	102	1963	288
1927	340	1949	292	1964	137
1931	328	1950	111	1965	320
1936	377	1951	230	1966	238
1937	100	1952	186	1967	287
1938	230	1953	378	1968	272
1939	186	1954	216		

$N = 31; J = 42; L = 32; r = 10;$ $x_h = 240.$

Table 2.62 Annual maximum floods: 69/2, Irwell at Adelphi Weir (cumecs).

	Parameter				Sample size
	α	u	$\bar{Q}$	$Q100$	
Historic data + recent data					
Estimate	68	177	216	648	31 recent values + 10 historic values
Large sample standard error	7	10	11	52	+ 32 censored values
Recent data alone					
Estimate	75	190	233	537	31 recent values
Large sample standard error	11	14	17	55	

Table 2.63 Parameter estimates: 69/2, Irwell at Adelphi Weir.

2.8.3 Censored series of peaks over a threshold

Missing peaks

The incorporation of missing peaks in a series of peaks over a threshold is illustrated by the estimation of the mean annual flood for 22/2, Coquet at Bygate. Almost 22 complete years of record (5 November 1947 to 30 September 1969) are available for this site, but the annual maxima for 14 of them are censored in that it is known that they exceed a chart limit of 26.3 cumecs; their exact values are unknown.

A total of 140 exceedances were read from the chart, 114 of which were uncensored. Estimation of the mean annual flood proceeds by a slight modification of the method presented in Section 2.7; essentially, the likelihood function is expressed as a product of two factors, one of which yields an estimate of the Poisson parameter describing the incidence of exceedances, the other an estimate of the parameter describing the exponential distribution of exceedance magnitudes. Estimation of the first parameter is unchanged by the censoring of exceedance data; estimation of the second parameter is as follows.

The density function of exceedances is

$$f(q/q_0) = \frac{1}{\beta} e^{-(q-q_0)/\beta} \tag{2.8.3.1}$$

where β is the parameter to be estimated and q_0 is the threshold value used to fix exceedances; $q_0 = 11.05$ cumecs for the Coquet at Bygate.

With n_1 uncensored exceedances, and n_2 censored, the likelihood is

$$L = \left(\frac{1}{\beta}\right)^{n_1} e^{-\sum_{i=1}^{n_1} \frac{1}{\beta}(q_i - q_0)} \left[e^{-\frac{1}{\beta}(q_c - q_0)n_2}\right], \qquad (q_c > q_0) \tag{2.8.3.2}$$

where q_c is the flow above which data are censored.

Solving $\partial \log L/\partial\beta = 0$ gives

$$\hat{\beta} = \left[\sum_{i=1}^{n_1} q_i - q_0(n_1 + n_2) + n_2 q_c\right] \Big/ n_1. \tag{2.8.3.3}$$

If $n_2 = 0$, so that no censored data occur, the estimate of β reduces to that in Equation (2.7.5.2).

For 22/2, Coquet at Bygate $n_1 = 114$, $n_2 = 26$, $q_c = 26.30$ cumecs, $q_0 = 11.05$ cumecs, $\sum_{i=1}^{114} q_i = 2014.16$.

Hence, $\hat{\beta} = 1150.96/114 = 10.096$.

From Equation (2.7.5.1), $\hat{\lambda} = M/N = 140/22 = 6.364$ and $\ln \hat{\lambda} = 1.8507$.

The parameter u of the Gumbel distribution, given in the line preceding Equation (2.7.3.17),

$u = q_0 + \beta \ln \lambda$

is therefore

$\hat{u} = 11.05 + (1.8507)(10.096) = 29.73$

while $\hat{\alpha} = \hat{\beta}$.

Hence the mean annual flood is

$\bar{Q} = \hat{u} + 0.577\hat{\alpha} = 29.73 + (0.577)(10.096) = 35.56.$

In 1966 the chart recorder was replaced by another capable of recording levels corresponding to flows greater than 26.30 cumecs; in 1967 exceedances of 28.71, 32.99, 27.93 and 31.42 cumecs occurred and in 1968 exceedances of 29.79, 51.03 and 30.45 cumecs occurred, all of which were greater than the former censoring point of 26.30. Other smaller exceedances were also observed. The estimate of the mean annual flood, 35.56 cumecs, is therefore not implausible.

Historic floods

Where a 'historic flood' record precedes a recent series of peaks over a threshold, then the early record may be regarded as a POT series above a high threshold, which may correspond to the bottom of the flood stone or which may be estimated from the historic series. Assuming that the recent series of n_1 exceedances of threshold q_0 is preceded by n_2 exceedances of threshold q_0', corresponding to the bottom of the stone, then the likelihood function from which the exponential parameter β is estimated becomes

$$L = \left(\frac{1}{\beta}\right)^{n_1 + n_2} e^{-\sum_{i=1}^{n_1}\left(\frac{q_i - q_0}{\beta}\right) - \sum_{i=1}^{n_2}\left(\frac{q_i - q_0'}{\beta}\right)} \tag{2.8.3.4}$$

Note errors 19/11/81

whence $\partial \log L/\partial\beta = 0$ gives

$$\hat{\beta} = \left[\sum_{i=1}^{n_1+n_2} \left(\frac{q_i - n_1 q_0 - n_2 q_0'}{n_1 + n_2}\right)\right] \Big/ (n_1 + n_2). \tag{2.8.3.5}$$

The Poisson parameter should be estimated from the mean number of exceedances per year in the recent POT series.

2.9 The time series approach illustrated by the shot noise model

2.9.1 Introduction

The time series approach to flood estimation was discussed in general terms in Section 2.2. A specific example of a time series model is known as the shot noise model, in which flows are treated as a series of impulses and decays and which is described in this section to illustrate this approach. It will be seen that the model has much in common with the peaks over a threshold approach, but makes use of the full range of flow records. The theory was developed by G. Weiss of Imperial College, London, working under the direction of Professors Cox and O'Donnell, and he gave much assistance with its adaptation to the problem of floods.

2.9.2 Theory of shot noise model

The flow is considered as the sum of a series of random impulses. Each impulse consists of a sudden random rise of height Y which decays exponentially, as shown in Figure 2.25(a). These impulses occur as a Poisson process (see Section 2.7.3); the probability of occurrence of an impulse in the time interval t to $t+\delta t$ depends only on the length of the interval δt and not on the time since the previous impulse. If impulses occur at the average rate v, then $v\delta t$ is the probability of an impulse occurring in the interval δt. The mean interval between impulses is $1/v$ and the intervals are exponentially distributed. Figure 2.25(b) shows how the impulses combine to produce the synthetic hydrograph. The flow rate at time t may be written as the sum of the impulses to date

$$X(t) = \sum_{m=0}^{N(t)} Y_m e^{-b(t-\tau_m)}$$

where $N(t)$ is the number of impulses in the interval $(0, t)$, and τ_m is the time of occurrence of the mth impulse.

After some initial transience the process soon becomes steady and can be regarded as stationary; its statistics do not change with time. Some useful expressions can be derived for the moments of the instantaneous flow rate, X:

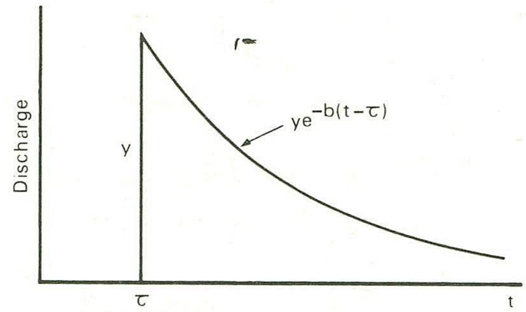
mean flow $\quad \mu_x = \frac{v}{b} E(Y),$

variance $\quad \sigma_x^2 = \frac{v}{2b} E(Y^2),$

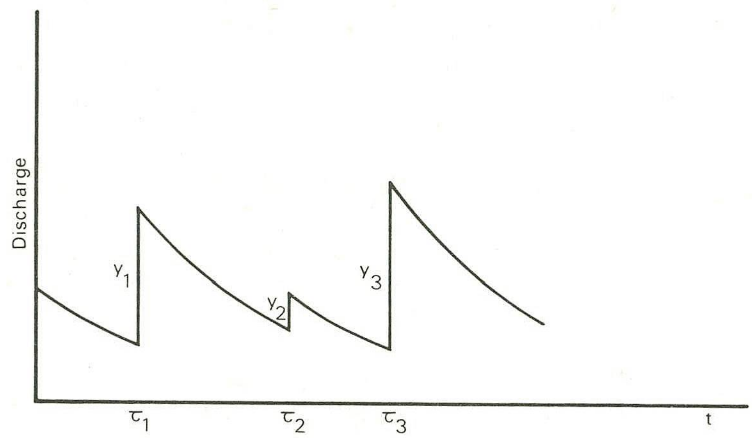
third moment $\quad g_x \sigma_x^3 = \frac{v}{3b} E(Y^3),$

serial correlation $\rho_x(s) = e^{-bs}.$

In the expression for the third moment g_x is the skewness while $E(.)$ denotes the expectation of the value in the brackets and depends on the



(a) A single shot noise pulse



(b) Flow hydrograph produced by combining pulses

Fig 2.25 The shot noise model as an approximation to river flow.

statistical distribution chosen for the impulse size. In simple shot noise an exponential distribution is used for impulses. That is

$$PR\,\{Y \leqslant y\} = 1 - \exp(-y/\beta).$$

For this distribution

$$E(Y) = \beta,\ E(Y^2) = 2\beta^2 \text{ and } E(Y^3) = 6\beta^2.$$

Thus

$$\mu_x = \frac{v\beta}{b} \tag{2.9.2.1}$$

$$\sigma_x{}^2 = \frac{v\beta^2}{b} \tag{2.9.2.2}$$

$$g_x\sigma_x{}^3 = \frac{2v\beta^3}{b} \tag{2.9.2.3}$$

and

$$\rho_x(s) = e^{-bs}. \tag{2.9.2.4}$$

Note that only the first two moments and the serial correlation are needed to derive all the parameters of the process (v, β, b). The serial correlation of the generated flow depends on b, the decay rate of the impulses. This result applies generally; the correlation structure of the flows depends only on the impulse shape.

These equations describe the instantaneous flow rate; of greater interest is the flow averaged over a period, for example the mean daily flow

$$Q(t, \Delta) = \frac{1}{\Delta}\int_t^{t+\Delta} X(v)\mathrm{d}v$$

is the mean flow over the period Δ.

The moments of the averaged flow, Q, can be expressed as the moments of the instantaneous flow multiplied by an averaging factor, which depends, as is to be expected, only on the pulse shape and the period Δ. The expressions are

$$\mu_Q = \mu_x, \tag{2.9.2.5}$$

$$\sigma_Q^2 = \sigma_x^2 \cdot \frac{2\{b\Delta - (1 - e^{-b\Delta})\}}{b^2\Delta^2}, \tag{2.9.2.6}$$

$$g_Q\sigma_Q^3 = g_x\sigma_x^3 \cdot \frac{3\{b\Delta - 2(1 - e^{-b\Delta}) + \frac{1}{2}(1 - e^{-2b\Delta})\}}{b^3\Delta^3} \tag{2.9.2.7}$$

$$\rho_Q(s) = \rho_x(s) \cdot \frac{(1 - e^{-b\Delta})^2}{2e^{-b\Delta}\{b\Delta - (1 - e^{-b\Delta})\}}. \tag{2.9.2.8}$$

These equations are used to derive estimates of the parameters of the shot noise process from the sample moments of the mean daily flow data. For the simple shot noise model Equations (2.9.2.3) and (2.9.2.7) are not needed as only three equations are needed for the three parameters.

Seasonality is introduced by using a different set of parameters for each month making 36 parameters in all for the simple shot noise process. If the data are mean daily flows and the lag one serial correlation is used the equations may be simplified by substituting $\Delta = 1$ and $s = 1$.

$$\mu_Q = \frac{v\beta}{b}, \tag{2.9.2.5a}$$

$$\sigma_Q^2 = \frac{v\beta^2}{b} \cdot \frac{2}{b^2}\{b - (1 - e^{-b})\}, \tag{2.9.2.6a}$$

$$g_Q\sigma_Q^3 = \frac{v\beta^3}{b} \cdot \frac{3\{b - 2(1 - e^{-b}) + \frac{1}{2}(1 - e^{-2b})\}}{b^3} \tag{2.9.2.7a}$$

$$\rho_Q(1) = \frac{(1 - e^{-b})^2}{2\{b - (1 - e^{-b})\}}. \tag{2.9.2.8a}$$

The method of solution is to use Equation (2.9.2.8a) to determine b and then to solve (2.9.2.5a) and (2.9.2.6a) simultaneously for β and v. When this model is applied to data it is found that the sample skewness is often very different from that calculated by Equation (2.9.2.7a). Three methods were tried to incorporate the skewness into the model.

The first method was to use the simple shot noise model with an added constant base flow. If a constant c is added to the flow, X or Q, given by the simple shot noise the mean flow becomes

$$\mu_Q = \frac{v\beta}{b} + c \tag{2.9.2.5b}$$

but the other moments are unchanged. The model now has four parameters per month v, β, b and c, making 48 in all. The parameter b is determined from Equation (2.9.2.8a) as above. Equations (2.9.2.6a) and (2.9.2.7a) are solved simultaneously to give β and v and the Equation (2.9.2.5b) is used to determine c.

The two other methods attempted to fit the model to the skewness by using a different distribution for the jumps. The gamma distribution is an obvious generalisation of the exponential distribution. It has the probability density function (see Section 1.2.3)

$$f(y) = \frac{y^{\gamma-1}e^{-y/\beta}}{\beta^{\gamma}\Gamma(\gamma)},$$

and the moments of the distribution are

$$E(Y) = \gamma\beta,$$

$$E(Y^2) = \gamma(\gamma+1)\beta^2$$

$$E(Y^3) = \gamma(\gamma+1)(\gamma+2)\beta^3.$$

These describe the distribution of the jumps. The mean daily flow moments are given by

$$\mu_Q = \frac{v\beta\gamma}{b} \tag{2.9.2.5c}$$

$$\sigma_Q^2 = \frac{v\gamma(\gamma+1)\beta^2}{2b}\cdot\frac{2}{b^2}\{b-(1-e^{-b})\} \tag{2.9.2.6c}$$

$$g_Q\sigma_Q^3 = \frac{v\gamma(\gamma+1)(\gamma+2)\beta^3}{2b}\cdot\frac{3}{b^2}\{b-2(1-e^{-b})+\tfrac{1}{2}(1-e^{-2b})\}. \tag{2.9.2.7c}$$

The serial correlation is unchanged as it depends only on the pulse shape.

Equation (2.9.2.8a) can be solved for the parameter b, the decay rate of the pulses. The averaging factors can then be calculated for the three moments in Equations (2.9.2.5c), (2.9.2.6c) and (2.9.2.7c) and the instantaneous moments can then be calculated. The instantaneous moments are then combined as

$$u = \frac{g_x\sigma_x^3\cdot\mu_x}{(\sigma_x^2)^2} = \frac{g_x\mu_x}{\sigma_x} = \frac{4}{3}\cdot\frac{\gamma+2}{\gamma+1},$$

or

$$\gamma = \frac{8-3u}{3u-4}.$$

Note that u is the ratio of the skewness of the instantaneous flow (g_x) to the coefficient of variation (σ_x/μ_x).

The form parameter γ of the gamma distribution must be positive and this imposes limits on u:

$$u = \frac{4}{3},\ \gamma\to\infty,$$

$$u = \frac{8}{3},\ \gamma = 0.$$

For values of u outside this range γ would be negative, which is impossible. If $u = 2$, $\gamma = 1$ and the gamma distribution becomes the exponential distribution.

The second jump distribution used was a special form of the Pareto distribution sometimes called the Lomax distribution. This has the distribution function

$$F(y) = 1-\left(1+\frac{y}{\theta}\right)^{-\alpha}.$$

The parameters α and θ must both be greater than zero and the distribution function is only defined for $y \geqslant 0$. This distribution is unusual in that moments of order greater than α do not exist. In this case the third moment is required and hence α must be greater than 3. The moments of the Lomax distribution are

$$E(Y) = \frac{\theta}{\alpha-1},$$

$$E(Y^2) = \frac{2\theta^2}{(\alpha-1)(\alpha-2)},$$

$$E(Y^3) = \frac{6\theta^3}{(\alpha-1)(\alpha-2)(\alpha-3)},$$

and the moments of the mean daily flows are

$$\mu_Q = \frac{v\theta}{b(\alpha-1)} \quad (2.9.2.5d)$$

$$\sigma_Q^2 = \frac{v\theta^2}{b(\alpha-1)(\alpha-2)}\cdot\frac{2}{b^2}\{b-(1-e^{-b})\} \quad (2.9.2.6d)$$

$$g_Q\sigma_Q^3 = \frac{2v\theta^3}{b(\alpha-1)(\alpha-2)(\alpha-3)}\cdot\frac{3}{b^2}\{b-2(1-e^{-b})+\tfrac{1}{2}(1-e^{-2b})\}. \quad (2.9.2.7d)$$

To solve these equations we use the averaging factors to get the continuous moments and again calculate the ratio u of skewness to cofficient of variation. Rearranging the above expressions gives

$$u = 2\frac{\alpha-2}{\alpha-3},$$

or

$$\alpha = \frac{3u-4}{u-2}.$$

For α to be in the permitted range (>3) u must be greater than a half.

This section has introduced four shot noise models. All four models use the same pulse shape, a sudden rise followed by an exponential decay; also in all of them the occurrence of pulses is a Poisson process. The simple shot noise model uses an exponential distribution of pulse heights. This enables the mean, variance and first serial correlation coefficient to be preserved. The base flow model is a simple attempt to add skewness to the preserved statistics. In this model the flow does not decay to zero but to a positive value, c. This effect is obtained by adding the constant c to the flow from the simple shot noise model. Other methods of preserving skewness involve the use of a two parameter distribution for pulse heights; the theory has been developed for two distributions, the gamma distribution and the Lomax distribution. As in the simple shot noise model no base flow is used in these cases. In the three last models there are restrictions on the values of certain parameters which restrict the range of certain moments of the data, and this may rule out their use in practice. All four models may be made seasonal by using different parameters for each month.

2.9.3 The use of the shot noise model

The steps in using any time series model are, firstly, to estimate the parameters of the model using the flow data from the gauging station under consideration and, secondly, to generate a long sequence of synthetic flows which can be used in the investigation. In this case the flows are to be used to estimate the frequency of occurrence of rare floods and for this purpose it is necessary to record the annual maximum flows and the highest few flows. The annual maxima can be analysed in the usual manner to give a frequency curve. In addition to this, the highest floods can be regarded as estimates of the magnitude of very rare floods. Frequencies can similarly be assigned to the second, third, etc. highest floods in the synthetic sequence.

For a reservoir design, the synthetic hydrograph may be routed through the reservoir. This will normally be done to estimate reservoir yield, but by small additions to the yield estimation computer program it should be possible to obtain spillway design information at the same time. It may be that the yield estimation program works on too long a time scale to yield useful flood information. In this case sequences with large reservoir spill could be reanalysed later at a shorter time interval in order to show how the proposed spillway system would deal with the flood.

For the purposes of this investigation it was decided to fit the shot noise model to a long record in order to see whether the distribution of synthetic annual maxima was the same as that of the original data. A computer program was written to estimate the shot noise parameters. For this purpose the mean, standard deviation, skewness and lag one serial correlation were required for each month in the year. The program computed these and other statistics and took, as input, mean daily flow data. After reading the data and calculating moments the program calculated estimates of the shot noise parameters. This program was reasonably simple and will not be described further.

These parameter estimates were used in the generating program to produce the synthetic flow sequences. This program is required to produce random impulses at random intervals. The method used was as follows. The program starts with a jump occurring. The height of this is decided by using a random number generator, giving numbers of the required distribution. The time to the next jump is then set using an exponentially distributed random number generator. The program then steps on through the days; as each day passes the instantaneous flow decreases by e^{-b}, where b is the decay coefficient and the time to the next jump is decreased by unity. When the day of the next jump is reached (residual time to jump <1), the jump is generated at the correct time and added to the flow already occurring. The process then continues as before. At the end of each month the parameters are changed to the value for the next month. As the flows are being produced, the mean, standard deviation, etc. are calculated and note is taken of any high flows. The FORTRAN subroutine (see below) shows the calculations involved in generating one day's flows. This subroutine generates simple shot noise, that is to say, it uses exponentially distributed jumps. In order to clarify the process involved, the coding for finding annual maxima and moments has been removed. Any intending user will be able to insert his own requirements. The program

```
TRG*DEMO.DAY

      SUBROUTINE DAY(X,Q,T,THETA,FNU,B,U)
C
C         SUBROUTINE TO GENERATE SHOT NOISE FLOW FOR ONE DAY
C     AT ENTRY
C      X        INSTANTANEOUS FLOW RATE AT START OF DAY
C      T        TIME LEFT TO NEXT JUMP
C      THETA )  3 PARAMETERS OF THE SHOT NOISE MODEL
C      FNU   )
C      B     )
C      U        RANDOM NUMBER SEED
C     AT EXIT
C      Q        DAILY MEAN DISCHARGE FOR DAY
C
C     THE SUBROUTINE RANDOM GENERATES A UNIFORM RANDOM NO. IN (0,1).
C
C  CALCULATE FLOW ASSUMING NO JUMP
C
C  INSTANTANEOUS FLOW
      EB = EXP(-B)
      X1= X
      X = X1*EB
C  DAILY MEAN DISCHARGE
      Q = X1*(1.-EB)/B
C
C  IS THERE A JUMP TODAY?
C
    1 IF(T.GT.1.0) GO TO 2
C
C  YES, CALCULATE RANDOM JUMP HEIGHT
C
      CALL RANDOM(U)
      Y = -THETA * ALOG(U)
C
C  CONTRIBUTIONS TO FLOW OF THIS JUMP
C
      DC = EXP(-B*(1.-T))
      DC1= (1.-DC)/B
      X = X + Y*DC
      Q = Q + Y*DC1
C
C  GET TIME TO NEXT JUMP (RANDOM)
C
      CALL RANDOM(U)
      DT = -ALOG(U)/FNU
      T = T + DT
      GO TO 1
C
C  NO MORE JUMPS TODAY   SUBTRACT DAY OFF TIME TO NEXT JUMP
C
    2 T = T-1.0
      RETURN
      END
```

requires a random number generator to produce exponentially distributed random numbers. If models other than the simple shot noise are used then generators for the gamma or Lomax distributions will also be required. Uniformly distributed numbers are most easily generated and can be transformed to the required distributions.

If the variate X is to have an exponential distribution then the probability of exceeding the value x is given by

$$PR\{X>x\} = 1-F(x) = \exp(-x/\beta).$$

This can be rewritten as

$$x = -\beta \ln(1-F(x)).$$

As $F(x)$ is a probability

$$0 \leqslant F(x) \leqslant 1$$

and similarly $1-F$ lies in the range (0, 1). If we generate a uniform random variable u, say, we can identify this with $1-F$, the probability that X exceeds the value x. The variate $x = -\beta \ln u$ will then have the required distribution. Note that we select a probability at random and calculate the variate corresponding to it. This is probably the fastest method of generating exponential random variates and the transformation is accurate (apart from any roundoff in the computations).

The uniformly distributed random numbers were generated by the subroutine FPMCRV from the ICL scientific subroutine library (International

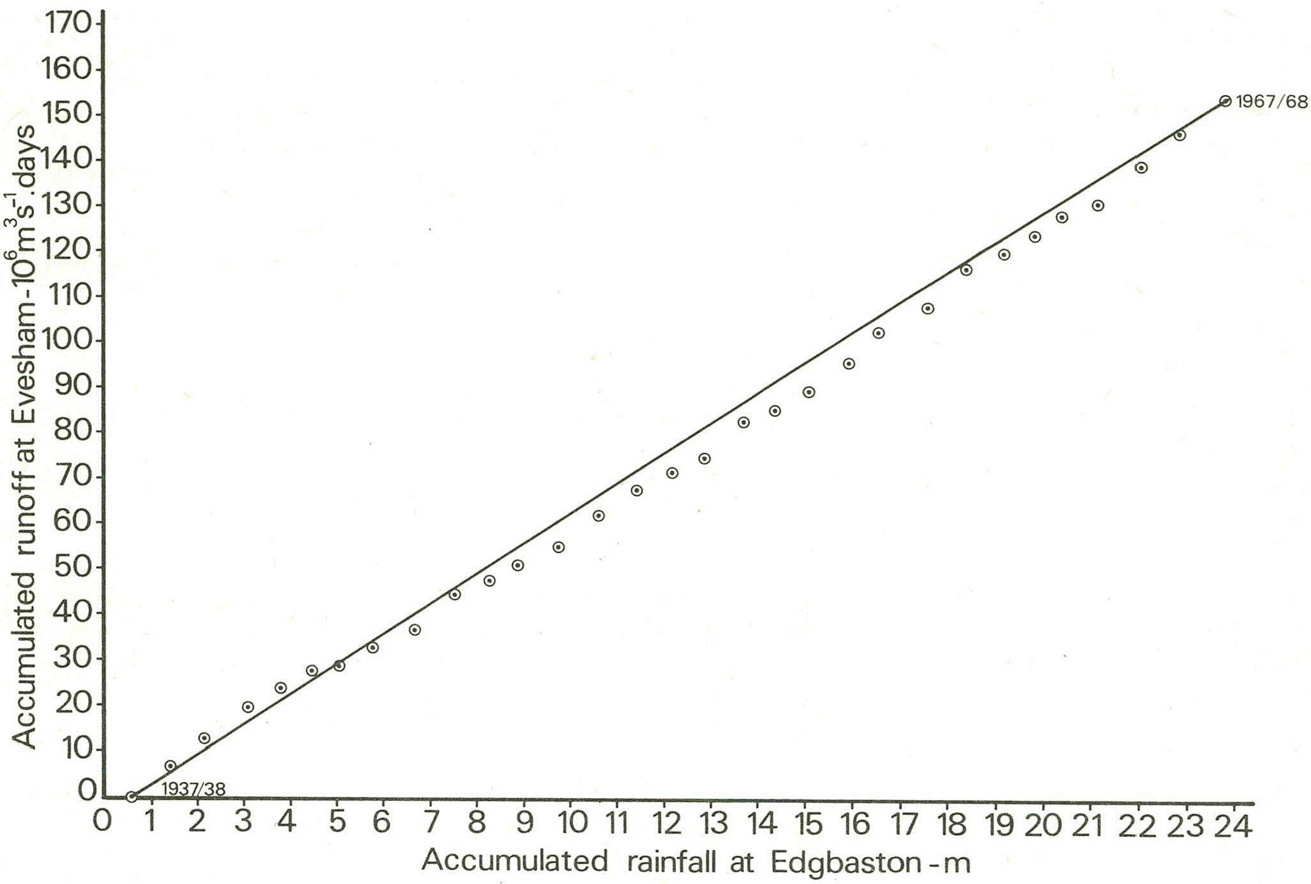


Fig 2.26 Double mass curve of Evesham discharge against Edgbaston rainfall.

Computers Limited, 1970). This subroutine uses the multiplicative congruential method, that is to say a new random number is generated from the current number by the relation

$$X_{n+1} = \alpha X_n \bmod M.$$

Several sequences of 20 000 numbers from this generator were tested for serial correlation and distribution, with satisfactory results.

2.9.4 Testing the model

Daily mean discharges from the Avon at Evesham were used in a trial of the flood reproducing properties of the shot noise model. The Warwickshire Avon is the major tributary of the Severn, which it joins at Tewkesbury. The Evesham gauge is the furthest downstream and records the runoff from 2210 km^2. The river rises near Naseby in Northamptonshire and is bounded on the south by the Cotswolds and the north by the Midland Plateau. Most of the catchment is good agricultural land, but it includes several large towns including Coventry and Rugby. Ledger (1972) gives a description of the hydrology and the water resources of the catchment. Evesham was selected as a long record of high quality with some supplementary historical flood information. Thirty two years of data from October 1938 to September 1970 were available for analysis. The measured flows were used without adjustments for abstractions. A double mass

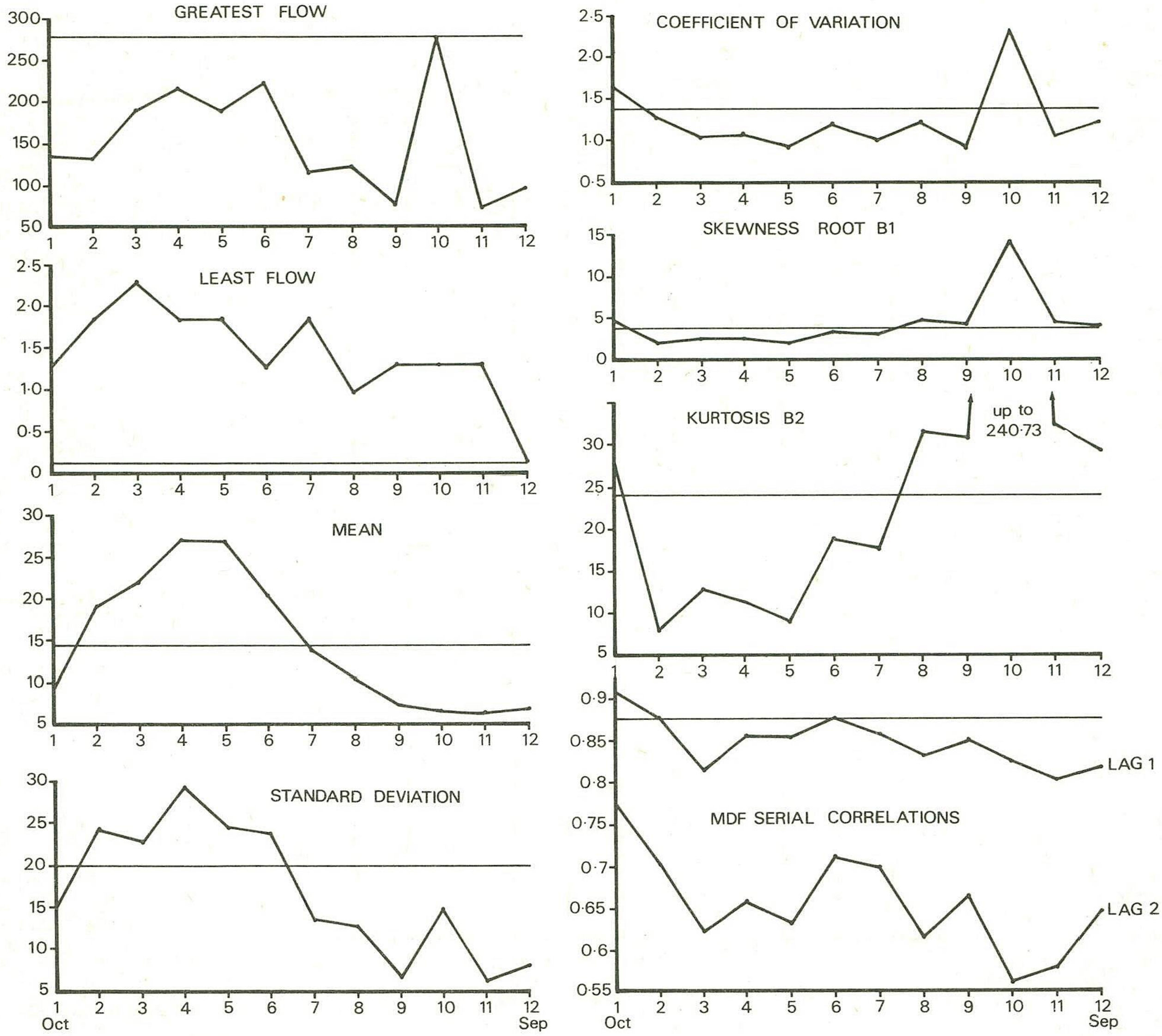


Fig 2.27 Monthly statistics of daily discharges at Evesham.

curve of accumulated flow at Evesham against accumulated rainfall at Edgbaston was used to check the record for homogeneity. The curve, shown in Figure 2.26, does not indicate any abrupt change in the regime of the river, though there may be slight evidence of a steady drift of conditions over the years.

The flows were analysed by the program described above to give monthly statistics and parameter estimates. Table 2.69 gives the monthly statistics which are plotted in Figure 2.27. The highest flood, which occurred in July, is the only flood of any size in July, and its effect can be seen in the various statistics for this month. The mean and standard deviation follow a nearly sinusoidal pattern; the coefficient of variation of daily flows is uniform throughout the year except for July where the value nearly doubles. The skewness behaves similarly and the kurtosis graph is dominated by the extremely high value for July. The lag one serial correlation is nearly uniform; the effect of the highest flood cannot be seen on this graph. Summer correlations tend to be a little lower than those in

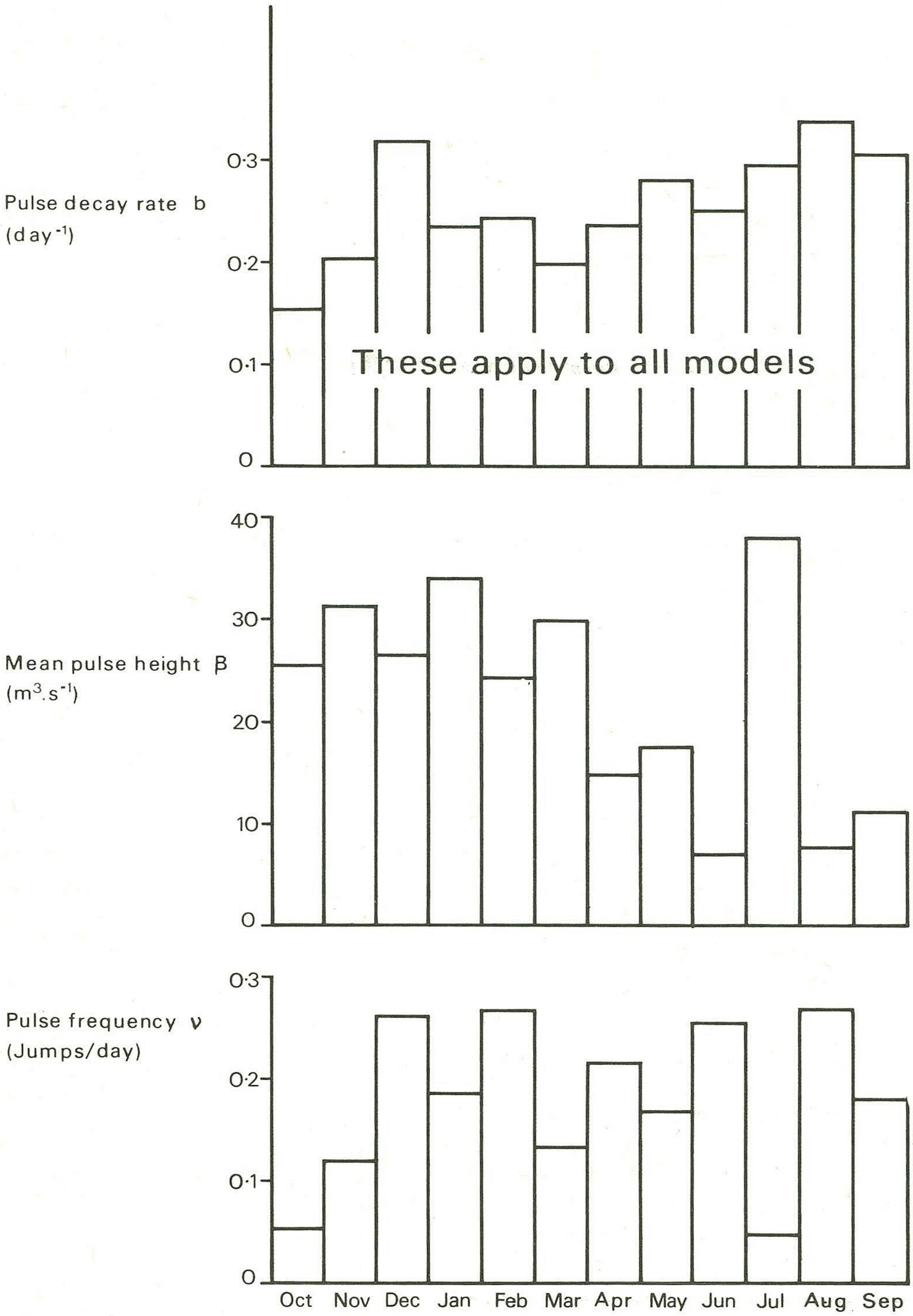


Fig 2.28(a) Parameters for simple shot noise.

winter. Parameter estimates for the shot noise models are shown in Table 2.65 and are plotted for each month in Figure 2.28. The pulse decay coefficient *b* is the same for all models as it is obtained from the serial correlation alone. The pulses decay slightly more rapidly in summer than

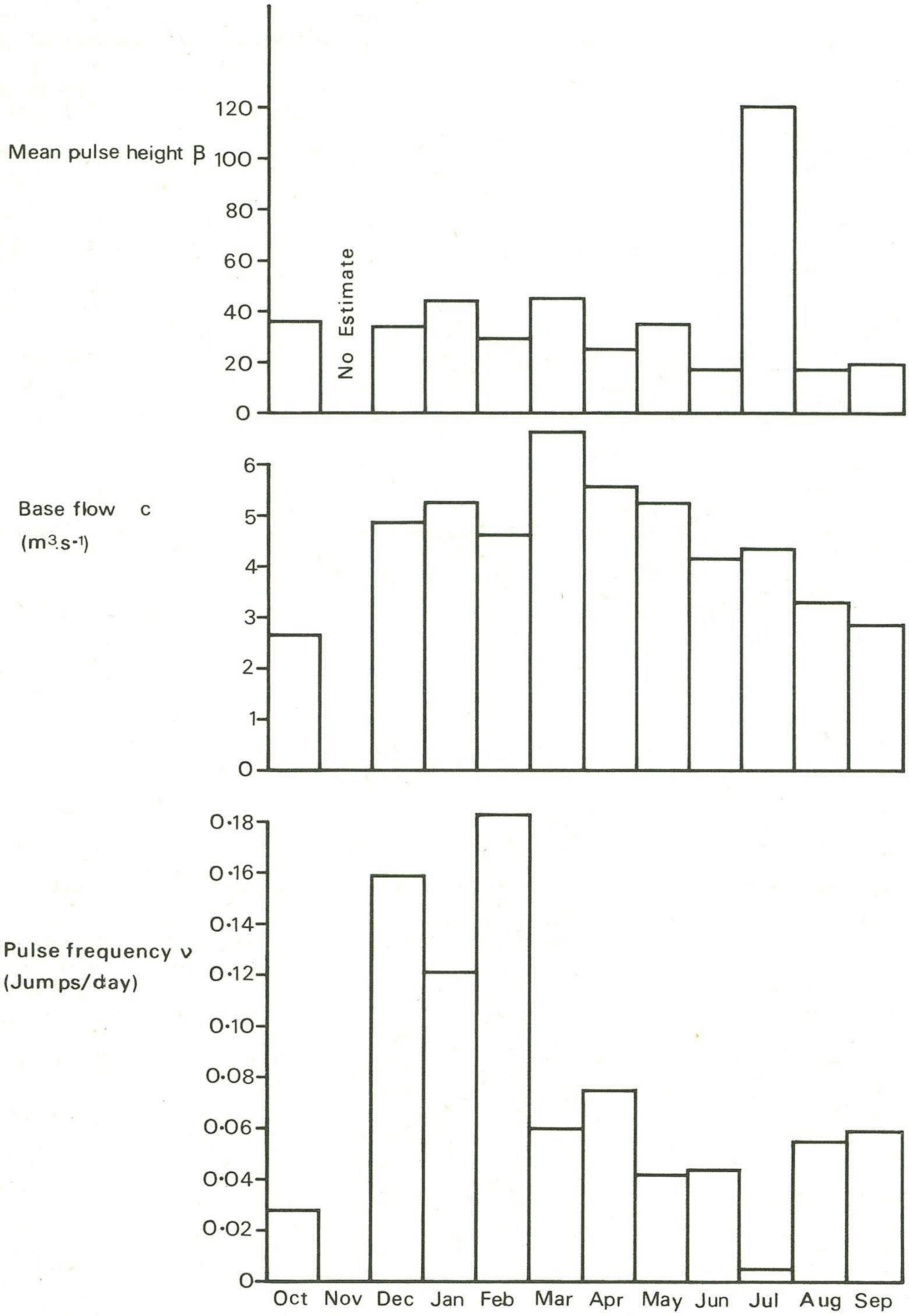


Fig. 2.28(b) Parameters for simple shot noise with base flow.

in winter, though there is considerable variation from month to month. The parameters for the simple shot noise with exponentially distributed pulse heights appear to have sensible values except for quite large variations from one month to the next. The effects of the single high flood in

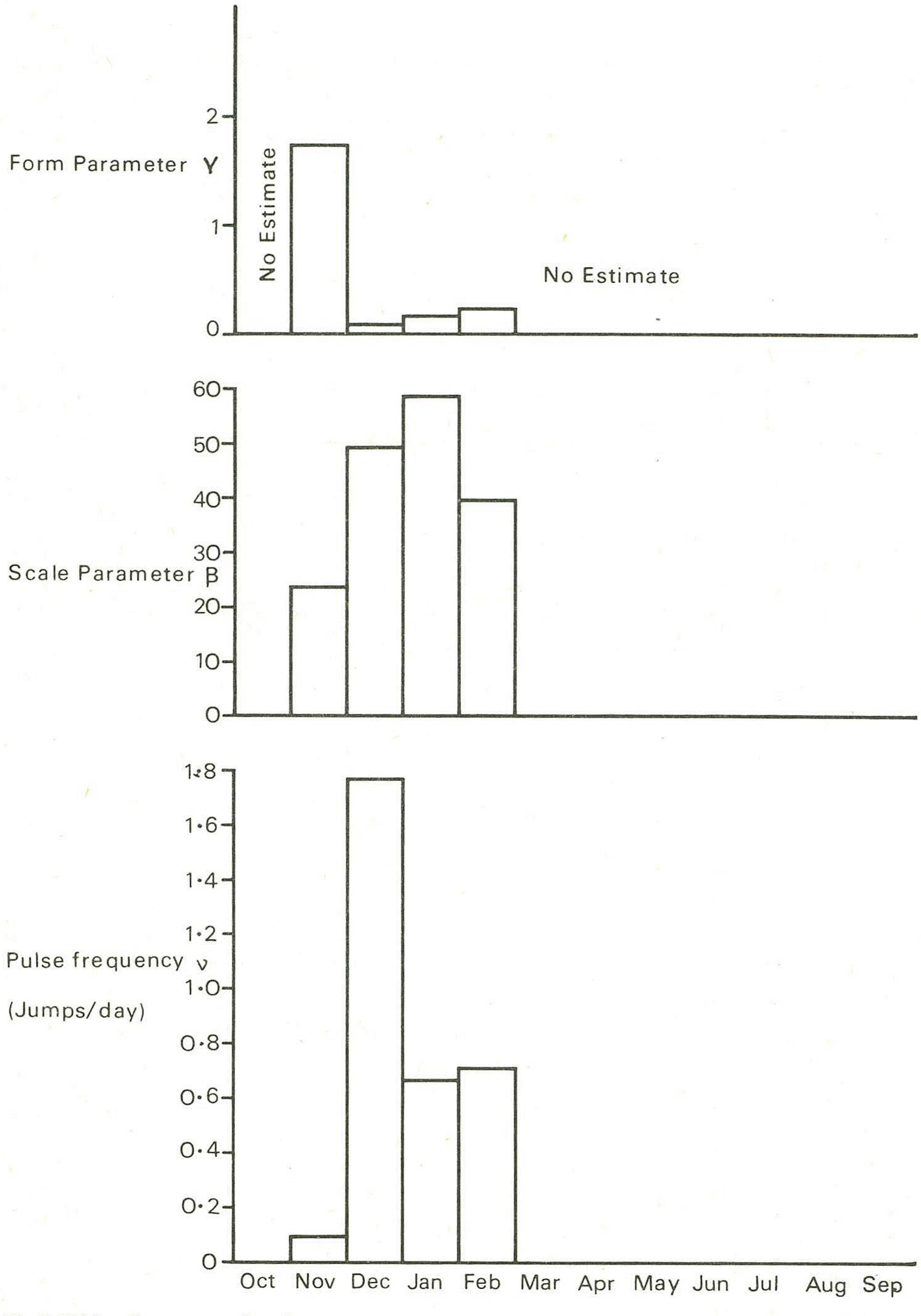


Fig 2.28(c) Parameters for shot noise model with a gamma distribution for jumps.

July has caused a very large mean pulse size and a low frequency for that month. When a base flow is added, physically unrealistic results are obtained; the base flow becomes negative in November and the other months' values are all greater than the minimum observed flow. When gamma distributed pulse heights are used worse results are obtained; the parameter *u* (skewness divided by CV) has values outside the permitted range for the month from March to October. The use of the Lomax distribution for pulse heights also gives unrealistic results, in this case in November.

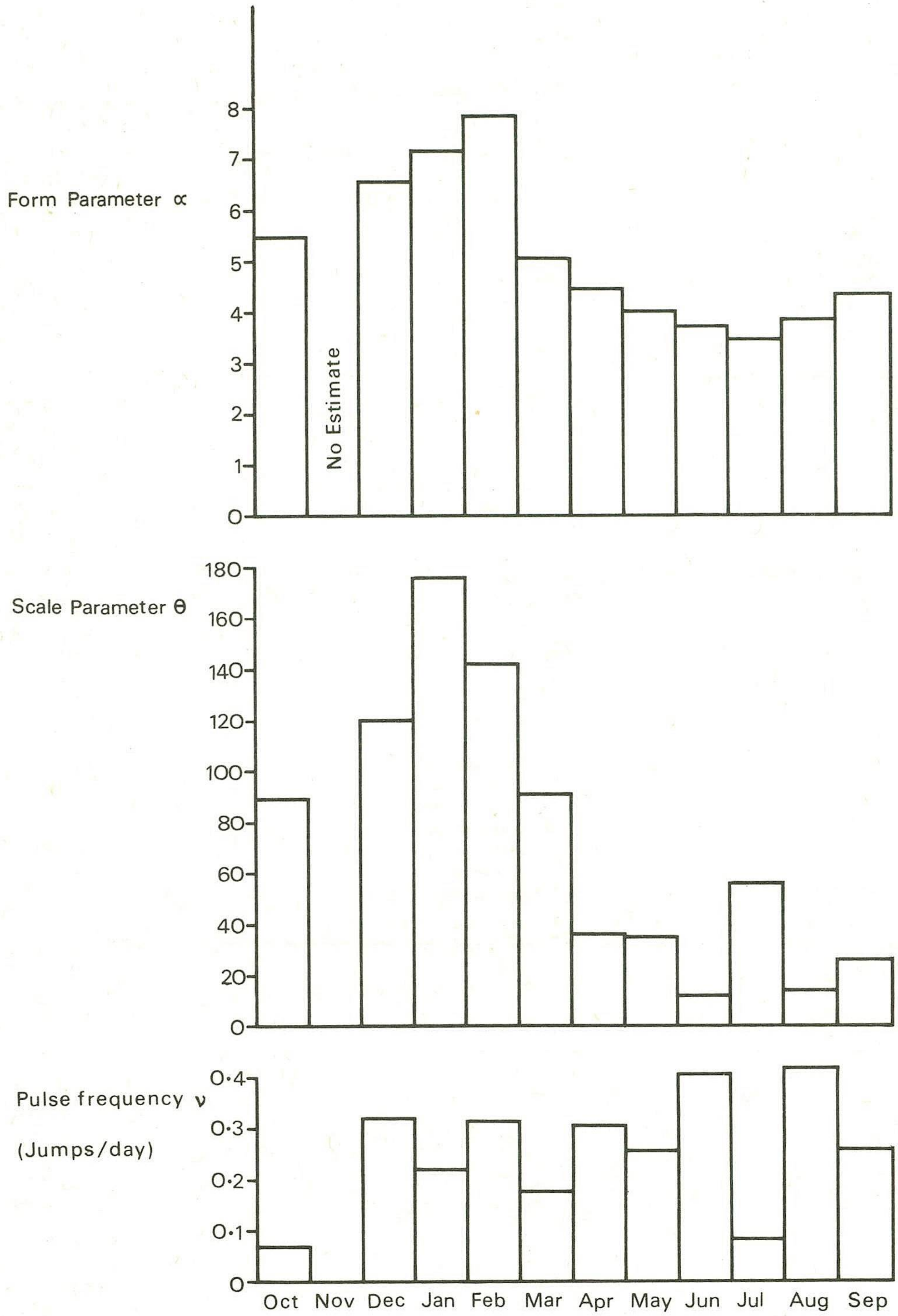


Fig 2.28(d) Parameters for shot noise model with a Lomax distribution for jumps.

The only model which can be used to fit the flow is thus the simplest shot noise with exponentially distributed jump heights. Accordingly, this model was used to generate long sequences of synthetic data. As a preliminary test several sequences of 32 years were generated; the recorded period at Evesham is 32 years. The synthetic data were found to reproduce closely the recorded mean annual flood, both for daily mean discharge and for instantaneous flow. These results were very encouraging, especially as

	Mean	Std dev.	cv	Skew	Kurtosis	Max.	Min.	ρ_1	b
Oct.	9.16	14.92	1.63	4.51	27.6	136	1.3	0.905	0.152
Nov.	19.18	24.08	1.26	2.25	7.9	133	1.8	0.876	0.202
Dec.	21.91	22.89	1.04	2.62	12.7	188	2.3	0.814	0.317
Jan.	27.08	29.21	1.08	2.62	11.3	216	1.8	0.858	0.234
Feb.	26.75	24.50	0.92	2.17	9.2	190	1.8	0.853	0.243
Mar.	20.14	23.77	1.18	3.46	13.9	222	1.3	0.878	0.198
Apr.	13.58	13.63	1.00	3.34	17.6	117	1.8	0.857	0.236
May	10.47	12.87	1.23	4.81	31.6	123	1.0	0.833	0.280
June	7.13	6.76	0.95	4.46	31.0	79	1.3	0.850	0.249
July	6.37	14.84	2.33	14.35	240.7	277	1.3	0.826	0.294
Aug.	6.07	6.41	1.06	4.55	32.5	73	1.3	0.804	0.337
Sept.	6.62	8.18	1.24	4.22	29.4	98	0.1	0.820	0.305
Year	14.48	19.95	1.38	3.82	24.2	277	0.1	0.877	0.200

Table 2.64 Statistics of daily flows, Avon at Evesham.

	Simple		With base flow			Gamma jumps			Lomax jumps		
	Mean pulse height β	Pulse frequency ν	Mean pulse height β	Frequency ν	Base flow c	Form para. γ	Scale para. β	Frequency ν	Form para. α	Scale para. θ	Frequency ν
Oct.	25.5	0.054	35.8	0.028	2.63	No estimate possible			5.49	89.2	0.070
Nov.	32.3	0.120	No estimate possible			1.73	23.6	0.095	No estimate possible		
Dec.	26.5	0.262	34.0	0.159	4.84	0.08	49.1	1.767	6.55	120.5	0.320
Jan.	34.0	0.186	42.2	0.121	5.24	0.16	58.5	0.664	7.16	175.6	0.222
Feb.	24.3	0.267	29.4	0.183	4.61	0.23	39.5	0.714	7.84	141.9	0.313
Mar.	29.9	0.133	44.6	0.060	6.61	No estimate possible			5.05	91.2	0.177
Apr.	14.8	0.216	25.1	0.075	5.57	No estimate possible			4.44	36.0	0.305
May	17.4	0.169	34.7	0.042	5.23	No estimate possible			4.00	34.7	0.254
June	7.0	0.255	16.7	0.044	4.16	No estimate possible			3.71	11.9	0.404
July	38.0	0.049	120.0	0.005	4.35	No estimate possible			3.46	55.7	0.083
Aug.	7.6	0.269	16.7	0.055	3.31	No estimate possible			3.83	13.8	0.418
Sept.	11.1	0.181	19.5	0.059	2.84	No estimate possible			4.33	26.0	0.259
Year	29.4	0.099	41.3	0.050	4.19	No estimate possible			—	—	—

Table 2.65 Shot noise parameter estimates.

Identification	No. of years	Instantaneous flow		Daily flow	
		Mean ann. flood cumecs	cv ann. flood (%)	Mean ann. flood	cv ann. flood (%)
Recorded data	32	156	55.7	127	42.4
Recorded data+30 historical floods since 1848		162	61.6	—	—
Synthetic 1	32	—	—	117	23.8
Synthetic 2	32	157	28.5	130	30.9
Synthetic 3	32	144	26.9	119	27.5
Synthetic 4	32	161	26.8	134	26.4
Synthetic 5	32	143	33.6	116	36.0
Synthetic 6	32	158	31.4	130	33.1
Synthetic 8	100	152	30.8	127	30.3
Synthetic 9	100	149	29.4	123	29.5
Synthetic 10	1000	135	29.7	110	29.8
Synthetic 11	1000	135	31.3	109.4	30.9

Table 2.66 Results of shot noise simulations. Station 54/2, Avon at Evesham.

the model was fitted using only daily mean discharges. The coefficients of variation of the floods were, however, rather low. Two sets of 1000 years each were then generated. Figure 2.29 shows the growth curves for the annual floods for these two sets compared with that for the actual flood data at the site. The curves of the synthetic data are coincident at the lower end but display a marked difference and unusual appearance at the upper end. Two lines are shown for the actual data, one using recorded data alone, and the other with historical flood information added. These two lines lie very close to each other and well above those for the shot noise synthetic data. Table 2.66 shows means and CVs for the actual and synthetic data. It is obvious that the historical variation of annual floods is not reproduced in the model. The region curve for Region 4 implies CV = 43.5%. It will be seen that this is nearer the shot noise prediction, but the discrepancy is still large.

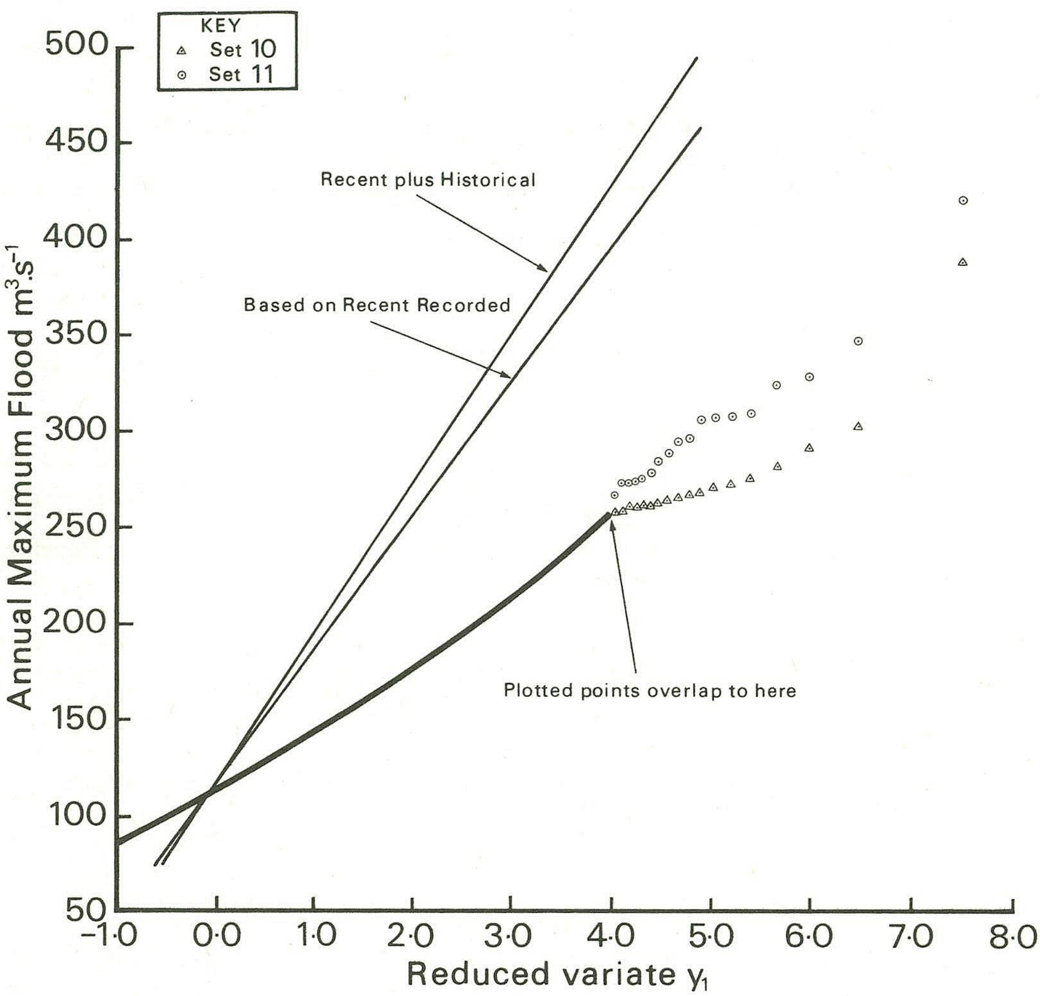


Fig 2.29 54/2, Avon at Evesham: flood frequency curves based on recorded floods and on synthetic records generated by shot noise model.

Possible improvements to the model

In his paper, Weiss (1973) found that the model described here fitted daily statistics (mean, variance and serial correlation) well but the synthetic flows averaged over a month had smaller variances and serial correlations than the historic data. To overcome this difficulty he used a double shot noise model. The flow is considered the sum of two shot noise processes; a 'fast' process such as that described here and a 'slow' process that describes variations on a time scale of a month. The problem here appears to be of the same type—the long term behaviour of the model is too regular and extra variation should be added. Weiss's double model is a possible method; alternatively a second process describing events on an annual, rather than monthly, time scale could be used.

Attempts to fit higher moments of the daily flows seem unlikely to be successful in improving the synthetic annual flood growth curves. The simple shot noise already predicts the mean annual flood well; any error arising from poor reproduction of higher moments should have shown up at this level. The shot noise model does not reproduce rare floods and hence cannot be used to predict the form of flood growth curves. The model shows some promise in that it reproduces the mean annual flood. Further work aimed at enhancing long term variability in the model may give useful results.

A general point about time series models should be made. These models generally have a large number of parameters (36 in this case) and to estimate this number of parameters long records are needed. When long records are available a conventional statistical analysis of annual maxima can be used. It may well be that the data requirements for a time series model are greater than for a conventional analysis.

2.10 Choice of design return period

2.10.1 Introduction

The relationships between design life, risk of failure and design return period are presented and tables relating them are given. A design life of 50 years is realistic for many projects because the present value of benefits enjoyed during the first 50 years of a project amounts to more than 99% of the total benefits possible in the most simplified situation, assuming a 10% discount rate. The risk of failure accompanying the use of the largest flood on record as a design rule is discussed and finally the variability of the design return period inherent in the ICE normal maximum flood is examined.

2.10.2 The design return period

The design return period cannot be realistically discussed outside the context of the benefits or losses to be incurred, whether these are capable of being expressed in monetary terms or not. Simple rules like 25, 50 and 100 year return periods for use in culvert, small bridge and large bridge schemes do not incorporate all the designer's needs. Neither does the use of a large recent flood as the design flood because such a rule gives no indication of the risk of failure.

Regardless of the design criterion, whether it be social, economic or rule of thumb, two of the three quantities design return period, T_D, design life, L, and risk of failure during the design life, r, are required to convey the full consequences of a design. These are related by

$$r = 1 - \left(1 - \frac{1}{T_D}\right)^L \tag{2.10.2.1}$$

so any two of them determine the third. Of course when $L = 1$, $r = 1/T_D$. It can be seen that since the risk depends on the life L as well as on T_D the latter is not sufficient to specify a design. In Table 2.67 r is tabulated against T_D and L, as in Borgman (1963). The concept of risk as a function of design return period and design life follows easily from the basic laws of probability and was discussed by Court (1952) and perhaps earlier by other writers. Equations (2.10.2.1) gives not the probability of just one exceedance but the probability of one or more exceedances. It can be recognised as the probability of at least one success in a sequence of L trials in each of which the probability of success is $1/T_D$. When $1/T_D$ is small the behaviour of such a sequence of trials should approximate a Poisson process with parameter $\lambda = 1/T_D$. In such a process the probability of at least one occurrence in L units of time is

$$r = 1 - e^{-\lambda L} = 1 - e^{-L/T_D}. \tag{2.10.2.2}$$

This accords with Equation (2.10.2.1) because as T_D gets large

$$\left(1 - \frac{1}{T_D}\right)^L = \left\{\left(1 - \frac{1}{T_D}\right)^{T_D}\right\}^{L/T_D} \simeq \{e^{-1}\}^{L/T_D}.$$

Hall & Howell (1963) used the Poisson distribution for computing risk and an expression similar to (2.10.2.2) was also quoted by Gumbel (1958, Equation 1.2.2.11). If T_D is very large in comparison to L, on expanding Equation (2.10.2.2), r is approximately given as

$$r = L/T_D, L \ll T_D. \tag{2.10.2.3}$$

This can be seen to hold in the upper right hand portion of Table 2.67.

Table 2.67 Risk of failure, r, as a function of design life L and design return period T_D (Equation 2.10.2.1).

Design life L	Design return period T_D						
	10	25	50	100	250	500	1000
5	0.410	0.185	0.096	0.049	0.020	0.010	0.00499
10	0.651	0.335	0.183	0.096	0.039	0.020	0.00996
20	0.878	0.558	0.332	0.182	0.077	0.039	0.01981
30	0.958	0.706	0.455	0.260	0.113	0.058	0.02957
40	0.985	0.805	0.554	0.331	0.148	0.077	0.03923
50	0.995	0.870	0.636	0.395	0.182	0.095	0.04884

The design return period as a function of risk and design life is, from Equation (2.10.2.1),

$$T_D = 1/\{1 - (1-r)^{1/L}\} \tag{2.10.2.4}$$

which for very small r is approximately

$$T_D \simeq 1/\{1 - e^{-r/L}\} \tag{2.10.2.5}$$

Table 2.68 Design return period T_D as function of risk of failure r and design life L (Equation 2.10.2.4).

Design life L	Risk of failure r						
	0.02	0.05	0.10	0.20	0.30	0.40	0.50
5	248	98	48	23	15	10	8
10	495	195	95	45	29	20	15
20	990	390	190	90	57	40	29
30	1485	585	285	135	85	59	44
40	1980	780	380	180	113	79	58
50	2475	975	475	225	141	98	73

and is tabulated in Table 2.68, a similar table having been given by Borgman (1963). Thus over a design life of 20 years a risk of 0.10 implies a design return period of 190 years.

2.10.3 The design life

This depends on social and economic factors rather than statistical or hydrological ones. In schemes where loss of life is not expected and all benefits and losses are expressible in monetary terms the present worth of the benefits to be gained during the first 50 years of the design life amounts to 99% of the present worth of the total benefits to be gained (Local Government OR Unit, 1973)) when the discount rate is about 10%. Therefore a design life of 50 years might be realistic in schemes dealing with urban and agricultural flooding but different considerations are applied in the reservoir spillway problem. The relation between risk r and design return period T_D for $L = 50$ is shown in Figure 2.30.

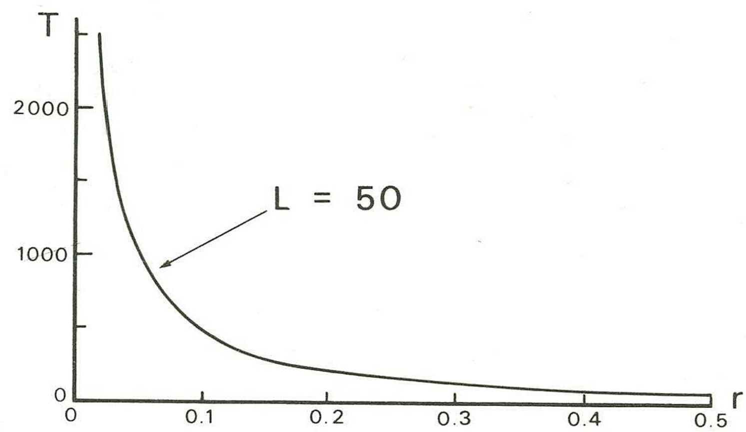


Fig 2.30 Design return period as a function of risk of failure during a 50 year design life (last row of Table 2.68).

2.10.4 Distribution of Q_{max} values

The largest flood during the design life of some hydrological works, the time to the economic horizon, has the same practical properties as the largest value in a record of the same length. It has often been the practice in the past to use the largest flood on record or in living memory as a design flood especially if it is remarkable and recent. However, since the largest flood in a record of length N years is a random variable the resulting design flood does not have a fixed return period or equivalently the probability of failure cannot be specified.

For a given N the largest value, Q_{max}, varies widely. If samples of equal size are drawn from an EV1 population with standard deviation σ, the resulting sequence of Q_{max} values also has standard deviation σ regardless of the value of N while if the population is GEV with $k<0$ the standard deviation of Q_{max} is greater than σ.

In addition the mean value of Q_{max} increases slowly with N, for instance

$$\text{Mean}(Q_{max}) = \text{Mean}(Q) + 0.78\,\sigma \ln N \qquad (2.10.4.1)$$

in the EV1 case. Therefore the difference between mean (Q_{max}) for samples of sizes, $2N$ and N, from the same EV1 parent is 0.54 σ which is about 38 % of the standard deviation of the difference between them.

It follows that an observed Q_{max} value in a short record could easily exceed many observed Q_{max} values in long records at the same site.

Study of standardised Q_{max} values

It is shown here that the use of Q_{max} as a design flood does not give a predetermined or designed risk of failure. The dotted curves on Figure 2.31 show relations between $Q_{max}/\bar{Q}$ and length of record N for three different levels of risk, r = 0.25, 0.50 and 0.75. These are derived from the distribution of Figure 2.16 and show, for instance, that in a 50 year design life a flood of $Q/\bar{Q} = 3$ has a 25 % chance of being exceeded at least once. The middle curve gives the median largest flood in an N year record and is the value of Q which satisfies

$$(F(Q))^N = 0.50. \qquad (2.10.4.2)$$

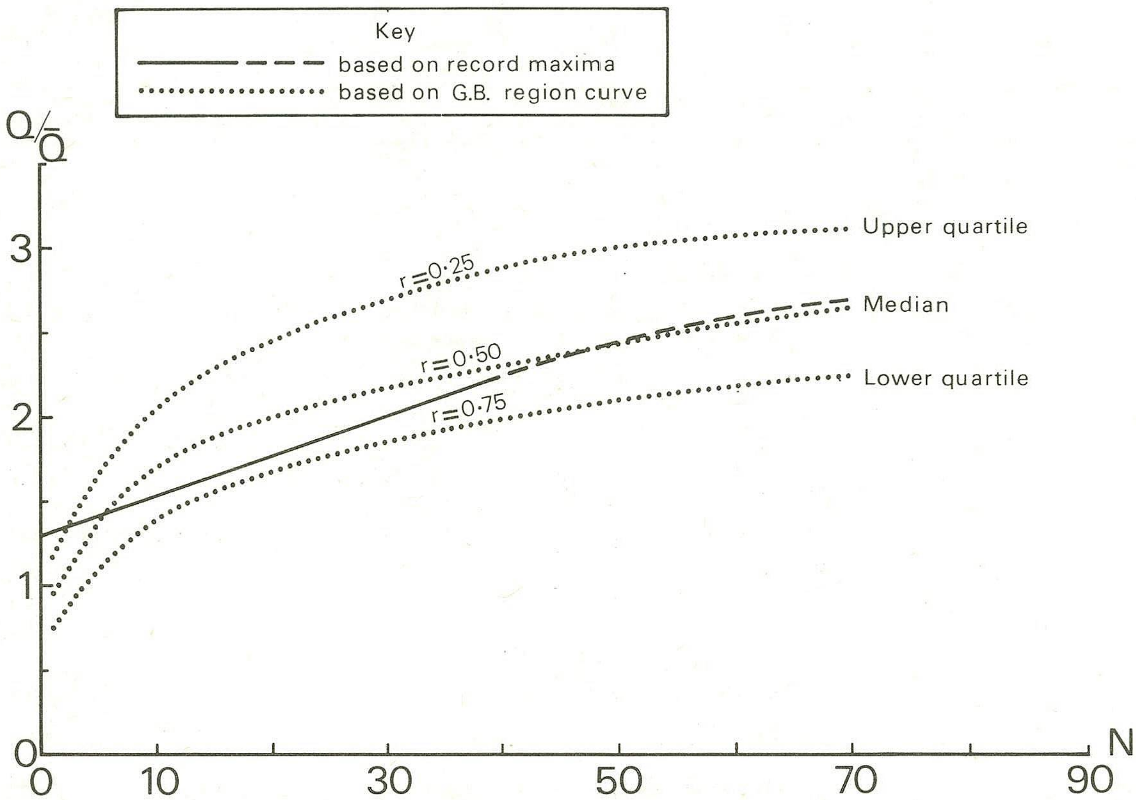
This is solved for a series of N values by computing

$$F(Q) = (0.50)^{1/N} \qquad (2.10.4.3)$$

and entering Figure 2.16 with $y = -\ln -\ln F(Q)$ to obtain $Q/\bar{Q}$. The outer dotted curves in Figure 2.31 are found in a similar manner with 0.50 replaced by 0.25 and 0.75 respectively.

If $F(Q)$ is unknown it should be possible to estimate such a family of curves from observed record maxima. Figure 2.32(a) to (c), based on the data summarised in Table 2.69, show attempts made to establish the

Fig 2.31 Probability or risk that a value $Q/\bar{Q}$ would be exceeded in N years. The solid line is from Figure 2.32(c) while the dotted curves are computed from the Great Britain region curve, Figure 2.16.



relation between median $Q_{max}/\bar{Q}$ and N by drawing smooth eye guided curves through the plots. For $N<40$ solid curves are used while dashed ones are used for $N>40$ to emphasise the sparseness of the data on which they are based. Due to the lack of enough long records attempts to show regional differences were unsuccessful as were efforts to derive quartile values.

The median curve of Figure 2.32c is superimposed on Figure 2.31 which shows that the two median curves agree only moderately well despite the fact that they are both derived from data of the same 450 stations. The dotted curve of Figure 2.31 is probably the more reliable because it is based on all available data whereas the solid line is based on record maxima alone.

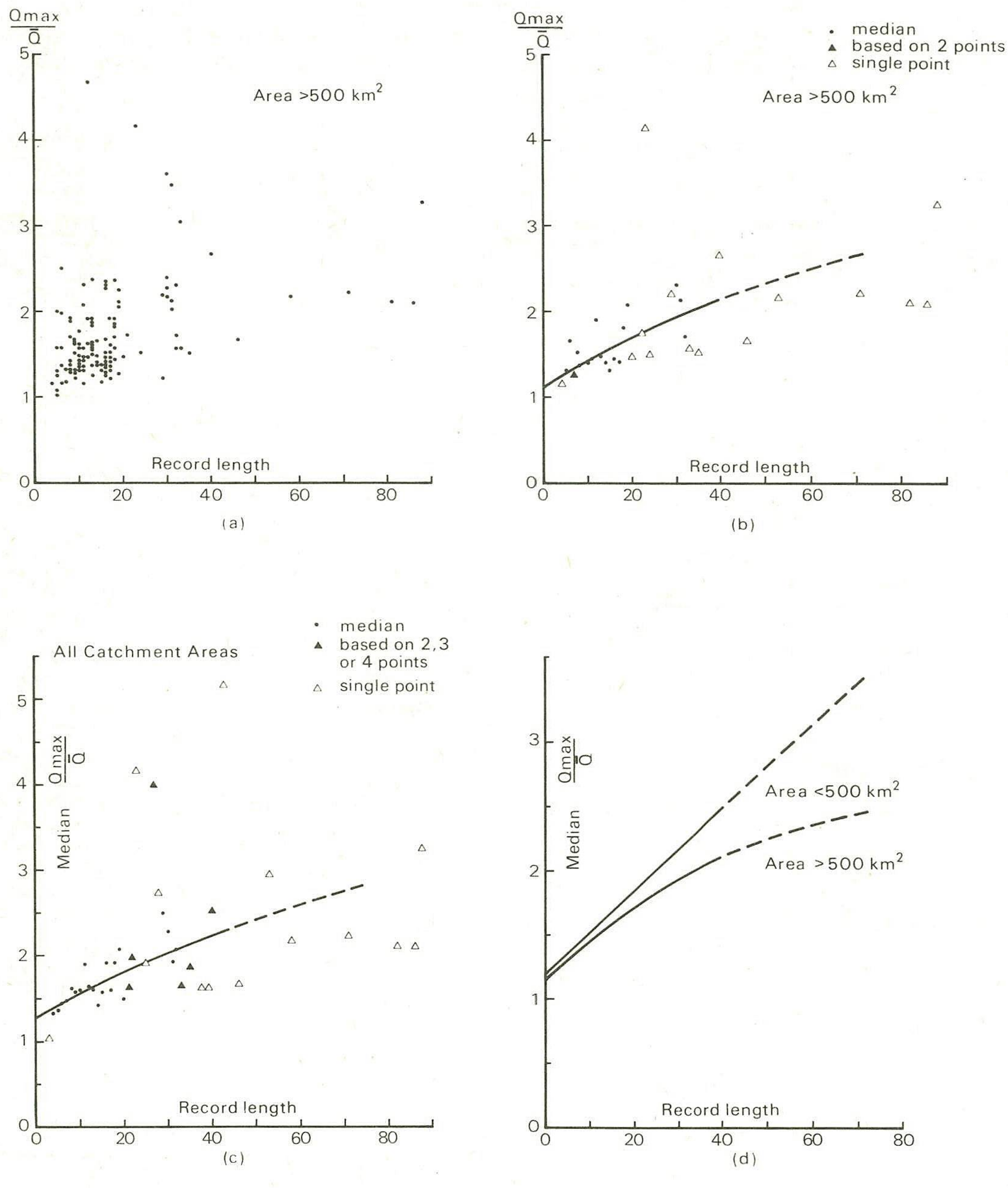


Fig 2.32 Empirical relation of $Q_{max}/\bar{Q}$ and median $Q_{max}/\bar{Q}$ to length of record.

Table 2.69 Summarised $Q_{max}/\bar{Q}$ values.

Years	Stations	Mean	Median	Quartiles		Limits	
				Lower	Upper	Min.	Max.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
3	1	1.275	1.275	—	—	—	—
4	15	1.376	1.325	1.207	1.458	1.033	2.175
5	45	1.499	1.363	1.249	1.598	1.014	4.037
6	46	1.535	1.445	1.275	1.715	1.094	2.505
7	41	1.622	1.478	1.312	1.844	1.100	3.299
8	48	1.696	1.615	1.337	1.815	1.164	4.358
9	37	1.725	1.583	1.436	2.082	1.149	2.659
10	31	1.862	1.595	1.447	1.944	1.304	5.025
11	18	2.179	1.895	1.532	2.276	1.342	4.979
12	19	1.919	1.646	1.539	1.943	1.254	4.676
13	20	1.655	1.604	1.502	1.819	1.258	2.395
14	10	1.605	1.434	1.416	1.656	1.233	2.302
15	9	1.534	1.572	1.414	1.635	1.171	1.944
16	13	1.873	1.925	1.716	2.023	1.381	2.591
17	11	1.767	1.598	1.444	1.775	1.224	3.747
18	9	1.948	1.934	1.834	2.112	1.280	2.682
19	9	1.998	2.067	1.737	2.250	1.295	2.883
20	9	1.575	1.497	1.483	1.740	1.245	1.870
21	3	1.659	1.636	1.615	1.692	1.594	1.747
22	4	2.074	1.994	1.863	2.205	1.528	2.778
23	1	4.164	4.164	—	—	—	—
25	1	1.919	1.919	—	—	—	—
27	2	3.992	3.992	3.092	4.891	2.192	5.791
28	1	2.737	2.737	—	—	—	—
29	8	2.437	2.054	1.875	2.882	1.228	4.615
30	5	2.495	2.288	2.186	2.403	1.984	3.616
31	6	2.117	1.939	1.721	2.035	1.632	3.475
32	6	2.475	2.030	1.722	2.747	1.576	4.603
33	4	1.930	1.660	1.526	2.064	1.340	3.061
35	2	1.883	1.883	1.706	2.060	1.529	2.237
38	1	1.626	1.626	—	—	—	—
39	1	1.631	1.631	—	—	—	—
40	2	2.536	2.536	2.468	2.602	2.401	2.670
43	1	5.159	5.159	—	—	—	—
46	1	1.690	1.690	—	—	—	—
53	1	2.958	2.958	—	—	—	—
58	1	2.175	2.175	—	—	—	—
71	1	2.235	2.235	—	—	—	—
82	1	2.120	2.120	—	—	—	—
86	1	2.103	2.103	—	—	—	—
88	1	3.258	3.258	—	—	—	—

2.10.5 Return period of ICE normal maximum flood

The Institution of Civil Engineers (1933) normal maximum flood, NMF, depends only on catchment area and no return period is attached to it. The return period of the NMF was discussed by Chapman & Buchanan (1966) who found that the return period was far from constant between catchments. The NMF was defined by a curve of flood intensity, cusecs/square mile, against catchment area, square miles, which would 'justly represent the flood intensities which may certainly be anticipated on any normal upland area, not less than 2 square miles in area'.

A selection of 52 catchments having areas between 5 km^2 and 100 km^2 and which have the greater part of their area at elevations above 250 m was made. For each of these catchments the NMF was read off in cumecs/km^2 from a NMF curve redrawn in these units on log-log paper. For each station the mean annual flood, MAF, expressed in cumecs/1000 km^2 is given in Table 2.70 as an index of comparison of the various catchments'

Station	Area (km²)	Record length	Mean annual flood $\bar{Q}$ (cumecs)	MAF ÷ AREA (cumecs/1000 km²)	Normal maximum flood NMF (cumecs)	NMF/$\bar{Q}$	T from region curve if <1000
3/801	72.3	8	73.9	1022	192.5	2.61	100
3/803	64.2	7	68.0	1060	187.0	2.66	110
6/906	27.5	7	17.7	643	115.0	6.50	—
15/1	70.7	22	50.2	710	191.6	3.81	446
15/2	15.4	20	7.8	506	84.7	10.86	—
15/4	24.7	39	6.4	260	108.7	16.93	—
15/5	40.9	38	15.2	373	143.2	9.39	—
15/808	17.8	9	12.7	714	91.4	7.21	—
15/809	16.5	17	8.3	505	87.5	10.50	—
16/802	99.5	9	156.9	1576	228.9	1.46	8.67
19/2	43.8	9	15.8	362	148.9	9.40	—
19/3	51.8	9	20.9	403	161.6	7.75	—
19/4	81.6	9	23.1	283	205.6	8.89	—
21/1	23.7	15	19.0	800	107.1	5.65	—
21/2	45.6	9	25.1	551	151.4	6.02	—
24/2	93.0	11	22.4	241	220.4	9.83	—
24/4	74.9	10	25.6	342	198.5	7.75	—
24/6	36.5	9	25.5	699	133.2	5.22	—
25/3	11.4	7	17.8	1559	73.0	4.11	—
25/6	86.2	9	78.9	915	213.8	2.71	—
27/10	18.9	32	10.7	568	94.5	8.81	—
27/12	36.0	16	14.6	406	133.2	9.10	—
27/852	21.1	22	19.8	938	100.2	5.06	—
28/801	9.1	43	5.4	593	64.6	11.99	—
45/6	20.4	5	11.4	559	98.9	8.67	—
46/5	21.5	5	51.0	2370	101.1	1.98	29
46/801	14.9	5	24.8	1667	83.4	3.36	270
46/806	14.0	17	26.4	1885	80.9	3.07	165
54/3	94.4	40	100.2	1061	222.9	2.22	34
54/13	57.0	6	72.8	1277	171.0	2.35	43
54/22	8.1	19	13.5	1678	60.4	4.47	—
55/4	72.8	31	57.8	793	192.9	3.34	—
55/8	10.4	19	16.9	1626	68.6	4.06	—
55/10	27.2	14	57.6	2118	114.2	1.98	74
55/15	24.6	16	21.1	856	108.7	5.16	—
55/808	95.3	21	89.2	936	224.0	2.51	425
56/3	62.2	6	30.5	490	180.4	5.92	—
58/3	62.9	8	19.3	307	182.5	9.43	—
58/4	85.7	8	96.4	1125	211.7	2.20	157
65/1	68.6	7	60.0	875	188.7	3.17	—
66/3	69.9	6	31.4	449	190.1	6.06	—
66/801	10.4	6	14.9	1424	69.9	4.70	—
67/3	20.2	5	13.4	662	98.6	7.37	—
67/9	79.5	12	8.9	112	202.7	22.82	—
69/802	13.0	23	14.7	1129	77.7	5.30	—
71/3	10.4	12	14.9	1431	68.6	4.61	—
71/5	10.6	9	17.8	1676	69.8	3.93	—
73/2	73.0	6	22.5	308	193.5	8.59	—
78/4	76.1	12	62.0	815	197.9	3.19	358
80/801	18.2	7	13.0	714	92.8	7.15	—
84/2	12.4	13	19.8	1600	75.6	3.81	715
91/802	6.5	29	7.0	1087	53.7	7.64	—

Table 2.70 Return periods of Institution of Civil Engineers normal maximum flood.

flood producing characteristics. The ratio of NMF to MAF is the dimensionless quantity $Q/\bar{Q}$ whose distribution is given by each region curve (Section 2.6); this ratio is also tabulated in Table 2.70. Only 14 of the 52 station ratios gave a return period in the region curve of less than 1000 years and these have been tabulated. Return periods of more than 1000 years have not been tabulated as the individual region curves do not extend beyond this point.

The length of record at these stations varies from 5 to 43 years, half of them being less than 10 years. The MAF values obtained from them are affected by sampling error and hence the NMF/MAF values are also affected. However, these sampling fluctuations are not solely responsible for the variation in NMF/MAF values which range from 1.46 to 22.82, the second smallest and second largest being 1.98 and 16.93. Five values are greater than 10.0, a value which has rarely been exceeded by actual $Q/\bar{Q}$ values (see Table 2.9). At the 14 stations where the return period of NMF/MAF measured from the region curves is less than 1000 years the return period of NMF/MAF varies from 9 years to 715 years. A 9 year return period as a design figure has about a 65% risk of failure in 10 years and a 99.5% risk in 50 years. On the other hand the values having return period of 1000 years or more have less than 1% risk of failure in 10 years and less than 5% in 50 years. In summary the normal maximum flood may give a flood of very high return period—a 'safe' design flood—but it does not guarantee it.

2.11 Recommendations and examples

2.11.1 Introduction

Since this report is the first based on the total collection of flood records in Great Britain and Ireland it was necessary to present full reviews of basic methods and results. This section collates the applicable results and specifies a strategy for estimating the T year flood and its associated standard error of estimate.

Bearing in mind the large sampling variation in any statistic, particularly a quantile, computed from a short record it is unreasonable to expect that an estimate derived from a short record should be identical to one based on a long record or taken from one of the region curves of Section 2.6. If $Q(T)$ is to be estimated at a site at which a flood record is available the fitted distribution should describe the distribution of the data closely. However, if other information is available about the hydrology of the region this is bound to be taken into account and therefore the best estimate does not necessarily describe the distribution of the sample values exactly.

2.11.2 Recommended distribution for annual maxima

Statistical tests do not point to a single distribution being certainly the correct one and therefore a choice must involve an element of subjectivity. The members of the U.S. Water Resources Council Committee (1967) found themselves faced with a similar choice; they recommended 'that the log Pearson Type 3 distribution be adopted as a base method for analysing flood flow frequencies' but 'that in such cases where investigation showed that other distributions or techniques would be better suited, these techniques should be used but justification for the departure from the base method should be documented' and 'that the choice of a base method should not be considered as final and should not freeze hydrologic practice into any set pattern either now or in the future' (Benson, 1968).

This report recommends the general extreme value (GEV) distribution because

it performed more consistently in the goodness of fit tests than either the log Pearson Type 3 or the Pearson Type 3 whose superiorities depend on which of two arbitrary criteria was used to determine the form of the goodness of fit index;

ii it describes the empirically derived region curves of Section 2.6 remarkably well;

iii it is in accord with the rainfall growth curves of Volume II;

iv estimates of its parameters by moments, sextiles or maximum likelihood are as easy to derive as the corresponding estimates in other three parameter distributions.

This judgement takes into account results based on all the flow and rainfall records of the country. Once the parameter values (u, α, k) are known the quantile value of return period T may be computed as

$$Q(T) = u + \alpha W(y_1; k) \quad (2.11.2.1)$$

where $W(y_1; k) = (1 - e^{-ky_1})/k$ is tabulated against T in Table 1.12. However, if Table 1.12 is not available the appropriate W value can be calculated using $y_1 \simeq \ln T$ for large T giving $W \simeq (1 - T^{-k})/k$ so that log tables or slide rule may be used to calculate the required quantile.

When only a small sample N, less than 25, is available it is recommended that the EV1 (Gumbel) distribution be fitted if an estimate based on the sample's data alone is required but this should not be used for gross extrapolation as on average this leads to underestimation at high return periods. The use of the EV1 distribution instead of the GEV is logical because it is a special case of it and can approximate to it over a limited range of return period.

For large return periods, $T > 2N$, it is recommended that the mean annual flood be computed from the record and this used to multiply the appropriate ordinate on the region curve. Despite the fact that the plotted data may deviate from the pattern of the region curve it should be remembered that large sampling variations are inherent in all records, particularly in short ones; see for instance Figure 1.18 and Table 2.26. For very large return periods, $T > 4N$, the extrapolation of a fitted distribution would be judged on present evidence to be very unwise; in such cases the use of the region curve, which is a synthesis of all the flood frequency information available, is the safest course of action.

The recommended course of action therefore depends on two factors (a) the length of record available, N years and (b) the return period for which the estimate is required, T years. The corresponding rules are given in outline in Table 2.71.

However, as in the case of the U.S. Water Resources Council recommendation the use of other distributions or strategies is not precluded; indeed analysts who are sufficiently well versed in statistical technique applied to hydrology might prefer to use their own strategy. In any case sensible data analysis which takes sound statistical theory and useful empirical evidence into account is desirable.

2.11.3 Differences between estimates

Each method of estimation provides a different value of $Q(T)$ and this is unavoidable. There is another source of differences at very high return periods based on the amount of extrapolation allowed in the region curves. The curves for individual regions are defined up to about $T = 200$ ($y \simeq 5.3$) but may be extrapolated over a range of one unit of y, say up to

1	No records or less than 10 years record at site	Estimate $\bar{Q}$ from catchment characteristics or Estimate $\bar{Q}$ from record either annual maximum or POT series	$Q(T)/\bar{Q}$ from region curve (see Figures 2.14 and 2.16 and Table 2.39) $Q(T)/\bar{Q}$ from region curve
2	N = 10–25 years record	or alternatively if $T<2N$ fit EV1 distribution to record to give $(\hat{u}, \hat{\alpha})$	$Q(T) = \hat{u}+\hat{\alpha}y_1$ where $y_1 = -\ln-\ln(1-1/T)$ $\simeq \ln(T-\frac{1}{2}), T > 5$ is given in Table 1.12
3	N>25 years record	Estimate $\bar{Q}$ from record or alternatively if $T<2N$ fit GEV distribution to record to give $(\hat{u}, \hat{\alpha}, \hat{k})$	$Q(T)/\bar{Q}$ from region curve $Q(T) = \hat{u}+\hat{\alpha}W$ where W is given in Table 1.12 for T and k $W = \left(\frac{1-e^{-ky_1}}{k}\right)$ and y_1 is as above
4	If $T>500$ use curve based on entire country		

Table 2.71 Scheme for estimating T year flood.

Notes: (i) If $Q_{max}/Q_{med}>3.0$, where Q_{max} and Q_{med} are the maximum and median flood on record use $1.07Q_{med}$ instead of $\bar{Q}$ (see Section 2.3.5).
(ii) If historical information exists then see Sections 2.6.3 and 2.8.

$T = 500$ ($y\simeq 6.2$). For higher return periods it is suggested that the countrywide curve, Figure 2.16, be used as the basis of the estimate but it is quite obvious from Figure 2.14 that there would then be a discontinuity in the $Q(T)$–T relation at $T = 500$. This of course is not real and some compromise between the two curves has to be adopted.

The possibility of different estimates is not new. The traditional attitude is that these should be 'compared' which means that the merits of each should be examined and the one which seems most reasonable in the light of known facts adopted. A more recent approach is to attach a degree of belief probability to each estimate and take their expectation. This of course may be viewed as a weighted average but is a particular aspect of Bayesian methods which is also closely linked with statistical decision theory. The theory is available but to be of wide use the degree of belief probabilities mentioned above must be based on some quantitative evidence such as standard errors and it is difficult with present knowledge to give even the order of magnitude of these with any assurance. Some of the more sophisticated or thorough methods involve large amounts of numerical integration; with modern computers these require minutes rather than hours of computing time but until the inputs to these computations are improved they will be no better than simple weighted averages.

2.11.4 Goodness of fit tests

Two well known goodness of fit tests, the χ^2 and the Kolmogorov–Smirnov, were described in Section 2.4 but it was concluded there that these are not sufficiently sensitive for the sample sizes usually available in hydrology. If one of these tests rejects a distribution it may be taken as positive evidence against the distribution; failure of either of these tests to reject it cannot however be interpreted as confirmation of the distribution. The other goodness of fit indices mentioned in Section 2.4 have a major weakness in failing to take into account that the values at both ends of the sample have larger sampling variances than values in the middle.

Because of the deficiencies of goodness of fit indices reliance has to be placed on the probability plots. (Plotting positions are given in Table 1.13 for Normal samples and in Table 1.16 for extreme value samples.) The probability plot contains all the information there is on the agreement between sample and distribution under test but it does not summarise it in a few figures; hence the judgement has to be visual and the plot has to be compared with plots of random samples of the same size from the distribution under test. This is a subjective procedure but the objective tests that are available are so ineffective that their objectivity alone is insufficient to recommend them. It is felt that an effective goodness of fit index should consist of two or more statistics computed from the probability plot rather than a single one.

2.11.5 Standard errors

The standard error of an estimate of $Q(T)$ obtained from a single record has been discussed in Section 2.5 and a single standard error formula was suggested for practical purposes regardless of the distribution being fitted or the method of fitting. It takes the form

$$\text{se}(\widehat{Q(T)}) = \frac{C\sigma}{\sqrt{N}} \qquad (2.11.5.1)$$

where C depends on return period T, σ is the population standard deviation and N is the sample size. The quantity σ is unknown and has to be estimated from the sample and as a result may be the subject of large sampling error. For this reason a constant value of coefficient of variation can be assumed in a given region and the sample mean employed instead of μ in the following formula

$$\text{se}(\widehat{Q(T)}) = \frac{D\mu}{\sqrt{N}} \qquad (2.11.5.2)$$

where $D = \text{cv}.C$. This formula should be used when the sample size N is less than 25, formula (2.11.5.1) above being used for larger sample sizes. The quantities C and D are tabulated in Table 2.37 of Section 2.5 and C is related to T by

$$C = 0.35+0.8y$$

$$\simeq 0.35+0.8\ \ln(T-\tfrac{1}{2}) \qquad (2.11.5.3)$$

for $5<T<1000$.

The standard error of a region curve ordinate used to estimate $Q(T)/\bar{Q}$ was discussed in Section 2.6 where it was suggested that the following relation be used

$$S_b = -3.5+7.7\ \ln T \qquad (2.11.5.4)$$

where

$$S_b = \frac{\text{standard error of region curve estimate of } Q(T)/\bar{Q}}{\text{region curve ordinate}} \cdot 100.$$

The sampling variance of a $Q(T)$ estimate obtained via a region curve is, assuming $E(\bar{Q})$ replaced by $\bar{Q}$,

$$\text{var}\ \widehat{Q(T)} = \bar{Q}^2\ \text{var}(Q/\bar{Q})+(Q/\bar{Q})^2\ \text{var}\ \bar{Q} \qquad (2.11.5.5)$$

where $Q/\bar{Q}$ is the region curve ordinate and

$$\text{var}(Q/\bar{Q}) = (0.01S_b . Q/\bar{Q})^2. \qquad (2.11.5.6)$$

2.11.6 Examples

This section will consider the gauging site 10/1, Ythan at Ardlethan, which has 31 years of record. The recommended methods will be demonstrated by estimating for this site the mean annual flood, the 25, 100 and 1000 year events on the assumptions that there are

a no records at the site;

b record for 7 years 1939–45;

c records for 15 years 1955–69;

d records for 31 years 1939–69.

a No records

The mean annual flood, $\bar{Q}$, is estimated from catchment characteristics and the other floods are estimated by multiplying the appropriate region curve ordinate by this $\bar{Q}$. Equation (2.3.3.1) gives $\bar{Q}$ as

$$\bar{Q} = C\ \text{AREA}^{0.94}\text{STMFRQ}^{0.27}\text{S1085}^{0.16}\text{SOIL}^{1.23}\text{RSMD}^{1.03}(1+\text{LAKE})^{-0.85}.$$

In this case when the catchment is in Region 1 the multiplier C is 0.0186 and the catchment characteristics are

AREA = 448 km^2

STMFRQ = Stream frequency = 0.38 junctions/km^2

S1085 = Slope from 10 to 85% of channel length = 3.48 m/km

SOIL = Index of runoff based on catchment soils' winter rain acceptance rates = 0.308

RSMD = Net 1 day rainfall of 5 year return period = 42.6 mm

LAKE = Proportion of catchment draining through a lake = 0.0

which give $\bar{Q} = 60.83$.

From Table 2.39 the region curve ordinates at $T = 25$ and 100 in Region 1 are

$$Q(25)/\bar{Q} = 1.81$$

$$Q(100)/\bar{Q} = 2.48.$$

Therefore, the estimates of $Q(25)$ and $Q(100)$ are

$$\widehat{Q(25)} = 1.81\bar{Q} = 1.81(60.83) = 110.1$$

$$\widehat{Q(100)} = 2.48\bar{Q} = 2.48(60.83) = 150.9.$$

The individual region curves cannot be extrapolated to $T = 1000$. The entire Great Britain curve with parameters $(u,\alpha,k) = (0.80, 0.24, -0.2)$ from Table 2.38 is used instead giving

$$Q/\bar{Q} = u+\alpha W(y_1;k)$$

where W depends on T and k and from Table 1.12 equals 14.91 at $T = 1000$ and $k = -0.2$. Therefore, the Great Britain region curve ordinate at $T = 1000$ is

$$Q/\bar{Q} = 0.80+0.24(14.91) = 4.38$$

and

$\widehat{Q(1000)} = 4.38(60.83) = 266.4.$

The standard errors of these estimates are obtained by using Equation (2.11.5.6) in each case. The region curve value of cv, Table 2.38, is 0.40 and therefore var($\bar{Q}$) lies between $(0.40\bar{Q})^2$ and $(0.40\bar{Q})^2/2$, that is between 296 and 592, say 440, so that the se of $\bar{Q}$ is between 17 and 24, say $21(\simeq\sqrt{440})$. For the other estimates the quantity S_b of Equation (2.11.5.4) is needed and the calculations are summarised in Table 2.72.

T	$Q/\bar{Q}$	$\widehat{Q(T)} = \bar{Q}.Q/\bar{Q}$	S_b Eqn (2.11.5.4)	Var($Q/\bar{Q}$) Eqn (2.11.5.6)	$\bar{Q}^2$var($Q/\bar{Q}$) (6)	$(Q/\bar{Q})^2$var$\bar{Q}$ (7)	var$\widehat{Q(T)}$ = (6)+(7)	se ($\widehat{Q(T)}$)	% of $\widehat{Q(T)}$
25	1.81	110	21.3	0.148	548	1441	1989	45	40
100	2.48	151	31.9	0.62	2294	2706	5000	71	47
1000	4.38	266	49.7	4.74	17 539	8441	25 980	161	61

Table 2.72 Flood estimates from site with no records.

Mean annual flood $\bar{Q} = 60.83$, se ($\bar{Q}$) $\simeq 21$.

b Seven years of record

The seven years 1939–45 have annual maxima

31.07, 48.55, 57.67, 39.62, 20.01, 46.56, 75.76 with mean $\bar{Q} = 45.61$.

With a region value of cv = 0.40 the variance of $\bar{Q}$ is

$\text{var}\bar{Q} = (\text{cv}.\bar{Q})^2/7 = 47.5$

$\text{se}\bar{Q} = 6.9.$

The calculation of the 25, 100 and 1000 year floods proceed as in the earlier case and are tabulated in Table 2.73.

T	$Q/\bar{Q}$	$\widehat{Q(T)} = \bar{Q}.Q/\bar{Q}$	S_b Eqn (2.11.5.4)	Var($Q/\bar{Q}$) Eqn (2.11.5.6)	$\bar{Q}^2$var($Q/\bar{Q}$) (6)	$(Q/\bar{Q})^2$var$\bar{Q}$ (7)	var$\widehat{Q(T)}$ = (6)+(7)	se $\widehat{Q(T)}$	% of $\widehat{Q(T)}$
25	1.81	82.5	21.3	0.148	307	156	463	22	26
100	2.48	113.1	31.9	0.62	1289	292	1581	40	35
1000	4.38	199.7	49.7	4.74	9856	911	10 767	104	52

Table 2.73 Flood estimates from site with 7 years of record.

Mean annual flood = $\bar{Q}$ = 45.6, var$\bar{Q}$ = 47.5, se $\bar{Q}$ = 6.9.

Care has to be exercised when comparing the standard errors in Tables 2.72 and 2.73. The reduction of the standard errors by about a third of their values at T = 100 and 1000 somewhat flatters the seven years of record. This reduction must be viewed in the light of the reduced estimate of $\bar{Q}$ which is used in these calculations rather than a population value of μ.

c 15 years of record

The 15 years 1955–69 have annual maxima

49.60, 55.64, 77.28, 48.15, 50.75, 58.43, 47.35, 52.74, 39.45, 37.87, 56.85, 53.86, 65.80, 54.59, 53.50.

The mean of these is $\bar{Q}$ = 53.46 and the variance is 92.2 which implies a very low value of CV $= \sqrt{92.2}/53.46 = 9.60/53.46 = 0.18$. The variance of $\bar{Q}$, using the region CV of 0.40, is

$$\text{var}\,\bar{Q} = (0.40\,\bar{Q})^2/15 = 30.5$$

$$\text{se}\,\bar{Q} = 5.52.$$

The 25 year flood is estimated by fitting an extreme value Type 1 distribution to the 15 annual maxima by the method outlined in Section 1.3; the maximum likelihood estimates of the parameters (u,α) are (49.11, 8.11). The EV1 reduced variate of return period 25 is, from Table 1.12, $y_1 = 3.20$. Therefore,

$$\widehat{Q(25)} = \hat{u}+3.20\,\hat{\alpha} = 49.11+(3.20)(8.11) = 75.07.$$

The data are shown plotted against EV1 reduced variate in Figure 2.33(a) where the agreement between data and fitted distribution is seen to be very acceptable. The standard error of $\widehat{Q(25)}$ is given by Equation (2.11.5.2) with $D = \text{CV}.C = 0.40C$ and $C = 0.35+0.8 \ln 24.5 = 2.91$ from Equation (2.11.5.3). That is

$$\text{se}\,\widehat{Q(25)} = (0.40)2.91\,\bar{Q}/\sqrt{15} = (0.40)(2.91)(53.46)/3.872 = 16.1.$$

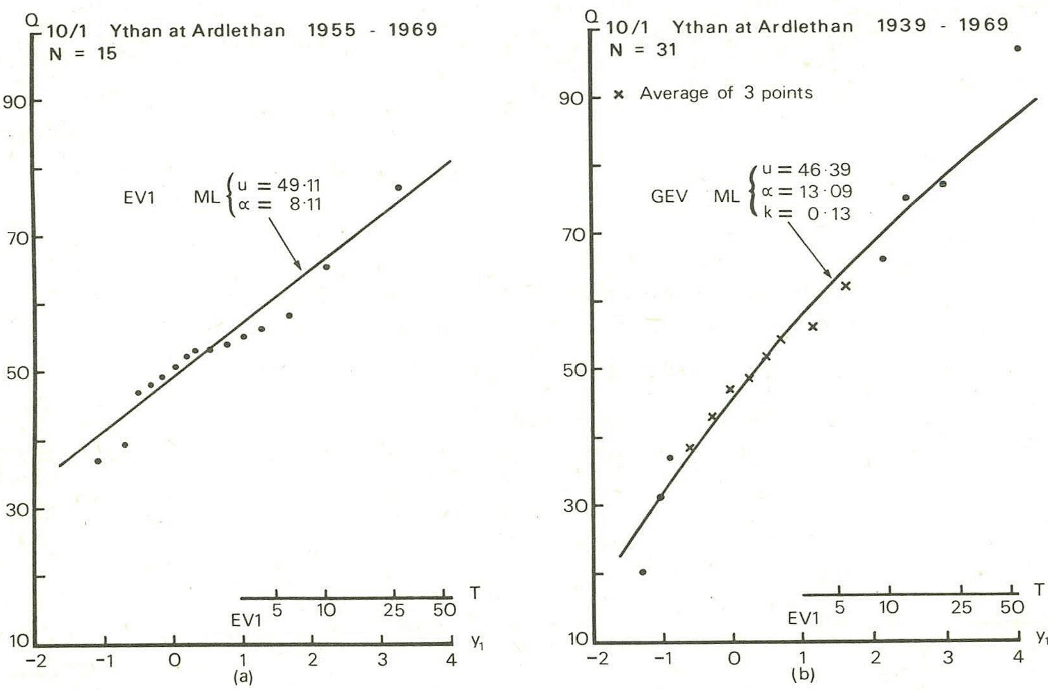


Fig 2.33 (a) Extreme value Type 1 distribution fitted to 15 years of data. (b) General extreme value distribution fitted to 31 years of data.

As before the 100 and 1000 year floods are estimated by using $Q/\bar{Q}$ from Region 1 and Great Britain curves respectively and the record mean; these and the standard error calculations are shown in tabular form in Table 2.74.

T	$Q/\bar{Q}$	$\widehat{Q(T)}$	S_b	var$(Q/\bar{Q})$	$\bar{Q}^2$var$(Q/\bar{Q})$	$(Q/\bar{Q})^2$var$\bar{Q}$	var$\widehat{Q(T)}$	se $\widehat{Q(T)}$	% of $\widehat{Q(T)}$
25	—	75.07	←		see above		→	16.1	22
100	2.48	132.6	31.9	0.62	1772	188	1960	44	33
1000	4.38	234.2	49.7	4.74	13 547	585	14 132	119	51

Table 2.74 Flood estimates from site with 15 years of record.

Mean annual flood $= \bar{Q} = 53.46$, var$\bar{Q} = 30.5$, se $\bar{Q} = 5.52$.

d 31 years of record

The 31 annual maxima from 1939–69 are

31.07, 48.55, 57.67, 39.62, 20.01, 46.56, 75.76, 56.21, 44.61, 47.88, 62.15, 42.08, 97.31, 43.33, 37.59, 66.86, 49.60, 55.64, 77.28, 48.15, 50.75, 58.43, 47.35, 52.74, 39.45, 37.87, 56.85, 53.86, 65.80, 54.59, 53.50

with mean $\bar{Q} = 52.23$ and variance = 215.1 which implies a cv value of 0.28 which is low by Region 1 standards. Using this, var $\bar{Q} = 215.1/31 = 6.94$ and se $\bar{Q} = 2.63$ but use of the Region 1 value of cv = 0.40 gives var $\bar{Q} = (0.40\bar{Q})^2/31 = 14.08$ and se $\bar{Q} = 3.75$. Experience with data suggests that the former are unrealistically small; therefore the latter will be adopted. According to the guidelines set out in Table 2.71 the three parameter general extreme value distribution may be fitted to this sample to estimate floods of return periods up to 60 years. The distribution fitted by maximum likelihood gives parameter estimates $(\hat{u},\hat{\alpha},\hat{k}) = (46.39, 13.09, +0.13)$. The 25 year flood is

$$\widehat{Q(25)} = u+\alpha W(y_1;k)$$

where $W(y_1;k)$ is obtained by interpolation in Table 1.12 or directly as $(1-e^{-ky_1})/k$ where $y_1 = 3.20$ when $T = 25$. The value of W is 2.62 and

$$\widehat{Q(25)} = 46.39+13.09(2.62) = 80.69.$$

The data are shown plotted on Figure 2.33(b) and it is seen that the fitted distribution describes the data well although the positive k value which is caused largely by the lower two points and gives a curve convex upwards would lead to underprediction at higher return periods. However, use of the rules in Table 2.71 would give protection as the fitted distribution would not have been used in that range. The standard error of $\widehat{Q(25)}$ is computed in exactly the same way as in *c* and is therefore

$$\text{se}\,\widehat{Q(25)} = (0.40)(2.91)(52.23)/\surd 31 = 10.92$$

the region value of cv = 0.40 being used because this sample coefficient of variation was judged to be unrealistically small for standard error computations.

The 100 and 1000 year events are computed as in *c* also except that $\bar{Q}$ is replaced by the new value. The estimate calculations are summarised in Table 2.75.

T	$Q/\bar{Q}$	$\widehat{Q(T)}$	S_b	var$(Q/\bar{Q})$	$\bar{Q}^2$var$(Q/\bar{Q})$	$(Q/\bar{Q})^2$var$\bar{Q}$	var$\widehat{Q(T)}$	se $\widehat{Q(T)}$	% of $\widehat{Q(T)}$
25	—	80.69	←		see above		→	11	14
100	2.48	129.5	31.9	0.62	1691	87	1778	42	32
1000	4.38	228.8	49.7	4.74	12 931	270	13 201	115	50

Table 2.75 Flood estimates from site with 31 years of record.

Mean annual flood = $\bar{Q}$ = 52.23, var $\bar{Q}$ = 14.08, se $\bar{Q}$ = 3.75.

Comparison of Table 2.72 with Tables 2.73, 2.74 and 2.75 shows that the introduction of the 7 years of record effects a greater relative increase in precision than does the increase from 7 to 15 years. This shows the importance of installing a gauge to collect even a short record while decisions about a project are being taken.

2.12 References

BENSON M.A. (1968) Uniform flood frequency estimating methods for Federal Agencies. *Water Resources Research*, **4**, 891–908.

BERNIER J. (1967) Sur la théorie du renouvellement et son application en hydrologie. *Electricité de France*, HYD67, **10.**

BLOM G. (1958) *Statistical estimates and transformed Beta variables*, pp 68–75 and pp 143–146. John Wiley, New York.

BORGMAN L.E. (1963) Risk criteria. *Journal of the Waterways and Harbours Division, Proceedings of the American Society of Civil Engineers*, **89,** 1–35.

CARTWRIGHT D.E. & LONGUET-HIGGINS M.S. (1956) The statistical distribution of the maxima of a random function. *Proceedings of the Royal Society, A*, **237,** 212–232.

CHAPMAN E.J.K. & BUCHANAN R.W. (1966) *Frequency of floods of 'Normal maximum intensity' in upland areas of Great Britain.* Proceedings of Symposium on River Flood Hydrology, Institution of Civil Engineers, London, 65–86.

CHOW V.T. (1954) The log-probability law and its engineering applications. *Proceedings of the American Society of Civil Engineers*, **80,** 1–25.

COURT A. (1952) Some new statistical techniques in geophysics. *Advances in Geophysics*, Ed. by H. E. Landsberg, Vol. 1. Academic Press, New York.

DALRYMPLE T. (1960) *Flood frequency analysis.* United States, Geological Survey Water-Supply Paper 1543 A, U.S. Government Printing Office, Washington, 1–51.

DAVENPORT A.G. (1964) *Note on the distribution of the largest value of a random function with application to gust loading.* Institution of Civil Engineers, Paper No. 1739, 187–196.

FISHER R.A. & TIPPETT L.H.C. (1928) Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Proceedings of the Cambridge Philosophical Society*, **24,** 180–190.

FISHER R.A. & YATES F. (1963) *Statistical tables for biological, agricultural and medical research*, p 47. Oliver & Boyd, Edinburgh.

FOSTER H.A. (1924) Theoretical frequency curves and their applications to engineering. *Transactions of the American Society of Civil Engineers*, **87,** 142–173.

GUMBEL E.J. (1935) Les valeurs extrêmes des distributions statistiques. *Annales de l'Institut Henri Poincaré*, **4,** 115.

GUMBEL E.J. (1937a) *La Durée Extreme de la Vie Humaine.* Hermann et Cie, Paris.

GUMBEL E.J. (1937b) Les intervalles extrêmes entre les emissions radioactives. *Journal de Physique* (series VII), VIII (No. 8), 321; (No. 11), 446.

GUMBEL E.J. (1941a) The return period of flood flows. *Annals of Mathematical Statistics*, **XII,** 163–190.

GUMBEL E.J. (1941b) Probability interpretation of the observed return period of floods. *Transactions of the American Geophysical Union*, Part III, 836–850.

GUMBEL E.J. (1942) Statistical control-curves for flood-discharges. *Transactions of the American Geophysical Union*, Part II, 489–500.

GUMBEL E.J. (1945) Floods estimated by probability methods. *Engineering News-Record*, **134,** 97.

GUMBEL E.J. (1958) *Statistics of Extremes*, pp 21–25. Columbia University Press, New York.

HALL W.A. & HOWELL D.T. (1963) Estimating flood probabilities within specific time intervals. *Journal of Hydrology*, **1,** 265–271.

HALL M.J. & O'CONNELL P.E. (1972) Time series analysis of mean daily river flows. *Water and Water Engineering*, **76,** 125–133.

HAZEN A. (1914) Storage to be provided in impounding reservoirs for municipal water supply. *Transactions of the American Society of Civil Engineers*, **78,** 1539–1641.

HAZEN A. (1930) *Flood flows*, pp 197. John Wiley, New York.

INSTITUTION OF CIVIL ENGINEERS (1933) *Report of the Committee on floods in relation to reservoir practice*, pp 39. London.

INTERNATIONAL COMPUTERS LIMITED (1970) *Scientific Subroutines 1900 Series, TP4096*, March.

JENKINSON A.F. (1969) Statistics of extremes. *Estimation of Maximum Floods*, Chap. 5. WMO Technical Note 98, 193–227.

KENDALL M.G. & STUART A. (1961) *The Advanced Theory of Statistics*, Vol. 1, pp 231–232, Vol. 2, pp 522–527. Charles Griffin and Co., London.

LANGBEIN W.B. (1949) Annual floods and the partial-duration flood series. *Transactions of the American Geophysical Union*, **30,** 879–881.

LEDGER D.C. (1972) The Warwickshire Avon. *Transactions of the Institute of British Geographers*, March, 83–109.

LEESE M.N. (1973) Use of censored data in the estimation of Gumbel distribution parameters for annual maximum flood series. *Water Resources Research*, **9,** 1534–1542.

LOCAL GOVERNMENT OR UNIT (1973) *The economics of flood alleviation.* Report No. C155, May 1973, Reading.

MATALAS N.C. & WALLIS J.R. (1972) *An approach to formulating strategies for flood frequency analysis.* Proceedings of the International Symposium on Uncertainties in Hydrologic and Water Resources Systems, Vol. 3, pp 940–961. University of Arizona, Tucson.

MEJIA J.M. (1971) *On the generation of multivariate sequences exhibiting the Hurst phenomenon and some flood frequency analysis.* Ph.D. Dissertation, Colorado State University.

MORAN P.A.P. (1957) The statistical treatment of flood flows. *Transactions of the American Geophysical Union*, **38,** 519–523.

PRASAD R. (1970) *Frequency analysis of hydrologic information.* ASCE Annual Conference of Hydraulics Division, at Minneapolis, Minnesota.

QUIMPO R.G. (1967) *Stochastic model of daily river flow sequences.* Hydrology Paper 18, p 30. Colorado State University, Fort Collins.

RICE S.O. (1954) Mathematical analysis of random noise. *Selected Papers on Noise in Stochastic Processes.* Dover, New York.

SHANE R.M. & LYNN W.R. (1964) Mathematical model for flood risk evaluation. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, HY6, **90,** November 1964, 1–20.

SIEGEL S. (1956) *Nonparametric Statistics for the Behavioural Sciences*, pp 312+xvii. McGraw-Hill Book Co., London.

SINGH K.P. & SINCLAIR R.A. (1972) Two distribution method for flood frequency analysis. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, HY1, **98,** 29–44.

SUKHATME P.V. (1937) Tests of significance for the χ^2 population with two degrees of freedom. *Annals of Eugenics*, **8,** 52–56.

SUKHATME P.V. (1938) On the distribution of χ^2 in samples of the Poisson series. *Journal of the Royal Statistical Society*, Suppl. **5,** 1.

THOM H.C.S. (1973) Distributions of extreme winds over oceans. *Journal of the Waterways, Harbours and Coastal Engineering Division, Proceedings of the American Society of Civil Engineers*, **99,** 1–17.

TODOROVIC P. & ROUSELLE J. (1971) Some problems of flood analysis. *Water Resources Research*, **7,** 1144–1150.

TODOROVIC P. & ZELENHASIC E. (1970) A stochastic model for flood analysis. *Water Resources Research*, **6,** 1641–1648.

U.S. WATER RESOURCES COUNCIL (1967) A uniform technique for determining flood flow frequencies. *Water Resources Council Bulletin No. 15*, December 1967.

WEATHERBURN C.E. (1962) *A First Course in Mathematical Statistics*, pp. 119–121. Cambridge University Press.

WEISS G. (1973) *Shot noise models for synthetic generation of multisite daily streamflow data.* IAHS Symposium on the Design of Water Resources Projects with Inadequate Data, Madrid.

YULE G.Y. & KENDALL M.G. (1950) *An Introduction to the Theory of Statistics*, pp 638–640. Charles Griffin & Co., London.

3 Methods of extension of short records

3.1 Introduction

Because river flows have only recently been measured on a large scale in the United Kingdom, the average flood record is at present very short. Full details of the numbers of gauging stations and their lengths of record are given in Volume IV; the average length of record used is only about 12 years, with most stations having less. Thus, the typical station used in this statistical analysis could be regarded as having only a decade of records, over the period 1959–69 including the major floods of 1960 and 1968.

This period is short for a reliable estimate of even the mean annual flood or its variability. Such estimates are subject to sampling error and in addition the mean itself is possibly subject to long term variations which would result in biassed estimates from certain periods. The sampling standard error of the mean annual flood, $\bar{Q}$, estimated from N years of record, expressed as a fraction of $\bar{Q}$, is $\text{cv}/\sqrt{N}$ where cv is the coefficient of variation. This is an underestimate if the bias referred to above is real. If one assumes that all the records are drawn from the same population it would be reasonable to substitute the average coefficient of variation. The mean value from 532 stations is 0.357, from which the sampling standard error of the mean annual flood estimated from a 10 year record might be expected to be about 11% of its mean.

The corresponding expression for the sampling error of the estimated coefficient of variation involves the third and fourth moments about the mean. The use of expressions for the moments of a double exponential distribution leads, with simplification, to an approximate estimate of this sampling error of 0.10, or 30% of the mean value of the coefficient of variation. The standard deviation of the observed 532 values of cv is 0.193.

In comparison with this it was found in Section 2.5 that the standard error of $\bar{Q}$ in individual decades of long records is 15% of the mean, while the standard error of the corresponding decade cv values was found to be 30% of their mean. Both these values are consistent with the deductions of the last two paragraphs.

When flood estimates are to be based on short records the sampling standard error can be reduced by extending the effective length of record. In this chapter techniques of data extension are developed or examined in order both to provide tools for this purpose, and also to provide more reliable estimates of flood statistics for subsequent correlation with catchment characteristics.

What information is usually available which might be used to extend short term records? There are likely to be adjacent long term flow records related to the short term record; in some cases long term daily or continuous rainfall records; other flood records from sites in the area with comparable catchment characteristics; long term river level records or historic flood records.

Long term flow or level records, supplemented by daily rainfall records, can be used to extend short term flow records by correlation in the common period; long term continuous rainfall records could be used with a conceptual rainfall–runoff model to generate a longer flow series; flood estimates based on catchment characteristics can be combined with short records, while the flood records of an area can be combined in a regional growth curve of floods with return period. These methods have in common the joint use of all available information in estimates at a site.

Various ways in which this additional information can be used to supplement short term records are described in following sections. The extension of short term records by correlation with nearby long term records is described in Section 3.2. The theoretical criteria for improved estimates of the mean and variance are given in terms of correlation coefficients between short term and long term stations (3.2.1). It was found in a pilot study (3.2.2) that these criteria could be met by using long term flow records, supplemented by daily rainfall records, and that monthly maxima of both records were well correlated between stations. The data were therefore obtained for a large scale record extension programme, in which correlation maps played a useful role. This technique is described in detail, with an example in 3.2.8, because it provides a direct means of correcting a short term record at a site where an estimate is required. It was found that long term water level records could be incorporated in this method; the records of isolated historic floods could not be so used, but their use in single station analysis is described in Section 2.8.

The extension of annual maximum flood series has been the main subject of study but a means of extending the partial duration series by adjacent records has also been derived and is described in Section 3.3.

Where long term continuous rainfall records are available, the use of a rainfall–runoff model to extend records is possible; work on this approach has been limited to the development of unit hydrograph techniques from discrete rainfall–runoff events (Chapter 6), supplemented by the development of a conceptual model using similar data (Chapter 7). The use of a conceptual model with continuous data is briefly described in Section 3.4, although continuous rainfall records are unlikely to cover a longer period than flow records. There are not in fact more long term recording raingauges than flow records in this country, and few of these raingauges are sited in upland areas.

Information derived from regional flood records by correlation of flood statistics with catchment characteristics, or by unit hydrograph techniques and rainfall frequency curves, can be used to supplement short term flood records. Means of combining estimates derived from different sources are discussed in Section 3.5.

The use of neighbouring long term records to complement a short term record is frequently seen as a filling in or extension procedure by which a long record is artificially produced for the site in question. The actual values of the extended portion need not be physically written down; the statistical parameters of the record could be estimated by considering all the records both long and short as a sample from a multivariate distribution.

3.2 Extension of annual maximum series by correlation with long term records

3.2.1 Theory of extension

This section outlines some statistical results relating to record extension. The theory assumes a multivariate Normal distribution. Unfortunately, untransformed flow data are not so distributed, but it is commonly assumed that their logarithms are. A logarithmic transformation has been used here without checks as to whether it yields the correct distribution.

Suppose there are a short record of flows, y, of length n_1 years and p nearby longer records with n_2 additional years of data in each. Let these longer records be $x_1, x_2, \ldots x_p$. The array of data may be shown as follows:

$$
\begin{array}{llll}
y_1 \; y_2 \; \ldots y_{n_1} & & & \\
x_{1,1} \; x_{1,2} \ldots \; x_{1,n_1+1} & & & \ldots x_{1,n_1+n_2} \\
 & & & \\
x_{p,1} \; x_{p,2} \ldots \; x_{p,n_1+1} \; x_{p,n_1+2} & & & \ldots x_{p,n_1+n_2}.
\end{array}
$$

Assume that the flow series y can be related to the longer records (x) by the equation

$$y = \beta_0+\beta_1 x_1+ \ldots \beta_p x_p+\varepsilon, \tag{3.1}$$

where the β_i are regression coefficients and ε is a random error used to account for the fact that y cannot be completely predicted from the x's. The n_1 years of common period can be used to make estimates (b_i) of the regression coefficients (β_i) and R^2 the coefficient of determination of the regression. Using the estimated equation,

$$y = b_0+b_1 x_1+ \ldots b_p x_p, \tag{3.2}$$

one can estimate the n_2 missing y values and these new values can be used with the existing n_1 values of y to estimate the mean, variance or any other statistic of interest. A question which now arises is whether the addition of these extended y values improves the estimates of the required statistics. If the regression coefficients (β_i) were known exactly, then using the extension would always improve the estimates. However, the regression coefficients are estimated and these estimates (b_i) may have large errors because they were estimated from the short common period n_1. It is therefore possible that adding the extended y values could give a worse estimate; hence it is necessary to know the relative variance of the two estimates. They can be compared by means of the relative information, I, which is defined as the variance of the record estimate divided by the variance of the estimate based on the extended record. When this is greater than unity it is worth extending the series. Fiering (1963) gives an estimator for the mean when only one long record is used:

$$\bar{y} = \bar{y}(n_1)+\frac{bn_1}{n_1+n_2}(\bar{x}(n_2)-\bar{x}(n_1)), \tag{3.3}$$

where $\bar{x}(n_1)$ and $\bar{y}(n_1)$ are the means of x and y respectively in the common period of length n_1 and $\bar{x}(n_2)$ is the mean of the n_2 values of x in the extension period. This estimator has variance

$$\text{var}(\bar{y}) = \frac{\sigma_y^2}{n_1}\left[1-\left(\frac{n_2}{n_1+n_2}\right)\left(\frac{\rho^2(n_1-2)-1}{n_1-3}\right)\right], \tag{3.4}$$

where σ_y^2 is the variance of y while σ_y^2/n_1 is the variance of $\bar{y}(n_1)$. Thus, the relative information is given by

$$\frac{1}{I} = 1-\left(\frac{n_2}{n_1+n_2}\right)\left(\frac{\rho^2(n_1-2)-1}{n_1-3}\right), \tag{3.5}$$

and I is greater than unity whenever

$$\rho^2 > 1/(n_1-2). \tag{3.6}$$

This critical value for ρ^2 is well known, but another consequence of this formula which is important is the small loss of information when ρ^2 is less than the critical value. For example, to extend a 5 year record of annual maxima to 10 years, ρ should be greater than 0.577. However, the relative information is always greater than two thirds and for $\rho = 0.5$ has the value 0.94. These figures would suggest that where a large bias in the short record is suspected from other evidence it may be advisable to extend even if the correlation coefficient (ρ) is a little less than the critical value.

In estimating the standard deviation of the annual floods (which is also needed either directly or indirectly to estimate the flood of return period T years) a difficulty arises. The regression equation does not explain all the variance of the short record (y) but only a proportion, R^2, of that variance. Thus, if one uses the regression formula to fill out the missing values, the standard deviation that is obtained will be biassed downwards.

This bias can be removed by adding in the extra variance (represented by the random error term, ε, in the equation above) to the extended series. A simple way to do this is to add to each extended flood a random number chosen from a distribution with mean zero and the appropriate variance. This method is extremely crude but was at the start of this study the only available method. (Better methods have since been described in the literature and will be discussed below.) In order to estimate the sampling properties of this method a simulation was undertaken.

In the simulation, sets of synthetic records were generated and the short records extended. Estimates were then made for the mean, standard deviation and the flood of return period T years (Q_T) using the formula, see Equation (1.2.3.34),

$$Q_T = \bar{Q} + K_T\sigma. \tag{3.7}$$

By repeating this process many times the sampling variance of Q_T etc. could be determined. Only the case of one long record was considered. The computer program generated n_1 pairs of correlated Normally distributed random numbers to represent the common period of record and derived the regression equation relating y (the short record) to x (the long record). A further n_2 random numbers for the x series were then generated and the corresponding values of y were calculated from the regression equation. Correlated variables x and y were generated from an uncorrelated pair (y,z) by using

$$x = y + \lambda z \tag{3.8}$$

which gives a correlation coefficient

$$\rho(x,y) = \frac{1}{\sqrt{1+\lambda^2}}. \tag{3.9}$$

As a test of the method the estimation of the mean annual flood was simulated, as in this particular case no extra random numbers need be added to make up the variance. Figure 3.1 shows the results of extending a 10 year record by 2, 20 and 50 years; there is a fair agreement with Fiering's formula.

Figure 3.2 shows typical results from a further simulation run to estimate the standard deviation of the annual flood by extending a 5 year record. Because of the methods used to economise in generation of random numbers, the sampling errors show up as shifts of the complete curve rather than as scatter of individual points from the curve. Figure 3.3 shows combined results from several simulations for the same problem.

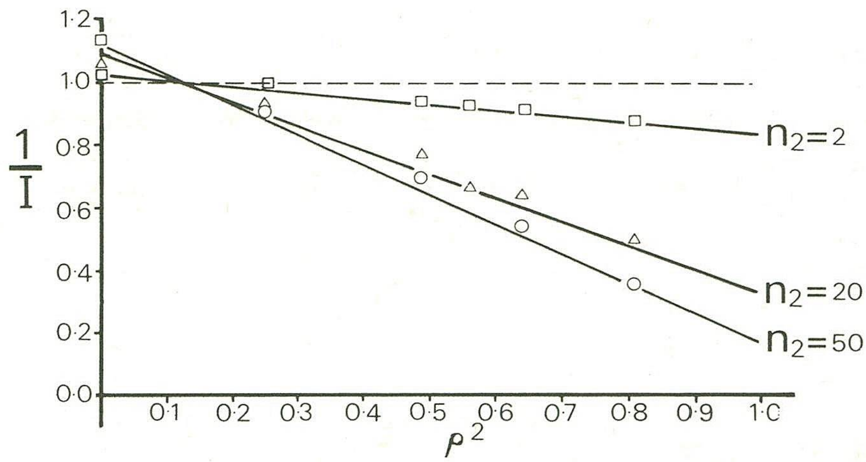


Fig 3.1 Information ratio (I) for estimating mean annual flood by extending a 10 year record by n_2 years. The lines show Fiering's formula, and the symbols the simulation results.

Here the relative information of estimating the standard deviation by extending a 5 year record for a further 5 years is plotted against the correlation coefficient. The curve drawn through the points by eye suggests that the correlation coefficient should be at least 0.84 before an extension should be made. The accuracy of this relationship can be judged from the scatter of the points. The correlation coefficient required to give equal information was estimated by fitting a polynomial in ρ^2 to the relative information data. The diagram in Figure 3.4 shows the results. The contours show the minimum correlation coefficient required before an extension can usefully be made. A certain amount of smoothing is incorporated in this diagram. It appears, from Figure 3.3, that the penalty for extending when correlation is slightly too small is not very severe.

Gilroy (1970) has formalised the idea of restoring the variance of the extended record and has calculated algebraically the effect of adding the random noise. As usual it is assumed that the flows are drawn from a multivariate Normal population, or that they can be transformed to conform with this distribution. A short record y of length n_1 years is to be extended by regression on p records x_i of length n_1+n_2. The formula

$$y_t' = b_0 + \sum_{i=1}^{p} b_i x_{ti} + \alpha\,\theta(1-R'^2)^{\frac{1}{2}} S_y(n_1) e_t \qquad (3.10)$$

is used to estimate the missing values. In this formula,

x_{ti} is the value of variate i at time t,
y_t' is the estimate for the missing value at time t,
b_0, b_i are the regression coefficients,
R' is the estimate of R, the multiple correlation coefficient, based on the n_1 common values,
$S_y(n_1)$ is the estimate of the standard deviation of y obtained from the n_1 recorded values,
e_t is a random value from a Normal distribution with mean zero and variance unity,
θ is 1 or 0 depending whether random noise is to be added or not,
α is used to remove bias in the estimator of the variance of y; an expression for α^2 involving n_1, n_2 and p is derived in the paper.

The n_1 recorded values and the n_2 extended values (y_t') are used to estimate the variance:

$$S_y^2 = \frac{1}{n_1+n_2-1}\left[\sum_{t=1}^{n_1}(y_t-\bar{y})^2 + \sum_{t=n_1+1}^{n_1+n_2}(y_t'-\bar{y})^2\right] \qquad (3.11)$$

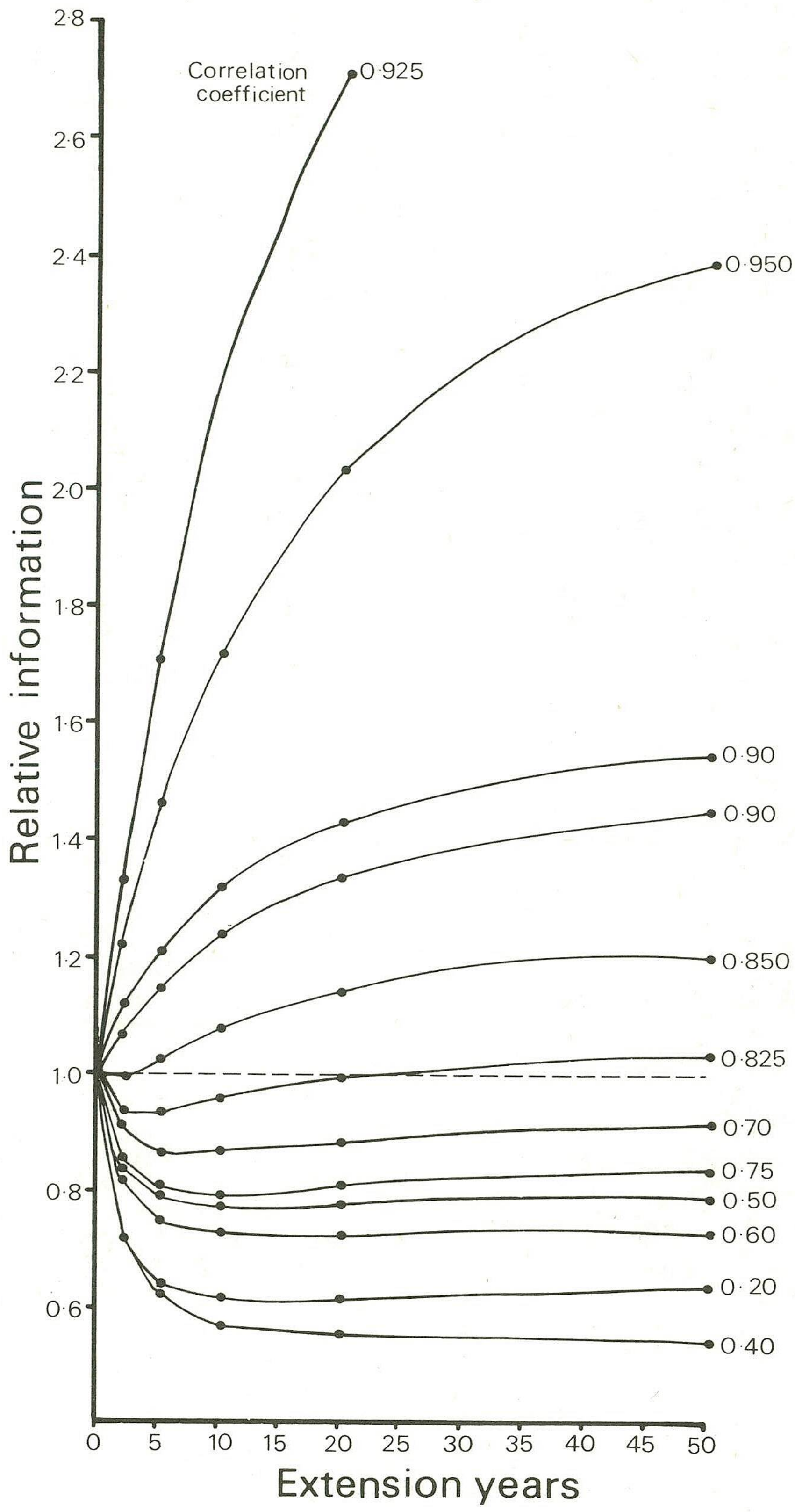


Fig 3.2 Relative information of estimates of the standard deviation of the annual flood made by extending a 5 year record.

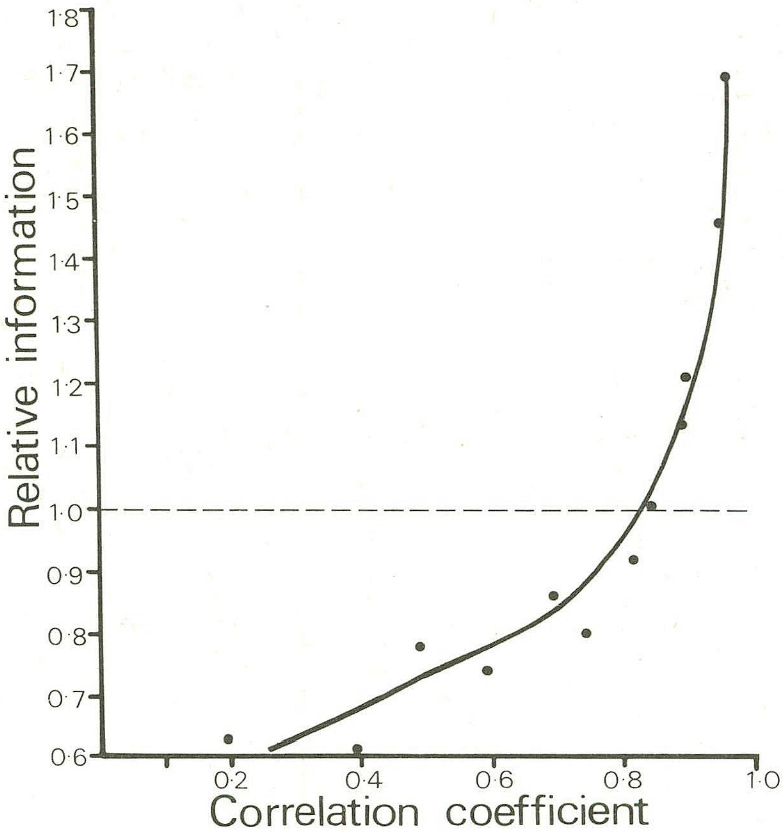


Fig 3.3 Relative information of estimates of the standard deviation made by extending a 5 year record by 5 years.

where $\bar{y}$ is estimated from the augmented series. Gilroy expands the estimator using the above formula for y' in terms of the random values e_t, and then substitutes expected values for the various sums and sums of products of e_t which occur. The variance of this estimator is derived in Gilroy's paper, to which the reader is referred for further details.

Moran (1974) adopts a different approach. He seeks a maximum likelihood estimator on the assumption of a multivariate Normal population. Once again, flood data would have to be transformed before this assumption would hold. He finds, in the case of one long record, that

$$S_y^2 = \hat{\beta}^2 S_x^2 + S_{y,x}^2 \tag{3.12}$$

where $\hat{\beta}$ is the estimated regression coefficient, S_x^2 is the variance of x calculated on all n_1+n_2 values and $S_{y,x}^2$ is the conditional variance of y given x, that is the variance about the regression line.

This estimator is biassed, but the bias can be calculated and hence allowed for. The unbiassed estimator of the variance y is

$$S_y^2 = \hat{\beta}^2 S_x^2 + \left[1 - \frac{(n_1+n_2-3)}{(n_1-3)\,(n_1+n_2-1)}\right] S_{y,x}^2, \tag{3.13}$$

with

$$S_x^2 = \frac{1}{(n_1+n_2-1)} \sum_{i=1}^{n_1+n_2} (x_i-\bar{x})^2, \tag{3.14}$$

and

$$S_{y.x}^2 = \frac{1}{n_1-2} \sum_{i=1}^{n_1} [y_i - \bar{y} - \hat{\beta}\,(x_i-\bar{x})]^2. \tag{3.15}$$

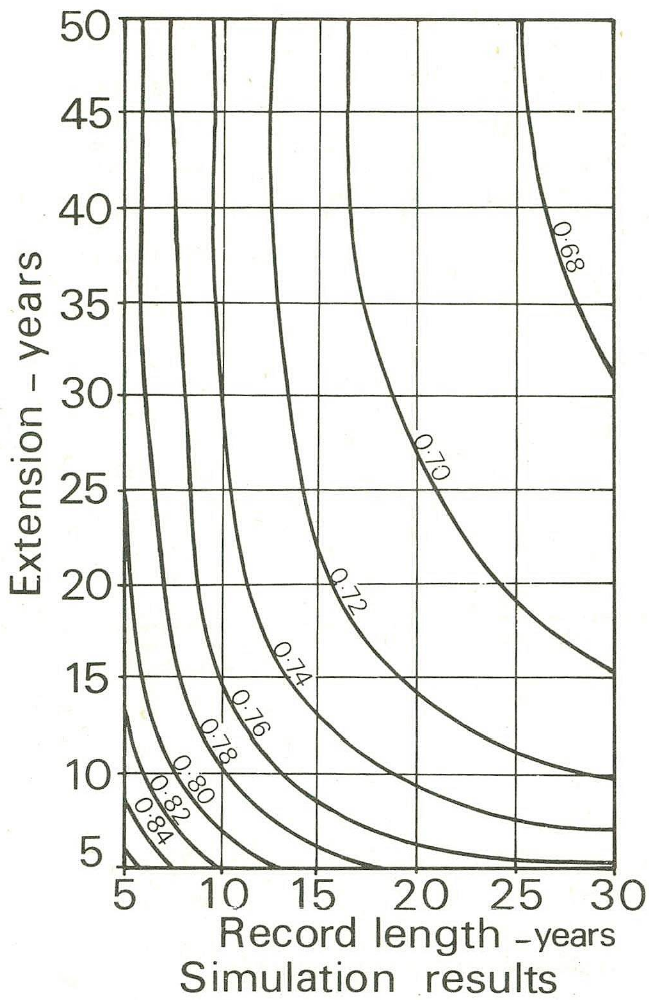


Fig 3.4 Minimum correlation coefficient for record extension to improve the estimate of the standard deviation.

This work can be extended to the case of *p* extending variables. The variance of this estimator has been derived and Figure 3.5 shows the value of *R* required to obtain a better variance estimator by use of concurrent data.

It is seen that higher values of *R* are required for estimation of variance from extended data than for estimation of the mean. In fact the lack of correlation between CV (the coefficient of variation of annual floods) and catchment characteristics found in a pilot study (Chapter 4), and the subsequent emphasis on regional curves (Chapter 2) based on the mean annual flood, led to a decision to concentrate on improving the estimates of the mean by extension.

3.2.2. Pilot study: choice of extending variables

At the same time as the statistical problems were being studied both theoretically and through a simulation study, a practical exercise in data extension was carried out to determine what records should be collected for large scale testing and use. In order to extend river flow records at a particular site it is customary to use records of flow at a number of adjacent gauging stations. However, there are other hydrological variables related to river flow which could be used, such as daily

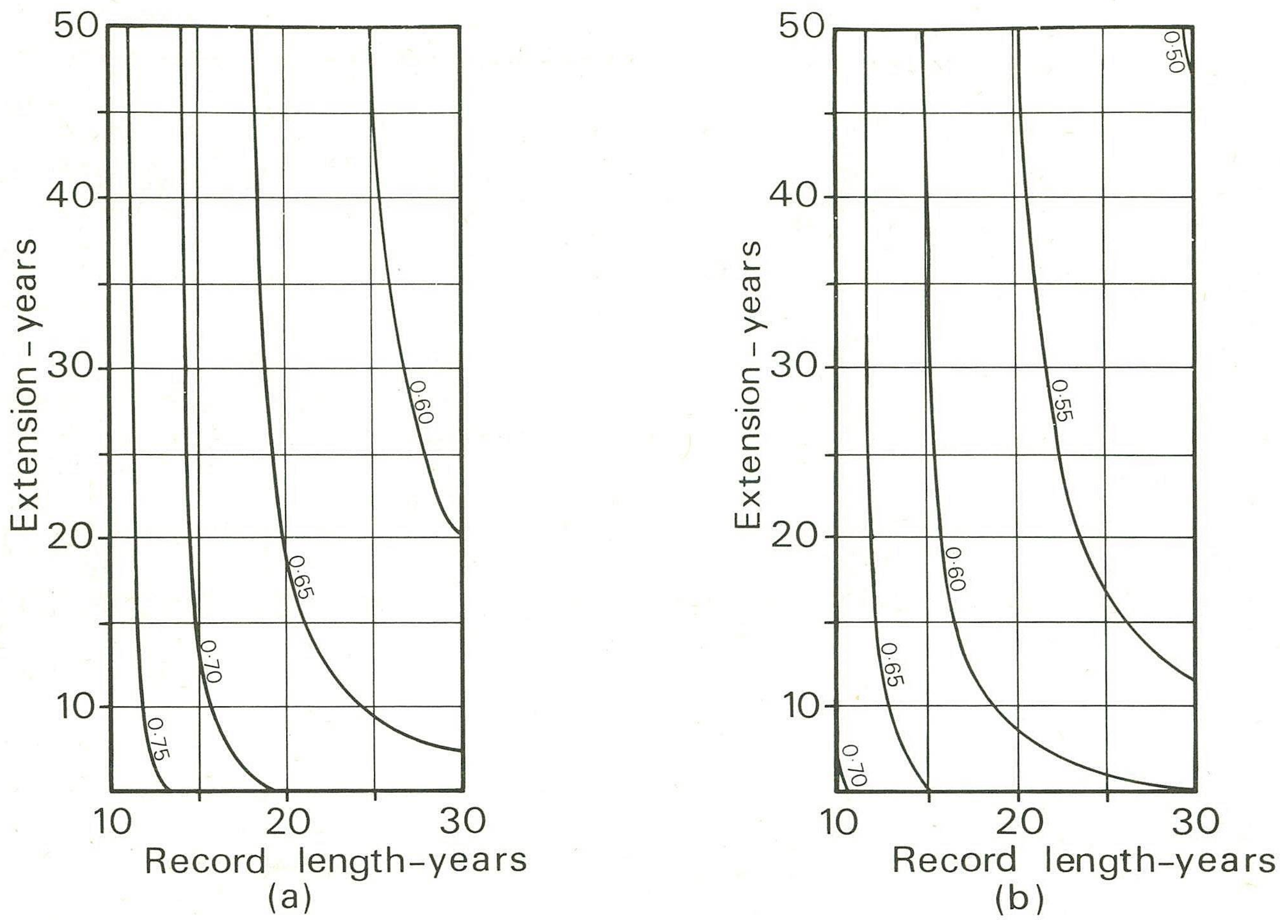


Fig 3.5 Minimum multiple correlation for extension to improve estimates of the variance using Moran's estimator. (a) Two extending records. (b) One extending record.

rainfall and soil moisture deficit. Having decided which variable to use the choice of time interval for comparison must be made. To extend flood records the annual maxima would appear the obvious choice for comparison between stations, but these are often poorly correlated and monthly maxima may be more useful. Relationships between data from the same hydrological event should be more stable and accurate than purely statistical ones between maxima. The shorter the sampling interval, the more frequently would maxima coincide in time at adjacent sites; thus monthly maxima might be better than annual maxima. A pilot study of record extension was made to see whether sufficiently high R^2 could be obtained in the regression and which combinations of variables would give the best R^2 values so that the necessary data could be assembled for a countrywide investigation of record extension. This pilot study was made before any data had been extracted from recorder charts and therefore mean daily flow data supplied by the Water Resources Board were used. The study was confined to a small area at the head of the Trent near Birmingham extending over the watershed into the Severn catchment. The river flow gauging stations used were as follows:

Number	Station	Grid reference
28/3	Tame at Water Orton	SP169914
28/4	Tame at Lea Marston	SP206935
28/5	Tame at Elford	SK173105
28/6	Trent at Great Haywood	SJ994231

28/8	Dove at Rocester Weir	SK112397
54/4	Sowe at Stoneleigh	SP332731
54/6	Stour at Kidderminster	SO828769
54/7	Arrow at Broom	SP087532

In addition monthly maximum daily rainfall totals were obtained from the Meteorological Office for the following stations:

Station	Grid reference
Birmingham (Edgbaston)	SP046286
Osbaston Hall	SK425045
Shobnall	SK234231
Stanley Moor	SK041712
Rainworth Pumping Station	SK587586
Waltham on the Wolds	SK804251
Milford Water Works	SO975212
Basford Water Works	SK565429

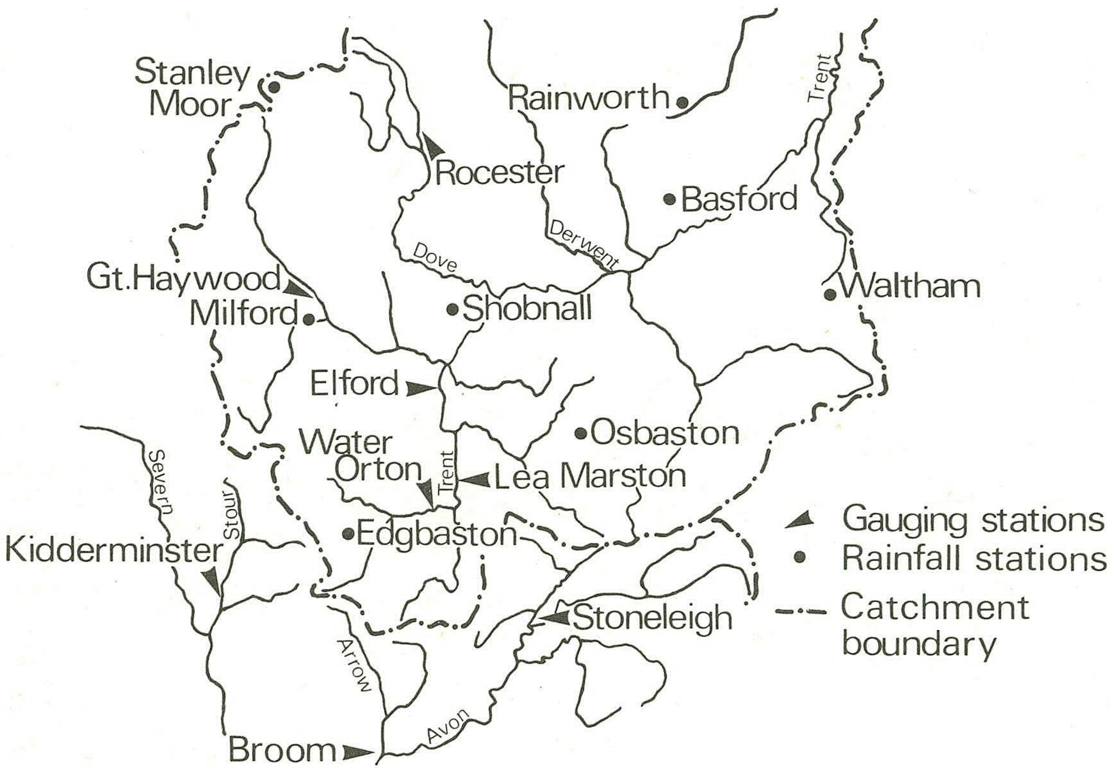


Fig 3.6 Location of stations used in pilot study for extension.

The location of these stations is shown in Figure 3.6. The correlation between annual maximum river flows was found to be too small to be useful and it was therefore decided to test monthly maxima. The first test was to extend the record of the Tame at Lea Marston. In order to ensure uniformity of conditions for comparison it was decided to use a regression of the Lea Marston maxima on the corresponding maxima of the best four variables. Three sets of variables were used; (i) river flows, (ii) mixed rainfall and river flow records, and (iii) rainfall records. The results were as follows:

Independent variables	R^2
(i) River flows only (four records)	0.883
(ii) Two river flow and two rainfall records	0.917
(iii) Rainfall only (four records)	0.582

In the first and third cases the coefficients of two of the records were not significantly different from zero (at the 5% level) and hence did not contribute to the extension but in the case of the mixed rainfall and river flow records all four records contributed.

It can be seen that river flow is the most important extending variable. Rainfall on its own is of little merit but it can provide a useful supplement to flow records.

The second test was intended to provide information on what might be called the successive extension method, where the extended record at a station is used as an independent variable in extending another station. In two trials the regressions were made both with and without missing variables being filled in. The stations extended were the Trent at Great Haywood and the Tame at Elford. Lea Marston was used as the incomplete station with missing variables being filled in from the best equation developed above. Once again four variables were used as independent variables with results as follows:

Station to be extended	Conditions	R^2
Trent at Great Haywood	Missing data filled in	0.850
	Observed data only	0.830
Tame at Elford	Missing data filled in	0.914
	Observed data only	0.948

It will be seen that the results are inconclusive. However, the successive extension method can be useful provided care is taken.

Flow records are often highly serially correlated and use of this is sometimes made in extension by using the flow in the previous period as a predicting variable. This technique was not found to be useful in the present case; monthly maxima were not sufficiently well serially correlated. In another test, soil moisture deficits estimated for the end of each month at Edgbaston were used but were not found to be useful in extending Lea Marston.

It was concluded that monthly maxima are better correlated than annual maxima and can be used to enable a more reliable extension to be made. Once the extended series of monthly maxima has been calculated, it is a simple matter to pick out the annual maxima. River flow records are best extended by using other river flows. It is therefore necessary to use whatever long records of river flow can be found. A number of long records of river level without reliable ratings exist on some rivers; it was decided that they were sufficiently similar to flow records to be collected and tested. Therefore, the raw material for record extension consisted of monthly maximum river flows and levels and daily rainfalls. Soil moisture data do not appear to be of use in extending flood records; this is probably because the deficits are zero in winter when floods are more common.

3.2.3 Preparation for the extension of monthly maxima

After assessing the statistical and practical problems, it was decided to use a multiple regression technique in a full scale investigation to extend wherever possible short records using adjacent long ones in order to improve the estimate of the mean annual flood. It was hoped by this means to reduce the sampling error associated with estimation from a short record and also to reduce the possible bias of the few recent years in which much of the recorded flow data was concentrated. By limiting the analysis to the mean the need to add a random deviate to estimate the variance was avoided.

It was felt that the use of monthly data represented a practical compromise between the use of daily data in a relationship between the same hydrological events and the use of annual maxima with only occasional coincidences. Nevertheless, in estimating the mean annual flood the criteria for extension were based on the correlation between recorded and estimated annual maxima and not with the higher correlations between the monthly maximum data used. All flow records of Grade A and B with 5 years of record or more were tried in the extension programme. Figure 3.7 shows the locations of stations whose monthly maxima were extracted; the distribution of stations is representative of the overall network. Data from some other stations which were not used in the statistical or catchment response analyses were also extracted. These included long term river gauges read once daily, records from sites below reservoirs and stations where gauging complications permitted only a daily mean discharge to be computed. These stations were

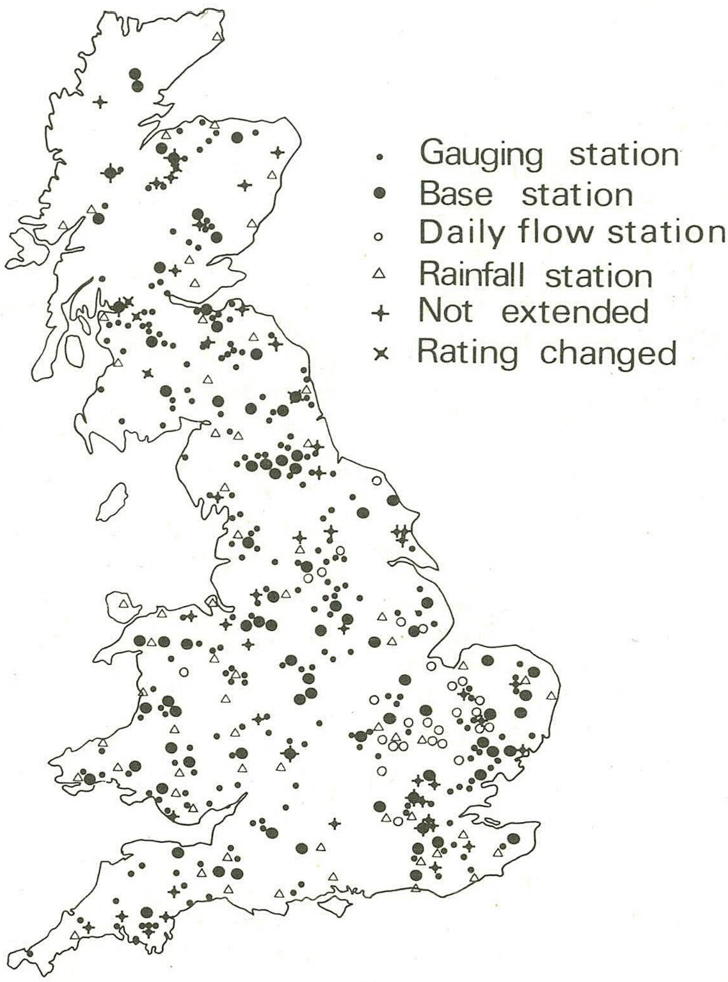


Fig 3.7 Location of stations used in data extension.

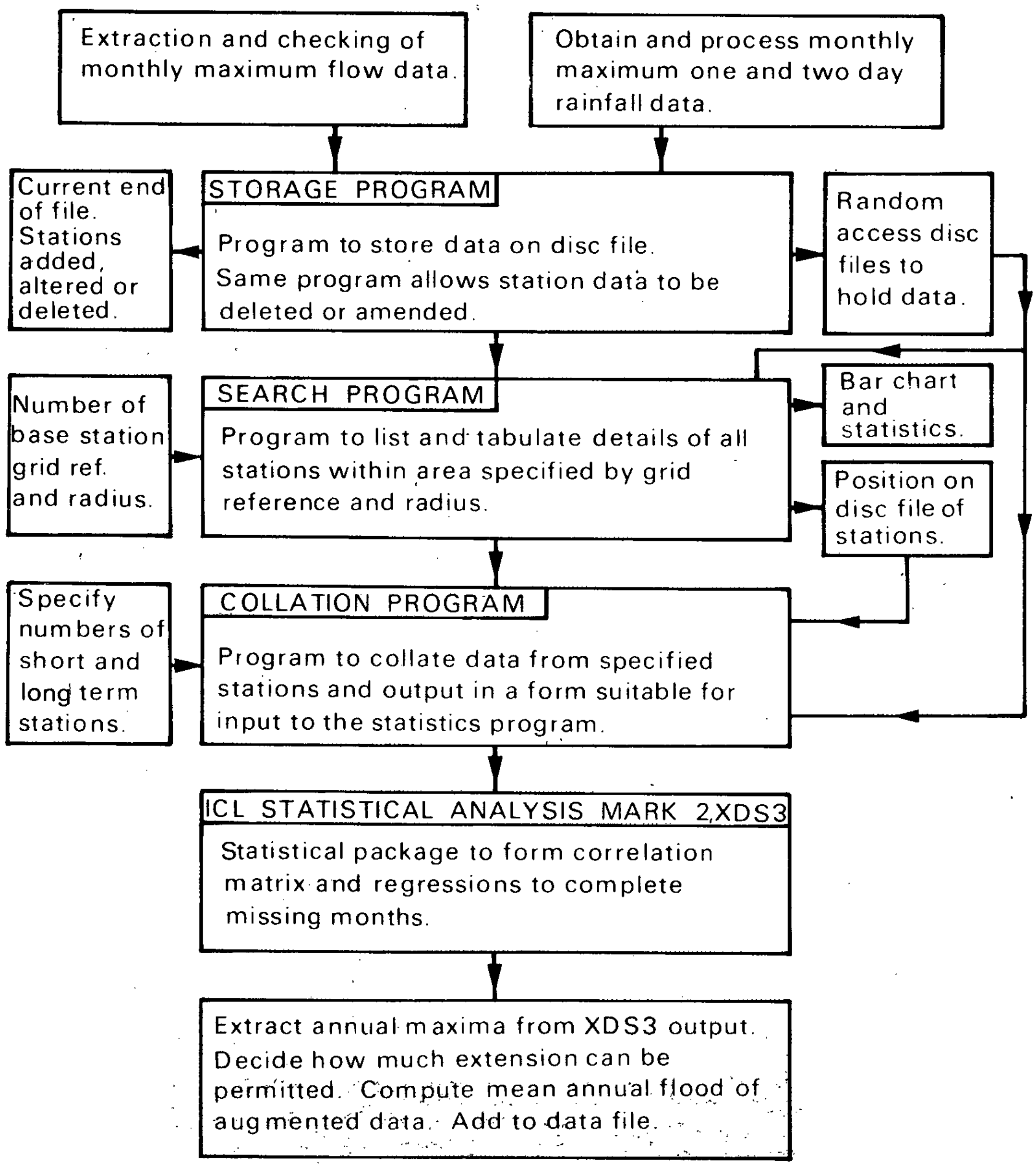
included in the hope that they could be used to extend the record of adjacent conventional stations. Stations in this category are also shown on Figure 3.7. Their greater concentration in the east is due to the flatter gradients and consequent regulation of eastern streams making for gauging complications. Some of these stations were maintained over long periods by navigation authorities.

Monthly maximum 1 and 2 day rainfalls were also included for possible use in extension. Data from MARAIN magnetic tape stations (Figure 3.7) were provided by the Meteorological Office for the period 1944–69.

The data extraction and checking processes have been fully described in Volume IV. The flow processing program provided a file retained within the computer system giving the monthly maximum discharges for each station in chronological order for use in the countrywide investigation.

The monthly maximum rainfall data were provided in the form of

Fig 3.8 Flow chart of selection and extension process.



paper tape from the Meteorological Office's KDF9 computer; the HRS computer centre provided a program to translate the code to one compatible with their ICL 1903A computer on which the data were processed.

3.2.4 Selection of stations for statistical analysis

Figure 3.8 illustrates the entire selection procedure which includes three computer programs and ends with a computer held file suitable for use with ICL's XDS3 statistics package (Statistical analysis Mark 2). The very large volume of data made it necessary to reduce the manual processing and selection. Batch processing was used at all stages. Even the eventual statistical analysis did not consist of individual regressions but used XDS3's missing value routine to extend a group of stations simultaneously.

The storage process

Batch processing required that the entire data set of 6880 station years of record from 628 flow and rainfall stations should be simultaneously accessible to the computer programs. A storage program therefore transferred the processed monthly data to a random access disc file. Two such files holding up to 120 000 separate numbers were required to hold the data set. The only visible output from this program was a list of the station numbers whose data had been added, deleted or amended and information on the amount of space left in the file.

The search process

This program was written to aid the preliminary selection of groups of stations for extension. It used the number of a base station and a grid reference and radius in kilometres (normally 100 km) to define a circular zone.

An example of the output of the program is given in Table 3.1, showing a bar chart illustrating record availability and also various statistics associated with each station within the circular zone. The statistics of the logarithms of the monthly maxima provide a comparison of their total period of record to the common period of each station with the base station, and enable anomalous records to be recognised. The final statistic is the correlation coefficient between each station and the base station calculated from their common period of record.

	STATION	BAR CHART (928 938 948 958 968 978)	OVERALL MEAN	OVERALL SDEV	OVERALL SKEW	OVERALL EXCESS	COMMON PERIOD MEAN	COMMON PERIOD SDEV	COMMON PERIOD CORRN	STATION NAME
0	27033	CCCC	2.2	1.1	-0.54	-0.3				SEA CUT AT SCARB
1	24002	CCCCCCCCCCC	1.5	1.0	-0.10	-0.7	1.9	0.8	0.56	GAUNLESS AT BISH
2	24004	CCCCCCCCCC	1.9	1.1	-1.10	0.4	2.0	1.0	0.48	BEDBURN BECK AT
3	24005	B7CCCCCCCCBCCCC	1.9	1.2	-0.41	-0.9	2.5	1.0	0.58	BROWNEY AT BURN
4	25001	CCCCCCCCCCCCC	4.8	1.1	-1.73	2.8	5.0	1.0	0.38	TEES AT BROKEN S
5	25004	1CCCCCCCCCCCC	1.9	0.8	-0.03	-0.5	2.3	0.7	0.60	SKERNE AT SOUTH
6	25005	4C CCCCCCCC	1.8	1.4	-0.61	-0.8	2.5	1.1	0.65	LEVEN AT LEVEN B
7	26001	7CCCCCCCCCCCCCCCCC	0.9	0.7	-0.08	-0.5	1.3	0.6	0.40	WEST BECK AT WAN
8	26002	3CCCCCCCC	1.7	0.7	-0.06	-1.0	2.1	0.5	0.66	HULL AT HEMPHOLM
9	26003	6CCCCCCCCCA	-0.3	0.6	0.05	-0.5	0.0	0.5	0.39	FOSTON BECK AT F
10	26801	7CCCC	-1.2	1.6	-1.95	-0.5	-1.0	1.4	0.79	CATCHWATER AT EL
11	27002	3BACCCCCCCBAC.CBCCCCCCCCCCCCCCCCCCC	4.3	0.9	-0.94	1.1	4.7	0.8	0.40	WHARFE AT FLINT
12	27008	BCCCCCCCCCCCCC	4.1	0.8	-0.42	-0.4	4.5	0.7	0.63	SWALE AT LECKBY
13	27010	4CCCCCCCBCB98.CCC.CBCCCCCBCCCCCCCC3	0.6	1.2	-0.91	-0.2	1.0	0.9	0.49	HODGE BECK AT BR
14	27014	7CCCCCCCCCCB	3.2	1.1	-0.50	-0.6	3.7	0.8	0.64	RYE AT LITTLE HA
15	27021	5CCCCCCCCCC	3.8	0.8	0.08	-0.5	4.1	0.7	0.77	DON AT DONCASTER
16	27024	4CCCCCCCCC	4.4	1.0	-0.95	1.2	4.6	0.9	0.33	SWALE AT RICHMON
17	27028	9CCCCCCCC	4.0	0.8	-0.52	0.1	4.3	0.7	0.53	AIRE AT ARMLEY
18	27810	5CCCCCCCCCCCCC	4.0	1.0	-0.33	-0.3	4.4	0.9	0.59	CALDER AT NEWLAN
19	27811	3CCCCB	5.1	0.9	-0.13	-0.8	5.3	0.8	0.63	AIRE AT BROTHERT
20	27846	4CCC9	4.9	0.8	-0.23	-0.7	5.0	0.7	0.64	AIRE AT ASH BRID
21	29001	1CCCCCACCC	-0.6	0.8	0.12	-0.7	-0.3	0.8	0.69	WAITHE BECK AT B
22	27007	CCCCCCCCCCCCCC	4.4	1.1	-3.00	8.7	4.7	0.8	0.39	URE AT WESTWICK
23	24724	9CCCCCCCCCCCCCCCCCCCCCCCCCC3	2.5	0.6	-0.16	0.2	2.7	0.5	0.08	ONE DAY
24	24724	9CCCCCCCCCCCCCCCCCCCCCCCCCC3	2.8	0.7	-0.22	0.5	3.0	0.5	0.11	TWO DAY
25	427802	3CCCCCCCCC9CCCBCCBC	2.7	1.3	0.22	-0.4	3.2	1.3	0.79	ESK AT SLEIGHTS
26	27001	4CCCCCBCCCCBCC.CCC.CCCCCCCCCCCCCCCCC	3.2	1.1	-0.33	-0.9	3.9	1.0	0.50	NIDD AT HUNSINGO
27	24001	CCCCCCCCCCCC	4.0	1.1	-1.07	0.2	4.2	1.0	0.55	WEAR AT SUNDERLA

Mean, standard deviation, skewness and excess kurtosis calculated from logarithmic data.
Bar chart symbols;
C, complete year;
B, 11 months;
A, 10 months;
9, 9 months etc.
Year number is start of water year, e.g. October 1928–September 1929.

Table 3.1 Bar chart and statistics.

Correlation maps

The value of flow stations for extension and the choice of a single batch for extension were primarily judged from their correlation coefficients. These were plotted on a location map and contours were drawn of equal correlation coefficient with each base station. Examples are shown in Figures 3.9(a)–(d). Distance from the base station is clearly the main influence on the contours but the rate of decay with distance varied markedly in different parts of the country. Topographical and geological factors also influenced the shape and size of contours. The coefficients usually fell more steeply inland than along the coast (compare Figure 3.9(a) and (b)). Over much of the Midlands and eastern England the 0.8 correlation contour was 50 km or more from the base station, while in some areas no adjacent station achieved a 0.8 correlation (Figure 3.9(d)). The contours were regular near the base station but became less regular as a background correlation of 0.4 or 0.3 was approached towards the edge of a region. This background level was presumably due to shared seasonal influences. Contours seldom were difficult to interpolate or required much smoothing.

Correlation of the base station with rainfall stations showed a similar pattern but the general correlation level was much lower; the maximum correlation was usually in the range 0.4–0.6. Exceptionally high correlations with rainfall were found in some small upland and also urban catchments; permeable catchments and the drier parts of the country showed the lowest correlations.

The collation process

This intermediate process was used to select the particular stations for inclusion as a batch in any one statistical analysis and also the potential period over which to extend. Each station of the search output table (Table 3.1) was placed in one of the following categories according to the evidence of the correlation coefficient maps.

Short term stations. The records of these stations which were to be extended as a batch with the base station were usually chosen from within the 0.8 correlation contour and from the immediate vicinity of the base station.

Long term stations. These stations were used to extend the short term stations and were usually chosen from within the 0.7 correlation contour unless they were particularly lengthy. The mean daily flow, reservoir and daily read gauges could be included in this category if their correlation and record length permitted. The best two rainfall variables were usually accepted whatever their correlation with the base station.

Rejected stations. In this group were stations too distant or too poorly correlated to be useful or with no common period of record with the included stations. Some apparently useful stations were rejected as long term stations if they duplicated others. Examples occur on the Wye, Nene and Mersey where several records were highly intercorrelated; more than two were seldom included in a single batch, the poorest being rejected.

The number of common months of record between the short term and long term stations was important because the analysis depended on regression equations based on that common period. To ensure stability

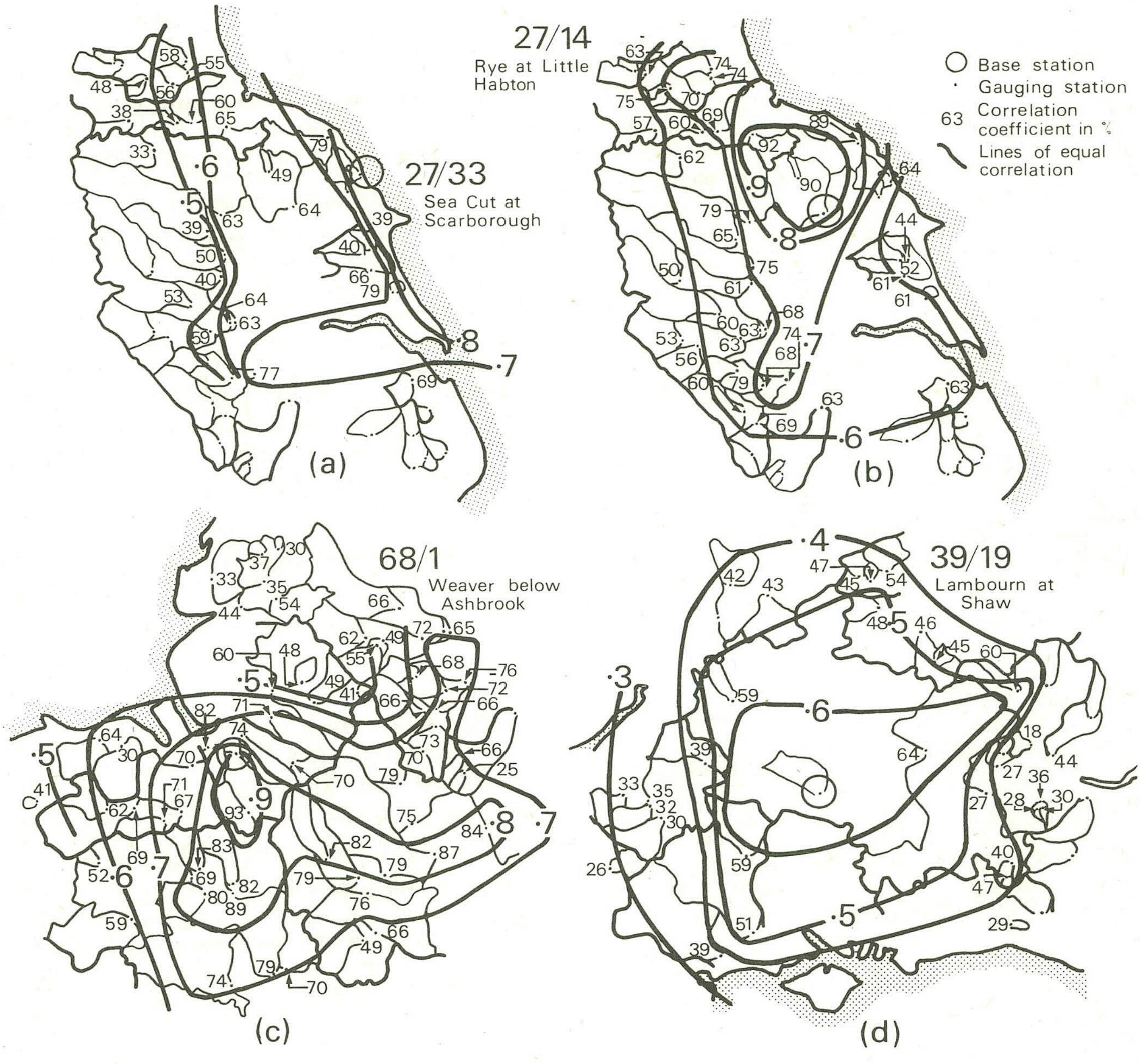


Fig 3.9 Examples of correlation maps.

at least 2 years, and usually 4 or 5 years, of common record were required. Stations with long missing periods of record required individual attention in order to satisfy this common record requirement. The extension usually started with the starting date of the longest record and finished in 1969. On average four short term stations were processed with each of the base stations shown on Figure 3.7. On average 10 long term stations were used and in some cases up to 20 were used in a run.

Regression and missing value program

The output from the collation program was ready for immediate input to ICL's statistics package XDS3. This permitted a number of statistical analyses including a missing value technique and regression analysis. The package was first used to calculate the cross product, covariance and correlation matrices from the common period of all the data and these formed the bases for subsequent analysis. The missing value technique, having identified a missing value shown by an asterisk '*' and also all variables recorded during that same month, derived a regression equation relating the missing variable to the variables present from the common period covariance matrix; this equation was then used to estimate the missing value. The missing value technique could in principle fit a different equation for every gap in the data but the equations were derived from the covariance matrix of the entire common period irrespective of month. This procedure was repeated until all missing values in the short term and long term stations were filled. A disadvantage was that no record was kept of the equations used or of the accuracy of their prediction, but the system enabled a very large number of regressions to be performed automatically.

As a guide to the overall precision of the method the optimum regression equation for the base station was printed with the significance of the regression coefficients and the multiple correlation coefficient; this equation was used to predict the base station maxima from the completed data set and these were compared with the actual records of the base station (Figure 3.11).

Choice of extended period

Once all the missing values had been estimated it was necessary to decide how much of the extension could be accepted for each short term variable of a batch. The criteria for acceptance were:

1 At least two stations should be used in estimating a missing value. The bar chart (Table 3.1) identified which stations were used.

2 At least one of the stations was a flow station. Rainfall records often provided useful second variables with significant partial correlation coefficients and with long periods of record.

3 The multiple correlation coefficient should exceed 0.85. As this information was not printed by the program this criterion required the use of a desk calculator.

In practice one or two long term stations predominated in each case and their record length determined the acceptable extension of the base station and other short term stations. The criteria for acceptable extension were not satisfied for about 48 stations. However, the extension of these stations would probably have been possible in an individual study. The final stage of the extension process was to extract the annual from the monthly maxima, to use these annual maxima to augment the recorded series and to recalculate the mean annual flood from the augmented series. Table 3.2 shows the recorded and extended mean annual flood of all stations used in the extension study.

Table 3.2 Results of data extension.

STATION NUMBER	STATION NAME	GRID REF	NUMBER OF YRS	MEAN ANNUAL FLOOD RECORDED	MEAN ANNUAL FLOOD EXTENDED	RECORDED PERIOD	CODE	EXTENDED PERIOD
3803	TIRRY AT RHIAN BRIDGE	NC553167	7	68.03	64.85	49-57;49,57	EP	51-68
3901	SHIN AT LAIRG	NC581062	6	62.71	42.56	49-56;49,56	EP	51-68
6903	MORISTON AT INVERMORISTON	NH416169	14	325.79	311.05	29-43;29	EP	29-42
6906	ALLT RHLARAIDH AT INVERMORISTON	NH377168	7	17.69	0.00	53-61;58,59	NW	
7001	FINDHORN AT SHENACHIE	NH828339	10	228.74	192.43	59-69;59	PO	51-69

Table 3.2—continued

STATION NUMBER	STATION NAME	GRID REF	NUMBER OF YRS	MEAN ANNUAL FLOOD RECORDED	EXTENDED	RECORDED PERIOD	CODE	EXTENDED PERIOD
7002	FINDHORN AT FORRES	NJ018583	12	513.76	452.93	58-69	EP	51-69
7003	LOSSIE AT SHERIFF MILLS	NJ198626	12	53.40	59.85	57-69:57	EP	44-69
8001	SPEY AT ABERLOUR	NJ278430	30	506.84	495.89	38-69:43,44	EP	38-69
8002	SPEY AT KINRARA	NH881382	10	144.64	0.00	51-69	NW	
8003	SPEY AT RUTHVEN BRIDGE	NN759996	10	108.11	0.00	51-69	NW	
8004	AVON AT DELNASHAUGH	NJ184352	18	275.15	261.47	51-69:51	EP	44-69
8005	SPEY AT BOAT OF GARTEN	NH946191	19	177.96	0.00	51-68	DE	
8006	SPEY AT BOAT O'BRIG	NJ318518	18	676.48	634.28	51-69:51	EP	44-69
8007	SPEY AT INVERTRUIM	NN688964	18	103.27	99.92	52-69	EP	51-69
8008	TROMIE AT TROMIE BRIDGE	NN788995	18	73.43	70.74	52-69	EP	51-69
8009	DULNAN AT BALNAAN BRIDGE	NH976247	17	107.09	0.00	52-69:69	NW	
8010	SPEY AT GRANTOWN	NJ034268	18	240.54	231.76	51-69:51	EP	44-69
9001	DEVERON AT AVOCHIE	NJ532464	10	151.25	158.80	59-69:59	EP	44-69
9002	DEVERON AT MUIRESK	NJ705498	10	292.41	311.30	59-69:59	EP	44-69
11001	YTHAN AT ARDLETHAN	NJ924308	31	52.23	0.00	39-69	NW	
12001	DEE AT CAIRNTON	NO632960	40	424.95	0.00	29-68	NW	
15001	ISLA AT FORTER	NO187647	22	50.19	0.00	47-68	NW	
15002	NEWTON AT NEWTON	NO230605	20	7.80	7.73	49-68	EP	47-68
15003	TAY AT CAPUTH	NO088395	18	775.66	789.02	51-69:51	EP	48-69
15004	INZION AT LOCH OF LINTRATHEN	NO280559	16	6.72	6.43	50-68:50,53,55	EP	49-68
15005	MELGAM AT LOCH OF LINTRATHEN	NO275558	38	15.24	14.84	26-66:27,53,54	EP	48-68
15006	TAY AT BALLATHIE	NO147367	17	997.04	997.52	52-69:52	EP	49-69
15007	TAY AT PITNACREE	NN924534	17	346.52	340.40	51-69:51,69	EP	48-68
15008	DEAN AT COOKSTON	NO340479	17	29.31	29.72	53-69	EP	50-69
15009	HUCKLE BURN AT EAST MILL	NO223604	17	8.33	8.13	49-68:50,53,54	EP	48-68
16001	EARN AT KINKELL BRIDGE	NN932166	20	189.00	0.00	48-69:48,54	NW	
16802	RUCHILL WATER AT CULTYBRAGGAN	NN764202	9	156.87	152.04	59-69:59,69	EP	48-68
18001	ALLAN WATER AT KINBUCK	NN792053	12	70.80	69.94	57-69:69	EP	48-68
18002	DEVON AT GLENOCHIL	NS858960	13	42.03	43.53	55-69:55,69	EP	48-68
18003	TEITH AT BRIDGE OF TEITH	NN725011	13	187.82	187.90	55-69:55,69	EP	48-68
19001	ALMOND AT CRAIGIE HALL	NT165752	14	123.50	132.70	55-69:55	EP	48-69
19002	ALMOND AT ALMOND WEIR	NT004652	0	15.84	15.02	60-69:60	EP	48-69
19003	BREICH WATER AT BREICH WEIR	NT014639	0	20.85	21.48	60-69:60	EP	48-69
19005	ALMOND AT ALMONDELL	NT086686	7	86.61	97.82	61-69:61,69	EP	48-69
19006	WATER OF LEITH AT MURRAYFIELD	NT228732	7	39.40	38.84	61-69:61,69	EP	48-68
19007	ESK AT MUSSELBURGH	NT339723	8	86.15	86.13	61-69:61	EP	48-69
19008	SOUTH ESK AT PRESTONHOLM	NT325623	7	20.31	31.51	63-69	EP	48-69
19802	NORTH ESK AT DALKEITH PALACE	NT333678	8	47.10	52.08	61-69:61	EP	48-69
20001	TYNE AT EAST LINTON	NT591768	10	60.25	0.00	59-68	NW	
20002	PEFFER WEST AT LUFFNESS	NT489811	4	3.37	4.21	65-68:65	EP	58-68
20003	TYNE AT SPILMERSFORD	NT456689	8	47.66	41.12	61-69:61	EP	59-69
21001	FRUID WATER AT FRUID	NT088228	15	18.95	19.65	47-61	EP	46-68
21003	TWEED AT PEEBLES	NT257400	23	259.17	0.00	39-69:40-47	NW	
21005	TWEED AT LYNE FORD	NT206397	8	137.37	162.04	60-68:60	EP	48-68
21006	TWEED AT BOLESIDE	NT498334	8	473.84	482.70	60-68:60	EP	48-68
21007	ETTRICK WATER AT LINDEAN	NT486315	8	230.02	226.65	61-68	EP	48-68
21008	TEVIOT AT ORMISTON MILL	NT702280	9	396.48	351.25	60-68	EP	48-68
21009	TWEED AT NORHAM	NT898477	11	888.60	944.34	59-68	EP	48-68
21010	TWEED AT DRYBURGH	NT588320	21	556.17	624.05	49-68	EP	48-68
21011	YARROW AT PHILLIPHAUGH	NT439277	7	79.00	79.25	61-68:61	EP	48-68
21012	TEVIOT AT HAWICK	NT522159	6	187.84	180.24	63-68	EP	48-68
21013	GALA WATER AT GALASHIELS	NT479374	6	48.28	60.74	63-68	EP	48-68
21031	TILL AT ETAL	NT927396	13	92.57	0.00	55-68:55	NW	
22001	COQUET AT MORWICK	NU234044	6	148.86	213.77	63-68	EP	55-68
22004	ALN AT HAWKHILL	NU213128	0	75.40	49.01	59-68:59	EP	55-68
22006	BLYTH AT HARTFORD	NZ245799	8	66.33	0.00	60-68:60	EP	60-68
22007	WANSBECK AT MITFORD FLUME	NZ181857	6	109.45	0.00	62-68:62	DE	
23002	DERWENT AT EDDYS BRIDGE	NZ041502	10	42.87	45.03	54-64:54	EP	56-68
23003	NORTH TYNE AT REAVERHILL	NY906732	10	486.94	489.09	59-68	EP	48-68
23004	SOUTH TYNE AT HAYDON BRIDGE	NY856647	10	390.97	409.56	59-68	EP	48-68
23902	NORTH TYNE AT BARRASFORD	NY924721	17	475.15	500.22	47-70:55,60,70	EP	47-69
24001	WEAR AT SUNDERLAND BRIDGE	NZ264376	12	195.40	200.58	57-68	EP	44-68
24002	GAUNLESS AT BISHOP AUCKLAND	NZ215306	18	22.41	21.39	50-68	EP	56-68
24003	WEAR AT STANHOPE	NY984391	11	139.53	126.70	58-68	EP	48-68:55-57
24004	BEDBURN BECK AT BEDBURN	NZ118322	10	25.60	25.18	59-68	EP	44-68
24005	BROWNEY AT BURN HALL	NZ259387	15	36.29	0.00	54-68	NW	
24006	ROOKHOPE BURN AT EASTGATE	NY953391	9	25.50	23.47	60-68	EP	55-68
25001	TEES AT BROKEN SCAR	NZ259137	13	419.16	397.43	56-68	EP	38-68
25002	TEES AT DENT BANK	NY932260	10	313.63	302.73	59-68	EP	57-68
25003	TROUT BECC AT MOOR HOUSE	NY759336	7	17.77	19.25	62-68	EP	59-68
25004	SKERNE AT SOUTH PARK	NZ284129	12	24.72	0.00	57-68	NW	
25005	LEVEN AT LEVEN BRIDGE	NZ445122	9	45.40	35.70	59-68:60	EP	36-68
25006	GRETA AT RUTHERFORD BRIDGE	NZ034122	9	78.86	74.91	60-68	EP	57-68
25008	TEES AT BARNARD CASTLE	NZ047166	5	322.28	259.03	64-68	EP	44-68
25808	BURNT WEIR AT MOOR HOUSE	NY752332	5	0.08	0.11	53-61:53,59,60,61	EP	53-68
25809	BOG WEIR AT MOOR HOUSE	NY773327	5	0.06	0.12	53-61:53,59,60,61	EP	53-68
25810	SYKE WEIR AT MOOR HOUSE	NY772333	3	0.10	0.10	53-61:55,59-61	EP	53-68
26001	WEST BECK AT WANSFORD BRIDGE	TA064560	16	6.76	0.00	53-68	NW	
26002	HULL AT HEMPHOLME LOCK	TA080498	20	12.73	0.00	49-68	NW	
26003	FOSTON BECK AT FOSTON MILL	TA093548	9	1.77	0.00	59-67	NW	
26801	CATCHWATER AT WITHERNWICK	TA171403	4	1.59	1.64	65-68	EP	61-68
27001	NIDD AT HUNSINGORE WEIR	SE428530	30	136.66	0.00	34-68	NW	
27002	WHARFE AT FLINT MILL WEIR	SE422473	32	264.81	260.47	36-68:39	EP	34-68
27006	DON AT HADFIELDS WEIR	SK389910	11	105.67	99.66	56-68:56,58	EP	44-68
27007	URE AT WESTWICK LOCK	SE356667	14	287.51	288.88	55-68	EP	34-68
27008	SWALE AT LECKBY GRANGE	SE415748	13	185.82	189.58	55-68:55	EP	28-68
27010	HODGE BECK AT BRANSDALE WEIR	SE627944	32	10.73	10.44	35-68:35,68	EP	34-68
27014	RYE AT LITTLE HABTON	SE743771	11	94.60	97.31	57-68:57	EP	28-68
27021	DON AT DONCASTER	SE569040	86	165.58	0.00	68-68	NW	
27022	DON AT ROTHERHAM	SK427928	8	147.49	126.45	60-68:66	EP	34-68
27023	DEARNE AT BARNSLEY	SE350073	16	23.77	24.40	60-68	EP	34-68
27024	SWALE AT RICHMOND	NZ146006	9	290.96	257.46	60-68	EP	38-68
27025	ROTHER AT WOODHOUSE MILL	SK432857	8	45.35	46.50	61-68	EP	44-68
27026	ROTHER AT WHITTINGTON	SK394744	9	37.97	35.80	59-68:59	EP	44-68
27027	WHARFE AT ILKLEY	SE112481	9	295.45	263.71	60-68	EP	36-68
27028	AIRE AT ARMLEY	SE281340	8	152.31	148.24	61-68	EP	34-68
27029	CALDER AT ELLAND	SE124219	16	176.65	152.69	53-68	EP	34-68
27030	DEARNE AT ADWICK	SE477020	5	44.86	31.23	63-68:63	EP	32-68
27031	COLNE AT COLNEBRIDGE	SE174199	5	102.82	150.61	64-68	EP	34-68
27033	SEA CUT AT SCARBOROUGH	TA028908	4	52.37	43.42	64-68:64	EP	60-68
27810	CALDER AT NEWLANDS	SE365220	12	230.51	220.79	57-68	EP	34-68
27811	AIRE AT BROTHERTON	SE495243	5	536.40	437.67	64-68	EP	34-68
27846	AIRE AT ASH BRIDGE	SE472266	5	391.28	343.67	63-68:63	EP	34-68
28003	TAME AT WATER ORTON	SP169914	14	70.93	71.97	55-68	EP	52-68
28004	TAME AT LEA MARSTON	SP206935	13	62.76	65.58	56-68	EP	53-68
28005	TAME AT ELFORD	SK173105	13	114.00	100.90	55-68:55	EP	40-68
28006	TRENT AT GREAT HAYWOOD	SJ994231	13	30.02	28.60	55-68:55	EP	44-68
28007	TRENT AT SHARDLOW	SK448299	14	270.92	266.55	55-68	EP	28-68
28009	TRENT AT COLWICK	SK620399	11	489.63	451.81	58-68	EP	28-68
28010	DERWENT AT LONGBRIDGE	SK357364	33	170.18	161.80	35-68:45	EP	28-68
28011	DERWENT AT MATLOCK	SK297586	11	114.28	105.87	57-68:57	EP	35-68
28012	TRENT AT YOXALL	SK131177	10	72.55	68.91	59-68	EP	44-68
28014	SOW AT MILFORD	SJ975215	10	31.39	27.28	59-68	EP	28-68
28015	IDLE AT MATTERSEY	SK690895	8	13.81	15.06	61-68	EP	38-68
28019	TRENT AT DRAKELOW PARK	SK239204	6	157.82	176.57	59-68:59-61,68	EP	28-68
28032	MEDEN AT CHURCH WARSOP	SK558680	5	5.88	4.34	64-68	EP	45-68
28801	BURBAGE BROOK AT BURBAGE	SK259804	43	5.39	0.00	25-68:25	NW	
28802	MAUN AT MANSFIELD	SK548621	5	9.26	0.00	63-68:63	NW	
29001	WAITHE BECK AT BRIGSLEY	TA253016	9	1.98	1.52	60-68	EP	38-68
29003	LUD AT LOUTH	TF336876	4	4.72	2.93	65-69:65	EP	59-69
30002	BARLINGS EAU AT LANGWORTH BRIDGE	TF066766	9	21.14	21.84	60-68	EP	51-68
30003	BAIN AT FULSBY LOCK	TF241611	7	16.64	16.55	62-68	EP	38-68
30004	PARTNEY LYMN AT PARTNEY MILL	TF402676	6	6.89	5.75	62-68:68	EP	38-68
30011	BAIN AT GOULCEBY BRIDGE	TF246794	3	6.61	5.52	65-68:65	EP	59-68
31002	GLEN AT KATES BRIDGE	TF106149	10	16.61	16.71	58-68:58	EP	39-68
31005	WELLAND AT TIXOVER	SP971998	7	35.14	33.56	62-68	EP	45-68
32002	WILLOW BROOK AT FOTHERINGHAY	TL067933	29	5.06	4.91	38-68:38,52	EP	32-68
32003	HARPERS BROOK AT OLD MILL BRIDGE	SP983799	28	6.92	6.53	38-68:38,48,65	EP	32-68
32007	BRAMPTON BRANCH AT ST ANDREWS MI	SP747617	29	16.54	16.40	40-68	EP	32-68
32008	WHILTON BRANCH AT DODFORD	SP627607	22	10.64	9.78	45-68:62,63	EP	32-68
32010	NENE AT WANSFORD	TL081996	30	70.53	66.91	39-68	EP	32-68
32801	FLORE EXPERIMENTAL CATCHMENT	SP660610	5	2.32	1.91	64-68	EP	45-68
33006	WISSEY AT NORTHWOLD	TL771965	13	7.36	8.02	55-68:55	EP	51-68
33009	GREAT OUSE AT HARROLD MILL	SP951565	18	98.06	90.87	51-68	EP	39-68
33011	LITTLE OUSE AT EUSTON COUNTY BRI	TL892801	8	2.56	2.70	60-68:60	EP	48-68
33012	KYM AT MEAGRE FARM	TL155631	9	20.56	30.77	60-68	EP	39-68
33013	SAPISTON AT RECTORY BRIDGE EUSTO	TL896791	9	5.87	5.20	60-68	EP	48-68
33015	OUZEL AT WILLEN WEIR	SP883409	7	16.73	16.75	61-68:61	EP	32-68
33018	TOVE AT CAPPENHAM	SP714488	7	18.72	22.85	62-68	EP	45-68
33019	THET AT MELFORD BRIDGE	TL880,30	8	7.00	0.00	60-68:61	NW	
33023	LEA BROOK AT BECK BRIDGE	TL662733	6	2.23	2.30	63-68	EP	48-68
33029	STRINGSIDE AT WHITE BRIDGE	TF716006	4	3.31	2.52	65-68	EP	39-68
33805	BEECHAMWELL BROOK AT BEECHAMWELL	TF738036	5	0.41	0.00	64-68	NW	
33809	BURY BROOK AT BURY WEIR	TL286837	6	8.82	8.41	63-68	EP	39-68
34001	YARE AT COLNEY	TG182082	11	10.86	10.43	57-68:57	EP	56-68

1 Number of years refer to record length available for calculation of AMAF.

2 Recorded and extended mean annual flood expressed in cubic metres per second (see note 7).

3 Dates before colon in recorded and extended period columns refer to the outside dates of the period.

4 All dates refer to starting year of a water year. For example 49 can be expanded to October 1949–September 1950.

5 Dates after the colon refer to years entirely or partially missing from which no annual maximum could be extracted.

6 Composite dates such as 61267 could be expanded to 61, 62, 66, 67.

7 Code column key: EP extended period; NW no extension worthwhile (0.00 quoted as extended mean annual flood in table); DE data error discovered at a late stage of the analysis.

Table 3.2—continued

STATION NUMBER	STATION NAME	GRID REF	NUMBER OF YRS	MEAN ANNUAL FLOOD RECORDED	MEAN ANNUAL FLOOD EXTENDED	RECORDED PERIOD	CODE	EXTENDED PERIOD
34002	TAS AT SHOTESHAM	TM226994	11	13.61	12.80	57-68;57	EP	56-68
34003	BURE AT INGWORTH	TG192296	10	6.02	4.77	59-68	EP	48-68
34004	WENSUM AT COSTESSEY MILL	TG177128	9	17.38	17.45	60-68	EP	56-68
34005	TUD AT COSTESSEY PARK	TG170113	8	2.82	0.00	61-68	NW	
34006	WAVENEY AT NEEDHAM MILL	TM229811	6	46.72	30.34	63-68	EP	49-68
34008	ANT AT HONING LOCK	TG331270	3	1.07	0.93	66-68	EP	59-68
34011	WENSUM AT FAKENHAM	TF919294	3	4.43	3.88	66-68	EP	39-68
35001	GIPPING AT CONSTANTINE WEIR	TM154441	8	20.72	21.93	61-68	EP	52-68
35003	ALDE AT FARNHAM	TM360601	8	7.20	7.30	61-68	EP	49-68
35008	GIPPING AT I.C.I. STOWMARKET	TM058578	5	14.96	10.59	63-68;63	EP	49-68
35801	BUCKLESHAM MILL RIVER AT NEWBOURN	TM270420	17	0.48	0.00	47-68;47,50-53	NW	
36002	GLEM AT GLEMSFORD	TL846472	6	10.57	8.12	62-68;62	EP	60-68
36003	BOX AT POLSTEAD	TL985378	6	3.20	2.37	63-68	EP	49-68
36006	STOUR AT LANGHAM	TM020344	6	35.93	26.11	63-68	EP	33-68
36007	BELCHAMP BROOK AT BARDFIELD BRIDGE	TL848421	5	3.82	2.70	64-68	EP	49-68
36008	STOUR AT WEST MILL	TL827463	8	23.56	18.44	60-68;60	EP	46-68
37001	RODING AT REDBRIDGE	TQ415884	19	23.68	22.40	49-68;49	EP	38-68
37003	TER AT CRABBS BRIDGE	TL786107	5	5.50	4.47	63-68;63	EP	51-68
37005	COLNE AT LEXDEN	TL962261	7	13.05	11.04	62-68	EP	51-68
37006	CAN AT BEACH'S MILL	TL690072	7	18.73	17.14	62-68	EP	33-68
37007	WID AT WRITTLE	TL686060	5	17.23	14.02	64-68	EP	33-68
37010	BLACKWATER AT APPLEFORD BRIDGE	TL845158	6	11.61	9.40	63-68	EP	49-68
37011	CHELMER AT CHURCHEND	TL629233	6	8.90	6.91	63-68	EP	33-68
37012	COLNE AT POOL STREET	TL771364	5	0.99	9.87	63-68;63	EP	33-68
37013	SANDON BROOK AT SANDON BRIDGE	TL755055	5	10.02	7.11	63-68;63	EP	33-68
37014	RODING AT HIGH ONGAR	TL561040	5	10.36	7.40	63-68;63	EP	33-68
38003	MIMRAM AT PANSHANGER	TL282132	16	1.81	0.00	52-68;52	NW	
38004	RIB AT WADESMILL	TL360174	10	15.76	11.95	59-68	EP	33-68
38007	CANONS BROOK AT ELIZABETH WAY HA	TL431103	19	6.27	0.00	50-68	NW	
38807	PYMMES BROOK AT EDMONTON	TQ340925	15	24.34	0.00	53-68;53	NW	
39019	LAMBOURN AT SHAW	SU470682	7	3.30	0.00	62-68	NW	
39020	COLN AT BIBURY	SP122062	6	3.51	3.88	63-68	EP	60-68
39023	WYE AT HEDSOR	SU896867	4	2.83	2.63	65-68	EP	62-68
39804	MOLE AT HORLEY MILL	TQ271434	7	22.64	24.32	61-68;61	EP	59-68
39813	MOLE AT IFIELD WEIR	TQ245365	10	4.73	4.59	58-68;58	EP	58-68
39820	DOLLIS BK AT HENDON LANE BRIDGE	TQ240895	14	7.36	0.00	51-68;51,54-56	NW	
39821	BRENT AT MONKS PARK	TQ202850	29	21.74	22.26	38-68;38,39	EP	39-68
39827	POOL AT SELWORTHY ROAD	TQ396722	6	6.24	0.00	61-68;63,64	NW	
39830	BECK AT RECTORY ROAD	TQ370697	7	2.51	0.00	62-69;69	NW	
39831	CHAFFINCH BROOK AT BECKENHAM	TQ359685	7	2.30	0.00	62-68	NW	
40004	ROTHER AT UDIAM	TQ773245	7	53.06	47.88	62-68	EP	38-68
40005	BEULT AT STILE BRIDGE	TQ758478	11	43.06	0.00	58-68	NW	
40006	BOURNE AT HADLOW	TQ632497	10	11.26	10.60	59-68	EP	58-68
40008	STOUR AT WYE	TR049470	7	23.86	19.61	60-68;62	EP	49-68
40009	TEISE AT STONE BRIDGE	TQ718399	8	34.02	33.76	61-68	EP	58-68
40011	STOUR AT HORTON	TR116554	5	19.71	16.31	64-68	EP	49-68
40012	DARENT AT HAWLEY	TQ551718	5	12.14	7.21	63-68;63	EP	58-68
40019	EDEN AT VEXOUR BRIDGE	TQ510455	8	48.65	62.19	61-68	EP	58-68
41005	OUSE AT GOLDBRIDGE	TQ428214	9	38.50	0.00	59-68;59	NW	
41006	UCK AT ISFIELD	TQ450189	5	36.07	31.12	64-68	EP	61-68
41811	CHESS ST AT CHESS BRIDGE	TQ216172	4	7.50	7.17	64-68;64	EP	58-68
43001	AVON AT RINGWOOD	SU142054	8	65.60	64.07	58-66;58	EP	57-68
43002	STOUR AT ENSBURY	SZ088965	9	127.02	132.00	59-68;59	EP	44-68
43005	AVON AT AMESBURY	SU151414	4	11.40	10.49	65-68	EP	40-68
45001	EXE AT THORVERTON	SS936016	13	190.67	0.00	55-68;55	NW	
45002	EXE AT STOODLEIGH	SS943178	9	163.15	146.49	59-68;59	EP	55-68
45003	CULM AT WOODMILL	ST021058	7	88.12	76.41	62-68	EP	55-68
45004	AXE AT WHITFORD	SY262953	4	140.46	106.07	64-68;64	EP	56-68
45005	OTTER AT DOTTON	SY087885	7	105.58	105.93	62-68	EP	56-68
45006	QUARME AT ENTERWELL	SS919356	5	11.41	8.80	64-68	EP	55-68
46002	TEIGN AT PRESTON	SX856746	13	204.37	0.00	55-68;55	NW	
46003	DART AT AUSTINS BRIDGE	SX751659	11	222.38	210.60	57-68;57	EP	56-68
46005	EAST DART AT BELLEVER BRIDGE	SX657775	5	50.95	58.68	64-68	EP	58-68
46801	ERME AT ERME INTAKE	SX640632	5	24.84	25.13	63-69;65,69	EP	60-68
46806	AVON AT AVON INTAKE	SX681641	17	26.39	0.00	39-56;56	NW	
47001	TAMAR AT GUNNISLAKE	SX426725	13	311.49	0.00	55-68;55	NW	
47004	LYNHER AT PILLATON MILL	SX368624	8	37.15	30.01	61-68	EP	56-68
47005	OTTERY AT WERRINGTON PARK	SX336866	8	50.57	52.93	60-68;60	EP	56-68
47007	YEALM AT PUSLINCH	SX574511	7	23.10	27.98	61-68;61	EP	58-68
48002	FOWEY AT RESTORMEL	SX108613	8	122.55	0.00	61-68	NW	
49001	CAMEL AT DENBY	SX017682	5	48.36	43.75	64-68	EP	56-68
49002	HAYLE AT ST ERTH	SW549342	10	7.55	6.98	57-68;66,67	EP	56-68
50001	TAW AT UMBERLEIGH	SS608237	11	277.85	255.01	58-68	EP	55-68
50002	TORRIDGE AT NEWBRIDGE	SS500185	8	248.91	242.04	59-68;59,61	EP	55-68
52004	ISLE AT ASHFORD MILL	ST361188	7	25.37	29.35	62-68	EP	44-68
52005	TONE AT BISHOPS HULL	ST206250	8	59.11	50.25	61-68	EP	55-68
52006	YEO AT PEN MILL	ST573162	7	44.38	47.85	62-68	EP	39-68
52009	SHEPPEY AT FENNY CASTLE	ST498439	5	7.53	6.43	64-68	EP	55-68
52010	BRUE AT LOVINGTON	ST590318	5	57.87	47.77	64-68	EP	39-68
53003	AVON AT BATH	ST752651	29	159.65	159.75	39-68;39	EP	39-68
53004	CHEW AT COMPTON DANDO	ST648647	11	45.48	26.59	57-68	EP	39-68
53005	MIDFORD BROOK AT MIDFORD	ST763611	8	31.22	27.70	61-68	EP	39-68
53007	FROME AT TELLISFORD	ST805564	8	64.02	66.79	60-68;60	EP	39-68
53008	AVON AT GT SOMERFORD	ST966832	5	56.20	34.00	63-68;63	EP	39-68
54001	SEVERN AT BEWDLEY	SO782762	46	397.10	0.00	23-69;69	NW	
54002	AVON AT EVESHAM	SP034431	32	155.95	0.00	37-68	NW	
54005	SEVERN AT MONTFORD	SJ413145	17	318.98	338.34	52-69;69	EP	30-68
54006	STOUR AT KIDDERMINSTER	SO828769	17	21.76	19.01	52-68	EP	37-68
54007	ARROW AT BROOM	SP087532	13	46.94	44.41	55-68;55	EP	37-68
54008	TEME AT TENBURY WELLS	SO598685	13	167.03	128.96	55-69;55,69	EP	29-68
54010	STOUR AT ALSCOT PARK	SP208507	10	30.01	33.38	58-69;58,69	EP	37-68
54011	SALWARPE AT HARFORD HILL	SO868619	11	24.03	17.18	57-68;57	EP	37-68
54012	TERN AT WALCOT	SJ592123	10	40.97	38.48	59-69;69	EP	37-68
54013	CLYWEDOG AT CRIBYNAU	SN944855	6	72.84	53.99	58-64;58	EP	30-68
54014	SEVERN AT ABERMULE	SO165958	8	341.75	284.33	59-69;59,67,69	EP	30-68
54016	RODEN AT RODINGTON	SJ589141	7	10.62	20.96	60-68;60,68	EP	37-68
54018	REA BROOK AT HOOKAGATE	SJ465092	6	24.83	28.32	62-68;68	EP	29-68
54019	AVON AT STARETON	SP333715	7	38.47	30.95	62-68	EP	37-68
54020	PERRY AT YEATON	SJ435193	6	10.62	9.36	63-69;69	EP	29-68
54022	SEVERN AT PLYNLIMON WEIR	SN850872	10	13.51	12.96	51-70;70	EP	28-69
55001	WYE AT CADORA	SO535090	32	539.49	519.28	36-68;36	EP	29-68
55002	WYE AT BELMONT	SO485388	58	440.59	441.68	08-68;09,19,31	EP	08-68
55003	LUGG AT LUGWARDINE	SO548405	29	50.23	50.04	39-68;39	EP	29-68
55004	IRFON AT ABERNANT	SN892460	31	57.75	65.57	37-68	EP	30-68
55007	WYE AT ERWOOD	SO076445	31	598.64	548.97	37-68;37	EP	30-68
55008	WYE AT CEFN BRWYN	SN829838	19	16.91	16.74	51-70;70	EP	44-69
55010	WYE AT PANT MAWR	SN843825	14	57.61	53.21	52-68;53,55,57	EP	44-68
56001	USK AT CHAIN BRIDGE	SO345056	12	444.98	365.59	56-68;56	EP	37-68
56002	EBBW AT RHIWDERYN	ST258888	12	102.38	87.09	37-69;69	EP	37-68
56003	HONDDU AT THE FORGE BRECON	SO051297	6	30.47	23.45	63-68	EP	37-68
56004	USK AT LLANDETTY	SO127203	3	417.19	341.15	65-68;65	EP	44-68
56006	USK AT TRALLONG	SN947295	6	190.29	162.31	63-68	EP	44-68
57003	TAFF AT TONGWYNLAIS	ST132818	10	359.97	285.16	60-70;70	EP	37-69
57004	CYNON AT ABERCYNON	ST079956	8	77.76	67.49	60-70;60,61,70	EP	37-69
58001	OGMORE AT BRIDGEND	SS904794	10	111.91	103.99	60-70;70	EP	58-69
58002	NEATH AT RESOLVEN	SN815017	10	210.24	215.62	60-70;70	EP	44-69
58003	EWENNY AT EWENNY PRIORY	SS914780	8	10.34	0.00	61-69;69	NW	
58004	AVAN AT CWMAVON	SS781919	8	96.39	67.84	61-70;61,70	EP	44-69
59001	TAWE AT YNYS TANGLWS	SS685998	12	203.63	196.35	56-68;56	EP	44-68
60001	TOWY AT TY CASTELL FARM	SN491204	11	369.63	361.32	57-68;57	EP	46-68
60002	COTHI AT FELIN MYNACHDY	SN506223	8	137.45	139.03	61-68	EP	38-68
60003	TAF AT CLOG Y FRAN	SN238160	5	62.83	53.46	64-68	EP	56-68
61001	WESTERN CLEDDAU AT PRENDERGAST MI	SM954177	8	42.73	44.44	61-68	EP	59-68
61002	EASTERN CLEDDAU AT CANASTON BRID	SN071152	9	76.84	70.74	59-68;59,66	EP	56-68
62001	TEIFI AT GLAN TEIFI	SN244416	10	192.94	177.59	59-68	EP	56-68
63001	YSTWYTH AT PONT LLOLWYN	SN591774	7	112.95	90.46	61-68;68	EP	49-68
63002	RHEIDOL AT LLANBADARN FAWR	SN601804	4	84.98	70.59	63-68;63,68	EP	60-68
64001	DOVEY AT DOVEY BRIDGE	SH745019	7	279.33	292.31	62-68	EP	52-68
65001	GLASLYN AT BEDDGELERT	SH592478	7	59.55	62.31	61-68;61	EP	52-68
66002	ELWY AT PANT YR ONEN	SJ021704	8	90.24	89.34	61-68	EP	30-68
66011	CONWAY AT CWM LLANERCH	SH802581	5	450.39	0.00	64-68	NW	
66801	CONWAY AT BLAEN Y COED	SH804452	6	14.87	18.33	50-57;50,57	EP	44-68
67002	DEE AT ERBISTOCK RECTORY	SJ358412	30	286.58	258.39	37-69;37,41,69	EP	30-68
67005	CEIRIOG AT BRYNKINALT WEIR	SJ295373	15	39.84	36.12	52-68;56,68	EP	44-68
67006	ALWEN AT DRUID	SJ042436	9	87.14	80.23	59-68;59	EP	30-68
67007	DEE AT GLYNDYFRDWY	SJ155428	5	277.12	204.20	64-68	EP	30-68
67803	DEE AT CHESTER WEIR	SJ418663	71	203.94	0.00	93-68;93-97	NW	
68001	WEAVER BELOW ASHBROOK	SJ670633	31	61.20	60.73	36-68;36,43	EP	37-68
68002	GOWY AT PICTON	SJ444714	20	16.05	17.61	49-68	EP	37-68
68003	DANE AT RUDHEATH	SJ668718	20	67.27	68.99	49-68	EP	36-68
68005	WEAVER AT AUDLEM	SJ652432	33	25.44	0.00	36-68	NW	
68006	DANE AT HULME WALFIELD	SJ844644	16	62.82	57.60	53-68	EP	36-68
68802	SANDERSONS BROOK AT SANDBACH	SJ753653	5	1.51	1.11	64-68	EP	36-68
69001	MERSEY AT IRLAM WEIR	SJ726937	35	174.00	0.00	34-68	NW	
69003	IRK AT SCOTLAND WEIR	SJ843992	14	39.66	43.45	49-68;50-53,57,62	EP	44-68
69006	BOLLIN AT DUNHAM MASSEY	SJ726876	33	41.27	40.62	36-68	EP	34-68

Table 3.2—continued

STATION NUMBER	STATION NAME	GRID REF	NUMBER OF YRS	MEAN ANNUAL FLOOD RECORDED	MEAN ANNUAL FLOOD EXTENDED	RECORDED PERIOD	CODE	EXTENDED PERIOD
69802	ETHEROW AT WOODHEAD	SK102998	23	14.68	14.71	36-68;36,423,5137,61267	EP	36-68
69804	TAME AT PORTWOOD	SJ906918	20	90.79	89.35	42-68;42,46-51	EP	34-68
71001	RIBBLE AT SAMLESBURY	SD589304	9	713.38	651.20	59-69;59,69	EP	36-68
71003	CROASDALE BECK AT CROASDALE FLUM	SD706346	12	14.88	13.51	56-68;56	EP	36-68
71004	CALDER AT WHALLEY	SD730360	6	172.83	176.63	61-68;61,65	EP	36-68
71005	BOTTOMS BECK AT BOTTOMS BECK FLU	SD745565	9	17.77	15.36	60-68	EP	36-68
71803	HODDER AT HIGHER HODDER BRIDGE	SD697411	9	361.14	368.76	60-68	EP	36-68
72001	LUNE AT HALTON	SD502647	10	671.72	656.58	59-68	EP	36-68
72002	WYRE AT ST MICHAELS ON WYRE	SD465411	7	151.97	152.35	61-68;61	EP	57-68
72807	WENNING AT HORNBY	SD586639	12	383.86	317.76	56-68;56	EP	36-68
73001	LEVEN AT NEWBY BRIDGE	SD371863	30	68.45	0.00	38-68;38	NW	
73002	CRAKE AT LOW NIBTHWAITE	SD294884	6	22.51	18.33	63-68	EP	44-68
73805	KENT AT KENDAL	SD516919	5	157.32	136.97	63-68;63	EP	44-68
75002	DERWENT AT CAMERTON	NY037305	9	185.94	169.89	60-68	EP	44-68
76002	EDEN AT WARWICK BRIDGE	NY471567	9	529.81	494.18	59-68;59	EP	44-68
76003	EAMONT AT UDFORD	NY575305	8	166.15	135.92	61-68	EP	44-68
76004	LOWTHER AT EAMONT BRIDGE	NY525285	7	155.80	137.50	61-68;61	EP	44-68
76005	EDEN AT TEMPLE SOWERBY	NY604282	5	303.91	255.93	64-68	EP	56-68
76011	COAL BURN AT COALBURN	NY693777	3	2.12	2.09	66-70;66,70	EP	48-69;55-57
77001	ESK AT NETHERBY	NY390718	8	791.44	711.91	61-68	EP	45-68
77011	ESK AT CANONBIE	NY397751	6	439.90	379.48	62-68;62	EP	45-68
78004	KINNEL AT REDHALL	NY078868	7	61.99	86.65	60-69;60,65,69	EP	46-68
79002	NITH AT FRIARS CARSE	NX923851	12	514.90	631.96	57-69;69	EP	46-68
79003	NITH AT HALL BRIDGE	NS684129	9	92.89	0.00	59-69;59,69	DE	
79004	SCAR AT CAPENOCK	NX843940	6	153.90	195.87	62-69;62,69	EP	46-68
79005	CLUDEN AT FIDDLERS FORD	NX928795	5	109.35	141.25	63-69;63,69	EP	46-68
80001	URR AT DALBEATTIE	NX822610	5	90.77	115.63	63-68;63	EP	46-68
81002	CREE AT NEWTON STEWART	NX412653	6	207.77	202.65	62-68;62	EP	46-68
82001	GIRVAN AT ROBSTONE	NX217997	6	88.28	87.63	63-69;69	EP	49-68
83002	GARNOCK AT DALRY	NS293488	10	56.82	53.07	59-69;59	EP	48-69
83802	IRVINE AT KILMARNOCK	NS430369	53	76.73	84.87	13-67;19,61	EP	53-68
84001	KELVIN AT KILLERMONT	NS558705	21	99.75	0.00	47-68;68	DE	
84002	CALDER WATER AT MUIRSHIEL	NS309638	13	19.84	18.71	51-64;51	EP	50-68
84003	CLYDE AT HAZELBANK	NS835452	13	307.22	367.69	55-68;68	EP	49-68
84004	CLYDE AT SILLS	NS927424	13	237.43	268.88	55-68;68	EP	49-68
84005	CLYDE AT BLAIRSTON	NS704579	14	407.74	489.38	55-68	EP	48-68
84006	KELVIN AT BRIDGEND	NS672749	12	13.97	14.88	56-68;68	EP	48-68
84011	GRYFFE AT CRAIGEND	NS414664	6	64.55	63.74	63-68	EP	48-68
84012	WHITE CART AT HAWKHEAD	NS499629	6	111.15	106.49	62-68;62	EP	48-68
84013	CLYDE AT DALDOWIE	NS672616	6	442.76	455.15	63-68	EP	48-68
84014	AVON AT FAIRHOLM	NS755518	5	212.84	200.23	63-68;63	EP	48-68
84015	KELVIN AT DRYFIELD	NS638739	15	35.93	58.02	46-68;46,50-56	EP	47-68
84803	NORTH CALDER AT CALDER PARK	NS681625	6	37.83	30.13	63-68	EP	54-68
84806	CLYDE AT CAMBUSNETHAN	NS786522	9	310.23	406.27	55-64;64	EP	48-68
85001	LEVEN AT LINNBRANE	NS394803	6	106.32	0.00	63-68	NW	
85002	ENDRICK WATER AT GAIDREW	NS485866	6	109.47	0.00	63-68	DE	
87801	ALLT UAINE AT INTAKE	NN263113	18	8.84	8.89	49-68;49,68	EP	50-68
90801	NEVIS AT ACHREOCH	NN167690	2	49.02	43.46	55-61;55689,61	EP	55-68
91901	MUCOMIR CUT AT GAIRLOCHY	NN179843	0	0.00	0.00	39-69;38,53,54	EP	38-69
95803	ABHAIN CUILEG AT BRAEMORE	NH193790	0	0.00	0.00	63-67;64,65,67	NW	

3.2.5 *Comparisons between recorded and extended flows*

It was hoped that extensions would improve the estimate of the mean annual flood by using long term regional information to reduce the sampling variance and also the bias resulting from sampling during the 1960s. Because the true values of the statistics are not known the improvement cannot be directly assessed. However, improvement may be revealed indirectly and the evidence is discussed in the following sections. After considering the effect of using monthly data to extend annual maxima the questions of sampling variance and bias are discussed separately.

Monthly and annual maximum correlation

The theoretical results of Section 3.2.1 express the information ratio in terms of the correlation coefficient and of recorded and extended sample size. These results are not directly applicable to the improved estimation of the annual maxima by comparison of monthly maxima. A scatter diagram between recorded and estimated monthly maxima (Figure 3.10) shows some tendency for the annual maxima, which are drawn from the higher monthly maxima, to cluster above the line. Table 3.3 shows the correlation between recorded annual maxima and estimates from the extended data set (Section 3.2.4). The data from all base stations with over 8 years of record were used in this table. The preponderance of slopes below one suggests a bias which probably springs from the use of an equation that is not entirely appropriate (Draper & Smith, 1968). Nevertheless, Table 3.4 shows that the procedure was successful in identifying the months in which the annual maxima occur. Scrutiny of the occasions when the correct month was not predicted showed that in general the monthly maxima were similar in size so that the error in the annual maximum would not be large.

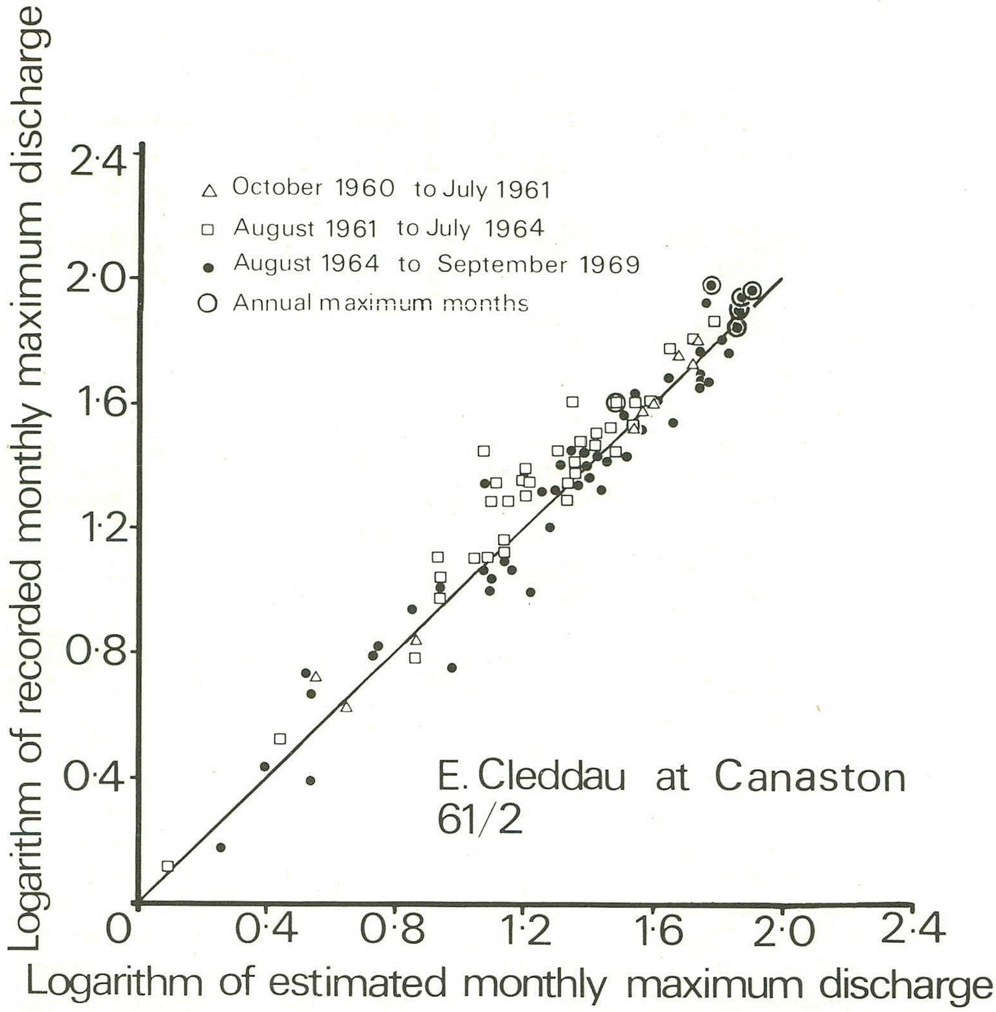


Fig 3.10 Prediction of monthly maximum discharge.

Improvement in sampling variance (direct evidence)

Table 3.3. shows that the correlation between recorded and estimated maxima usually exceeds the criterion (Section 3.2.1) for improved estimation of the mean. Figure 3.11 shows examples of the scatter diagrams of annual maxima; some poor correlations were observed, particularly in cases where no extension was thought worthwhile (Figure 3.11c). Because this information was only available for base stations with longer records it can only be used to test the judgement made on the basis of the correlation between monthly maxima. Figure 3.12 shows that the monthly and annual maximum correlation coefficients are not closely related. The ratio between the two is about 0.9 which suggests that the graphs and tables of Section 3.2.1 should be entered with a monthly maximum R decreased by a tenth.

Improvement in sampling variance (indirect evidence)

By conservative choice of extension criteria the theoretical requirements for information gain were satisfied but there is no way of testing whether

Table 3.3 Comparison between annual and monthly maximum correlation.

Station number	Coefficient of determination		Regression coefficients	
	Annual maxima	Monthly maxima	Intercept	Slope
6/903	0.591	0.582	0.261	0.966
7/1	0.908	0.931	0.246	0.861
8/9	0.917	0.876	0.501	0.745
9/1	0.905	0.931	0.899	0.585
15/1	0.799	0.821	0.406	0.791
15/8	0.727	0.776	0.478	0.693
16/802	0.796	0.945	−0.588	1.300
21/8	0.515	0.954	0.164	0.946
23/902	0.971	0.996	0.212	0.922
24/4	0.825	0.884	0.716	0.481
25/2	0.763	0.893	0.621	0.740
27/14	0.652	0.960	1.177	0.414
27/25	0.859	0.897	0.679	0.538
28/3	0.760	0.684	1.008	0.458
28/4	0.878	0.893	0.555	0.679
28/5	0.897	0.794	−1.430	1.751
28/6	0.942	0.901	−0.102	1.107
28/11	0.916	0.939	0.235	0.894
30/1	0.861	0.974	0.074	0.930
31/2	0.948	0.776	−0.817	1.701
32/3	0.923	0.880	0.083	0.795
33/19	0.807	0.548	0.224	0.705
34/2	0.936	0.801	−0.340	1.527
35/1	0.861	0.947	0.526	0.604
38/4	0.882	0.870	0.276	0.818
39/802	0.000	0.477	1.054	−0.420
40/19	0.955	0.902	0.442	0.605
43/1	0.782	0.676	−0.491	1.286
49/2	0.000	0.621	1.264	− 0.727
53/7	0.482	0.945	0.457	0.731
54/2	0.983	0.964	0.001	1.005
54/14	0.790	0.970	0.064	0.968
54/22	0.857	0.970	0.061	0.931
55/3	0.793	0.920	0.979	0.409
55/4	0.769	0.835	0.663	0.621
58/2	0.869	0.805	0.200	0.897
60/2	0.872	0.937	0.275	0.879
61/2	0.878	0.939	0.303	0.869
67/5	0.486	0.819	0.943	0.421
67/6	0.908	0.922	0.176	0.895
68/1	0.935	0.914	−0.361	1.218
68/6	0.892	0.891	−0.038	1.052
69/804	0.815	0.897	0.094	0.930
71/803	0.812	0.945	0.346	0.851
84/806	0.978	0.994	0.052	0.972

Table 3.4 Prediction of month of occurrence of annual maximum.

	Number of occasions when predicted annual maximum occured in month with given observed rank in year								
	Correct	Incorrect							Total incorrect
Rank	1	2	3	4	5	6	7	8	
Number	613	88	35	12	12	3	3	2	155

in a particular case the extended mean is closer to the true value than the recorded mean. However, the use of a more reliable set of dependent variables such as the extended mean annual flood should lead to improved regression equations with independent variables such as catchment and climate characteristics.

A full account of the regressions of flood statistics on catchment

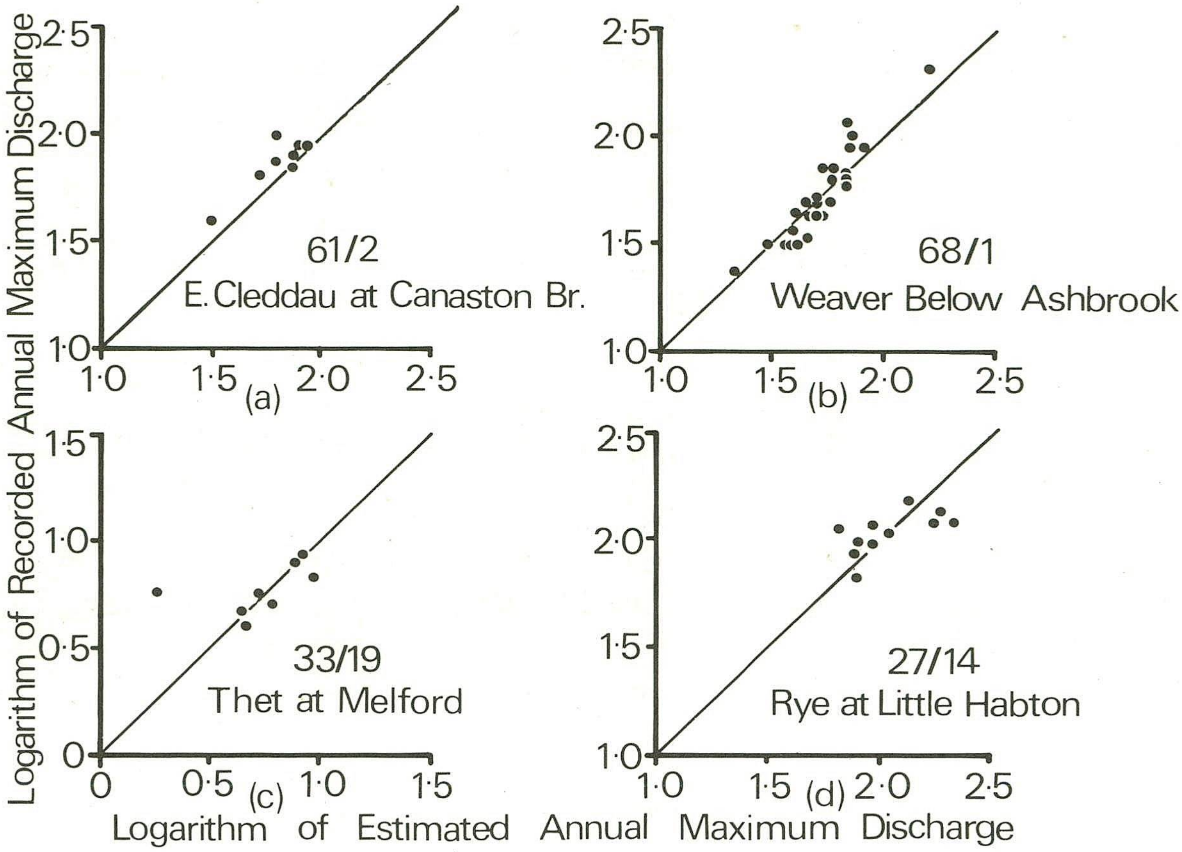


Fig 3.11 Prediction of annual maximum discharge.

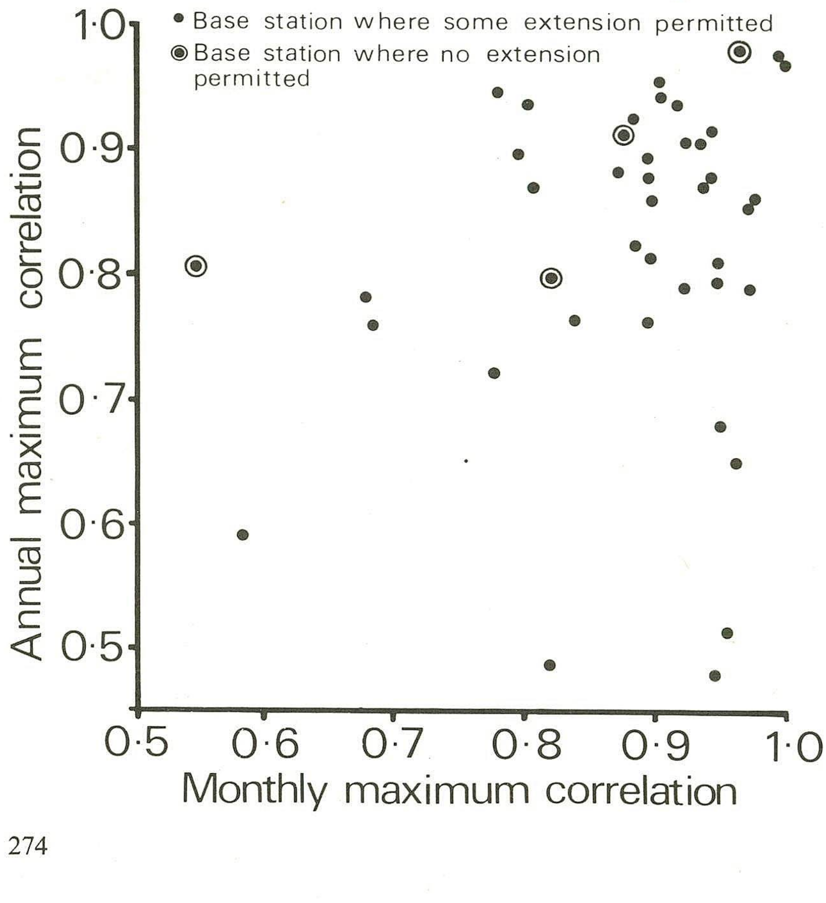


Fig 3.12 Relationship between monthly and annual maximum correlations.

Independent variable	Coefficients and statistics							
	Countrywide		Hydrometric areas					
			28–39		45–53		54–65	
	AMAF	EXTMAF	AMAF	EXTMAF	AMAF	EXTMAF	AMAF	EXTMAF
Intercept	−2.43	−2.52	−1.40	−1.73	−2.59	−3.01	−1.98	−2.26
AREA	0.99	0.99	0.97	0.98	0.91	0.97	0.94	0.94
STMFRQ	0.32	0.32	0.33	0.28	0.28	0.27	0.20	0.16
RSMD	1.17	1.23	0.32	0.55	1.56	1.72	0.87	1.03
TAYSLO	0.22	0.21	0.40	0.33	−0.08	−0.01	0.41	0.37
SOIL	0.81	0.90	0.92	1.07	0.60	0.95	0.70	0.73
URBAN	0.62	0.78	1.89	2.11	−1.97	4.14	−1.13	−1.84
LAKE	−0.74	−0.94	1.06	1.25	—	—	0.62	−0.01
Residual error	0.19	0.18	0.15	0.16	0.16	0.16	0.15	0.14
Factorial error (%) +	53.8	52.8	42.6	46.2	43.9	43.5	40.9	37.1
−	35.0	34.5	29.9	31.6	30.5	30.3	30.0	27.1
Multiple correlation	0.96	0.97	0.96	0.96	0.95	0.95	0.96	0.97
Multiple determination	0.93	0.93	0.92	0.92	0.90	0.90	0.92	0.93

Table 3.5 Countrywide and regional regression comparisons of AMAF and EXTMAF.

characteristics is given in Chapter 4 where a definition of the catchment characteristics is given. Because the improvement in regression is a test of the extension a brief account of this comparison is given here. The improvement was tested by forming regressions of AMAF and EXTMAF (arithmetic mean annual flood and extended mean annual flood), on certain catchment characteristics—AREA, STMFRQ (stream frequency), TAYSLO (channel slope), SOIL (soil index), RSMD (rainfall minus soil moisture deficit or short term net rainfall), URBAN (urban fraction) and LAKE (lake fraction). Only those stations were used for which both AMAF and EXTMAF were measured. Table 3.5 shows that the improvement was slight. There was also little tendency to break up areas of a consistently high or low estimation on a map of residuals. To test whether this disappointing result might be due to the limitations of a countrywide regression equation the effects of adjusted estimation were tested by breaking down the data set into regions. Table 3.5 shows the results from three regions; Trent to Thames, Dorset to Bristol Avon and Severn to Gwynedd. The use of EXTMAF instead of AMAF resulted in a marked improvement only in the third region; the pattern of mapped residuals was little changed.

Bias reduction

An approximate index of change showed the areas (Figure 3.13) where extension led to marked increases or decreases in the mean annual flood. These might reflect bias in the recorded data. In many areas this map agreed with the pattern of residuals from countrywide regression equations (Figure 4.23); in other areas, particularly the south eastern peninsula and over much of the north west, the two maps did not agree. Greater detail was sought by a series of maps showing the ratio (EXTMAF − AMAF)/AMAF for stations whose recorded and extended periods began in certain years. In three regions a large concentration of extended records began during 1961–65; Essex and Kent, the south west peninsula, and the border area. Short extension (after 1951) tended to produce a small decrease in the mean annual flood in the south east but a mixed response in the northern and south west groups. Longer extension was possible only in the north and in Essex and in both cases the estimated mean annual flood was increased by record extension. Isolated records elsewhere showed similar trends.

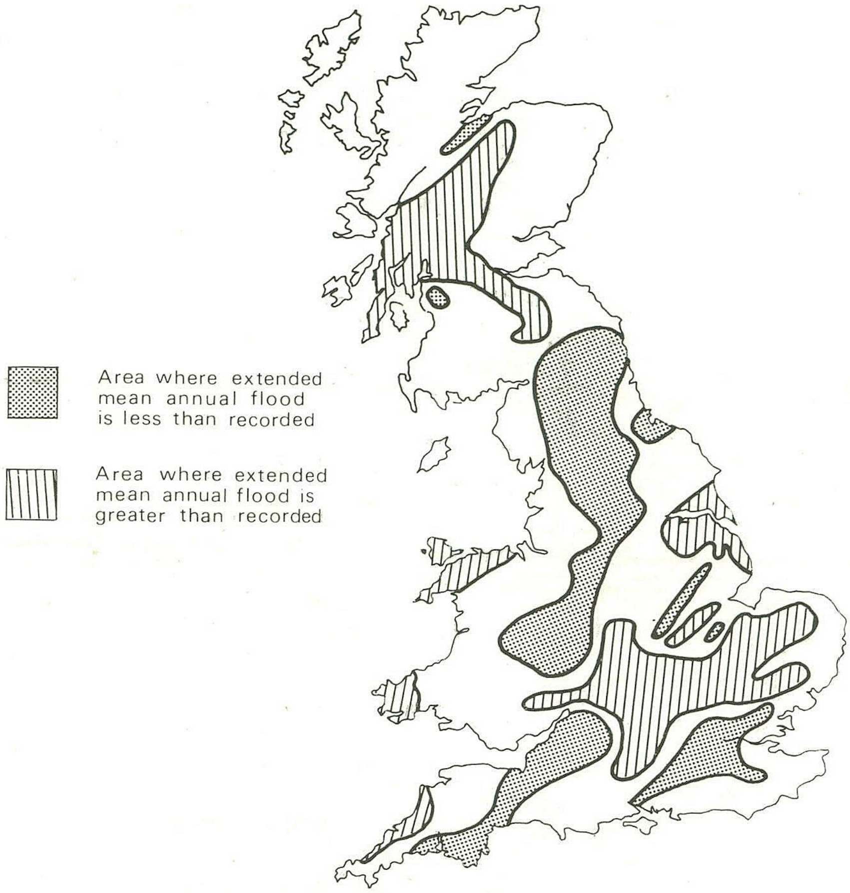


Fig 3.13 Effect of extension on mean annual flood.

Stations with a medium length of record (commencing between 1951 and 1960) were more evenly distributed than those with short records. Short extension of medium length records produced an extended mean annual flood between 90 and 104% of the recorded mean, with only four exceptions. Stations whose extended period began earlier (1941–50) showed similar changes with a more even distribution about the 100% point. The longest extensions to medium record stations were possible in the Midlands and Pennine rivers. There were some large adjustments to these stations; the use of long established Severn and Wye records for extension led to sizeable reductions in the mean annual flood. In other cases a small reduction was common; only in Yorkshire were increases widespread.

There were eleven long established stations where record extension was worthwhile; the average increase in estimated mean annual flood was about 4%.

Where any regionally consistent pattern of change has resulted from data extension the effect has been to lower the estimate of mean annual flood. Increases in mean annual flood were more common in Scotland but, there as elsewhere, were distributed throughout the region rather than concentrated in any area. The effect of extension was less marked as the recorded period lengthened, although data extension could quite dramatically alter the estimated mean of even 20 year stations.

3.2.6 Summary of conclusions

The monthly maximum data of 346 stations were subjected to a series of automatic procedures including regression analysis in order to extend their record and improve the estimates of mean annual floods. The actual relations used to extend the records were not printed but by adopting conservative criteria the theoretical correlation threshold for useful extension of the annual maxima was achieved with few exceptions. 48 stations were rejected on these criteria which may have been too conservative. The applicability of the monthly maximum criteria to annual maximum extension was tested. It was found that about 4% of the cases tested were overextended and possibly a higher proportion were incorrectly rejected. The mean annual flood was reduced in most cases. The reduction was greatest in the shorter term stations though the number of reductions was similar in medium term stations. Increases in the mean annual flood tended not to cluster in regions but were scattered throughout the country, particularly in Scotland.

Although the theoretical criteria for useful extension were satisfied in most cases, the results were disappointing when the extended mean annual flood was substituted for the recorded mean in regression on catchment characteristics. The improvement in multiple correlation coefficient was small both in the countrywide and regional regressions. Responsibility for this may however lie with the limitations of the assumed relationship.

The prediction of the month in which the annual maximum occurred was 80% successful and detailed investigation of unsuccessful predictions showed that their peaks were in most cases not much lower than the actual annual maximum.

3.2.7 Recommendations

While entirely automatic procedures were necessary in this large scale study, it is recommended that all regressions should be examined when extending single or small groups of stations. The use of correlation maps in variable selection was very valuable and should be retained in any future or further study. The requirements for monthly maximum correlation to improve estimates of the mean annual flood merit further theoretical investigation. It is expected that the problem could best be studied through simulation although a very considerable number of samples would be required before stable statistics would be found. It is felt that the best approach to extension is through monthly maxima rather than through the direct use of annual maxima. If missing value analyses are to be used, there now exist a number of methods for unbiassed estimation which are not based on the common period of record among all variables (Afifi & Elashoff, 1966); this removes a considerable restriction on the sample size.

3.2.8 Example

The Sea Cut at Scarborough (27/33) has been chosen to illustrate the use of data extension at a single station. Only 4 years of complete record were available (1965/66–1968/69); Table 3.1 shows a summary of the bar chart obtained from all records of monthly maximum rainfall and discharge within a 100 km radius. Figure 3.9(a) shows the plotted correlation contours. Useful variables appear to be:

25/5 complete record back to 1961/62 and incomplete before that to 1958/59;
26/2 complete record back to 1961/62;
26/801 good correlation but only 7 extra months of record;
27/14 correlation a little low but geographically adjacent;
27/21 record back to 1959/60;
29/1 record back to 1960/61;
24724 rainfall correlation very poor;
27/802 daily read record almost complete to 1952/53 and good correlation.

It was decided to break the analysis into four periods. As the records of all stations extended over Scarborough's period of record (but for a few months) the covariance matrix derived from the period 1965 to 1969 was used as the basis for all regressions.

1 1961/62–1964/65 25/5, 26/2, 27/14, 27/21, 29/1, 24724, 27/802;
2 1960/61 27/14, 27/21, 29/1, 24724, 27/802;
3 1959/60 27/14, 27/21, 24724, 27/802;
4 1952/53–1958/59 24724, 27/802.

Table 3.6 shows the regression equations and other relevant details for each prediction equation.

Table 3.6 Scarborough record extension regressions.

Possible extension	Coefficients 1 1961/62	2 1960/61	3 1959/60	4 1952/53	Correlation with 27/33
25/5	0.005	—	—	—	0.62
26/2	0.698	—	—	—	0.64
27/14	−0.492	−0.330	−0.315	—	0.61
27/21	0.367	0.239	0.609	—	0.76
29/1	0.119	0.462	—	—	0.69
24724	−0.048	−0.292	−0.300	−0.408	0.08
27/802	0.613	0.638	0.619	0.795	0.79
Intercept	−0.322	0.615	−0.097	0.352	—
R	0.887	0.871	0.845	0.828	—
R^2	0.787	0.759	0.714	0.686	—
see %	+79, −44	+83, −45	+91, −48	+101, −51	—

The prediction error increases steadily as the number of stations available for extension decreases. It was not thought advisable to use relations 3 and 4 based on only four and two stations, as in both cases most of the variance accounted for was due to the record of 27/802 alone (this is a daily read staff gauge on the Yorkshire Esk).

The regression equations were applied in the relevant period to produce an extended record of annual maxima during the recorded period. Both equations overestimate in this 4 year period but the effect of including the predicted 1960/61–1964/65 data is to reduce the mean from 52.4 cumecs to 35.3 cumecs.

Table 3.8 shows the complete list of Sea Cut's annual maxima. The records of two neighbouring stations are also included and show that the lower values of earlier years are not unreasonable.

Table 3.7 Comparison of recorded and predicted annual maxima.

Year	Equation 1	Equation 2	Recorded
1965/66	33.18	35.00	42.46
1966/67	61.66	55.98	59.43
1967/68	63.11	58.75	50.93
1968/69	87.18	117.06	56.62
Mean	61.28	66.70	52.36

Table 3.8 Annual maximum discharges.

Water year	Sea Cut 27/33	Bransdale 27/10	Leven Bridge 25/5
1960/61	39.9†	10.3	—
1961/62	14.9†	14.3	22.2
1962/63	30.2†	7.4	52.2
1963/64	13.0†	9.2	30.1
1964/65	10.7†	6.1	26.9
1965/66	42.5	8.7	36.1
1966/67	59.4	5.5	39.0
1967/68	50.9	17.3	68.4
1968/69	56.6	16.8	98.9

†Estimated values.

3.3 Extension of partial duration (POT) series

3.3.1 The need for extension

The use of a partial duration series of floods, or the series of peaks over a threshold, is described in Section 2.7. Because the typical British flood record is short, the partial duration or POT series was extracted for all stations where this was possible, with an arbitrary threshold exceeded on average about five times per year. The magnitudes of these exceedances, and the times at which they occurred, can be used to estimate the flood frequency distribution once a model of these events is accepted. A simple model which has been suggested for this analysis is a combination of a random (or Poisson) distribution in time of the exceedance events, and an exponential distribution of their magnitudes. These are together equivalent to a double exponential distribution of the annual maxima, and the parameters of this distribution can be deduced from the POT series.

The validity of this model is discussed in Chapter 2, where the statistical efficiencies of the estimates of the T year flood from the annual maximum and the POT series are compared. The limiting factor in the use of the POT series for estimating the mean annual flood or its standard deviation is likely to be the fit of the data to the model rather than the amount of data. Nevertheless, if the record at a site has been collected during a period during which there is evidence that floods were markedly different from the average in number or size, then it may be thought necessary to extend a POT series using a nearby long term station in order to improve the estimated flood statistics. A method of extension of the records of a POT series developed during the present investigation has been published (Frost & Clarke, 1972); a summary of this method follows.

3.3.2 *Method of extension*

Suppose that there exists a short series of exceedances, or peak discharges exceeding a threshold q_0, and the times at which they occurred, for gauging site A. Denote this series by $(t_1, q_1; t_2, q_2; \ldots\ldots t_{n_1}, q_{n_1})$, where t_i and q_i are the time at which the ith exceedance occurred, and its magnitude, respectively. For a nearby gauging station B, a longer record of exceedances is available, denoted by $(t_1', q_1', t_2', q_2'; \ldots\ldots t_{n_1+n_2}', q_{n_1+n_2}')$ where the primes indicate the exceedances at site B which may not coincide with those at site A. Those exceedances that do occur simultaneously (i.e. within a short time interval) at both gauging sites are correlated in that high values of q_i correspond to high values of q_i'. Can the longer record of exceedances at site B, correlated with those at site A, be used to improve the estimate of the T year flood at site A?

Methods of estimating the T year flood from a single series of exceedances have been described in Chapter 2. Two assumptions are commonly made: first, that the time interval between successive exceedances is a random variable with an exponential distribution. This is equivalent to assuming that exceedances follow a Poisson process, such that the probability is $\lambda \Delta t$ that an exceedance occurs in any time interval Δt, that the probability that more than one event occurs in Δt is negligible, and that what happens in the interval $(t, t+\Delta t)$ is independent of what has happened up to time t. Secondly, the assumption, based on observation, is made that the magnitudes of exceedances follow an exponential distribution with parameters β and q_0. With these two assumptions, the distribution of annual maximum discharges is shown to have the distribution function

$$F(q) = \exp\{-\exp(-(q-u)/\beta)\} \tag{3.16}$$

where $u = q_0+\beta(\ln \lambda)$, λ is the average number per year, and q_0 is the threshold. The two assumptions (Poisson process for the incidence of exceedances, exponential distribution of their magnitudes) are carried over to the next section in which two correlated sequences of exceedances are considered.

One begins by dividing the time scale into intervals of equal length Δt, small enough for no more than one exceedance to occur in each. Corresponding to each Δt, zero is written if no exceedance occurs, unity otherwise. If the records at gauging sites A and B are of lengths $N_1 \Delta t$ and $(N_1+N_2)\,\Delta t$ respectively, each record is thus transformed into a sequence of noughts or ones, that at site A containing N_1 elements and that at site B (N_1+N_2) elements. For any one interval Δt in the common period of record, one of four types of event may be obtained: (0,0), corresponding to no exceedance at site A and none at site B; (1,0), (0,1), and (1,1). Denote the probabilities of these events by p_{00}, p_{10}, p_{01}, and p_{11} whose sum of course is unity.

The magnitudes of the exceedances, where these occur together in both sequences, are assumed to follow a bivariate exponential distribution defined by three parameters: μ_q, $\mu_{q'}$ and ρ $(0<\rho<1)$ where ρ is a measure of the correlation between exceedance magnitudes that occur in the same Δt interval. Use of the two POT series to estimate the T year flood therefore requires knowledge of six parameters: p_{00}, p_{10}, p_{01}; and μ_q, $\mu_{q'}$, ρ. Only three of these enter the expression for the T year flood, which can be shown to be:

$$Q(T) = \frac{1}{\hat{\mu}_q} \ln\left[\frac{1-\hat{p}_{00}-\hat{p}_{01}}{1-(1-1/T)^{1/N}}\right] \tag{3.17}$$

where $\hat{\mu}_q$, $\hat{p}_{00}$, and $\hat{p}_{01}$ are the (maximum likelihood) estimates of μ_q, p_{00}, and p_{01}, and N is the number of time intervals Δt in a year.

3.3.3 Numerical example

This is based on data from two small streams, the Kennett or Lea Brook and Sapiston. Approximately 7 years of record were available at Beck Bridge (33/23) on the former stream, and rather more than 9 years at Euston Rectory (33/13) on the latter, giving $N_1 = 361$, $N_2 = 135$, using $\Delta t = 1$ week. Taking thresholds at 0.63 and 2.01 cumecs respectively, there were $n_{11} = 22$ events in which exceedances were recorded at both gauging sites, $n_{00} = 320$ in which no exceedance was recorded at either, $n_{10} = 12$ in which an exceedance occurred at Beck Bridge but none occurred at Euston Rectory, and $n_{01} = 7$ in which the converse occurred. For the period in which only the Euston Rectory record is available, there were $m = 125$ events in which no exceedance occurred, and $N_2 - m = 10$ exceedances. For these data the maximum likelihood estimates of p_{00}, p_{01}, p_{10} and p_{11} were found to be 0.8881, 0.0190, 0.0333 and 0.0596 respectively, values lying very close to the simple proportions n_{ij}/N_1 which had the values 0.8864, 0.0194, 0.0332 and 0.0609. Maximum likelihood estimates of μ_q, $\mu_{q'}$, ρ were $\hat{\mu}_q = 0.5847$, $\hat{\mu}_{q'} = 1.1698$ and $\hat{\rho} = 0.8409$; substitution of these values in the equation for values of $T = 10$ and 100 gave 3.945 and 5.913 cumecs respectively for the 10 and 100 year floods at Beck Bridge. The corresponding estimates derived from the Beck Bridge data alone were 3.823 and 5.712 cumecs.

3.4 Extension of records by means of a rainfall–runoff model

3.4.1 General considerations

In this country rainfall records are often much longer than river flow records. If a reliable conceptual model existed for a catchment the actual rainfall record could be used to generate a runoff record for the missing years from which annual maxima or other flood data could be extracted. A limitation to this approach for smaller catchments must be the relatively short period of most recording raingauge records. However, the fitting of a conceptual model to a short period of rainfall/evaporation/runoff records at a site could provide more reliable estimates of flood statistics by using a longer period of rainfall records, either actual or generated. A number of models have been developed which might be used for this purpose. The use of rainfall–runoff models for flood prediction depends on the reliability of the model outside the range in which it has been fitted either subjectively or objectively, and thus on the way in which the structure of the model fits the physics of the process.

It has not been possible to do work on this approach during the time available in the present study. The alternative approach described in Chapter 6 has been to split the problem into two—the prediction of the volume of quick response runoff from rainfall and antecedent conditions, and the shape of the unit hydrograph giving the distribution of this runoff.

Both these processes have been examined for a number of individual events, and the nature of the rainfall–runoff volume relationship has been studied empirically as has the variation of the response. In the unit hydrograph approach, the rainfall–runoff response is examined directly either as a percentage runoff or as a subtractive loss rate, and the ratio of the peak flow to the total runoff is also compared with the volume of runoff. This enabled the assumptions on which extrapolation to the design flood is based, to be stated simply and tested as far as the data permit. The combination of design rainfall with the unit hydrograph response model has been examined by a simulation study and this is described in Chapter 6.

With more sophisticated models which are fitted to moderate floods the structure of the model determines the nonlinearity of the response and the effect on extrapolation is more difficult to assess. The reliability of the conceptual model must therefore be assessed carefully. If the purpose of the study is to generate and use annual maxima, then it is essential to ensure that they are properly generated by the model. A method of doing this would be to regard the annual maxima predicted by the model as a predicting variable and to ensure that they are sufficiently well correlated with the actual maxima in the calibration period for an extension to be worthwhile.

The use of a model to convert long rainfall records into discharge has been described elsewhere (Clarke, 1973, Chapter 6). A brief description of the technique follows.

3.4.2 Procedure for extension

First, a continuous series (as a rough rule, say no less than 3 years) of streamflow values must be obtained such that the nth value in the sequence is the accumulated streamflow in the time interval defined by times $n-1$ and n. (For the Ray catchment at Grendon Underwood, the duration of this interval was 3 hours, the response time between peak rainfall and peak discharge being roughly 10 hours; for catchments exhibiting more rapid response, accumulation of runoff over a shorter time interval would be necessary.) Parallel series of precipitation and evaporation must also be obtained for the same period, the nth value of the precipitation sequence being an estimate of the mean areal precipitation between times $n-1$ and n; similarly, the nth value of the evaporation sequence is an estimate of the cumulative potential evaporation between times $n-1$ and n.

Second, the model itself must be defined by a set of rules forming an accounting procedure relating runoff in the nth interval to the precipitation falling on the catchment, the evaporation and other losses leaving it, and the change in water content of the soil layers. The accounting procedure is a practical expression of the 'continuity equation' which, together with others that are invariably much simplified in practical rainfall–runoff modelling, affords a mathematical description of the behaviour of the catchment as a deterministic system. Before the water budgeting procedure can be used to relate runoff to precipitation and losses, numerical values must be derived for parameters in the model, such as those in the assumed relation between actual evaporation and soil moisture deficit, or the storage coefficients of reservoirs through which direct runoff is routed.

Third, a measure must be decided that expresses the goodness of fit of the streamflow given by the model to the observed streamflow sequence. A measure or objective function commonly used is the least squares criterion

$$\Sigma(q_{\text{observed}} - q_{\text{fitted}})^2 \tag{3.18}$$

summed over the number of values in the streamflow sequence. Different numerical values for the model parameters will clearly give different values in the sequence $\{q_{\text{fitted}}\}$, and hence different values of the objective function; those numerical values must therefore be used which give the objective function optimum properties (in this case a minimum value).

Fourth, these parameter values must be estimated, and it is for this purpose that the data sequences, described above, are used. In many applications, a particular set of parameter values is assumed in the first instance; using the water budgeting procedure that constitutes the model, the precipitation and evaporation sequences are used to derive a streamflow sequence $\{q_{\text{fitted}}\}$. This sequence is taken together with the observed streamflow sequence $\{q_{\text{observed}}\}$, to obtain the value of the objective function for the particular set of model parameters used. New sets of parameter values are taken and the process is repeated, perhaps many times, until that set of parameter values is found which minimises the objective function; the search for the minimum may be by trial and error (as with the Stanford Watershed Model), or conducted according to a prestated set of rules ('algorithm'). The parameter set minimising the particular objective function used represents the 'best fitting' set, in that values of the set $\{q_{\text{fitted}}\}$ agree most closely with those of $\{q_{\text{observed}}\}$. There is, of course, no guarantee that the particular minimum value of the objective function that is found is unique; furthermore, a different choice of objective function may well lead to different parameter values in the best fitting set.

Fifth, the fitted model is used to transform the long precipitation record and the long record of potential evaporation—if it exists—into a long sequence of streamflow estimates. Long precipitation records are likely to be more readily available than long evaporation records; if the latter are not available, it must be decided whether or not it will be adequate to substitute the sequence of mean values derived from the period for which records exist.

Sixth, the transformation of precipitation and potential evaporation to a streamflow sequence gives a sequence of estimated accumulated runoff, usually in units of millimetre depth over the whole catchment. This must then be transformed to a volume (units m^3) leaving the catchment as runoff during each time increment. If interest centres on the volume of flood flows, the volumes of discharge must be summed over the number of time increments for which flood conditions obtained; if, on the other hand, interest centres on peak instantaneous discharge, some device is necessary whereby this may be estimated from a knowledge of accumulated discharge. This device could be a regression of peak discharge on accumulated 3 hour discharge. Such a procedure is likely to underestimate the variability of peak flows, for the same reason that the extension of annual maximum instantaneous discharges by regression methods leads to estimates of the annual maxima having underestimated variance. Methods of allowing for this underestimation require further study before recommendations can be made.

Lastly, the sequence of estimated peak flows, flood volumes or flood durations in the extended sequence must enter the frequency analysis for the estimation of the T year flood.

3.4.3 Use of a statistical model of rainfall

The procedure described above can be carried out to generate flows where long rainfall records exist, using a deterministic rainfall response model to convert actual rainfall records to runoff, in order to extend a short flow record. Whereas time series methods or statistical models of the flow sequence require large amounts of data (20 years or more is a common rule) and therefore appear to be of little use in extending flow records, they might be used to generate rainfall sequences. Rainfall records are often of adequate length and a rainfall time series model could be used together with a reliable conceptual model to produce synthetic flows from which flood statistics could be estimated. Given adequate models for the rainfall and for the catchment response this approach may well be an effective way of making use of the available river flow data. However, no work on this approach has been done during the present study, and the following account of the problems involved illustrates the need for research.

It may be possible, by introducing many assumptions, to postulate a statistical model of the rainfall sequences observed during the period of rainfall record. This new model can be used to generate artificial ('synthetic') rainfall sequences which might have been observed if the statistical model were a true description of the rainfall process. Typical assumptions required are those concerning the probability distribution of time intervals between successive storms; the probability distribution of storm durations; the statistical relation between storm duration and total storm depth; the relation between total storm depth and its distribution over the time increments within the storm; the probability distribution of random errors in the rainfall recorded during each time increment of a storm; and seasonal variations in these distributions and relations.

Clearly, the use of synthetic rainfall sequences requires the introduction of many further assumptions. For the simple models used in the Floods Study, it has been demonstrated that the hydrographs derived by routing synthetic storms through them are relatively insensitive to certain of the above assumptions, particularly those concerning within storm distribution of precipitation; this need not necessarily be true for all rainfall–runoff models. Wherever assumptions enter the method, they must be tested by examining the sensitivity of the conclusions (in this case the estimate of the T year flood) to relaxation of the assumptions, and the greater the number of assumptions (and this number is very great, where synthetic precipitation sequences are generated) the greater the effort required to explore their effect on the conclusions.

3.5 Extension of records at a site by means of regional estimate

3.5.1 Introduction

Attempts to solve the problem of flood frequency prediction, i.e. estimation of the relationship between magnitude Q and the return period T, are usually based on one of three methods. These are (a) the analysis of a record of discharge at the site in question, (b) use of previously established relationships between the characteristics of other catchments in the region and measures of the flood magnitude frequency relationships, or (c) an attempt to transform the problem into one of rainfall frequency prediction through a study of the relationship between rainfall and discharge. All three of these methods have been discussed in different chapters of the report but are discussed here as a means of extending the record at a site by means of regional information. When the number of years of record at a site is not large, then a useful amount of additional information about the flood regime can be added through catchment characteristics. This information is never negligible but is relatively more useful when the period of record is short. The information which catchment characteristics give about the mean annual flood has been described as equivalent to about a year's record at the site. The direct comparison between the two is described in Chapter 4. The estimates obtained are subject to large sampling variance. The synthesis of a rainfall depth/duration/frequency relationship and a method of converting rainfall into discharge have been approached through a simulation study in Chapter 6. These provide a means of flood frequency prediction either from catchment characteristics or from a short period of record.

A combination of estimates of the flood regime derived from catchment characteristics with those obtained from a record of some years' duration can be considered as an extension of the record. The knowledge from catchment characteristics can be regarded as prior information which is modified by successive samples at the site to provide a posterior estimate. Bayes' theorem has provided a means of combining such information in the form which the hydrologist requires.

3.5.2 Bayesian estimation of flood frequency distribution

For a gauged catchment with N years of record the most obvious source of information on the flood distribution is the observed series of annual maximum floods $Q_1, Q_2, \ldots. Q_N$. It might be assumed that this is a random sample from a double exponential (Gumbel) distribution, whose probability density function is denoted by $f(Q)$. Estimates of the mean ($\bar{Q}$) and coefficient of variation (CV) could be found by the method of maximum likelihood and hence an estimate $\widehat{Q(T)}$ found of $Q(T)$ for any return period T. Further, a standard error σ_T of this estimate may be computed together with a confidence interval corresponding to any desired level of probability. However, the nature of this confidence interval is such that it does not assert that the true value of $Q(T)$ lies within it with a stated probability but only that if a large number of samples were available and such a confidence interval were computed from each, then a certain proportion of the intervals so computed would contain the true value. The method does not allow the combination of a pair of numbers (Q_A, Q_B) with a statement of 'the probability that $Q(T)$ lies between Q_A and Q_B'. This is really the type of statement which is required in practice.

Whereas the above theory considers any quantity to be estimated as a fixed but unknown value, the method of Bayesian estimation considers the unknown quantity as having a probability distribution which is modified by extra information. Accordingly, as it becomes modified the variance of the distribution is decreased; that is, knowledge of the parameter is becoming more precise. The fact that all the knowledge available about $Q(T)$ is expressed as a probability distribution does allow the required type of statement about $Q(T)$ to be made, *viz.* $PR(Q_A \leq Q(T) \leq Q_B)$ is some stated amount.

When the results of a flood frequency analysis are to be used in a further decision analysis in which costs and monetarily expressed benefits also appear, the estimates of flood magnitudes are best entered into the analysis as probability distributions.

Whereas Bayesian estimation allows the combination of several sources of information to provide a final estimate, this combination is not the same as taking a weighted average of the various individual estimates. The two methods may not differ greatly numerically but they differ in meaning.

An algebraic expression for the posterior distribution function of $Q(T)$ has been derived by Cunnane & Nash (1971). They also present a method of numerical analysis for estimating the flood frequency relationship. However, because the emphasis of this report has been on the separate estimation of the mean annual flood and a regional curve of Q against T, it will be more helpful to describe the Bayesian estimation of the mean annual flood.

3.5.3 Bayesian estimation of mean annual flood

At any station let the annual maximum flood Q have distribution function $F(Q;\mu,\sigma\ \ldots)$ where μ and σ are the population mean and standard deviation. The notation $F(Q;\mu,\sigma\ \ldots)$ indicates that the distribution depends on parameters μ and σ and others unspecified indicated by the dots; in practice only one other parameter would be considered which would be a shape parameter.

The Bayesian estimate of μ is the result of successive modification of one estimate in the light of additional information. An estimate $\hat{\mu}_0$ is obtained from the regression equation relating sample mean annual flood to catchment characteristics. Additional information in the form of a sample $Q_1, Q_2, \ldots Q_N$ is then used to modify $\hat{\mu}_0$ to give a new estimate $\hat{\mu}_1$.

An estimate has a standard error of estimate; let s_0 and s_1 be the standard errors of $\hat{\mu}_0$ and $\hat{\mu}_1$. Define the precision h_0 and h_1 by

precision = reciprocal of variance,

$$h_0 = 1/s_0^2 \text{ and } h_1 = 1/s_1^2. \tag{3.19}$$

This definition accords with common sense because high precision is equivalent to small variance.

The sample estimate of μ is the sample mean $\bar{Q} = \Sigma Q_i/N$. Regardless of the form of $F(Q)$, the expressions for the expected value of $\bar{Q}$ and its variance are

$$\begin{aligned} E(\bar{Q}) &= \mu \\ \text{var}(\bar{Q}) &= \sigma^2/N. \end{aligned} \tag{3.20}$$

Therefore, the precision of $\bar{Q}$ as an estimator of μ is N/σ^2. A special case is when $N = 1$; the precision of a single year's data as an estimate of the mean annual flood is $1/\sigma^2$.

It has been estimated in a previous study (Nash & Shaw, 1965) that the precision of $\hat{\mu}_0$ is about that of 1 year's data, i.e. $1/\sigma^2$. The precision of estimates based on the regressions presented in Chapter 4 varies between regional and countrywide equations, but is of the same order.

Frequency and degree of belief probabilities

Given these estimates which are to be combined, it is appropriate to return to the distinction between the two types of probability statement.

Consider a long sequence of annual maximum flood magnitudes Q. Let the event A be the occurrence of $Q \leq q'$ where q' is any arbitrary value. Each item in the sequence can be examined for the occurrence of A or not. In a sequence of M items let $m(A)$ be the number of occurrences of A. Then the frequency probability of A is $\lim_{M \to \infty} m(A)/M$. The essential feature of this frequency probability is that some condition (occurrence of Q) can be repeated and examined over and over again for the occurrence of another condition (that of A). Denote such repeatable conditions by *events*.

Now consider another type of occurrence which may be called a *proposition*, such as 'my neighbour is $180 \pm \frac{1}{2}$ cm tall'. This proposition is either true or false; measuring his height would show whether it is true or false. At this moment on the basis of information available we can express our degree of belief in the proposition as a number between 0 and 1. If we *know* him to be 180.3 cm tall then our degree of belief is 1; if we know him to be 181 cm tall then our degree of belief is 0. Otherwise our degree of belief is somewhere between 0 and 1. We also call this degree of belief a probability or by a more descriptive name, subjective or personal probability. The information available to us is not the result of repeated observation of events produced at random and consequently a probability based on it is not a frequency probability. Thus we speak of degree of belief or subjective probabilities where we could not properly speak of frequency probabilities.

Prior and posterior distributions

At a given station the mean annual flood μ is a fixed quantity. If $\mu = 100$ then in the frequency sense 'the probability that $\mu = 90$' is zero. But in the degree of belief sense a non zero probability may be attached to the *proposition* or *hypothesis* that $\mu = 90$ and it is upon this that Bayesian estimation depends. All the information about μ obtained from catchment characteristics is expressed as a degree of belief probability distribution. Its mean is $\hat{\mu}_0$ as remarked earlier. This distribution is taken as the distribution of residuals in the regression analysis upon which the relation between mean annual flood and catchment characteristics is based. This distribution is called the prior distribution. It is the degree of belief probability distribution of μ based on information independent of the sample of measured annual maxima—or prior to the sample. Denote the density of the prior distribution by $g(\mu)$.

This distribution $g(\mu)$ is modified by the new information contained in the sample. The result is a new degree of belief probability distribution for μ, denoted by $p(\mu)$. This is called the posterior distribution—posterior to use of the sample information—and its mean $\hat{\mu}_1$ is taken as the best point estimate of μ. The link between the prior and posterior distributions is made by Bayes' theorem.

Bayes' theorem

Denote the observed sample $Q_1 Q_2 \ldots . Q_N$ by **Q**. Thus **Q** is the sample information and in the same way let C be the information derived from catchment characteristics. Then the prior distribution is more properly denoted $g(\mu/C)$, or the distribution of μ given C. Now consider some value of μ, say μ'. If μ' were the true value of μ then the probability of observing the sample $\mathbf{Q}$ is proportional to $\prod_{i=1}^{N} f(Q_i/\mu') = L(\mathbf{Q}/\mu')$ say. In the present context if $\mathbf{Q}$ is treated as constant and μ' varied $L(\mathbf{Q}/\mu')$ is referred to as the likelihood of $\mathbf{Q}$ given μ'. It can be manipulated according to the rules of probability.

Then the posterior probability is

$$p(\mu/\mathbf{Q},C) = \frac{PR(\mu,\mathbf{Q}/C)}{PR(\mathbf{Q}/C)} = \frac{PR(\mathbf{Q}/\mu,C)\,.\,PR(\mu/C)}{PR(\mathbf{Q}/C)}. \qquad (3.21)$$

Substituting $L(\mathbf{Q}/\mu,C)$ for $PR(\mathbf{Q}/\mu,C)$ and $g(\mu/C)$ for $PR(\mu/C)$ and noting that $PR(\mathbf{Q}/C) = \int_\mu PR(\mu,\mathbf{Q}/C)\,\mathrm{d}\mu = \int_\mu PR(\mathbf{Q}/\mu,C).PR(\mu/C)\mathrm{d}\mu$ leads to

$$p(\mu/\mathbf{Q},C) = \frac{L(\mathbf{Q}/\mu,C)\,.\,g(\mu/C)}{\int_\mu L(\mathbf{Q}/\mu,C)\,.\,g(\mu/C)\mathrm{d}\mu}. \qquad (3.22)$$

This is the mathematical expression of Bayes' theorem which says that posterior probability is proportional to the product of the likelihood and the prior probability.

The denominator is the subjective probability that the sample $\mathbf{Q}$ would occur regardless of the value of μ. In the Bayesian framework μ is estimated at each stage by a probability distribution. Any measure of central tendency such as the mean, median or mode of this distribution may be used as a point estimate of μ.

Special case of Bayes' theorem

Suppose that the prior distribution of μ is Normal with mean $\hat{\mu}_0$ and variance σ_0^2 (precision $h_0 = 1/\sigma_0^2$) and that the data $Q_1\ Q_2\ \ldots.Q_N$ are Normally distributed with mean μ and variance σ^2 (precision $h = 1/\sigma^2$). Then the posterior distribution of μ is also Normal with mean

$$\hat{\mu}_1 = \{(Nh)\,\bar{Q}+h_0\,\hat{\mu}_0\}/(Nh+h_0) \qquad (3.23)$$

and precision

$$h_1' = Nh+h_0. \qquad (3.24)$$

In other words $\hat{\mu}_1$ is the weighted mean of the sample mean and the prior mean, weighted with their precision, and the posterior precision is the sum of the precision of the prior and sample means.

This is a quantitative expression of ideas which would be accepted intuitively. The result is exact under the stated conditions of Normality or symmetry but one would intuitively expect that the result for the mean would remain close to the truth under departure from Normality. In the present application the prior distribution is lognormal (or almost equivalently gamma) while the sample data also have a skew distribution. The robustness of the result for the mean should be tested analytically but in the meantime it will be used as it is without modification. It is likely that the result for the precision is less robust than that for the mean and its truth should not be assumed. It should be noted that the result for the mean, although it may not be exact in the skew case, is nevertheless the linear combination of the prior and sample point estimates which has maximum precision or minimum variance.

Columns 7 and 8 of Table 3.9 show the posterior mean annual floods for 18 stations in the Trent River Authority area assuming that the prior estimate (from catchment characteristics) is worth (a) 1 year's data (column 7) and (b) 2 years' data (column 8).

Station	log(AMAF)	$\widehat{\log(\text{QBAR})}$	AMAF data mean	$\widehat{\text{QBAR}}$ regression estimate	N	$\frac{N(\text{AMAF})+\text{QBAR}}{N+1}$	$\frac{N(\text{AMAF})+2\text{QBAR}}{N+2}$
28/2	1.42193	1.40937	26.420	25.667	15	26.372	26.331
28/3	1.85083	1.52586	70.930	33.563	14	68.439	66.259
28/4	1.79768	1.83589	62.760	68.532	13	63.172	63.530
28/5	2.05690	2.05496	114.000	113.490	13	113.964	113.932
28/7	2.43284	2.60225	270.920	400.174	14	279.537	287.077
28/8	1.99721	1.86789	99.360	73.772	16	97.855	96.517
28/9	2.68987	2.74587	489.630	557.013	11	495.245	500.304
28/10	2.23091	2.25235	170.180	178.792	33	170.433	170.672
28/11	2.05797	2.14561	114.280	139.834	11	116.410	118.227
28/12	1.86069	1.98973	72.550	97.664	10	74.833	76.736
28/14	1.49679	1.67257	31.390	47.051	10	32.814	34.000
28/15	1.14019	1.27482	13.810	18.829	8	14.368	14.814
28/16	1.11294	1.39653	13.970	24.919	7	14.464	15.625
28/18	2.11089	2.21694	129.090	164.794	8	133.057	136.231
28/19	2.19816	2.34363	157.820	220.611	6	166.790	193.245
28/32	0.76910	0.73717	5.876	5.460	5	5.807	5.757
28/801	0.73161	0.81890	5.390	6.590	43	5.417	5.443
28/802	0.96658	0.45533	9.259	2.853	5	8.191	7.429

Table 3.9 Bayesian estimates of mean annual flood.

3.5.4 Bayesian estimation of $Q(T)$

A method is available (3.5.2) for combined estimates of the flood frequency distribution using information derived from catchment characteristics and a sample of records. The approach developed in this study has involved the separate estimation of $\bar{Q}$ from catchment characteristics (Section 4.3) or from records, and of $Q(T)/\bar{Q}$ from regional curves built up from a number of records (Section 2.6). The simplified approach described in the last section can provide a combined estimate of the mean annual flood from two or more sources of information whose precision can be assessed. The example illustrated estimates based on catchment characteristics with records at a site. There is a corresponding problem in combining an estimate of $Q(T)$ based on the records at a site with that derived from an estimate of $\bar{Q}$ and an empirical regional curve relating $Q(T)$ to $\bar{Q}$. The precision of these estimates is required if they are to be combined; this problem is discussed in Chapter 2, but it is clear that further research is required.

The background to the use of regional growth curves is described in Chapter 2. It is shown in Chapter 4 that only a small proportion of the variance of the coefficient of variation of annual maximum floods (CV) can be related to catchment characteristics. This is probably due to the large sampling error of individual estimates of CV; this is illustrated by the variations between decades of sample estimates at single long term stations (Section 2.5), and also by the fact that the proportion of the variance of CV which can be related to catchment characteristics increases steadily with length of record. It is noted that the catchment characteristics best related to CV, or the increase of flood magnitude with return period, are those of the climate group rather than geometrical or soils characteristics. Regional growth curves relating $Q(T)$ to Q, analogous to the rainfall growth curves described in Volume II, have therefore been built up from the records of all stations within regions or groups of hydrometric areas.

3.6 References

AFIFI A.A & ELASHOFF R.M. (1966) Missing observations in multi-variate statistics. I: Review of the literature. *Journal of the American Statistical Association*, **61**, 595–604.

CLARKE R.T. (1973) *Mathematical models in hydrology*. FAO, Rome, Irrigation and drainage paper 19.

CUNNANE C. & NASH J.E. (1971) Bayesian estimation of frequency of hydrological events. Proceedings of the Warsaw Symposium, Mathematical models in hydrology, IAHS Publication 100, (published 1973) **1**, 45–53.

DRAPER N.R. & SMITH H. (1968) *Applied Regression Analysis*. John Wiley and Sons, New York.

FIERING M.B. (1963) Use of correlation to improve estimates of the mean and variance. United States Geological Survey, Professional Paper 434C.

FROST J. & CLARKE R.T. (1972) Estimating the *T* year flood by the extension of records of partial duration series. *Bulletin of the International Association of Hydrological Sciences*, **17**, 209–217.

GILROY E.J. (1970) Reliability of a variance estimate obtained from a sample augmented by multiple regression. *Water Resources Research*, **6**, 1595–1600.

MORAN M.A. (1974) On estimators obtained from a sample augmented by multiple regression. *Water Resources Research*, **10**, 81–85.

NASH J.E. & SHAW B.L. (1965) *Flood frequency as a function of catchment characteristics*. Institution of Civil Engineers, Symposium on River Flood Hydrology, (published 1966), 115–136.

4 Estimation of flood peaks from catchment characteristics

4.1 Introduction

Where no flow records exist at a site, a preliminary estimate of the mean annual flood or another flood statistic may be obtained by using a relationship between floods and catchment characteristics obtained from maps. The derivation of such a relationship is one reason for the study of flood records on a large scale. In this investigation a joint study of British and Irish records has been carried out in order to increase the number of records and range of catchment characteristics on which the relationship is based.

However, the results of such an indirect method of estimation are generally less reliable than an estimate based on direct analysis of records at a site, and it is recommended that a river flow measuring station be installed as soon as the need for a flood estimate at a site is foreseen. There is usually sufficient time between the conception of a project and its final design for the installation of a gauge and the collection of a few years of records. These may be extended by correlation (Chapter 3) and used to estimate the mean annual flood, from which the flood of a given return period may be deduced by means of a region curve (Chapter 2). Alternatively, a unit hydrograph may be derived and used to convert a rainfall estimate to a flood estimate (Chapter 6). The indirect estimate described in this chapter may be useful in a feasibility study, or as the basis for the design of a gauging structure.

Previous investigations in this country have provided a set of equations relating mean annual flood to catchment area alone, with geographical variations allowed for in a regional factor (Cole, 1965). Other studies have taken account of an index of climate or slope (Nash & Shaw, 1965). The choice of catchment characteristics for use in this investigation is described in Section 4.2. Morphometric characteristics included area, channel slope, channel network, and a lake index; other characteristics described soils and urban development; climate characteristics included annual average rainfall and short term rainfall.

The use of multiple regression techniques to study the relationships between the mean annual flood and its coefficient of variation and catchment characteristics is described in Section 4.3. First tests were carried out to choose the measure of the mean annual flood; to see whether the investigation should include short term stations and those of low reliability; whether division should be made on the basis of catchment size; and whether division into regions was worthwhile. It was decided to use the mean annual flood estimated from the extended annual maximum series (Chapter 3), and to use all stations with five or more annual maxima, and of grades A–D. It did not appear useful to divide stations by catchment size, but it did appear worth dividing the country into regions. However, for most of the country the regional equations differed significantly in their intercepts but not in the coefficients of the variables; one region was found to require a different form of equation. The residual errors from these equations are larger than would be accounted for by sampling error, and maps of residuals from the countrywide and regional equations show clusters of positive or negative residuals. Residual maps of this nature may in certain cases be used to adjust estimates from equations.

The coefficient of variation did not prove well correlated with catchment characteristics, and it is suggested that region curves (Section 2.6) should be used to derive the flood of a given return period from the mean annual flood.

A number of maps illustrating this chapter (Figures 4.18–4.23) are contained in Volume V.

4.2 The choice of catchment characteristics for trial

4.2.1 Introduction

A detailed review of possible catchment characteristics would be lengthy. Boulton (1965) lists 13 indices of morphometry alone and many more are available. Morphometric characteristics describe the geometry of the basin. Further characteristics are required to describe the cover type or climate. A fuller account of work on catchment characteristics is available (Newson, 1974).

Morphometric characteristics are dealt with before indices of soils, land use and climate. Although the method of choice was similar in each case, a small number of morphometric indices were selected from a large number which could be measured from published maps, whereas investigation was required to provide indices of soil and climate.

4.2.2 Morphometric characteristics

The choice of characteristics depended on the following criteria:
i Judgement of the likely predictive success of variables, based on the results of other studies and on theory.
ii The numerical calculation of regression equations and the interpretation of their coefficients is simpler when independent variables are uncorrelated. Because catchment characteristics are dogged by correlations careful selection is necessary.

Author	Date	Flood parameter	Catchment characteristics
Armentrout & Bissell	1970	Q_{10}	S (at lower end), A
Bell & Om Kar	1969	Lag time	L, S, Lca
Benson	1962	$Q_{2.33}$	A, S, Ri, storage
Carlston	1963	$Q_{2.33}$	Dd, A
Chow	1962	Lag time	L, S
Gray	1964	Lag time	L, S
Gupta, Bhattacharya & Jindal	1969	Q	A, L, Lca, S
Hickok, Keppel & Rafferty	1959	Lag time	L, Dd, S (average basin slope)
Kennedy & Watt	1969	Lag time	L, S, storage, population
Kinnison & Colby	1945	Q	A, Lca
Kirpich (reporting Ramser)	1940	Time of rise	S, Ld
Lynn	1971	$Q_{2.33}$	A, S, R
Nash	1960	UH parameters	L, S
Nash & Shaw	1965	$Q_{2.33}$	A, R, S (average basin slope)
Potter	1953	Q_{10}	S, A
Reich	1970	Q_{10}	A, Physiographic regions (analysis of variance)
Rodda	1967	$Q_{2.33}$	A, Dd, Ri
Snyder	1938	Q	A, lag time
		Lag time	L, Lca
Taylor & Schwarz	1952	Lag time	L, Lca, S

A = area of catchment, S = stream slope, L = length of main stream, Lca = length to centroid of area, Dd = drainage density, Ld = direct length from furthest point of catchment, R = annual rainfall, Ri = rainfall intensity, Q = peak discharge, $Q_{2.33}$ = mean annual flood, Q_{10} = 10 year return period flood, UH = unit hydrograph.

Table 4.1 Examples of variables used in predictive equations for flood parameters (early references from Gray (1964)).

iii In the present study variables must be capable of simple measurement from widely available material; this may require a simpler characteristic which correlates well with a more complex one.

i Previous experience

Gray (1964) summarises the successful variables applied in statistical flood studies. He distinguishes five groups of factors: size and shape of drainage area; density and distribution of watercourses; general land slope; slope of channels; storage.

In describing the significance of each heading he ascribes to size the scale of the flood process in each basin, to the stream network a time distribution of floods, to slopes the infiltration and velocity performance, and considers that storage will modify the effect of all these. Table 4.1 shows the variables used in some flood studies; size and slope variables are most common and drainage-network variables are also represented.

ii Statistical considerations

In the use of multiple regressions in hydrology correlation between the independent variables often makes interpretation of the resulting regression coefficients difficult. One reason why the area variable has been so useful in statistical hydrology is its association with other significant variables, e.g. slope and channel network properties. Wong (1963), after studying mean annual floods for New England, attempted to overcome the objections to multiple regression by means of principal component analysis. This transformed 12 catchment variables to an orthogonal set of which two

Table 4.2 Portion of correlation matrix showing only moderate and high values of r. (Negative values shown as 0.$\bar{7}$2.) Values required for significance at the 5 and 1 % probability levels with $n = 16$ are 0.497 and 0.623 respectively.

Variables	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		0.96	0.97	0.98				0.89		0.61				0.52		
2			0.96	0.98				0.92						0.57		
3				0.99				0.95						0.53		
4								0.96						0.52		
5						0.89			0.70	0.63				0.48		0.44
6									0.91					0.67	0.51	
7			'stream density'												0.67	
8																0.51
9													0.62	0.73		
10				'shape'							0.72	0.71				
11												0.97	0.55			
12													0.63			
13				'slope'										0.72	0.54	
14															0.70	
15																0.52
16				'rock'												

1 = area of catchment, 2 = length of main stream (blue lines on 1:25 000 map), 3 = length of all streams (blue lines on 1:25 000 map), 4 = length of all streams (extended to dry valley), 5 = drainage density (blue lines), 6 = drainage density (extended), 7 = 6÷5, 8 = no. of stream lengths (between junctions), 9 = 8÷area, i.e. quickly sampled drainage density, 10 = 2÷area, 11 = altitude of watershed (highest point), 12 = highest point in catchment AOD minus altitude of gauging station, 13 = basin slope from grid sample (m/km), 14 = stream slope of main stream between 10 and 85 percentiles of length (m/km), 15 = hypsometric integral, 16 = % of permeable rock (limestones and sandstones as opposed to clays and igneous rocks).

Table 4.3 Significant principal components of Bristol Avon trial data (logtransformed).

	Variables	Principal components 1	2	3	4
1	Area	−0.022	−0.003	−0.104	−0.049
2	L. main	0.013	−0.045	−0.321	−0.014
3	L. total	0.036	−0.121	−0.174	−0.135
4	L. total (ext.)	0.068	−0.050	−0.192	−0.037
5	Drainage density	0.230	−0.387	−0.202	−0.382
6	Drainage density (ext.)	0.397*	−0.146	−0.320	−0.030
7	6÷5	0.224	0.444	−0.171	0.622*
8	Links	0.184	−0.100	−0.150	−0.008
9	Frequency (Links÷area)	0.461*	−0.185	−0.087	0.055
10	Elongation	0.060	−0.004	−0.105	0.093
11	Altitude	0.171	−0.081	0.444*	−0.094
12	Relief	0.211	−0.116	0.399*	0.049
13	Basin slope	0.433*	−0.034	0.338*	0.064
14	Mainstream slope	0.298	0.247	0.285	−0.157
15	Hypsometric integral	0.363*	0.421*	−0.234	−0.144
16	Permeable rock %	−0.024	0.555*	−0.007	−0.609*
Variance accounted for by above principal components.					
	Component variance	7.705	3.693	2.031	1.385
	Proportion	0.48	0.23	0.13	0.09
	Total proportion	0.48	0.71	0.84	0.93

Variables are described in Table 4.2. Asterisks show higher weightings.

contributed 85% to the total variance; these two were largely measures of main stream length and average land slope. Wong did not use these principal components for regression because of problems in interpretation and use of the resulting equations. Thus, the technique of principal component analysis can be viewed as a means of obtaining uncorrelated linear combinations of variables which, by grouping related variables, permits selection of a small number of dominant catchment characteristics.

For selection of catchment characteristcs the only data available for multivariate analysis were 16 variables on 16 catchments in the Bristol

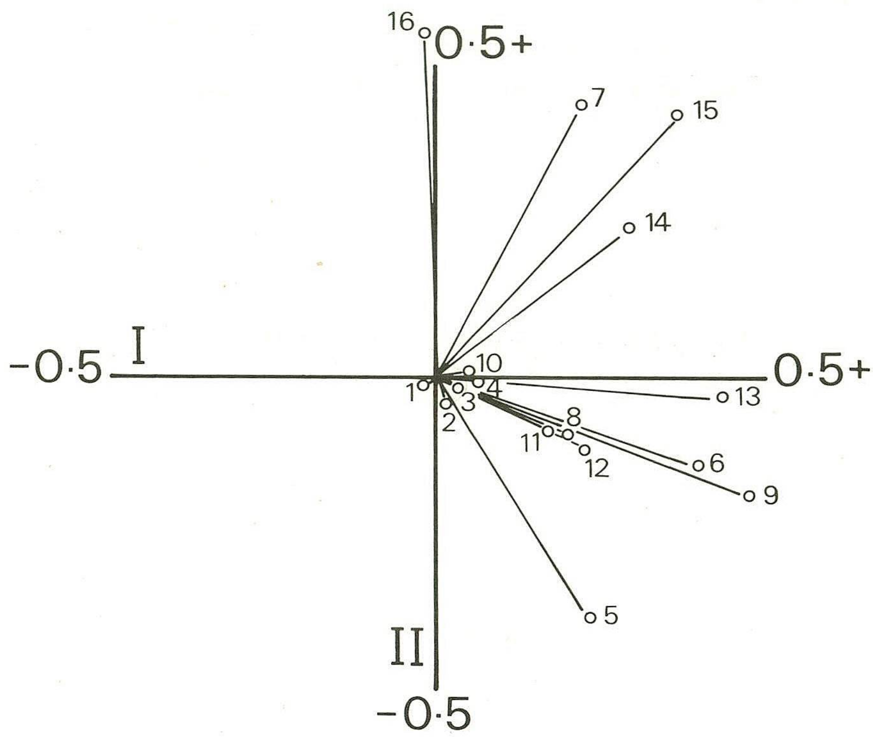


Fig 4.1 Weights of individual variables in first and second principal components.

Avon area. Tables 4.2 and 4.3 show summaries of the correlation matrix and principal component analysis of this set. Comparison of these tables shows that the variables dominant in each of the four major principal components are highly correlated; in a study relating floods to these variables, only one of each correlated set need be considered. Figure 4.1 shows the first and second component vectors; on this graph the close component vectors are collinear. The four size factors illustrate this. The first four principal components account for 92% of the variance and suggest the following groups of variables:

a Size. This includes basin area, mainstream length and total stream length. The shape parameter 'elongation' appears in this group because of its strong negative correlation with area.

b Slope. While basin slope has the highest weighting, stream slope, the hypsometric integral and drainage density are also represented.

c Geology. Percentage permeable rock dominates; the extent of dry valleys and the hypsometric integral also appear with geology.

d Drainage density.

This preliminary grouping of morphometric variables corresponds largely with that of Gray and the components are similar to those obtained by Wong. Representative indices of these groups were derived for the flood study. Retrospective analysis of catchment data for over 500 basins is discussed later.

iii Practical considerations

Advanced techniques for the analysis of maps are now available but are not yet widely used. Thus, the use of automatic cartography was restricted to the derivation of main channel lengths and slopes by d-mac digitiser. Even in this case the use of simple instruments like dividers was simulated in the programming. A careful choice of map scales and editions was made so that the basic information would be available not only to the flood study team but to users throughout the British Isles. There are no British maps of computed morphometric indices and all data must be measured from the original topographic sheets. There was an early attempt to encourage river authorities to measure morphometric variables for gauged catchments (Boulton, 1965) but only figures for catchment area are widely available. Since measured values of morphometric indices, especially of the stream network, are affected by the topographic map scale, the map usually chosen for deriving indices is that on which perennial streams are most completely and conveniently depicted. In Britain this is the Ordnance Survey 1:25 000 map. This map has the advantage over larger scales that both streams and contours are distinctively coloured. There are two main editions of the map, the First Series or Provisional Edition, and the Second Series. The latter has been compiled with photogrammetric aid and shows more channels than the former. If the Second Series becomes widely available, revision may be necessary but for this study measurements were made from the First Series (Provisional Edition). The availability of this edition is illustrated in Figure 4.2. For those parts of northern Scotland where 1:25 000 maps are not published, 1:10 560 (6 inch) maps reduced to 1:25 000 were consulted at the Map Research and Library Group of the Ministry of Defence at Tolworth. In Ireland the 1:126 720 ($\frac{1}{2}$ inch to

1 mile) coloured map showing streams and contours is widely available and convenient. However, the 1:63 360 (1 inch) map was used for network indices and a conversion factor derived by comparison between British 1:25 000 and 1:63 360 data. The effect of scale on other indices is less important than on network properties but the presence of 25 ft interpolated form line contours on the 1:25 000 map is useful for slope measurements in low lying catchments.

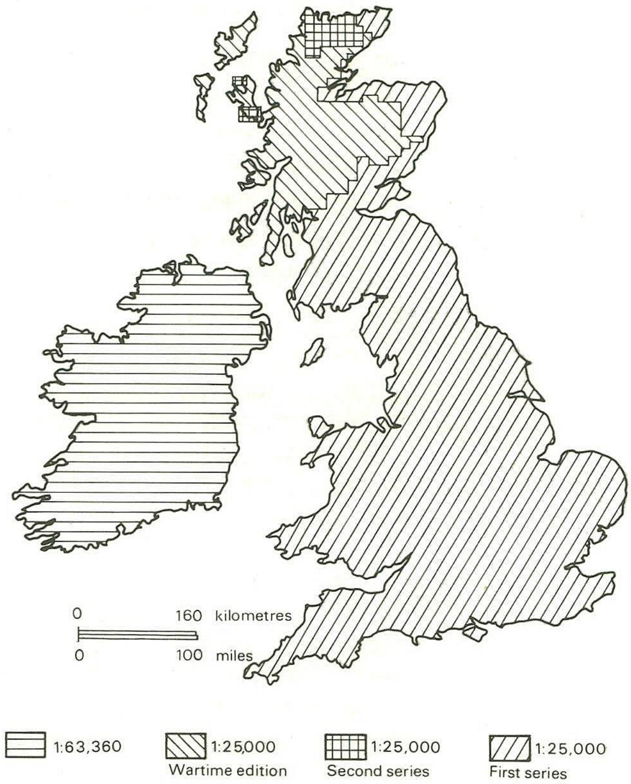


Fig 4.2 Availability of medium scale maps for deriving morphometric catchment characteristics.

Size. Size variables like stream length and area are highly correlated. However, since the computation of mainstream slope required length, both area and mainstream length were available.

The areas of most catchments were already available to three significant figures from the gauging authority or the Water Resources Board. Two other methods were used to measure areas and subareas; large areas were measured by the d-mac digitiser, while for small subareas (for example, urban areas or soils) squared paper overlays were used. In the measurement of mainstream length a simulated chord length of 0.1 km was used which can be replicated with dividers. Whereas there was little problem in defining catchments in Britain, detailed inspection of large scale maps was necessary in Ireland where large areas of indeterminate drainage exist. In Britain catchwaters for reservoired basins which transfer water across catchment boundaries need attention; such problems illustrate the need for a visual inspection of the catchment and site.

Slope. Basin overland slope can only be measured by grid sampling. Stream channel slopes can, however, be measured precisely. Benson (1959) has shown that only mainstream slope need be measured because there is a close correlation with tributary slopes. Strahler (1950) has demonstrated the correlation of channel and valley side slope from field measurements. The United States Geological Survey defines mainstream slope between the 10 and 85 percentiles of mainstream length (upstream from the gauging station) in order to exclude the highest and lowest gradients. Benson (1959) found that these percentiles gave the best prediction of mean annual flood. This involves less work than grid sampling, is more precise and is related to basin slope. There are several other indices of mainstream slope, notably that of Taylor & Schwarz (1952) which is based on the square root of gradients. Although this slope proved to be closely correlated with the U.S. Geological Survey index, both measures were obtained from the STREAMSLOPES program. Taylor/Schwarz slope (TAYSLO) uses the fact that velocity in each reach of a subdivided mainstream is related in the Manning equation to the square root of slope. The index is equivalent to the slope of a uniform channel having the same length as the longest watercourse and an equal time of travel.

Determination of equivalent mainstream slope

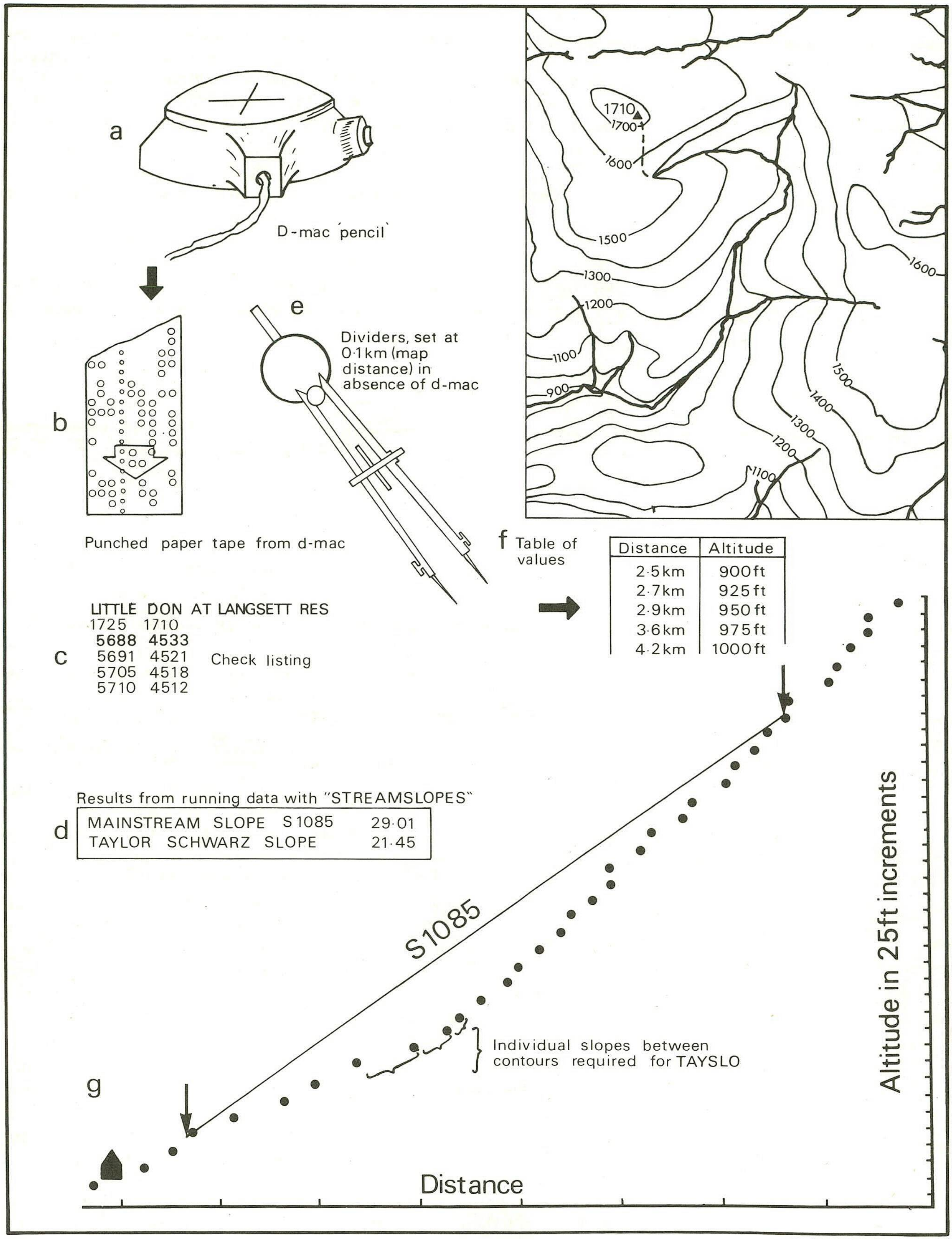
Elevation (ft)	Δ elevation (ft)	Length (ft)	S_i	$\sqrt{S_i}$	$1/\sqrt{S_i}$
515					
590	75	52 800	0 001420	0 0377	26 15
610	20	52 800	0.000379	0.0195	51.28
630	20	52 800	0.000379	0.0195	51.28
670	40	52 800	0.000758	0.0275	36.36
740	70	52 800	0.001328	0.0364	27.47
910	170	52 800	0.003220	0.0567	17.64
					210.54

TAYSLO = $\{\text{Number of equal length reaches}/\Sigma(1/\sqrt{S_i})\}^2 = (6/211)^2 = 0.00081$.

In the present study the stream was not divided up into equal lengths as advocated by Taylor & Schwarz. Because the distances between 25 ft contours were available, these were used for Taylor/Schwarz calculations. Instead of reaches of equal length and varying elevation, reaches of equal elevation and varying length were used; in practice there is little difference between the two methods. The calculation employs a total length of the stream instead of a number of equal reaches; so that the corresponding expression becomes TAYSLO = $\{L/\Sigma(L_i/\sqrt{S_i})\}^2$. The U.S. Geological Survey slope (S1085) was calculated from d-mac digitiser output by the same program. The input data consist of closely spaced coordinates along the stream channel together with codes at appropriate intervals for the end of mainstream, station, and every contour. The operation is illustrated in Figure 4.3. Reach lengths between contours are calculated from the coordinates with a step of 0.1 km introduced to allow comparability with a convenient and accurate setting for dividers.

Users of the equations in this report are recommended to derive the raw data for both S1085 and TAYSLO as follows. Choose the mainstream from maps. In most cases this is simply the longest stream in the basin; in cases of difficulty, work upstream and at every junction follow the stream draining the larger area. Distances are measured upstream from the station with precision dividers set at 0.1 km (4 mm on the 1:25 000 map) and the distance to each contour is tabulated against its elevation. Once the total length to the end of the stream is known, the lengths and elevations of the 10 and 85% points are used to calculate slope; the units used are parts per thousand or metres per kilometre. Benson describes the

Fig 4.3 Guide to estimation of channel slope (S1085).



length as that 'along the main channel between the gage and the divide' which implies the length to the watershed. In the present study the channel was defined as the blue line on the 1:25 000 map.

The program gives not only mainstream length and two slope measures but also uses the distance from the end of the stream to the watershed to calculate the dry valley factor which is defined as the length between stream head and watershed divided by the mainstream length. The program also provides a graph of the channel profile. A map indicating S1085 values is given in Figure 4.4.

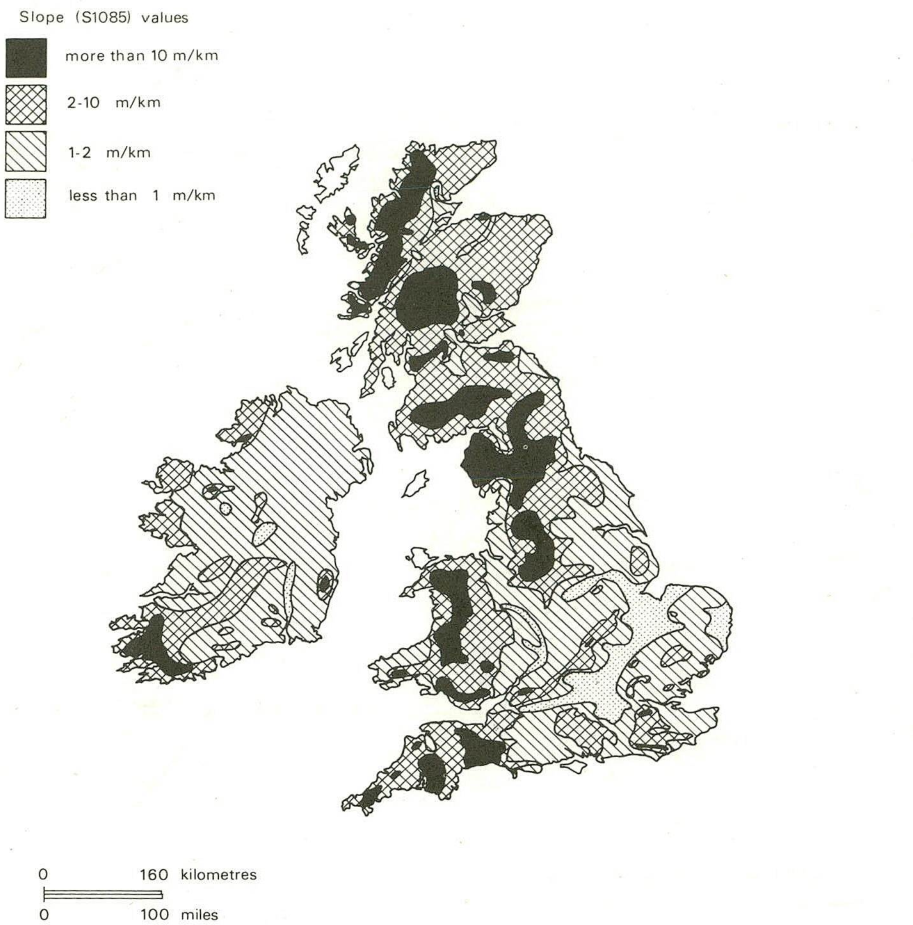


Fig 4.4 Sketch map indicating values of channel slope (S1085).

Channel network. Drainage density cannot be reliably sampled by grid or other methods. Its measurement requires the total channel length in the catchment which is divided by the catchment area. Because stream frequency is simpler to measure, use was made of the high correlation which it has with drainage density (Melton, 1958).

Stream frequency was simply measured by counting channel junctions on 1:25 000 maps and dividing by basin area. The precise technique is described as follows:

Once the necessary maps are assembled in a logical order, the stations or sites for estimation are marked to avoid duplication. The number of natural stream junctions is counted upstream from the lowest site, which is also included as a junction. It is best to work progressively up each tributary; the running total is noted at each major junction and at additional gauges. Artificial channels in fenland or flood plain situations and also canals are ignored.

Where natural channels exist, but are not shown on the map, for instance in urban areas, or where junctions occur in a lake or reservoir, the missing junctions are counted.

In catchments under 0.2 km^2 or at sites where the 1:25 000 series does not show a stream, the following procedure should be adopted to avoid exaggerated estimates: Move downstream to the nearest third order stream (see Figure 4.5) and measure the stream frequency of its basin.

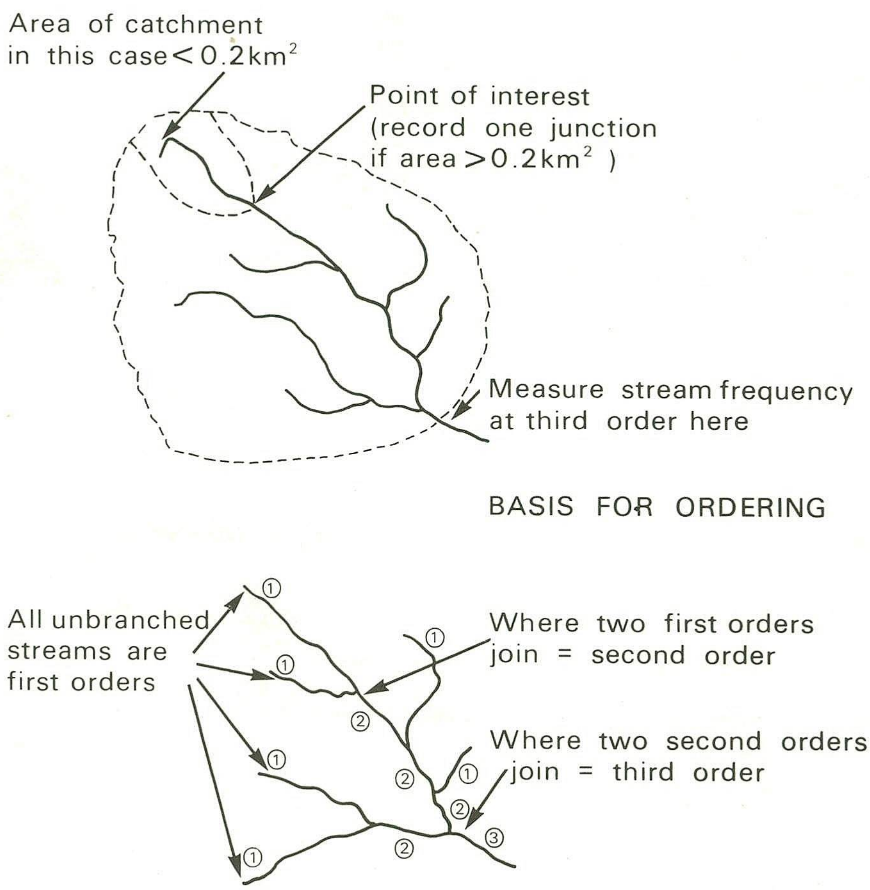


Fig 4.5 Measurement of stream frequency for very small catchments.

For the Republic of Ireland junctions were counted on the 1 inch map of the Irish Ordnance Survey and stream frequency was converted by a factor obtained from 55 British catchments (Figure 4.6) to equivalent 1:25 000 values. This conversion gives similar stream frequencies for the wetter parts of Ireland and Great Britain. Figure 4.7 can be used as a rough check on the stream frequencies obtained by the user of this report. For the two catchments in Northern Ireland 1:10 560 maps were used.

Where both First and Second Series 1:25 000 maps are available, the First Series should be used.

Storage index. Although an index of the channel or flood plain storage upstream of the gauging station should be a useful catchment characteristic

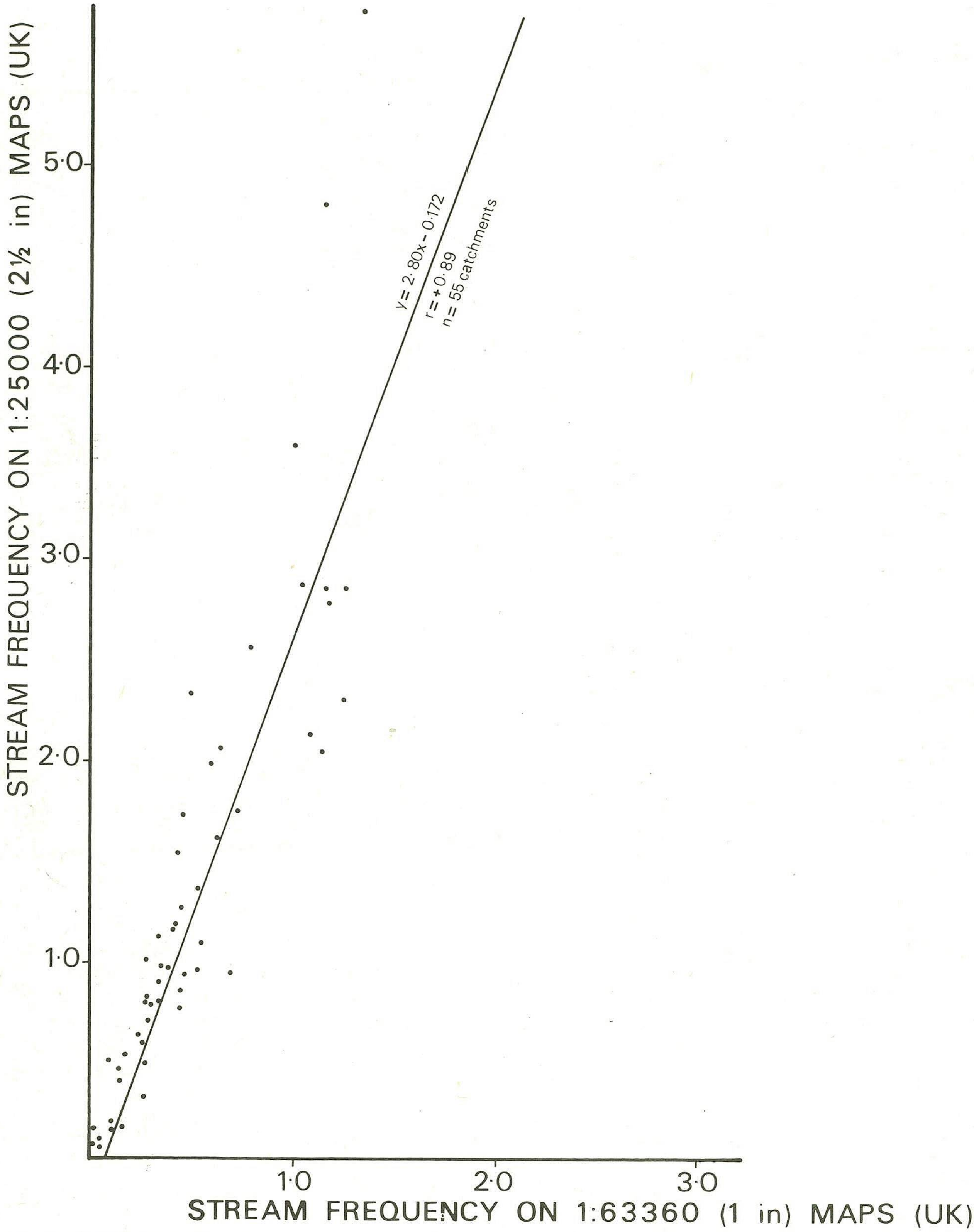


Fig 4.6 Relationship between stream frequency measurements from 1 : 25 000 and from 1 : 63 360 maps.

as a measure of flood attenuation, it was not possible to choose an index which could be measured from maps for a large number of sites. The flood

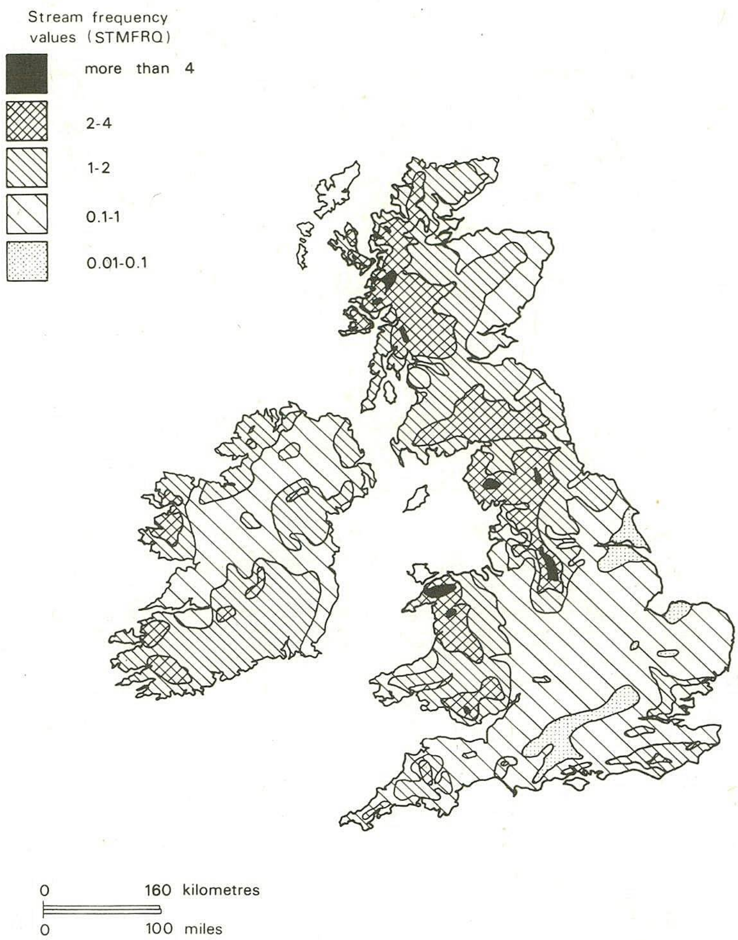


Fig 4.7 Sketch map indicating values of stream frequency (STMFRQ).

routing study (Volume III) has shown that accurate prediction of flood attenuation requires some knowledge of flow records together with map analysis. While the extent or width of flood plain might prove a useful independent variable, problems of definition and measurement prevented its use in the regression study. It is probable that channel slope is related to flood plain storage, but a storage index could be tested in further research.

Because a number of gauging stations contained lakes which would provide storage, an index of lakes and reservoirs was derived for use in the analysis. An index of the fraction of the catchment draining through lakes or reservoirs was derived from the 1:250 000 outline maps. For consistency certain rules were adopted. All the lakes or reservoirs whose surface area was less than 1% of the area contributing to that lake were ignored. In practice one follows up each tributary until one meets a lake or reservoir whose area is greater than 1% of the area contributing and records the contributing area. This is repeated on all other tributaries within the gauged catchment and all the contributing areas are totalled to give the total area contributing to lakes or reservoirs. This total contributing area is divided by the total area of the gauged catchment to give a lake index (LAKE). This index has the disadvantage, when used in a multiplicative equation, that the effect of increasing the lake fed fraction from 0.01 to 0.1 is the same as from 0.1 to 1.0. To overcome this and to provide an index which vanishes as a product when no lakes are present, the index was transformed from LAKE to (1 + LAKE) when used in regression analysis.

4.2.3 *Soils and land use*

Soils—the winter rain acceptance rate

It has been widely anticipated that one, as yet largely untried, catchment characteristic for flood prediction is related to the infiltration capacity of the soils on the catchment. While some qualitative idea of infiltration capacity can be obtained from maps of solid geology the situation is complicated in Britain and Ireland by the widespread deposits of glacial drift. While drift maps are fairly widely available it was considered that soil surveyors have the most complete information about the ground surface of catchments. The United States has gained a lead in the hydrological interpretation of soils through Musgrave (1955) and the Soils Conservation Service (1964). Later the minimum infiltration rates established for American soils were applied to the soils of England and Wales by Painter (1971).

Until now the reason for the absence of a national scale map of hydrological soil groups has been the somewhat sparse nature of completed soil surveys. However, to contribute to the soil map of Europe the Soil Surveys of England and Wales, Scotland, Northern Ireland and the Republic of Ireland have interpolated between existing surveys and produced national soil maps at a scale of 1 : 1 000 000. From these the regional surveyors were asked to interpret the soil classification in terms of 'winter rain acceptance' according to a scheme drawn up by D. Mackney, P. D. Smith and A. J. Thomasson of the Soil Survey of England and Wales. Since few measurements are available a purely ordinal scale was preferred to an interval scale based on measurements abroad. Thus, the approach is similar to Chiang & Peterson (1970) rather than Painter.

The criteria for the classification are as follows:

Winter rain acceptance is broadly infiltration potential and the reverse of runoff potential. It is favoured by combinations of high permeability, low groundwater levels, and level or gently sloping terrain. Rapid movements of water, likely to cause severe flooding during and after heavy rain, tend to occur near or over the soil surface; movements through deeper soil, or permeable geological deposits downward or horizontally, tend to affect streamflow later, or not at all.

Five classes of winter rain acceptance are depicted on the map, Figure 4.18 in Volume V:

Class	Acceptance (infiltration potential)	Runoff
1	Very High	Very low
2	High	Low
3	Moderate	Moderate
4	Low	High
5	Very low	Very high

The guidelines for allotting classes are based on properties of soils more or less at field capacity condition (see Table 4.4). Upland and peaty soils, despite their special hydrological properties, were placed in Class 5, awaiting information on the residual variance from regression analyses to decide whether a Class 6 was justified. This was not adopted for the present study.

Property		Classes
A Drainage group	1	Rarely waterlogged within 60 cm at any time (well and moderately well drained)
	2	Commonly waterlogged within 60 cm during winter (imperfect and poor)
	3	Commonly waterlogged within 60 cm during winter and summer (very poorly drained)
B Depth to 'impermeable' layers	1	>80 cm
	2	80–40 cm
	3	<40 cm
C Permeability group (above 'impermeable' layers or to 80 cm)	1	Rapid
	2	Medium
	3	Slow
D Slope	1	0–2°
	2	2–8°
	3	>8°

Table 4.4 Classification of soil factors.

Having decided all four parameters, Table 4.5 was used to reach the index of 'winter rain acceptance'.

Table 4.5 The classification of soils by winter rain acceptance rate from soil survey data.

Drainage class	Depth to impermeable layer (cm)	Slope classes: 0 - 2°			2 - 8°			>8°		
		Permeability rates above impermeable layers								
		Rapid (1)	Medium (2)	Slow (3)	Rapid (1)	Medium (2)	Slow (3)	Rapid (1)	Medium (2)	Slow (3)
1	>80	1	1	2	1	2	2	1	2	3
1	40 - 80	1	1	2	2	2	3	3	3	4
1	<40	—	—	—	—	—	—	—	—	—
2	>80	2	2	3	3	3	4	—	4	5
2	40 - 80	2	3	3	3	4	4	4	4	5
2	<40	3	4	4	4	4	4	4	5	5
3	>80	4	4	5	5	5	5	—	5	5
3	40 - 80	4	5	5	5	5	5	—	5	5
3	<40	5	5	5	5	5	5	5	5	5

Winter rain acceptance indices: 1, very high; 2, high; 3, moderate; 4, low; 5, very low. Upland peat and peaty soils are in Class 5. Urban areas are unclassified.

Geological and landscape considerations

In some areas soil properties do not give acceptance rate indices which correlate well with actual catchment behaviour. Thus, plateau soils in permeable geological formations—chalk, carboniferous limestone, oolite limestone—may receive a classification higher than Class 1. Ponding may occur during and immediately after heavy rain, but water normally either sinks slowly into the profile, or moves laterally to more permeable sites and enters the soil. In neither case is there much opportunity for overland flow moving directly to a permanent stream. For these reasons such areas are given a winter rain acceptance index of 1. In the case of chalky boulder clay over chalk, integrated ditch systems reduce the effect of the permeable substratum, and a normal grading is satisfactory.

Intensively drained fenland, with pumps and/or tidal sluices, should not normally have an acceptance index above 3. Such areas may occasionally be flooded, but do not contribute to flooding elsewhere. Undrained clayey alluvium (e.g. Fladbury Series) should normally be in either index 4 or 5.

Consideration may also be given in hill areas to the distance to permanent watercourses. A close stream pattern favours low acceptance, a widely spaced pattern favours higher acceptance.

Land use

The maps compiled by the Land Use Survey of Great Britain and the tabulated data of the Agricultural Census and the Ministry of Agriculture's Survey of Land Use Capability were examined for possible use in flood studies. It was, however, decided that forestry and urban development were the most likely land use classes to affect floods. Accordingly, the Forestry Commission and the Department of the Environment were asked to provide recent 1:625 000 maps of these areas. The percentage forest cover for each catchment was used experimentally in regressions with mean annual floods for north west England but did not prove useful. However, early trials showed the significance of the fraction of each catchment under 'built-up area' which is the term used by the Department of the Environment and the Scottish Development Department. The urban fraction of each catchment was measured by using a catchment overlay on the 1:625 000 map. It is realised that this is a crude index but some account must be taken of the unknown effects of urbanisation. It should be stressed, however, that the drainage of urban areas and the effect on runoff of urban development were not investigated in this study; the aim was to test whether flood estimates for catchments with some urban development would be improved by allowing for the urban fraction. As with the lake index, the urban index was transformed for use in regression by adding 1 to the urban fraction.

Certain additional factors were measured for the 150 catchments used in unit hydrograph analyses; these are described in Volume IV, Chapter 3.

4.2.4 Climate characteristics

Besides physical catchment characteristics (Sections 4.2.2 and 4.2.3) climate characteristics were also required for use in predicting equations. Previous studies have made use of mean annual rainfall (for example, Nash & Shaw, 1965) and this was provided by the Meteorological Office for every catchment for the 1916–50 standard period. In order to test climate characteristics more directly related to high flows (Rodda, 1967) a short period rainfall of a given return period was extracted, in this case 2 day 5 year rainfall or M52D (Figure II.3.2). A further step was to take account also of the antecedent conditions that control the portion of the rainfall which runs off.

Experience with conceptual models has shown that, in Britain, runoff volume can be forecast with fair accuracy by subtracting the antecedent soil moisture deficit from storm rainfall (Mandeville *et al.*, 1970). In this study the excess of a given return period was estimated for all catchments as an index of potential runoff, rainfall excess (Q) being given by the formula

$$Q = R - \text{SMD} \quad (R \geqslant \text{SMD}) \tag{4.1}$$

where R is daily rainfall and SMD is soil moisture deficit.

Data

The Meteorological Office provided daily rainfall frequencies by months for 400 long term rainfall stations over the United Kingdom; data for 13 Irish stations were obtained from the Irish Meteorological Service.

The Meteorological Office calculate SMD for a number of rainfall stations in Great Britain as a regular service (Grindley, 1970); this analysis was extended in Scotland and Northern Ireland for this study, and further

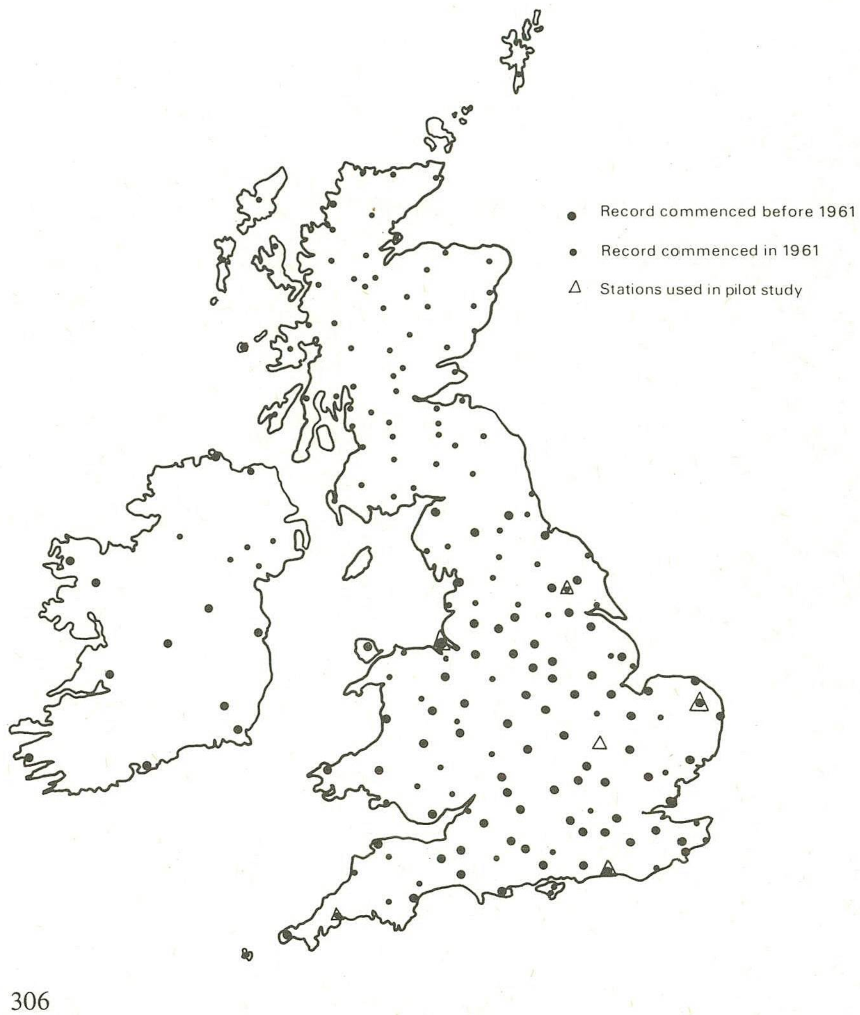


Fig 4.8 Location of soil moisture deficit stations.

stations were analysed by the Irish Meteorological Service. The locations of 200 stations used in this study are shown in Figure 4.8. End-of-month values have been separately tabulated for each station's entire period of record.

SMD is calculated from a water balance between daily rainfall and a Penman estimate of actual transpiration, assuming a notional catchment under short rooted vegetation (50 %), long rooted vegetation (30 %) and riparian areas (20 %), and estimates of the root constant for these three zones. Rainfall is added to the soil moisture reservoir, and the potential evaporation is subtracted when the deficit lies between zero and the root constant. When the deficit is zero, the excess rainfall is assumed to run off, and when the deficit exceeds the root constant evaporation is assumed to occur at a reduced rate. Thus, the assumptions behind the computed SMD figures conform well to the assumptions behind the rainfall excess model.

Rainfall–SMD dependence

While it would be possible to study the rainfall excess directly by considering only sites and periods where the two data sources coincide, this would severely limit the number of sites and lose the benefit of the great number and length of the rainfall records. The alternative approach adopted in this study was to describe rainfall and SMD separately with suitable statistical distributions and to combine the two to deduce the distribution of their difference. Unfortunately, rainfall and antecedent SMD are correlated, and a pilot study was undertaken to determine how to render them independent; and also to investigate whether the readily available end-of-month data could be substituted for the SMD on the days of rainfall, which would require extraction from lineprinter output. Data were obtained from six paired sites representing a variety of climatic conditions (Figure 4.8).

Possible differences between end-of-month and wet day data and correlation between rainfall and SMD were felt to be due to

a serial correlation between daily rainfall totals (climatic persistence) which results in rainfall towards the end of a wet spell encountering a smaller than characteristic SMD;

b a shared seasonal variation, SMD being strongly seasonal with high deficits in summer and low in winter, with rainfall also displaying a tendency to summer maxima.

There were significant differences between end-of-month and wet day deficits when rainfalls above a low threshold were considered on an annual basis. However, the effect of rainfall persistence was removed when the threshold was raised to include only storm events exceeded on average 10 times a year, and the tendency for high rainfalls to be associated with large deficits was eliminated by dividing the year into seasons. This pilot study indicated that end-of-month SMD data could adequately represent the seasonal distribution of the SMD values actually experienced on days of heavy rain.

Distribution fitting

a Rainfall. Having verified that the seasonal rainfall and SMD could be separately treated, the next stage was to study their statistical distributions. It was proposed initially to fit the gamma distribution to daily rainfall totals greater than the given threshold, but the monthly data of the pilot study stations proved consistent with the exponential distribution (Beran & Sutcliffe, 1972). This distribution was assumed in subsequent analysis with considerable computational advantage.

The parameters required for a complete description of the seasonal rainfall distribution were the threshold rainfall (r_0) and the proportion of days (p) within each season when the rainfall exceeded r_0, as well as the exponential parameter β. These were estimated directly from the data, and their seasonal values were mapped in order to interpolate to SMD stations.

The probability that any given rainfall r will be exceeded is

$$PR(r) = \exp\{-(r-r_0)/\beta\}.p \qquad (4.2)$$

and the probability that any given N days contains at least one exceedance is

$$PR_{\mathrm{M}}(r) = 1-\{1-PR(r)\}^N \qquad (4.3)$$

where $PR(r)$ is the probability of an event drawn from the 'exceedance' series and $PR_{\mathrm{M}}(r)$ is the probability in the 'maximum' series (Chapter 2).

b Soil moisture deficit. Soil moisture deficit is strongly seasonal, with low deficits predominating in winter and higher deficits in summer. It was intended to fit a beta distribution as the simplest of the common distributions capable of displaying the observed variety of skewness and shape. However, the analysis described below showed that the raw SMD could be used; this avoided all need to smooth these data through fitting a statistical distribution to the SMD values (s).

The combination of rainfall and soil moisture deficit

a Theory. The density function of the difference between two independent variables can be deduced using the convolution theorem. Thus, given that $k>0$, and replacing the double inequality ($k<r-s<k+\delta k$) for typographic convenience by the equality ($r-s = k$), we may write that

$$\begin{aligned} PR\,(r-s = k) &= \int_0^\infty PR\,(r = k+a)\, PR\,(s = a)\,\mathrm{d}a \\ &= \int_0^\infty \frac{1}{\beta} e^{-((k+a)/\beta)}\, PR\,(s = a)\,\mathrm{d}a \\ &= \frac{1}{\beta} e^{-k/\beta} \int_0^\infty e^{-a/\beta}\, PR\,(s = a)\,\mathrm{d}a \\ &= PR\,(r = k) \int_0^\infty e^{-s/\beta} f(s)\,\mathrm{d}s. \end{aligned} \qquad (4.4)$$

Because the integral term is a constant 'c' which is independent of k, it follows that if PR $(r = k)$ is exponential, then PR $(r-s = k)$ is also exponential with the same slope. Also the probability of the gross rainfall exceeding any given amount can be converted to the probability of the excess rainfall exceeding the same amount by multiplying it by the constant factor 'c', which is independent of the probability.

It follows from the geometry of the exponential curve that the difference between the gross rainfall and the rainfall excess of the same return period, R_T and $(R-\text{SMD})_T$, is a constant which is independent of the particular return period. In other words for all return periods T, $(R-\text{SMD})_T = R_T - d$, where d is a constant weighted mean SMD with greater weight attached to small deficits and the weighting dependent on β,

$$d = -\beta \ln(c). \tag{4.5}$$

b Preparation of rainfall excess statistics. As the theoretical results arise only from the properties of the exponential rainfall distribution and are independent of the form of the SMD distribution, monthly values of c and hence d were calculated from the raw data of each station using

$$c = \sum_{i=1}^{n} \exp(-s_i/\beta).s_i$$

$$d = -\beta \ln(c).$$

The s_i were the n SMD values using for each month the current and preceding end-of-month values, and the values of β were interpolated in the appropriate seasonal map.

Table 4.6 Daily probability of exceeding given rainfall and rainfall excess. Station—Cheltenham, threshold rainfall—12.6 mm.

Month	β (mm)	p (%)	c (%)	d (mm)		% probability of exceedance of stated values of k (mm)					
						20	30	40	50	60	80
Jan.	5.70	3.20	75.36	1.61	$PR(r)$	0.874	0.151	0.026	0.005	0.001	0.000
					$PR(k)$	0.658	0.114	0.020	0.003	0.001	0.000
Feb.	5.70	3.20	69.61	2.06	$PR(r)$	0.874	0.151	0.026	0.005	0.001	0.000
					$PR(k)$	0.608	0.105	0.018	0.003	0.001	0.000
Mar.	3.80	1.80	38.22	3.66	$PR(r)$	0.257	0.018	0.001	0.000	0.000	0.000
					$PR(k)$	0.098	0.007	0.001	0.000	0.000	0.000
Apr.	3.80	1.80	18.87	6.34	$PR(r)$	0.257	0.018	0.001	0.000	0.000	0.000
					$PR(k)$	0.048	0.003	0.000	0.000	0.000	0.000
May	7.70	3.30	13.13	15.64	$PR(r)$	1.26	0.344	0.094	0.026	0.007	0.001
					$PR(k)$	0.166	0.045	0.012	0.003	0.001	0.000
June	7.70	3.30	5.06	22.97	$PR(r)$	1.26	0.344	0.094	0.026	0.007	0.001
					$PR(k)$	0.064	0.017	0.005	0.001	0.000	0.000
July	7.70	3.30	0.96	35.74	$PR(r)$	1.26	0.344	0.094	0.026	0.007	0.001
					$PR(k)$	0.012	0.003	0.001	0.000	0.000	0.000
Aug.	7.70	3.30	0.46	41.46	$PR(r)$	1.26	0.344	0.094	0.026	0.007	0.001
					$PR(k)$	0.006	0.002	0.000	0.000	0.000	0.000
Sept.	6.50	3.80	3.29	22.18	$PR(r)$	1.22	0.261	0.056	0.012	0.003	0.000
					$PR(k)$	0.040	0.002	0.000	0.000	0.000	0.000
Oct.	6.50	3.80	8.51	16.02	$PR(r)$	1.22	0.261	0.056	0.012	0.003	0.000
					$PR(k)$	0.104	0.022	0.005	0.001	0.000	0.000
Nov.	5.70	3.20	26.01	7.68	$PR(r)$	0.874	0.151	0.026	0.005	0.001	0.000
					$PR(k)$	0.227	0.039	0.007	0.001	0.000	0.000
Dec.	5.70	3.20	55.08	3.40	$PR(r)$	0.874	0.151	0.026	0.005	0.001	0.000
					$PR(k)$	0.481	0.083	0.014	0.002	0.000	0.000
Year					$PR(r)$	0.959	0.212	0.050	0.012	0.003	0.000
					$PR(k)$	0.208	0.037	0.007	0.001	0.000	0.000

The probability $PR(k)$ of exceeding any rainfall excess k was then calculated from

$PR(k) = c.PR(r)$.

These calculations were made for each month of the year for each SMD station; Table 4.6 shows the results for Cheltenham. For this station the daily rainfall exceeded on average 10 times per year was 12.6 mm. Fewer threshold exceedances occur in March and April than during the rest of the year; this season has a low β and thus the steepest slope of any season's rainfall distribution. Thus, the probability that any March day has a rainfall exceeding 20 mm is 0.257 % while the corresponding probability for July is 1.26 %.

However, the multiplying factor c to obtain rainfall excess probabilities from rainfall probabilities is much larger in winter (e.g. 38.2 % in March) than in summer, when high deficits eliminate all but the rarest rainfalls. The corresponding weighted mean SMD, or the difference between rainfall and rainfall excess is also shown; this varies from less than 2 mm in winter to over 40 mm in summer. Thus, the probability of a given rainfall excess is much greater in winter (e.g. 0.098 % in March) than in summer.

The average of the 12 monthly exceedance probabilities gave the yearly exceedance probabilities for rainfall and rainfall excess. Table 4.6 shows that over a period of years 0.212 % of days experience a rainfall of 30 mm, while 0.208 % experience a rainfall excess of 20 mm.

c *Estimation of annual mean soil moisture deficit.* The daily probabilities of Table 4.6 were converted to annual probabilities, and interpolation gave the rainfall and rainfall excess that is exceeded with any given probability as shown in Table 4.7. The mean SMD values were calculated from the differences between rainfall and rainfall excess at specific exceedance probabilities.

Table 4.7 shows that rainfall and rainfall excess diverge with increasing return period, because the difference at low return periods gives greater weight to winter d values than at high return periods where summer values have more influence. However, examination of many sites showed that the difference between rainfall and rainfall excess at the same probability varied only very slowly over a large probability range, and that the value at the 5 year return period level could be taken as representative over the range from the mean annual to the 100 year event. This value could be viewed as an effective soil moisture deficit representing typical antecedent conditions at any site. A map was prepared of effective SMD over the British Isles (Figure 4.19 in Volume V, of which Figure 4.9 is a smaller version).

Table 4.7 Rainfall, rainfall excess and mean SMD corresponding to various probabilities. Station—Cheltenham.

Exceedance probability (%)	Return period (years)	Rainfall (mm)	Rainfall excess (mm)	Mean SMD (mm)
50	2	32.8	22.4	10.4
20	5	39.3	27.7	11.6
10	10	44.3	31.8	12.5
5	20	49.2	36.0	13.2
4	25	50.8	37.3	13.5
2	50	55.8	41.5	14.3
1	100	60.8	45.8	15.0
0.5	200	65.9	50.1	15.8
0.2	500	72.6	55.9	16.7
0.1	1000	77.8	60.3	17.5

The use of effective SMD

The average effective SMD (SMDBAR) was measured for each catchment from Figure 4.19, and used in its own right as a catchment climate characteristic. It was combined with short term rainfall in a second variable—the 5 year return period 1 day rainfall excess (RSMD). The 2 day rainfall for each catchment (M52D) was converted to 1 day rainfall (Volume II, Chapter 3), corrected for catchment area using the areal reduction factor (Volume II, Chapter 5), and finally the average effective SMD was subtracted to give the 5 year return period 1 day rainfall excess. Both SMDBAR and RSMD are given for each catchment in the list of catchment characteristics in Volume IV, Chapter 5.

Fig 4.9 Effective soil moisture deficit (mm) for conversion of daily rainfall to rainfall excess (see Figure 4.19 in Volume V for larger scale version).

4.2.5 *Measurement from maps of catchment characteristics requiring fractions or average values*

The derivation of urban, lake and soil indices for a catchment, or the estimation of an average rainfall or climate index, requires the transposition of the catchment outline from a topographic map and the measurement of subarea fractions or areas between isopleths on a suitable map.

The lake and urban fractions were derived from the 1:250 000 Ordnance Survey 'water' sheets which also show settlements and roads in feint. The soil and climate maps (Volume V) have been drawn on the 1:625 000 scale at which standard topographical maps, including the Water Resources Board map of gauging sites, show relief and streams and make delimitation of the catchment quite accurate. Areas can be measured by planimeter, or by counting squares on transparent millimetre square grids which is as accurate for small irregular areas. When subareas have been converted by the appropriate scale, fractions or catchment averages can be calculated.

4.2.6 *Comparison of measured catchment variables*

Further analysis of the relationships between catchment characteristics followed their measurement for over 500 basins in Great Britain and Ireland.

Derivation of soil index

Before these catchment characteristics were used in analysis, a single numerical soil index for each catchment was derived. The fraction of each catchment within each soil class was measured from an early edition of the soil map (Figure 4.18). The five soil class fractions are listed in Volume IV. Because the map has been revised, these fractions may differ from those derived from the published map. In some cases the five fractions do not add up to 1, where the catchment includes unclassified soils in major urban areas or in large lakes. The five fractions were tested individually as independent variables in regressions of mean annual flood on catchment characteristics; this produced a progression of exponents for the five variables which was consistent with the constants derived in a regression of percentage runoff on the individual soil fractions for catchments used in unit hydrograph analysis (Section 6.5.7). A weighted mean of the soil fractions was adopted as a soil index; this index—$(0.15S_1+0.30S_2+0.40S_3+0.45S_4+0.50S_5)/(S_1+S_2+S_3+S_4+S_5)$—has a range from 0.15 to 0.50. This index assumes that unclassified soils are distributed in the same proportion as classified soils within each catchment; in a small number of cases where a large proportion of a catchment was unclassified the soil index was individually assessed.

Correlation structure

A correlation matrix based on 533 stations is shown in Table 4.8. The highest correlation coefficients among the morphometric variables are between the two measures of mainstream slope, but slope and area are

Table 4.8 Correlation matrix of log transformed catchment characteristics for 533 catchments in Great Britain and Ireland.

	BESMAF	CV	AREA	STMFRQ	TAYSLO	S1085	SOIL	URBAN	LAKE	SAAR	RSMD	M52D	SMDBAR
BESMAF	1.000	−0.172	0.750	0.424	−0.280	−0.180	0.415	−0.085	0.085	0.397	0.394	0.376	−0.397
CV	−0.172	1.000	−0.069	−0.240	0.001	−0.065	−0.130	0.068	−0.095	−0.341	−0.300	−0.243	0.377
AREA	0.750	−0.069	1.000	−0.076	−0.717	−0.675	−0.022	−0.088	0.106	−0.122	−0.122	−0.135	−0.007
STMFRQ	0.424	−0.240	−0.076	1.000	0.345	0.410	0.592	−0.179	0.139	0.603	0.567	0.545	−0.500
TAYSLO	−0.280	0.001	−0.717	0.345	1.000	0.888	0.242	0.042	−0.171	0.390	0.429	0.444	−0.192
S1085	−0.180	−0.065	−0.675	0.410	0.888	1.000	0.309	0.014	−0.104	0.514	0.548	0.560	−0.289
SOIL	0.415	−0.130	−0.022	0.592	0.242	0.309	1.000	−0.094	0.150	0.478	0.450	0.456	−0.368
URBAN	−0.085	0.068	−0.088	−0.179	0.042	0.014	−0.094	1.000	−0.087	−0.213	−0.210	−0.197	0.204
LAKE	0.085	−0.095	0.106	0.139	−0.171	−0.104	0.150	−0.087	1.000	0.252	0.218	0.177	−0.273
SAAR	0.397	−0.341	−0.122	0.603	0.390	0.514	0.478	−0.213	0.252	1.000	0.928	0.915	−0.809
RSMD	0.394	−0.300	−0.122	0.567	0.429	0.548	0.450	−0.210	0.218	0.928	1.000	0.952	−0.785
M52D	0.376	−0.243	−0.135	0.545	0.444	0.560	0.456	−0.197	0.177	0.915	0.952	1.000	−0.685
SMDBAR	−0.397	0.377	−0.007	−0.500	−0.192	−0.289	−0.368	0.204	−0.273	−0.809	−0.785	−0.685	1.000

inversely related. Among the climate variables, mean annual rainfall is closely linked to short term rainfall. The rainfall variables are linked with stream frequency and channel slope, while the soil index is related to stream frequency.

Principal component analysis

A selection of the same set of logarithmically transformed catchment characteristics was subjected to principal component analysis. The first two components are dominant (see Table 4.9). The weights of the original variables are shown in Table 4.9. The first component is dominated by those variables which are high in wet upland Britain. The next four components are dominated in turn by size, urban fraction, lake index, and by soil and climate.

Table 4.9 Principal component analysis of logarithmically transformed catchment characteristics for 533 catchments in Great Britain and Ireland.

Eigenvectors	Component No. 1	Component No. 2	Component No. 3	Component No. 4	Component No. 4	Component No. 6	Component No. 7
Variance	2.61677	1.54151	0.92411	0.86008	0.51698	0.37390	0.16665
AREA	0.25068	0.62026	0.04361	0.37060	0.38707	0.07856	0.50715
STMFRQ	−0.49125	0.20439	0.03616	0.27014	−0.04750	−0.79880	−0.04769
S1085	−0.47426	−0.44002	−0.04460	−0.10076	0.12842	0.09129	0.73789
SOIL	−0.43204	0.23762	0.20694	0.39108	−0.55373	0.50320	−0.03596
URBAN	0.13298	−0.34338	0.86395	0.25853	0.21117	−0.05196	−0.06209
LAKE	−0.11049	0.43306	0.45294	−0.74294	−0.14023	−0.05225	0.14403
RSMD	−0.50433	0.13402	−0.02029	−0.08826	0.67865	0.29796	−0.41248

4.3 Regressions of flood statistics on catchment characteristics

4.3.1 Introduction

In this section an attempt is made to derive equations relating useful flood statistics to catchment characteristics. If a particular statistical distribution had been shown to be most suitable to describe flood frequencies, it would have been appropriate and convenient to try to relate the two or three parameters of this distribution to catchment characteristics. In the absence of such a finding (Chapter 2), it is preferable to choose distribution free measures such as the mean annual flood and either its standard deviation or the coefficient of variation. As it turns out, useful relationships were not found between the coefficient of variation and catchment characteristics, and the mean annual flood is probably the most useful flood statistic. Although this flood has a low return period (2.33 years if the annual floods follow a Gumbel distribution), it can be used to scale the regional frequency curves of Chapter 2 in order to permit the estimation of floods of any return period.

The method used to derive the equations is multiple regression. In this method a linear equation is formed, the coefficients of which are chosen so as to minimise the sum of squares of differences between the measured and estimated mean annual floods. Multiple regression is an obvious extension of the simple regression used to fit straight lines to experimental data. The theory of multiple regression will not be discussed here; the interested reader is referred to statistical texts such as Draper & Smith (1966).

Although the equations are linear, that is of the form

$$y = b_0 + b_1 x_1 + b_2 x_2 + \dots, \tag{4.6}$$

the variables may be transformed before use. In particular the common logarithms of the variables are often used in regressions for hydrological purposes. The equation could then be written as

$$\log y = b_0 + b_1 \log x_1 + b_2 \log x_2 + \dots \tag{4.7}$$

or alternatively

$$y = 10^{b_0} x_1^{b_1} x_2^{b_2} \dots \tag{4.8}$$

This transformation is appropriate when the error of estimation of y approximates a constant percentage for all observations. In order to define some terms used later in this section consider the following regression equation

$$\log y = b_0 + b_1 \log x_1 + b_2 \log x_2 + \varepsilon. \tag{4.9}$$

Here ε is called the *residual* and measures the difference between the measured $\log y$ and the value predicted by the equation. A measure of the 'average' size of residuals is the *standard error of estimate* defined as

$$s = \sqrt{\frac{\Sigma \varepsilon_i^2}{n-m-1}} \tag{4.10}$$

where n is the number of observations used in deriving the equation, m the number of variables on the right hand side of the equation called *independent variables*, and the sum extends over all observations. y, or more properly $\log y$, is the *dependent variable*. The regression equation is the same as

$$y = 10^{b_0} x_1^{b_1} x_2^{b_2} 10^{\varepsilon}. \tag{4.11}$$

10^{b_0} and 10^{ε} are, of course, the antilogarithms of b_0 and ε. 10^{ε} gives the factor by which the prediction must be multiplied to give the actual measurement. The antilog of the standard error is called the *factorial standard error*, and may be regarded as an 'average' of these factors. b_1 and b_2 are called *coefficients* or *exponents*; b_0 is referred to as the *constant term*, or the *intercept*, and its antilogarithm is called the *multiplier*.

The technique of multiple regression is a purely statistical technique. Although care is taken to choose independent variables on hydrological grounds the results must not be interpreted as implying some causative relation between the independent variables and the dependent variable. Difficulty can arise when a regression equation is applied to new data. It is obvious that an equation derived in Britain is unlikely to be of any use in an arid climate such as central Australia. However, less obvious differences of hydrological properties may cause errors in prediction. For example, a new catchment may involve a combination of independent variables completely different from those used to derive the equation. In this case errors in prediction may arise.

4.3.2 Organisation of data

The flood statistics and catchment characteristics required for regressions were stored in a computerised data bank called the Master List. This grew from a complete list of all the gauging stations with their positions, record dates and other details. The data occupied about half a million words of disc, though about a third of this space is free to allow for future expansion. The list is classified in two ways, by gauging station and by item. An item is a unit of information of varying length, examples being catchment area and mean annual flood; a station's data consist of a heading, containing station number, name and location, and values for the items or blank entries where the items are not available. The locations of stations used in analysis are shown in Figure 4.20, which is included in Volume V.

Two programs were written to manipulate the Master List, one to read data from the list and the second to write to it. The writing program was used to set values for items for stations on the list, to correct an item value on the list by overwriting with the correct value, and to add new stations. The reading program was used to provide a card file containing selected items for particular stations. Stations could be specified by individual station number or by ranges of station numbers. This file was read by other programs for further processing. Because of the large space occupied by the list, a copy of the disc file was held on magnetic tape and only written to the disc when it was needed. Before a run of the writing program, a second copy was made on another magnetic tape to guard against corruption of the Master List.

A program was provided to read the output from the reading program and to prepare data for the ICL Statistical Analysis Program, Mark 2. Facilities were provided in this program to select stations by grade, length of record and by value of certain variables. For example, A grade stations with 10–20 years of record and area under 500 km^2 could readily be chosen. The program checked each station's data for completeness and reported any missing items. Stations with incomplete data were excluded from the output.

The system took a considerable time to develop but the following advantages outweighed this:

i Particular combinations of stations or variables could readily be produced.

ii Data could be validated as they were added to the list and subsequent analyses could then be performed without fear of error.

iii Tabulations of data or lists of stations could easily be made for publication (see Volume IV).

4.3.3 Computations

All computations were made using the ICL Statistical Analysis Program, Mark 2 (International Computers Limited, 1969). This program provides facilities for a number of statistical analyses including multiple regression. In addition there are comprehensive facilities for transforming the data. Two multiple regression algorithms are available; optimal regression (Beale, Kendall & Mann, 1967) and stepwise regression (Efroymson, 1960).

In optimal regression, the number of independent variables required is specified and the program searches to find the set of independent variables that best describes the variation in the dependent variable. The stepwise method adds variables to the regression set (or removes them) on the basis of the t statistic of their estimated coefficient. The process stops when all variables are included which are significant at a specified level. The output produced by the program is the same in both cases; coefficients with their standard errors, the error sum of squares, the standard error of estimate and the multiple correlation coefficient. No analysis of variance of the regression is given but the information for performing one is readily available. The regression may be graduated; that is, a tabulation is made for each observation of the independent variable, the value predicted by the regression and the difference of these two or the residual from the regression. This tabulation was not felt to be sufficiently comprehensive so a program was written to take the data for the statistical analysis program and the regression equations and to produce statistics of the residuals, histogram plots and a listing of the data and results in order of the size of the residuals. One version of this program plotted the coded residuals on a lineprinter map of Great Britain. This feature could be of use in determining regions with uniform flood characteristics.

4.3.4 *Choice of preliminary independent variable set*

Several tests were made to derive a standard independent variable set. This set was needed at first for use in further experiments designed to decide such questions as:

i Which measure of the mean annual flood is best related to catchment characteristics?

ii Should all the data be used for estimating the equations or do short term low grade stations introduce too much error?

iii Can a division be made on the basis of catchment size?

iv Should all the data be treated as one or can regional divisions be made with advantage?

Following the discussion in Section 4.2 the following catchment indices were considered for use:

Size. AREA was the main size index considered (see Table 4.19 for definition of variable names).

Slope. The Taylor–Schwarz slope (TAYSLO) and the 10–85% slope (S1085) were both tried. S1085 is more easily calculated but as it depends on only two points on the profile it may be more subject to measurement error.

Drainage network. The stream frequency (STMFRQ) defined as the number of junctions per square kilometre of catchment was used.

Climate. Three rainfall indices were available. The annual average rainfall for the standard period 1916–50 (SAAR) is easily measured from published maps. This index, however, has the conceptual disadvantage that it is not directly related to short term hydrological events such as floods. The two other indices were developed to overcome these objections. The 2 day rainfall of 5 year return period (M52D), shown on a map published with this report (Figure II.3.2) is a more direct index of flood producing rainfall. The third index (RSMD) is described above

(4.2.4). These three indices were all found to be closely correlated, so only SAAR and RSMD were considered.

Soil. The proportion of the catchment covered by each class of soil was determined from the soil map (Figure 4.18) and a composite runoff index derived from the formula

$$\text{SOIL} = (0.15\,S_1 + 0.30\,S_2 + 0.40\,S_3 + 0.45\,S_4 + 0.50\,S_5)/(S_1 + S_2 + S_3 + S_4 + S_5)$$

where S_1, S_2, S_3, S_4, S_5 denote the proportions of the catchment covered by each of the soil classes 1–5. The factors in this formula were derived from a consideration of the runoff percentages in the events used in the unit hydrograph analyses (see Sections 4.2.6 and 6.5.7).

Land use. The proportion of the catchment in an urban area (URBAN) was used.

Lakes. LAKE, the proportion of the catchment draining through a lake (see Section 4.2.2) was used as the index of the effect of lakes.

There are only two items in this list which require a choice of index, slope and climate. Tests were made on all four combinations using the mean annual flood as independent variable. The mean annual flood was calculated from extended records, where they were available.

All variables were transformed by taking the common logarithm of the variable. URBAN and LAKE often have zero values so for these variables the logarithm of one plus the variable was used; thus if URBAN or LAKE is zero, the transformed variable is also zero. These transformations imply a multiplicative formula of the form

$$y = A_0\, x_1^{b_1}\, x_2^{b_2} \ldots,$$

which is derived by fitting

$$\log y = b_0 + b_1 \log x_1 + b_2 \log x_2 + \ldots,$$

$$(b_0 = \log A_0).$$

This form is dictated by the large range of values for most of the variables, particularly the flood variables, (mean annual flood 0.058–997 cumecs, AREA 0.048–9868 km^2, TAYSLO 0.26–94 etc.). All catchments with at least 5 years of record were used in the tests. The correlation matrix of the transformed data is given as Table 4.8. The original and transformed variables are usually referred to in this section by the same name, it being obvious from the context which form is intended.

Tables 4.10a, b, c and d show the results of these regressions. The optimal method was used in each case and the tables show the results of fitting the best set of one to seven independent variables. There are many similarities in these results. The standard error of estimate (see) reduces with an increasing number of independent variables at the same rate for all sets, as is also shown in Figure 4.10. AREA, STMFRQ and the climate index are always the first three variables to enter the set. When SAAR is used as the climate index, slope measure is the fourth variable; however, if RSMD is used the slope measures are higher than those of SAAR, but the differences are not pronounced. The soil variable enters fourth when RSMD is used; otherwise it enters fifth after the slope variable. URBAN and LAKE are less important but make a useful contribution to explaining the variation of the mean annual flood.

It is evident that there is little difference between the four sets of equations; the two measures of slope were equally successful and RSMD was slightly better than SAAR. It was decided for the present tests to use S1085 as the slope measure and SAAR as the rainfall measure. The full equation for this variable set is then:

$$\text{BESMAF} = 1.285\times 10^{-4}\text{AREA}^{0.96}\text{STMFRQ}^{0.34}\text{S1085}^{0.23}\text{SOIL}^{0.90}\text{URBAN}^{1.9}\text{LAKE}^{-1.1}\text{SAAR}^{1.18} \qquad (4.12)$$

The standard error of estimate for this equation is 0.194. The antilog of this standard error, the factorial error, is 1.56. This implies that in about two thirds of predictions by this equation the actual value will be in the range from −36% to +56% of the prediction. The 95% limits on a prediction are −58%, +140%. It will be seen that predictions from this equation are very crude and give little more than the order of magnitude of the answer.

When the residuals from these regression equations were examined, it was found that catchments with lakes had some of the largest negative residuals. (A negative residual means the measured mean annual flood is lower than the prediction.) This still holds when LAKE is used in the equation. It would appear, therefore, that the flood reducing effects of a lake are not fully accounted for by the index LAKE. An alternative procedure would be to discard from the analysis catchments with a large proportion of their area draining through a lake. This is a somewhat unsatisfactory procedure as flood estimates cannot then be made for

Table 4.10a Effect of S1085, SAAR combination on BESMAF regressions.

No. var.	Name	Coeff.	seb	t	R	R^2	see	Const.	Multiplier
1	AREA	0.77	0.030	26.1	0.750	0.562	0.436	−0.170	0.676
2	AREA	0.84	0.020	41.9	0.897	0.805	0.292	−6.620	2.399×10^{-7}
	SAAR	2.09	0.081	25.6	—	—	—	—	—
3	AREA	0.84	0.017	49.4	0.927	0.859	0.248	−4.344	4.529×10^{-5}
	STMFRQ	0.51	0.035	14.4	—	—	—	—	—
	SAAR	1.34	0.086	15.6	—	—	—	—	—
4	AREA	0.99	0.023	43.6	0.938	0.880	0.229	−3.836	1.459×10^{-4}
	STMFRQ	0.44	0.033	13.2	—	—	—	—	—
	S1085	0.35	0.036	9.5	—	—	—	—	—
	SAAR	0.99	0.088	11.2	—	—	—	—	—
5	AREA	0.85	0.015	57.4	0.946	0.895	0.214	−3.976	1.057×10^{-4}
	STMFRQ	0.35	0.034	10.4	—	—	—	—	—
	SOIL	1.00	0.099	10.0	—	—	—	—	—
	LAKE	−1.32	0.139	9.5	—	—	—	—	—
	SAAR	1.37	0.078	17.5	—	—	—	—	—
6	AREA	0.86	0.014	60.9	0.952	0.906	0.203	−4.291	5.117×10^{-5}
	STMFRQ	0.37	0.032	11.5	—	—	—	—	—
	SOIL	0.96	0.094	10.2	—	—	—	—	—
	URBAN	2.04	0.269	7.6	—	—	—	—	—
	LAKE	−1.30	0.132	9.8	—	—	—	—	—
	SAAR	1.45	0.075	19.4	—	—	—	—	—
7	AREA	0.96	0.020	49.2	0.956	0.914	0.194	−3.891	1.285×10^{-4}
	STMFRQ	0.34	0.031	10.8	—	—	—	—	—
	S1085	0.23	0.032	7.1	—	—	—	—	—
	SOIL	0.90	0.091	9.9	—	—	—	—	—
	URBAN	1.86	0.259	7.2	—	—	—	—	—
	LAKE	−1.05	0.131	8.0	—	—	—	—	—
	SAAR	1.18	0.081	14.7	—	—	—	—	—

Coeff. is the regression coefficient b_i in an equation of the form: BESMAF = multiplier. $\text{AREA}^{b_1}\,\text{SAAR}^{b_2}$; seb is the standard error of estimate of b; t is the Student's t statistic enabling the significance of b to be judged; R, R^2 are the coefficients of multiple correlation and determination of the relationship; see is the standard error of estimate of the relationship. The factorial standard error of estimate is the antilogarithm of the see; Const. is the intercept of the regression equation (b_0 of Section 4.3.1); Multiplier is the antilogarithm of 'const'. for use in the multiplicative form. All equations are optimal (Section 4.3.3).

Table 4.10b Effect of S1085, RSMD combination on BESMAF regressions.

No. var.	Name	Coeff.	seb	t	R	R^2	see	Const.	Multiplier
1	AREA	0.77	0.030	26.1	0.750	0.562	0.436	−0.170	0.676
2	AREA	0.84	0.020	41.6	0.896	0.803	0.294	−3.567	2.710×10^{-4}
	RSMD	2.08	0.082	25.3	—	—	—	—	—
3	AREA	0.84	0.017	50.4	0.930	0.865	0.243	12.407	3.917×10^{-3}
	STMFRQ	0.52	0.033	15.6	—	—	—	—	—
	RSMD	1.35	0.082	16.5	—	—	—	—	—
4	AREA	0.83	0.015	53.7	0.940	0.884	0.226	−1.797	1.596×10^{-2}
	STMFRQ	0.38	0.035	10.7	—	—	—	—	—
	SOIL	0.96	0.104	9.2	—	—	—	—	—
	RSMD	1.23	0.078	15.8	—	—	—	—	—
5	AREA	0.85	0.014	58.4	0.948	0.899	0.210	−1.952	1.117×10^{-2}
	STMFRQ	0.37	0.033	11.4	—	—	—	—	—
	SOIL	1.01	0.097	10.4	—	—	—	—	—
	LAKE	−1.23	0.136	9.1	—	—	—	—	—
	RSMD	1.34	0.073	18.3	—	—	—	—	—
6	AREA	0.86	0.014	62.1	0.954	0.910	0.199	−2.144	7.178×10^{-3}
	STMFRQ	0.39	0.031	12.7	—	—	—	—	—
	SOIL	0.98	0.092	10.6	—	—	—	—	—
	URBAN	2.06	0.264	7.8	—	—	—	—	—
	LAKE	−1.20	0.128	9.3	—	—	—	—	—
	RSMD	1.42	0.070	20.2	—	—	—	—	—
7	AREA	0.94	0.020	47.1	0.956	0.914	0.194	−2.093	8.072×10^{-3}
	STMFRQ	0.37	0.031	12.0	—	—	—	—	—
	S1085	0.18	0.034	5.4	—	—	—	—	—
	SOIL	0.93	0.091	10.3	—	—	—	—	—
	URBAN	1.90	0.259	7.3	—	—	—	—	—
	LAKE	−1.01	0.130	7.7	—	—	—	—	—
	RSMD	1.19	0.081	14.7	—	—	—	—	—

For explanation of headings see footnote to Table 4.10a.

Table 4.10c Effect of TAYSLO, SAAR combination on BESMAF regressions.

No. var.	Name	Coeff.	seb	t	R	R^2	see	Const.	Multiplier
1	AREA	0.77	0.030	26.1	0.750	0.562	0.436	−0.170	0.676
2	AREA	0.84	0.020	41.9	0.897	0.805	0.292	−6.620	2.400×10^{-7}
	SAAR	2.09	0.081	25.6	—	—	—	—	—
3	AREA	0.84	0.017	49.4	0.927	0.859	0.248	−4.344	4.529×10^{-5}
	STMFRQ	0.51	0 035	14.4	—	—	—	—	—
	SAAR	1.34	0.086	15.6	—	—	—	—	—
4	AREA	1.01	0.023	44.0	0.940	0.884	0.226	−4.194	6.397×10^{-5}
	STMFRQ	0.44	0.033	13.3	—	—	—	—	—
	TAYSLO	0.39	0.038	10.4	—	—	—	—	—
	SAAR	1.09	0.083	13.1	—	—	—	—	—
5	AREA	1.00	0.022	46.2	0.948	0.899	0.211	−3.442	3.614×10^{-4}
	STMFRQ	0.31	0.034	9.2	—	—	—	—	—
	TAYSLO	0.37	0.035	10.5	—	—	—	—	—
	SOIL	0.87	0.098	8.9	—	—	—	—	—
	SAAR	0.98	0.078	12.5	—	—	—	—	—
6	AREA	0.97	0.021	46.4	0.952	0.906	0.202	−3.789	1.626×10^{-4}
	STMFRQ	0.31	0.032	9.7	—	—	—	—	—
	TAYSLO	0.29	0.036	8.0	—	—	—	—	—
	SOIL	0.93	0.094	9.8	—	—	—	—	—
	LAKE	−0.95	0.139	6.8	—	—	—	—	—
	SAAR	1.14	0.079	14.5	—	—	—	—	—
7	AREA	0.97	0.020	48.5	0.957	0.916	0.193	−4.088	8.166×10^{-5}
	STMFRQ	0.34	0.031	10.8	—	—	—	—	—
	TAYSLO	0.26	0.035	7.5	—	—	—	—	—
	SOIL	0.90	0.090	10.0	—	—	—	—	—
	URBAN	1.82	0.258	7.1	—	—	—	—	—
	LAKE	−0.97	0.133	7.3	—	—	—	—	—
	SAAR	1.24	0.076	16.1	—	—	—	—	—

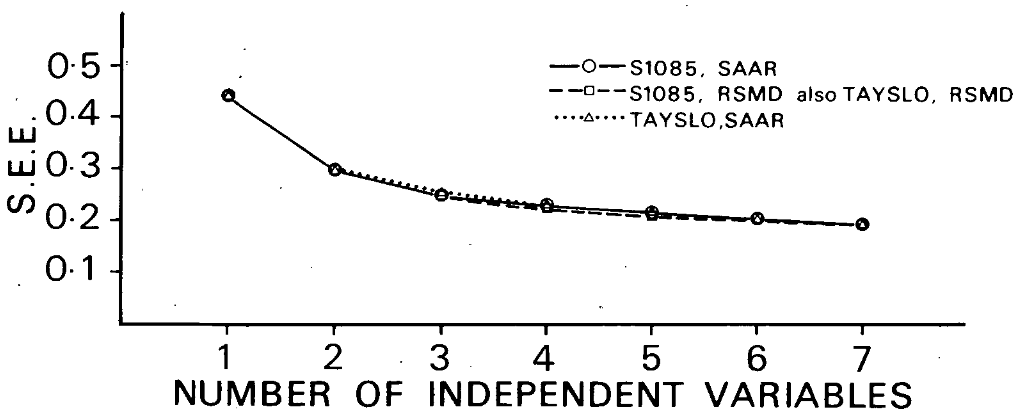
For explanation of headings see footnote to Table 4.10a.

Table 4.10d Effect of TAYSLO, RSMD combination on BESMAF regressions.

No. var.	Name	Coeff.	seb	t	R	R^2	see	Const.	Multiplier
1	AREA	0.77	0.030	26.1	0.750	0.562	0.436	−0.170	0.676
2	AREA	0.84	0.020	41.6	0.896	0.803	0.294	−3.567	2.710×10^{-4}
	RSMD	2.08	0.082	25.3	—	—	—	—	—
3	AREA	0.84	0.017	50.4	0.930	0.865	0.243	−2.407	3.917×10^{-3}
	STMFRQ	0.52	0.033	15.6	—	—	—	—	—
	RSMD	1.35	0.082	16.5	—	—	—	—	—
4	AREA	0.83	0.015	53.7	0.940	0.884	0.226	−1.797	1.596×10^{-2}
	STMFRQ	0.38	0.035	10.7	—	—	—	—	—
	SOIL	0.96	0.104	9.2	—	—	—	—	—
	RSMD	1.23	0.078	15.8	—	—	—	—	—
5	AREA	0.85	0.014	58.4	0.948	0.899	0.210	−1.952	1.117×10^{-2}
	STMFRQ	0.37	0.033	11.4	—	—	—	—	—
	SOIL	1.01	0.097	10.4	—	—	—	—	—
	LAKE	−1.23	0.136	9.1	—	—	—	—	—
	RSMD	1.34	0.073	18.3	—	—	—	—	—
6	AREA	0.86	0.014	62.1	0.954	0.910	0.199	−2.144	7.178×10^{-3}
	STMFRQ	0.39	0.031	12.7	—	—	—	—	—
	SOIL	0.98	0.092	10.6	—	—	—	—	—
	URBAN	2.06	0.264	7.8	—	—	—	—	—
	LAKE	−1.20	0.128	9.3	—	—	—	—	—
	RSMD	1.42	0.070	20.2	—	—	—	—	—
7	AREA	0.94	0.021	45.8	0.956	0.914	0.194	−2.167	6.808×10^{-3}
	STMFRQ	0.97	0.031	12.1	—	—	—	—	—
	TAYSLO	0.20	0.036	5.4	—	—	—	—	—
	SOIL	0.94	0.090	10.4	—	—	—	—	—
	URBAN	1.87	0.260	7.2	—	—	—	—	—
	LAKE	−0.95	0.134	7.1	—	—	—	—	—
	RSMD	1.23	0.077	15.9	—	—	—	—	—

For explanation of headings see footnote to Table 4.10a.

Fig 4.10 Effect of different rainfall and slope characteristics on regression error.



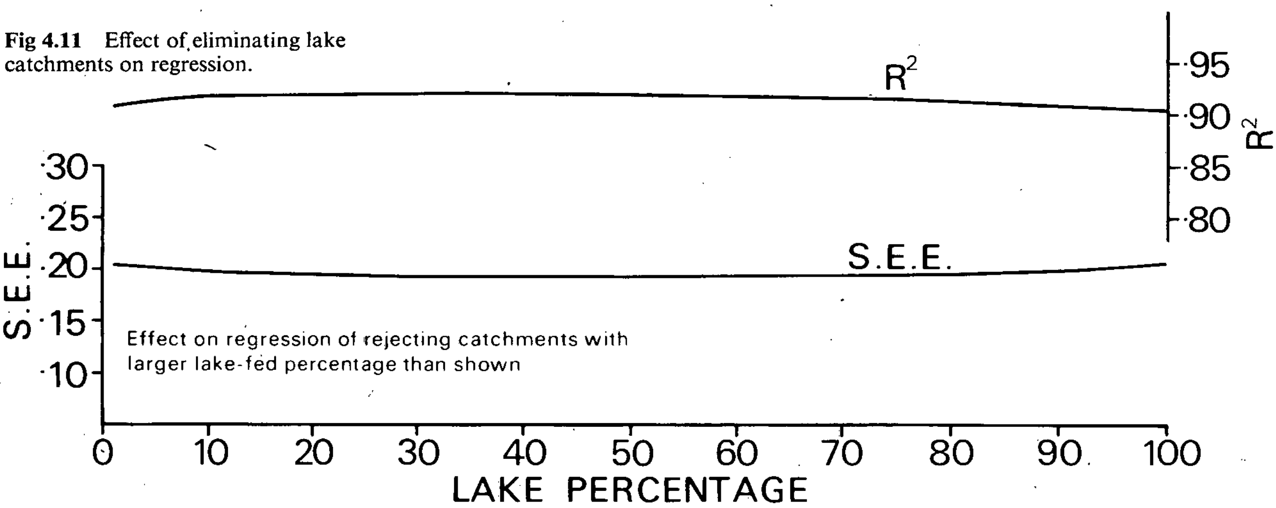
catchments with lakes. However, if the precision of the prediction can be increased this disadvantage could be ignored. Tests were made excluding catchments with LAKE greater than 1, 10, 33, 50, 67, 75 and 90% to see whether the standard error of estimate could be reduced. Table 4.11 gives the standard errors for one to seven independent variables; the variables used at each stage were the optimal sets when any lakes are excluded. It will be seen, by comparing the 6 and 7 variable lines, that one can either exclude catchments with lakes or use LAKE and obtain similar results. Figure 4.11 shows the variation of standard error and R^2 for the six variable case. As there was no real improvement in the standard error, there would appear to be no reason to exclude the lake catchments from the analysis. Figure 4.12 shows how the regression coefficients varied as lake catchments were removed; the variations were again insignificant.

Table 4.11 Effect of eliminating lake catchments on standard error of estimate.

No. var.	Name	Lake cutoff (%)							
		1	10	33	50	67	75	90	All catchments
1	AREA	0.463	0.452	0.445	0.441	0.440	0.439	0.438	0.437
2	AREA	0.303	0.286	0.280	0.280	0.280	0.282	0.284	0.292
	SAAR	—	—	—	—	—	—	—	—
3	AREA	0.245	0.235	0.232	0.232	0.233	0.234	0.239	0.248
	STMFRQ	—	—	—	—	—	—	—	—
	SAAR	—	—	—	—	—	—	—	—
4	AREA	0.225	0.216	0.213	0.215	0.216	0.217	0.222	0.232
	STMFRQ	—	—	—	—	—	—	—	—
	SOIL	—	—	—	—	—	—	—	—
	SAAR	—	—	—	—	—	—	—	—
5	AREA	0.211	0.204	0.202	0.204	0.204	0.205	0.209	0.221
	STMFRQ	—	—	—	—	—	—	—	—
	SOIL	—	—	—	—	—	—	—	—
	URBAN	—	—	—	—	—	—	—	—
	SAAR	—	—	—	—	—	—	—	—
6	AREA	0.204	0.197	0.194	0.194	0.195	0.196	0.200	0.206
	STMFRQ	—	—	—	—	—	—	—	—
	SOIL	—	—	—	—	—	—	—	—
	URBAN	—	—	—	—	—	—	—	—
	S1085	—	—	—	—	—	—	—	—
	SAAR	—	—	—	—	—	—	—	—
7	AREA	—	0.197	0.194	0.193	0.192	0.192	0.195	0.195
	STMFRQ	—	—	—	—	—	—	—	—
	S1085	—	—	—	—	—	—	—	—
	SOIL	—	—	—	—	—	—	—	—
	URBAN	—	—	—	—	—	—	—	—
	LAKE	—	—	—	—	—	—	—	—
	SAAR	—	—	—	—	—	—	—	—
No. catchments		321	418	481	496	502	507	517	531
No. missing		210	113	50	35	29	24	14	0

The table shows the see of the regressions of BESMAF on the variables shown after eliminating catchments with a LAKE index larger than the given percentage.

Fig 4.11 Effect of eliminating lake catchments on regression.



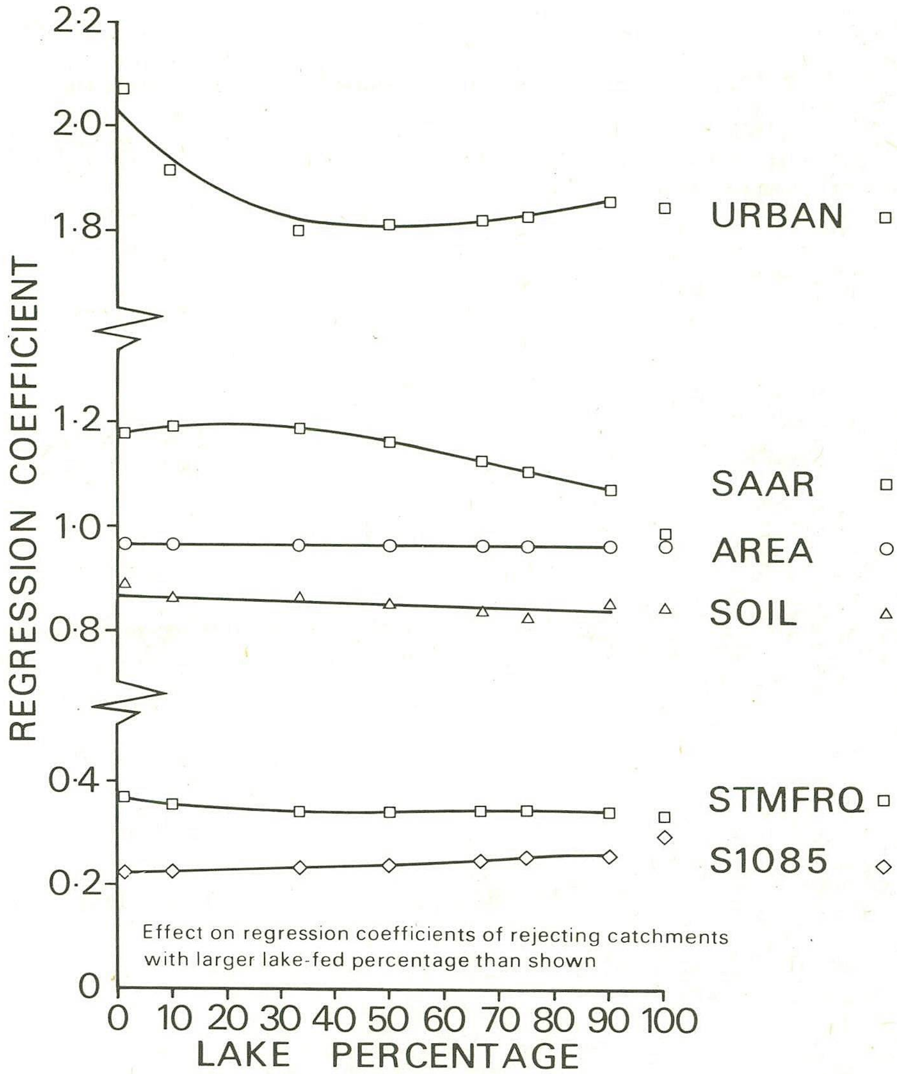


Fig 4.12 Effect of eliminating lake catchments on regression coefficients.

4.3.5 Choice of dependent variable

The estimation of the mean annual flood is discussed in Chapter 2. For the purpose of this section the methods discussed there can be considered to be of two types. The first method is the obvious one of taking the average of the annual maxima. This number was computed for each catchment studied and stored on the Master List as AMAF (arithmetic mean annual flood). More reliable estimates of the mean can be made by extending the record (see Chapter 3) and a mean calculated from an extended record was available for many catchments. This estimate, called EXTMAF (for extended mean annual flood), was also stored on the Master List. As EXTMAF was not available for all stations, a third mean annual flood estimate combining these was stored on the Master List. This was called BESMAF (best estimate of the mean annual flood) and was defined to be EXTMAF if it existed, or AMAF in other cases.

Station 22/2, the Coquet at Bygate, was so truncated that the mean annual flood could not be determined normally. However, an estimate was made using the censored data available (see Section 2.8.3) and this was stored on the list as BESMAF.

The other method of estimating the mean annual flood is to use the model of Section 2.7 and a partial duration series. It was thought that this method might be more reliable than the arithmetic mean for short records, particularly for short records which include a very large flood. The mean, estimated using one, two and three floods per year on average, was calculated and stored on the Master List. These estimates were called PT1MAF, PT2MAF and PT3MAF respectively.

The potential advantages of these estimates were considered to be as follows:

i The partial duration series comprises all the floods over a particular threshold and is thus a more homogeneous set than the annual maximum series which can include some very low floods;

ii A single large flood in a short annual maximum series can cause overestimation of the mean. For example, the September 1968 flood in south east England is sometimes several times greater than the next highest recorded flood. Many of the records in this area are short, which must cause frequent overestimation of the mean. A partial duration estimate, using several floods per year, should be more reliable as the single outlier would be outweighed by the information from the extra floods;

iii For the data which fit the model, it can be shown that these estimates of the mean have lower variance (see Section 2.7.10).

The disadvantages of these methods of estimation are:

i The data do not fit the model of Section 2.7 very well;

ii The effects of isolated outliers can be reduced by record extension;

iii The addition of extra, and hence smaller, floods to the data is unlikely to improve estimates of the return period of large floods.

It was decided to relate PT1MAF, PT2MAF and PT3MAF to catchment characteristics in order to test whether they are predicted more accurately than other measures. If this were so it would lend support to the arguments for the stability of these measures.

The standard set of independent variables (4.3.4) was used. BESMAF was taken as dependent variable for comparison with PT1MAF, PT2MAF and PT3MAF. All four dependent variables were available for 501 stations, as the partial duration series could not be obtained for all stations and at four stations less than three peaks a year could be obtained. Table 4.12 summarises the results; the partial duration estimates have a slightly larger R^2 and smaller standard error than BESMAF. The three estimates from the partial duration series are virtually identical, and they are closely

Estimate	Original standard deviation	R^2	Standard error
BESMAF	0.625	0.910	0.188
PT3MAF	0.620	0.914	0.184
PT2MAF	0.619	0.914	0.185
PT1MAF	0.620	0.914	0.184

501 stations used in regression, 7 variable equation using AREA, STMFRQ, S1085, SAAR, SOIL, LAKE, URBAN.

Table 4.12 Comparative prediction of annual maximum and partial duration series estimates.

Table 4.13a Regression of BESMAF on catchment characteristics (501 stations).

No. var.	Name	Coeff.	seb	t	R	R^2	see	Const.	Multiplier
1	AREA	0.76	0.028	26.9	0.769	0.591	0.400	−0.096	0.802
2	AREA	0.82	0.019	44.2	0.908	0.824	0.262	−6.217	6.06×10^{-7}
	SAAR	1.98	0.077	25.8	—	—	—	—	—
3	AREA	1.01	0.024	42.6	0.928	0.861	0.234	−5.303	4.98×10^{-6}
	S1085	0.41	0.037	11.2	—	—	—	—	—
	SAAR	1.45	0.083	17.4	—	—	—	—	—
4	AREA	0.99	0.022	45.0	0.939	0.882	0.215	−4.050	8.92×10^{-5}
	S1085	0.37	0.034	10.8	—	—	—	—	—
	SAAR	1.19	0.081	14.7	—	—	—	—	—
	SOIL	0.98	0.102	9.6	—	—	—	—	—
5	AREA	0.97	0.021	46.1	0.945	0.893	0.206	−4.453	3.52×10^{-5}
	S1085	0.31	0.034	8.8	—	—	—	—	—
	SAAR	1.37	0.081	16.8	—	—	—	—	—
	SOIL	1.02	0.097	10.4	—	—	—	—	—
	LAKE	−1.02	0.145	7.0	—	—	—	—	—
6	AREA	0.96	0.020	47.4	0.950	0.902	0.196	−3.822	1.51×10^{-4}
	STMFRQ	0.25	0.035	7.2	—	—	—	—	—
	S1085	0.26	0.033	7.9	—	—	—	—	—
	SAAR	1.15	0.083	13.8	—	—	—	—	—
	SOIL	0.79	0.098	8.0	—	—	—	—	—
	LAKE	−1.02	0.138	7.4	—	—	—	—	—
7	AREA	0.96	0.019	49.3	0.954	0.910	0.188	−4.085	8.22×10^{-5}
	STMFRQ	0.28	0.034	8.2	—	—	—	—	—
	S1085	0.24	0.032	7.5	—	—	—	—	—
	SAAR	1.23	0.081	15.2	—	—	—	—	—
	SOIL	0.77	0.094	8.2	—	—	—	—	—
	LAKE	−1.02	0.133	7.7	—	—	—	—	—
	URBAN	1.64	0.260	6.3	—	—	—	—	—

Table 4.13b Regression of PT3MAF on catchment characteristics (501 stations).

No. var.	Name	Coeff.	seb	t	R	R^2	see	Const.	Multiplier
1	AREA	0.76	0.027	28.1	0.782	0.612	0.386	−0.087	0.819
2	AREA	0.82	0.018	45.4	0.911	0.830	0.256	−5.940	1.15×10^{-6}
	SAAR	1.89	0.075	25.2	—	—	—	—	—
3	AREA	1.01	0.023	43.8	0.930	0.865	0.228	−5.027	9.41×10^{-6}
	S1085	0.41	0.036	11.5	—	—	—	—	—
	SAAR	1.36	0.081	16.8	—	—	—	—	—
4	AREA	0.99	0.021	46.4	0.942	0.887	0.210	−3.797	1.59×10^{-4}
	S1085	0.37	0.033	11.1	—	—	—	—	—
	SAAR	1.11	0.079	14.1	—	—	—	—	—
	SOIL	0.96	0.099	9.6	—	—	—	—	—
5	AREA	0.98	0.020	47.6	0.947	0.897	0.199	−4.203	6.27×10^{-5}
	S1085	0.30	0.033	9.1	—	—	—	—	—
	SAAR	1.29	0.079	16.4	—	—	—	—	—
	SOIL	1.00	0.094	10.6	—	—	—	—	—
	LAKE	−1.02	0.140	7.3	—	—	—	—	—
6	AREA	0.96	0.020	49.0	0.952	0.906	0.190	−3.608	2.47×10^{-4}
	STMFRQ	0.24	0.034	7.0	—	—	—	—	—
	S1085	0.26	0.032	8.2	—	—	—	—	—
	SAAR	1.08	0.081	13.4	—	—	—	—	—
	SOIL	0.78	0.095	8.2	—	—	—	—	—
	LAKE	−1.03	0.134	7.7	—	—	—	—	—
7	AREA	0.96	0.019	50.8	0.956	0.914	0.184	−3.859	1.38×10^{-4}
	STMFRQ	0.26	0.033	7.9	—	—	—	—	—
	S1085	0.24	0.031	7.8	—	—	—	—	—
	SAAR	1.16	0.079	14.7	—	—	—	—	—
	SOIL	0.77	0.092	8.3	—	—	—	—	—
	LAKE	−1.03	0.129	8.0	—	—	—	—	—
	URBAN	1.57	0.253	6.2	—	—	—	—	—

related to the estimates from the annual maximum series (see 2.7.9). Details of the regressions using BESMAF and PT3MAF are given in Tables 4.13a and b; the similarity of the coefficients and results at each stage show that the estimates can be regarded as interchangeable.

Because the exclusive use of partial duration series estimates would have reduced the number of stations which could be used, and the stations for which only annual maximum series could be obtained included a number of unusual stations where the response was damped by lake storage or chalk, it was decided to use BESMAF as the measure of mean annual flood in the rest of the regressions. These estimates are illustrated in Figure 4.21, where the mean annual flood BESMAF is shown divided by the area of the catchment. The variability of annual floods is illustrated by Figure 4.22 showing CV. Both these maps are included in Volume V.

4.3.6 The effects of errors in the data

In selecting data for the regressions, the main concern was to use as many gauging stations as possible. To this end the minimum record length requirement was set at 5 years and all stations with grades A–D were used. These requirements allowed some very low quality data to be used, which might have been considered a cause of the poor predictive accuracy of the derived regression equations.

Tests were therefore made in which the shorter and lower grade records were omitted from the data using three minimum periods and three minimum grades. The minimum periods used were 5, 10 and 20 years and the minimum grades were D, B and A grades. The numbers of stations available were as shown below:

Grades	Minimum record (years)		
	5	10	20
A–D	532	313	72
A and B	465	275	57
A only	280	161	23

It will be seen that there are many stations with 10 years of annual maxima or more, but that 20 year stations are rare. Overall, just over half the stations are grade A and 87% are grade B or better. These ratios still hold for the 10 year stations but at 20 years less than a third of the stations are A grade but 79% are B or better.

The standard variable set was used in the tests, and regressions were made using five, six and seven independent variables. The five independent variable set was AREA, STMFRQ, S1085, SOIL and SAAR, with URBAN and LAKE added as the sixth and seventh variables respectively. Table 4.14 shows the standard errors of estimate from these regressions, and illustrates the improvement in standard error with record length. Generally there is no improvement when better graded stations are used, except for the A grade, 20 year stations. However, in this case there are only 23 stations and consequently the results must be considered suspect.

The residuals, or differences between the predicted and measured mean annual flood, arise from two causes: error in estimating the true mean annual flood and lack of fit in the regression equations. The estimating error can be further subdivided into measurement error and sampling error. The measurement errors were intended to be categorised by the

Number of variables	Period of record (years)	Grades		
		A–D	A and B	A
5	5	0.214	0.218	0.231
6		0.206	0.209	0.214
7		0.194	0.197	0.198
5	10	0.191	0.197	0.208
6		0.185	0.192	0.197
7		0.174	0.179	0.184
5	20	0.160	0.166	0.137
6		0.155	0.163	0.111
7		0.147	0.157	0.109

Five variables AREA, STMFRQ, S1085, SOIL, SAAR
Six variables AREA, STMFRQ, S1085, SOIL, SAAR, URBAN
Seven variables AREA, STMFRQ, S1085, SOIL, SAAR, URBAN, LAKE

Table 4.14 Variation of regression standard error with station grade and period of record.

The table shows the standard error of estimate of the regression calculated from graded stations with records longer than periods shown.

grading system; sampling errors occur because records are short and also diminish as $1/\sqrt{N}$ where N is the record length. These results suggest that the difference in measurement errors between the various grades are of small importance compared with lack of fit or sampling error. When a derived equation is used to make a prediction, the prediction will be in error because of lack of fit in the regression model. The user of the equation requires some estimate of the size of this error; a rough estimate can be derived from these results.

Period of record (years)	Variable name	Grade					
		A–D		A and B		A	
		Coeff.	seb	Coeff.	seb	Coeff.	seb
5	AREA	0.98	0.021	0.98	0.024	1.00	0.031
	STMFRQ	0.32	0.034	0.33	0.037	0.35	0.046
	S1085	0.32	0.034	0.33	0.037	0.35	0.051
	SOIL	0.87	0.100	0.85	0.110	0.69	0.141
	SAAR	0.89	0.083	0.86	0.091	0.91	0.120
	see	0.214	—	0.218	—	0.231	—
	R	0.946	—	0.944	—	0.947	—
	R^2	0.894	—	0.891	—	0.897	—
	Const.	−3.117	—	−3.061	—	−3.335	—
10	AREA	0.99	0.026	1.00	0.028	1.01	0.037
	STMFRQ	0.28	0.040	0.27	0.043	0.26	0.055
	S1085	0.28	0.038	0.29	0.040	0.32	0.055
	SOIL	0.97	0.126	0.99	0.142	0.85	0.176
	SAAR	0.97	0.103	0.95	0.113	0.94	0.152
	see	0.191	—	0.197	—	0.208	—
	R	0.951	—	0.950	—	0.952	—
	R^2	0.904	—	0.903	—	0.906	—
	Const.	−3.387	—	−3.323	—	−3.371	—
20	AREA	0.99	0.055	0.98	0.061	1.06	0.088
	STMFRQ	0.30	0.060	0.29	0.070	0.45	0.088
	S1085	0.28	0.085	0.29	0.096	0.52	0.127
	SOIL	0.26†	0.265	0.36†	0.306	−0.43†	0.359
	SAAR	1.11	0.170	0.94	0.204	0.77	0.272
	see	0.160	—	0.166	—	0.137	—
	R	0.970	—	0.971	—	0.986	—
	R^2	0.941	—	0.943	—	0.972	—
	Const.	−4.038	—	−3.465	—	−3.554	—

Table 4.15a Variation of regression coefficients with station grade and record length (5 variable equation).

†Coefficient has $t<2$.

The sampling error of a log mean annual flood is given by cv/$\sqrt{N}$ where CV is the coefficient of variation of the annual floods. Table 4.14 shows that increasing the minimum record length from 5 to 10 years reduces the standard error by 10%. The mean record length when 5 year stations are included is 13.5 years, and when only 10 year (and longer) stations are used is 18.1 years; thus the sampling error should decrease to $\sqrt{(13.5/18.1)}$ or 0.84, a 16% reduction. As the observed reduction is only 10%, it can be estimated that the sampling error is about 10/16 or 62% of the total residual error. This figure can be estimated in another way. The mean CV for all catchments is 0.357 which gives 0.097 as the sampling error of an average record, which is half the standard error for the 7 variable equation (Table 4.15). These two error analyses are very crude but they both suggest that sampling error is an important component of the observed residual error from the regressions.

Table 4.15a, b and c gives the coefficients for the regressions, and illustrates the small changes in the coefficients, particularly in the 5 and 10 year sections of the table. This indicates that the short stations are evenly distributed on either side of the regression surface, but tend to be further from the surface than the longer term stations. With the 20 year stations some coefficient changes can be seen, probably because of the small numbers of stations left and the consequent uneven distribution over the country. Note, in particular, that the soil coefficient loses significance. An examination of the distribution of SOIL values shows the same

Table 4.15b Variation of regression coefficients with station grade and record length (6 variable equation).

Period of record (years)	Variable name	Grade					
		A–D		A and B		A	
		Coeff.	seb	Coeff.	seb	Coeff.	seb
5	AREA	0.94	0.021	0.99	0.023	1.01	0.029
	STMFRQ	0.34	0.033	0.34	0.035	0.37	0.043
	S1085	0.30	0.033	0.30	0.035	0.32	0.047
	SOIL	0.84	0.096	0.84	0.106	0.72	0.131
	URBAN	1.84	0.274	1.95	0.300	2.62	0.381
	SAAR	0.99	0.081	0.97	0.088	1.04	0.112
	see	0.206	—	0.209	—	0.214	—
	R	0.951	—	0.949	—	0.955	—
	R^2	0.904	—	0.901	—	0.912	—
	Const.	−3.437	—	−3.404	—	−3.747	—
10	AREA	1.00	0.025	1.00	0.027	1.02	0.036
	STMFRQ	0.31	0.040	0.31	0.043	0.32	0.054
	S1085	0.27	0.037	0.28	0.039	0.31	0.052
	SOIL	0.86	0.125	0.90	0.140	0.73	0.170
	URBAN	1.64	0.354	1.61	0.418	2.13	0.490
	SAAR	1.06	0.101	1.03	0.112	1.07	0.147
	see	0.185	—	0.192	—	0.197	—
	R	0.954	—	0.952	—	0.957	—
	R^2	0.910	—	0.906	—	0.916	—
	Const.	−3.721	—	−3.626	—	−3.882	—
20	AREA	0.98	0.054	0.98	0.060	1.11	0.073
	STMFRQ	0.34	0.063	0.34	0.073	0.52	0.075
	S1085	0.27	0.082	0.29	0.095	0.57	0.104
	SOIL	0.18†	0.259	0.28†	0.306	−0.59†	0.295
	URBAN	1.23	0.546	1.04†	0.607	1.50	0.476
	SAAR	1.13	0.165	0.98	0.202	0.83	0.222
	see	0.155	—	0.163	—	0.111	—
	R	0.972	—	0.972	—	0.991	—
	R^2	0.945	—	0.945	—	0.982	—
	Const.	−4.114	—	−3.654	—	−3.989	—

†Coefficient has $t<2$.

Period of record (years)	Variable name	Grade					
		A–D		A and B		A	
		Coeff.	seb	Coeff.	seb	Coeff.	seb
5	AREA	0.96	0.020	0.97	0.021	0.98	0.027
	STMFRQ	0.34	0.031	0.35	0.033	0.38	0.040
	S1085	0.23	0.032	0.23	0.035	0.24	0.045
	SOIL	0.90	0.091	0.90	0.100	0.76	0.121
	URBAN	1.86	0.259	1.98	0.282	2.63	0.352
	LAKE	−1.05	0.130	−1.07	0.138	−1.26	0.183
	SAAR	1.18	0.081	1.17	0.087	1.28	0.110
	see	0.194	—	0.197	—	0.198	—
	R	0.956	—	0.955	—	0.962	—
	R^2	0.914	—	0.912	—	0.925	—
	Const.	−3.891	—	−3.874	—	−4.318	—
10	AREA	0.98	0.024	0.98	0.026	0.99	0.034
	STMFRQ	0.31	0.037	0.30	0.040	0.32	0.050
	S1085	0.21	0.036	0.21	0.038	0.23	0.051
	SOIL	0.94	0.118	0.98	0.132	0.81	0.160
	URBAN	1.53	0.333	1.52	0.392	1.98	0.580
	LAKE	−0.91	0.140	−0.94	0.150	−0.96	0.194
	SAAR	1.25	0.099	1.22	0.109	1.24	0.141
	see	0.174	—	0.179	—	0.183	—
	R	0.960	—	0.959	—	0.963	—
	R^2	0.922	—	0.920	—	0.927	—
	Const.	−4.117	—	−4.049	—	−4.207	—
20	AREA	0.96	0.051	0.95	0.060	1.03	0.096
	STMFRQ	0.34	0.059	0.33	0.070	0.53	0.074
	S1085	0.19	0.083	0.18†	0.104	0.44	0.156
	SOIL	0.30†	0.248	0.39†	0.298	−0.53†	0.297
	URBAN	1.26	0.513	1.12†	0.584	1.54	0.471
	LAKE	−0.83	0.288	−0.85	0.275	−0.88†	0.744
	SAAR	1.37	0.177	1.29	0.238	1.16	0.351
	see	0.147	—	0.157	—	0.109	—
	R	0.975	—	0.975	—	0.992	—
	R^2	0.951	—	0.951	—	0.984	—
	Const.	−4.660	—	−4.367	—	−4.648	—

Table 4.15c Variation of regression coefficients with station grade and record length (7 variable equation).

†Coefficient has $t<2$.

range of soil classes, though the average SOIL index is a little higher than when the shorter period stations are included. The variance of transformed soil values is reduced for the 20 year stations, indicating a lower spread about the mean, which could reduce its significance as a variable but not by the amount found here. When the distribution of 20 year stations is examined on a map it is noticeable that there are very few stations on the chalk of southern England. This suggests that the soil index may not differentiate enough between the permeability of the chalk and of other soils grouped with it in Class 1.

On the results of this study there are some grounds for using only stations over 10 years. However, as there are small differences in coefficients between the 5 and 10 year results it was decided to use the 5 year stations, even though the resulting standard error is exaggerated by sampling error. A crude error analysis suggests that sampling error could account for about half of the observed standard error.

4.3.7 Grouping stations by area

Floods on very large catchments can arise from a storm covering only part of the catchment; with smaller catchments, the rainfall is more likely to be uniform. These effects may be reflected in the regression relationship between flood statistics and catchment characteristics. It might be expected that the mean annual flood will increase more slowly than catchment area. However in these investigations it has been found that when other catchment characteristics are taken into account the exponent of AREA in the equations is usually below but very close to unity. It was decided to see whether any differences in the equations arose when the available data were split into two groups of large and small catchments. Four tests were conducted with the division at 20, 50, 100 and 500 km^2. BESMAF was used as the dependent variable and the five independent variables were AREA, STMFRQ, S1085, SOIL and SAAR. The results may be summarised below:

Less than:		Dividing area	Greater than:		Combined standard error
no.	see		no.	see	
32	0.211	20 km^2	500	0.209	0.210
67	0.285	50 km^2	465	0.198	0.210
129	0.276	100 km^2	403	0.188	0.212
394	0.228	500 km^2	138	0.169	0.215
All catchments					0.214

Note: no. is the number of catchments in the group; see is the standard error of estimate.

The combined standard error is calculated by

$$s = \sqrt{\frac{\text{SS}}{n-2m-2}} \qquad (4.13)$$

where SS is the sum of squares of the residuals over both sections, n is the number of stations (532 here), and m is the number of independent variables in each equation.

It will be seen that the overall improvement, as shown by the combined error, is small. In each case the larger catchments are fitted better by the derived equation, but usually at the expense of a worse fit for the smaller catchments. In no case are the improvements as great as can be obtained by dividing into regions as shown below, though the catchments over 500 km^2 on their own give as good a result as regionalisation. Tabulations of the equation coefficients etc. are given in Table 4.16. There are some differences in coefficients between the various equations but these are usually less than the standard errors of the coefficients themselves and in no case are they sufficiently great to be significant.

It was, therefore, decided on the basis of these tests that catchments of all sizes should be taken together. An interesting byproduct of these tests is the poor fit obtained for smaller catchments; larger catchments would appear to behave more regularly.

4.3.8 Use of regions

In this regression study, so far, Great Britain and Ireland have been considered as one homogeneous whole. It might be that this was too ambitious an undertaking and it might have been better to have used a number of regions with a separate regression equation for each. The regions would ideally be selected so that each was homogeneous with respect to its hydrological properties. In practice, the regions would probably be selected on the basis of general geographical similarity. After such region-

Table 4.16 Comparison of regressions using stations split into size groups.

Name	Areas less than 20 km²			Areas greater than 20 km²		
	Coeff.	seb	t	Coeff.	seb	t
AREA	0.97	0.063	15.3	0.98	0.026	38.0
STMFRQ	−0.07	0.142	0.5	0.31	0.035	8.9
S1085	0.16	0.162	1.0	0.32	0.035	9.2
SOIL	0.28	0.291	1.0	0.95	0.106	9.0
SAAR	1.21	0.313	3.9	0.93	0.087	10.7
	$R = 0.960$, $R^2 = 0.922$			$R = 0.940$, $R^2 = 0.884$		
	see = 0.211, const. = −4.044			see = 0.209, const. = −3.232		
	multiplier = 9.036×10^{-5}			multiplier = 5.861×10^{-4}		
	Areas less than 50 km²			**Areas greater than 50 km²**		
AREA	0.94	0.073	12.9	1.01	0.027	37.7
STMFRQ	0.31	0.113	2.8	0.30	0.036	8.3
S1085	0.15	0.159	0.9	0.34	0.034	10.0
SOIL	1.03	0.280	3.7	0.93	0.107	8.7
SAAR	0.60	0.294	2.0	1.00	0.087	11.5
	$R = 0.900$, $R^2 = 0.810$			$R = 0.941$, $R^2 = 0.885$		
	see = 0.285, const. = −1.863			see = 0.198, const. = −3.515		
	multiplier = 1.371×10^{-2}			multiplier = 3.055×10^{-4}		
	Areas less than 100 km²			**Areas greater than 100 km²**		
AREA	0.89	0.056	15.8	1.01	0.029	34.6
STMFRQ	0.32	0.078	4.1	0.29	0.037	7.8
S1085	0.19	0.105	1.8	0.37	0.034	10.9
SOIL	0.86	0.209	4.1	0.92	0.111	8.2
SAAR	0.84	0.206	4.1	1.01	0.092	11.0
	$R = 0.890$, $R^2 = 0.792$			$R = 0.939$, $R^2 = 0.882$		
	see = 0.276, const. = −2.704			see = 0.188, const. = −3.585		
	multiplier = 1.977×10^{-3}			multiplier = 2.600×10^{-4}		
	Areas less than 500 km²			**Areas greater than 500 km²**		
AREA	0.96	0.028	33.8	0.94	0.062	15.2
STMFRQ	0.30	0.039	7.8	0.42	0.081	5.2
S1085	0.30	0.042	7.2	0.38	0.056	6.8
SOIL	0.90	0.113	7.9	0.58	0.234	2.5
SAAR	0.89	0.100	9.0	0.87	0.163	5.3
	$R = 0.922$, $R^2 = 0.850$			$R = 0.904$, $R^2 = 0.817$		
	see = 0.228, const. = −3.088			see = 0.169, const. = −3.073		
	multiplier = 8.166×10^{-4}			multiplier = 8.453×10^{-4}		

alisation it would be hoped that the resulting regression equations would be a better fit to the data because any extraneous factors affecting floods, which were not included in the regression, would be nearly constant in the region and hence would not contribute to the variability of the mean annual flood. Thus, we may regard the regionalisation as an attempt to account for neglected factors.

There are a number of disadvantages in regionalisation. It is a simple matter to group the data into regions, but there may be difficulty in assigning a new catchment to a particular region. Any rules for defining regions will be somewhat arbitrary and 'borderline' problems will occur. These could be especially severe if two adjoining regional equations were to give widely different estimates for the mean annual flood for a particular catchment. A second disadvantage is that some of the regions may have records from few gauging stations which would make it difficult to estimate the regression equations for these regions. Finally, a system which requires a number of equations is less convenient to use than a single equation.

Before the use of regions can be justified it must be shown that better predictions result. The reduction in error should not only be statistically significant but should also be of sufficient size to justify the use of regions despite the disadvantages outlined above. The second criterion is the more difficult of the two to apply and is discussed at the end of this section.

The statistical test can conveniently be considered in two parts: a test for differences in regression coefficients (that is, the slopes of the regression surface) can be followed, if the slopes are found to be the same, by a test for differences of the constant terms or intercepts. The test is illustrated in Figure 4.13(a) which shows a regression of y on x. The two closed curves represent the spread of data points for two regions A and B. The centroids for the two groups of data points are marked. The separate regression lines for regions A and B are shown, passing through the centroids. When the data are taken together, the regression line is shown by the dashed line marked as normal regression. For the slope test the regional means are subtracted from the variables and when these data are plotted Figure 4.13(b) is obtained.

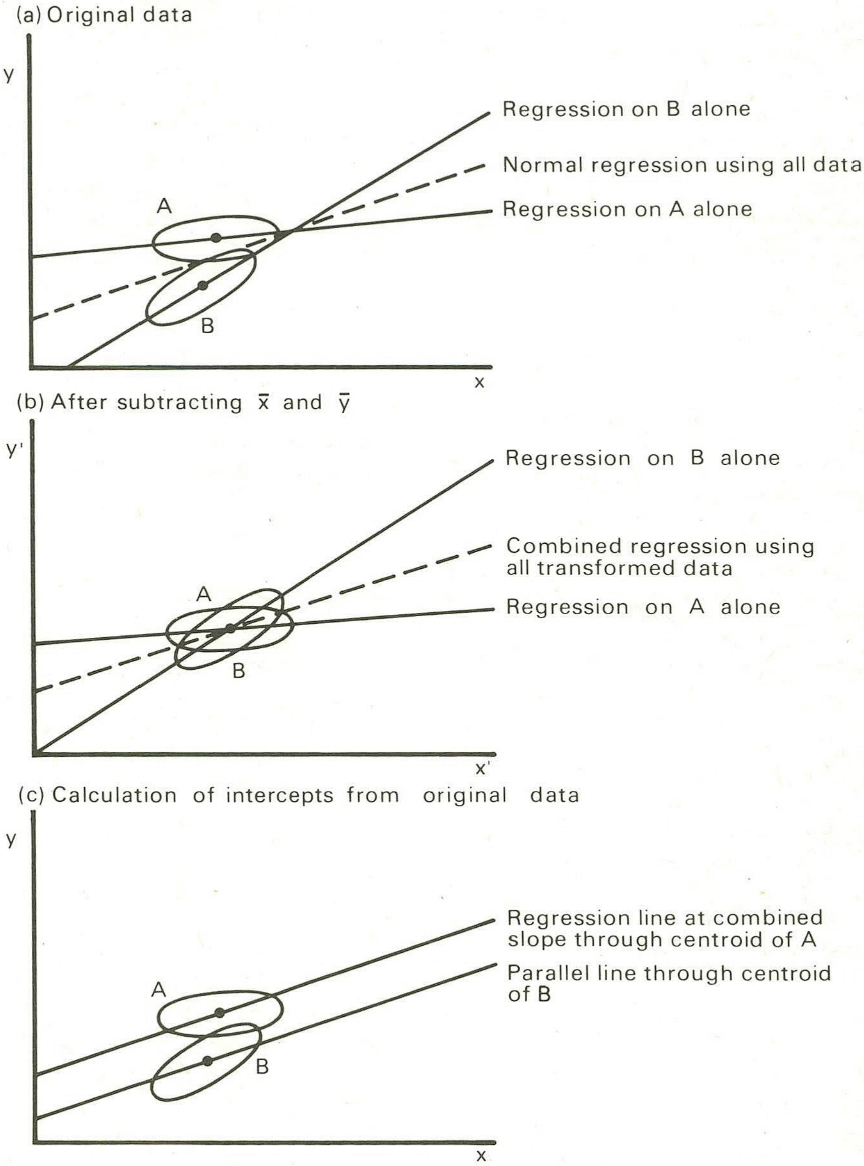


Fig 4.13 Representation of tests for slope and intercept differences.

Three regressions are needed; two using the regions A and B separately, and the combined regression using the data as shown in Figure 4.13(b). If SSA and SSB denote the regression sum of squares for regions A and B respectively and SSC the sum of squares explained by the combined regression, the analysis of variance table for slope differences may be set out as:

	Source	df	Sum of squares
I	Combined regression	1	SSC
II	Differences between slopes	1	SSA+SSB−SSC
III	Pooled within region residuals	$(n_A-2)+(n_B-2)$	P (see note)
	Total	(n_A+n_B-2)	$\Sigma(y-\bar{y})^2$

The pooled within region residual sum of squares, P, can be obtained by summing the residual sum of squares from the two regions A and B. The total sum of squares is calculated about the regional means.

To test the significance of the sum of squares in line II we make the null hypothesis that the slopes are the same in the two regions. Under this hypothesis the ratio of the mean squares from lines II and III from this table is distributed as F with 1 and (n_A+n_B-4) degrees of freedom. (The use of an F test requires the assumption that the residuals from the regression are Normally distributed.)

This test can readily be extended to several independent variables and several regions. A computer program was written to perform the calculations for this test. The input required for each region was the covariance matrix of the variables in the regression, the number of stations and residual sum of squares. These data were all extracted from the output from the ICL Statistical Analysis Mark 2 programs.

If it is decided that the slopes in the two regions are not significantly different the second test can be made. The situation is as shown in Figure 4.13(c). The two regions are shown well separated for clarity. Regression lines A and B at the combined slope have their intercepts shown; the overall regression is not shown. The analysis of variance for the two region, one variable case is:

	Source	df	Sum of squares
IV	Overall regression	1	Regression ss when all data are used
V	Intercept differences	1	By subtraction
VI	Differences between coefficients	1	As in line II in previous table
VII	Pooled within region residuals	n_A+n_B-4	As in line III in previous table
	Total	n_A+n_B-1	$\Sigma(y-\bar{y})^2$ (sum over all data)

Under the null hypothesis that the two intercepts are the same, the ratio of mean squares from lines V and VII is distributed as F with 1 and n_A+n_B-4 degrees of freedom. Once again the test can be extended to several variables and several regions.

The first test compared regressions in Great Britain and Ireland using BESMAF as the independent variable and AREA, STMFRQ, SOIL, S1085 and SAAR as independent variables. URBAN had to be omitted as it is zero for all Irish catchments and LAKE was omitted because only a few British catchments have lakes. There were 418 catchments in Great Britain and 114 in Ireland. Details of the regressions are shown in Table 4.17. The analysis of variance for coefficient differences is given in Table 4.18.

Name	Great Britain			Ireland			Great Britain and Ireland		
	Coeff.	seb	t	Coeff.	seb	t	Coeff.	seb	t
AREA	0.963	0.025	38.6	0.970	0.043	22.5	0.976	0.021	45.6
STMFRQ	0.360	0.038	9.5	0.234	0.084	2.8	0.315	0.034	9.2
S1085	0.296	0.046	6.5	0.176	0.054	3.3	0.320	0.034	9.4
SOIL	0.776	0.114	6.8	0.569	0.201	2.8	0.868	0.100	8.7
SAAR	0.909	0.094	9.6	1.549	0.248	6.2	0.888	0.083	10.6
R	0.952	—	—	0.937	—	—	0.946	—	—
R^2	0.906	—	—	0.878	—	—	0.895	—	—
see	0.217	—	—	0.168	—	—	0.214	—	—
Const.	−3.148	—	—	−5.280	—	—	−3.117	—	—
Multiplier	0.711×10^{-3}	—	—	0.525×10^{-5}	—	—	0.765×10^{-3}	—	—

Table 4.17 Details of regressions, Great Britain and Ireland.

Source	df	SS	MS
i Differences between coefficients—analysis of variance			
Combined fit	5	207.71	—
Coefficient differences	5	0.394	0.0788
Pooled within region residuals	520	22.453	0.0432

$F = \frac{0.0788}{0.0432} = 1.82$ with 5 and 520 df and is not significant at 5% level (5% level of F is 2.21)

Source	df	SS	MS
ii Differences between intercepts—analysis of variance			
Overall regression	5	206.532	—
Intercept differences	1	1.306	1.306
Coefficient differences	5	0.394	—
Pooled within region residuals	520	22.454	0.0432
Totals	531	230.686	—

$F = \frac{1.306}{0.0432} = 30.25$ with 1 and 520 df. This value of F is highly significant; the 1% point is 6.64, and thus the null hypothesis of no intercept differences must be rejected

iii Combined equation under assumption of equal coefficients and different intercepts

$\text{BESMAF} = b_0\ \text{AREA}^{0.95}\ \text{STMFRQ}^{0.35}\ \text{S1085}^{0.25}\ \text{SOIL}^{0.78}\ \text{SAAR}^{1.00}$

	Intercept	Multiplier b_0
Great Britain	−3.363	4.335×10^{-4}
Ireland	−3.496	3.188×10^{-4}

Table 4.18 Test for regression equation differences between Great Britain and Ireland.

This is not significant; at the 5% level F is 2.21. (This value, 2.21, would be exceeded by chance in 5% of random trials where there was no coefficient difference.) Thus, the null hypothesis that there are no coefficient differences can be accepted, and one can proceed to a test of intercept differences. The analysis of variance table is as shown in Table 4.18. This value of F is highly significant; the 1% point is 6.64, and thus the null hypothesis of no intercept differences must be rejected.

The regression equations for Ireland and Great Britain can be regarded as describing two parallel hyperplanes in the variable space. The combined slope estimates are also given in Table 4.18. Using the mean values of the variables and these slopes, two intercepts were obtained as shown in Table 4.18.

Note that a catchment in Great Britain with exactly the same catchment characteristics as one in Ireland would be expected to have a mean annual flood 36% higher.

It has been shown that there is a significant improvement in the fit when two equations with different intercepts but the same slopes are used for Great Britain and Ireland. The question now arises whether the improvement is sufficient to warrant the use of two equations on what might be called 'engineering grounds'. Consider first the disadvantages enumerated above. The regional boundary, the Irish Sea, appears definite in this case, but note that Northern Ireland has only two gauging stations and parts of Northern Ireland are closer to Scotland than they are to the bulk of the Irish gauging stations. As for the other two disadvantages, there are over 100 gauging stations in the smaller region which causes no trouble with estimation and there is only one parameter different in the two equations which is scarcely an inconvenience. Thus, there are few disadvantages in using this scheme; what is the size of the improvement in fit obtained? The table below compares the single equation and two equation systems. The two equations give improvements in Ireland, but hardly any improvement in Great Britain. Overall, there is a small improvement; the factorial error is reduced from 1.64 to 1.62. The improvement in Ireland is largely in the reduced bias in prediction. On balance the two equations are worth using, largely because of the improved prediction in Ireland.

	All catchments	Great Britain	Ireland
Using a single equation			
Mean residual	0.0	0.0234	−0.0858
Corresponding % error	0.0	10%	18%
Standard error	0.214	0.217	0.183
% stations overestimated	51%	45%	70%
Using the two equations			
Mean residual	0.0	0.0	0.0
Corresponding % error	0.0	0.0	0.0
Standard error	0.209	0.217	0.176
% stations overestimated	58%	60%	50%

A negative residual denotes overestimation by the equation.

4.3.9 *Extensions to the use of regions—further division of Great Britain*

The separation of Great Britain and Ireland into two regions with regression equations using jointly estimated slopes and separate intercepts seemed successful, so it was decided to attempt to divide Great Britain into regions.

As a start ten basic regions were chosen from geographical groups of hydrometric areas with a reasonable number of gauging stations. These basic regions were intended to be combined to form a few large regions. The ten regions were:

No.	Hydrometric areas	Description
1	1–16, 88–97, 104–108	Northern Scotland
2	17–21, 77–87	Southern Scotland and Borders
3	22–27	North east England
4	28 and 54	Severn and Trent catchments
5	29–35	East Anglia
6	36–39	Essex, Lee and Thames
7	40–44, 101	South coast
8	45–53	South west peninsula
9	55–67, 102	Wales except Severn
10	68–76	North west England

These regions proved to have differing coefficients when tested together. Various subsets were tried in an attempt to obtain groupings that could be regarded as hydrologically homogeneous regions. Though adjacent regions could be joined, it was unusual for larger groupings to prove homogeneous. Region 4 was divided into 4S (the Severn) and 4T (the Trent) in order to see whether these subregions would go better with adjacent regions and similarly Region 3 was divided into a northern part (3N) comprising hydrometric areas 22–25 and a southern part (3S) comprising hydrometric areas 26–27. Figure 4.14 shows the results of paired comparisons of basic regions. It will be seen that there is a central core of Regions 2, 3, 4, 9 and 10 which can be linked and a number of peripheral Regions 1, 5, 6, 7 and 8 which are different. These diagrams are based on paired comparisons; if all the central Regions (2, 3, 4, 9 and 10) are tried together differences between them are found. Nevertheless, it was decided to adopt this scheme of one large central region with several small regions around it. Regions 6 and 7 were joined because there are only a few gauging stations in Region 7.

Tests with this grouping gave differences significant at the 0.1 % level. A close examination of the results showed that the combined Region 6 and 7 accounted for much of the observed difference sum of squares. Most of this sum of squares is contributed by Region 6, the Thames, Lee and Essex. The Thames catchment, in particular, is very heterogeneous. There are very impervious catchments in the Greater London area and on the Oxford Clay which contrast with the pervious catchments on the Chalk and Oolite. Catchments from hydrometric area 39 (the Thames) are among those with the largest residuals, both negative and positive, in any regression. A further test was therefore made after discarding Region 6 from the data.

The coefficient differences for this set were considerably reduced and are just significant at the 1 % level. Despite this it was decided to use the following grouping:

Six regions with the same coefficients:

i Combined Region 2, 3, 4, 9 and 10
ii Northern Scotland Region 1
iii South Coast Region 7
iv East Anglia Region 5
v South west Peninsula Region 8
vi Ireland

A region with different coefficients:

vii Thames, Lee and Essex Region 6

The intercept differences for the group *i–vi* proved to be highly significant. The complete analysis of variance table is given below:

Source	df	SS	MS
Overall regression	5	167.6625	—
Intercept differences	5	3.7113	0.7422
Coefficient differences	25	1.3421	0.05368
Pooled within region residuals	446	12.9802	0.02914
Total corrected ss	481	185.6960	—

F test for coefficient differences:
$F = 1.84$ with 25 and 446 df (the 1 % value for F is 1.8).
F test for intercept differences:
$F = 25.4$ with 5 and 446 df (the 1 % value for F is 4.1).

4.3.10 Conclusions and final equations

As a result of the tests described in Sections 4.3.4–4.3.9 the following decisions were made:

i BESMAF was to be used as a measure of the mean annual flood where available and the normal arithmetic mean in other cases;

ii Short term and low grade stations should be included in the data as their exclusion did not change the results achieved;

iii There was no case for treating small and large catchments separately;

iv The study area was to be divided into seven regions, six of which would have the same coefficients but different intercepts. The seventh region was to be dealt with separately.

It now remained to develop the final equations for operational use. As a first task the equations for the six regions with the same coefficients were developed. The independent variable set chosen originally was a tentative set designed to decide the four points set out above. When these decisions had been made the independent variable set was reconsidered. The criteria for the choice of independent variables were

i Statistical significance—the variables chosen should make a contribution to explaining the variance of the mean annual flood;

AREA	Catchment area in km^2
STMFRQ	The number of stream junctions as shown on the 1:25 000 map, divided by the catchment area
S1085	The stream channel slope measured between two points 10 and 85% of the stream length from gauge expressed in m/km
TAYSLO	The Taylor–Schwarz slope of the channel. The stream is divided into n elementary reaches each with slope S_i. Where the reaches are of equal length, the Taylor–Schwarz slope S is calculated from $\frac{1}{\sqrt{S}} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\sqrt{S_i}}$ and is expressed in m/km. (S is the slope of the uniform channel which, assuming the Manning formula, has the same length and travel time as the original)
MSL	The length of the main stream in km
DVF	Dry valley factor. The length of dry valley from the stream head to the watershed divided by the total length to the divide
SAAR	Standard period annual average rainfall in mm. The standard period used is 1916–50 (1931–60 in Ireland)
NUMAM	Number of annual maxima available for analysis
AMAF	Arithmetic mean of the annual maximum floods
CV	Coefficient of variation of the annual maximum floods
M52D	The catchment average 2 day rainfall of 5 year return period
SMDBAR	A weighted mean soil moisture deficit
RSMD	A net 1 day rainfall of 5 year return period. The rainfall is net in the sense that SMDBAR is subtracted from it
URBAN	The urban fraction of the catchment
LAKE	The fraction of the catchment draining through a lake or reservoir
BESMAF	An estimate of the mean annual flood derived wherever possible by extending the record by correlation with nearby records. If extension was not possible, BESMAF = AMAF
SOIL	A soil index with values in the range 0.15–0.50. Soils were classified into five classes. If S_i is the fraction of the catchment covered by soil class i the index is given by $(0.15\ S_1+0.30\ S_2+0.40\ S_3+0.45\ S_4+0.50\ S_5)/(S_1+S_2+S_3+S_4+S_5)$
PT1MAF, PT2MAF, PT3MAF	The mean annual flood, estimated from a POT series using one, two or three floods per year

Table 4.19 Definitions of catchment characteristics.

ii Physical reality—variables should be chosen on hydrological grounds. It is hoped that the application of this criterion will ensure that the equations can be applied to new catchments without excessive error;

iii Uniformity—where it is possible, without sacrificing the predictive power of the equations, the variable set should be the same as that used for other predicting equations (see Chapter 6).

Using these criteria it was decided to try the following independent variables:

AREA, STMFRQ, S1085, SOIL, RSMD, SAAR, LAKE and URBAN (see Table 4.19 for definitions of these variable names)

Of this set SAAR and RSMD are alternative measures of the climate of the catchment. SAAR, the standard period annual average rainfall, is marginally better than RSMD, the net 1 day rainfall of 5 year return period, in reducing the residuals from the regression. Table 4.20 compares these two in the 6 independent variable case. In the 5 variable case the standard errors are 0.174 and 0.177 while in the 6 variable case they are 0.163 and 0.168. It will be seen that the differences in standard error are small, and RSMD had previously been slightly better. RSMD is a measure of short term

Table 4.20a Regional regression analysis using one independent variable

Variable	Coefficient	Standard error	Student's t
Regression coefficients			
AREA	0.73	0.023	32.2
Region name		Intercept	Multiplier
Regional intercepts			
Ireland (Region 11)		−0.179	0.663
N. Scotland (Region 1)		0.208	1.615
East Anglia (Region 5)		−0.658	0.220
S. coast (Region 7)		−0.251	0.561
S.W. England (Region 8)		0.121	1.320
Central region (Regions 2, 3, 4, 9, 10)		0.146	1.398
Average		−0.017	0.962
Accuracy of prediction			
Standard error of estimate		0.314	—
Factorial standard error of estimate		2.060	—
Determination coefficient (R^2)		0.686	—

Table 4.20b Regional regression analysis using two independent variables.

Variable	Coefficient	Standard error	Student's t
Regression coefficients			
AREA	0.80	0.017	46.1
STMFRQ	0.74	0.038	19.3
Region name		Intercept	Multiplier
Regional intercepts			
Ireland (Region 11)		−0.348	0.449
N. Scotland (Region 1)		0.069	1.172
East Anglia (Region 5)		−0.469	0.340
S. coast (Region 7)		−0.278	0.528
S.W. England (Region 8)		0.077	1.194
Central region (Regions 2, 3, 4, 9, 10)		−0.101	0.793
Average		−0.172	0.672
Accuracy of prediction			
Standard error of estimate		0.235	—
Factorial standard error of estimate		1.720	—
Determination coefficient (R^2)		0.824	—

Table 4.20c Regional regression analysis using three independent variables.

Variable	Coefficient	Standard error	Student's t
Regression coefficients			
AREA	0.85	0.015	56.0
SOIL	1.53	0.099	15.4
RSMD	1.38	0.090	15.4
Region name		**Intercept**	**Multiplier**
Regional intercepts			
Ireland (Region 11)		−1.971	0.0107
N. Scotland (Region 1)		−1.949	0.0113
East Anglia (Region 5)		−2.029	0.0094
S. coast (Region 7)		−1.786	0.0164
S.W. England (Region 8)		−1.656	0.0221
Central region (Regions 2, 3, 4, 9, 10)		−1.789	0.0163
Average		−1.856	0.0139
Accuracy of prediction			
Standard error of estimate		0.197	—
Factorial standard error of estimate		1.574	—
Determination coefficient (R^2)		0.877	—

Table 4.20d Regional regression analysis using four independent variables.

Variable	Coefficient	Standard error	Student's t
Regression coefficients			
AREA	0.85	0.014	60.4
STMFRQ	0.32	0.038	8.5
RSMD	1.06	0.092	11.6
SOIL	1.17	0.101	11.6
Region name		**Intercept**	**Multiplier**
Regional intercepts			
Ireland (Region 11)		−1.633	0.0233
N. Scotland (Region 1)		−1.513	0.0307
East Anglia (Region 5)		−1.630	0.0234
S. coast (Region 7)		−1.444	0.0360
S.W. England (Region 8)		−1.266	0.0542
Central region (Regions 2, 3, 4, 9, 10)		−1.452	0.0353
Average		−1.501	0.0315
Accuracy of prediction			
Standard error of estimate		0.184	—
Factorial standard error of estimate		1.526	—
Determination coefficient (R^2)		0.893	—

Table 4.20e Regional regression analysis using five independent variables.

Variable	Coefficient	Standard error	Student's t
Regression coefficients			
AREA	0.87	0.013	64.9
STMFRQ	0.31	0.036	8.8
LAKE	−0.97	0.119	8.1
SOIL	1.23	0.095	12.9
RSMD	1.17	0.087	13.4
Region name		**Intercept**	**Multiplier**
Regional intercepts			
Ireland (Region 11)		−1.737	0.0183
N. Scotland (Region 1)		−1.649	0.0224
East Anglia (Region 5)		−1.772	0.0169
S. coast (Region 7)		−1.599	0.0252
S.W. England (Region 8)		−1.441	0.0362
Central region (Regions 2, 3, 4, 9, 10)		−1.606	0.0248
Average		−1.642	0.0228
Accuracy of prediction			
Standard error of estimate		0.172	—
Factorial standard error of estimate		1.486	—
Determination coefficient (R^2)		0.906	—

Table 4.20f Regional regression analysis using six independent variables.

Variable	Coefficient	Standard error	Student's t
Regression coefficients			
AREA	0.95	0.019	50.0
STMFRQ	0.22	0.036	6.1
S1085	0.19	0.031	6.1
SOIL	1.18	0.091	13.0
SAAR	1.05	0.081	13.0
LAKE	−0.93	0.116	8.0
Region name		**Intercept**	**Multiplier**
Regional intercepts			
Ireland (Region 11)		−3.376	0.000420
N. Scotland (Region 1)		−3.317	0.000482
East Anglia (Region 5)		−3.457	0.000349
S. coast (Region 7)		−3.223	0.000598
S.W. England (Region 8)		−3.115	0.000767
Central region (Regions 2, 3, 4, 9, 10)		−3.270	0.000537
Average		−3.302	0.000498
Accuracy of prediction			
Standard error of estimate		0.163	—
Factorial standard error of estimate		1.456	—
Determination coefficient (R^2)		0.916	—

rainfall and is hydrologically the more direct measure. It was found to be more stable in ridge regression tests of variable sets. Accordingly, it was decided to adopt RSMD in preference to SAAR as the independent variable for climate. Ridge regression is described in Section 5.3.2.

The Taylor–Schwarz slope (TAYSLO) had previously been considered as an alternative to the 10–85% slope (S1085). S1085 was found to be the more stable in ridge regression tests and in the investigations of Chapter 6 it was found to be better than TAYSLO. Therefore, only S1085 was considered here.

Table 4.20h shows the 7 variable equation, including the intercepts for the six regions. It will be seen that the coefficient of the variable URBAN is no longer significantly different from zero and thus it should be dropped from the set. This result is not too surprising as many of the highly urban catchments are in the London area which has been excluded from these tests, along with the rest of the Thames, Lee and Essex areas.

Table 4.20g Regional regression analysis using six independent variables.

Variable	Coefficient	Standard error	Student's t
Regression coefficients			
AREA	0.94	0.020	48.1
STMFRQ	0.27	0.036	7.6
S1085	0.16	0.032	5.0
SOIL	1.23	0.093	13.2
RSMD	1.03	0.089	11.5
LAKE	−0.85	0.119	7.2
Region name		**Intercept**	**Multiplier**
Regional intercepts			
Ireland (Region 11)		−1.764	0.0172
N. Scotland (Region 1)		−1.731	0.0186
East Anglia (Region 5)		−1.815	0.0153
S. coast (Region 7)		−1.630	0.0234
S.W. England (Region 8)		−1.501	0.0315
Central region (Regions 2, 3, 4, 9, 10)		−1.671	0.0213
Average		−1.696	0.0201
Accuracy of prediction			
Standard error of estimate		0.168	—
Factorial standard error of estimate		1.472	—
Determination coefficient (R^2)		0.911	—

Table 4.20h Regional regression analysis using seven independent variables.

Variable	Coefficient	Standard error	Student's t
Regression coefficients			
AREA	0.94	0.020	48.1
STMFRQ	0.28	0.037	7.7
S1085	0.16	0.032	4.9
SOIL	1.21	0.094	12.9
RSMD	1.05	0.090	11.6
LAKE	−0.86	0.119	7.3
URBAN	0.47	0.371	1.3
Region name		**Intercept**	**Multiplier**
Regional intercepts			
Ireland (Region 11)		−1.796	0.0160
N. Scotland (Region 1)		−1.763	0.0173
East Anglia (Region 5)		−1.841	0.0144
S. coast (Region 7)		−1.663	0.0217
S.W. England (Region 8)		−1.534	0.0293
Central region (Regions 2, 3, 4, 9, 10)		−1.711	0.0195
Average		−1.732	0.0185
Accuracy of prediction			
Standard error of estimate		0.168	—
Factorial standard error of estimate		1.471	—
Determination coefficient (R^2)		0.911	—

In this equation the exponent of AREA is 0.94, which implies that the mean annual flood increases nearly in proportion to catchment area. Compared with other investigations this exponent seems rather high (though Benson's investigation gave an even higher exponent). This investigation has made use of a large number of independent variables, some of which are well correlated with AREA. When they are excluded the

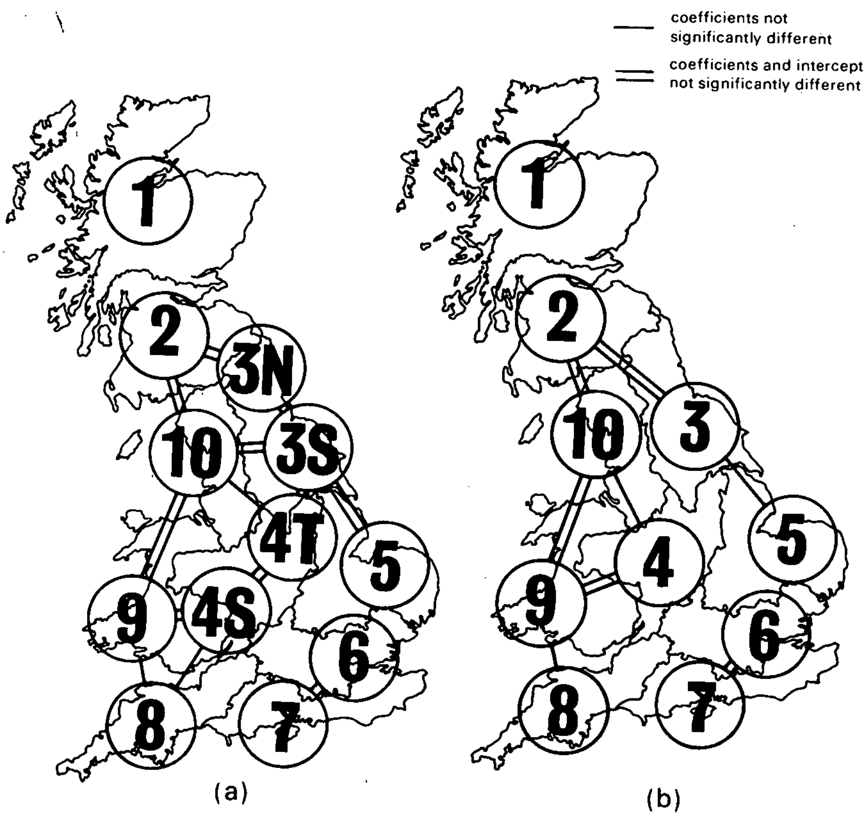


Fig 4.14 Comparisons of adjacent regions. (a) Divided basic regions. (b) Basic regions.

exponent of AREA is reduced. Table 4.20 gives details of regressions with one to six independent variables, and shows how the exponent of AREA increases as extra variables are added. When AREA is used on its own the exponent is 0.73; when STMFRQ is added the exponent increases to 0.80. AREA and STMFRQ are negatively correlated because large catchments tend to have a small stream frequency and *vice versa*. In the best 3 variable equation the exponent of AREA is 0.85, the other two variables SOIL and RSMD being negatively correlated with AREA, RSMD more so than SOIL. In the 4 variable equation the exponent of AREA is again 0.85 and in the 5 variable equation it is 0.87. The jump to 0.94 in the recommended 6 variable equation appears to be caused by the addition of the slope measure S1085 to the set. Because large catchments tend to be flat and many of the small ones are steep, there is a negative correlation between S1085 and AREA; hence, when S1085 is added as an independent variable, the exponent of AREA increases.

The recommended 6 variable equation for the central region is

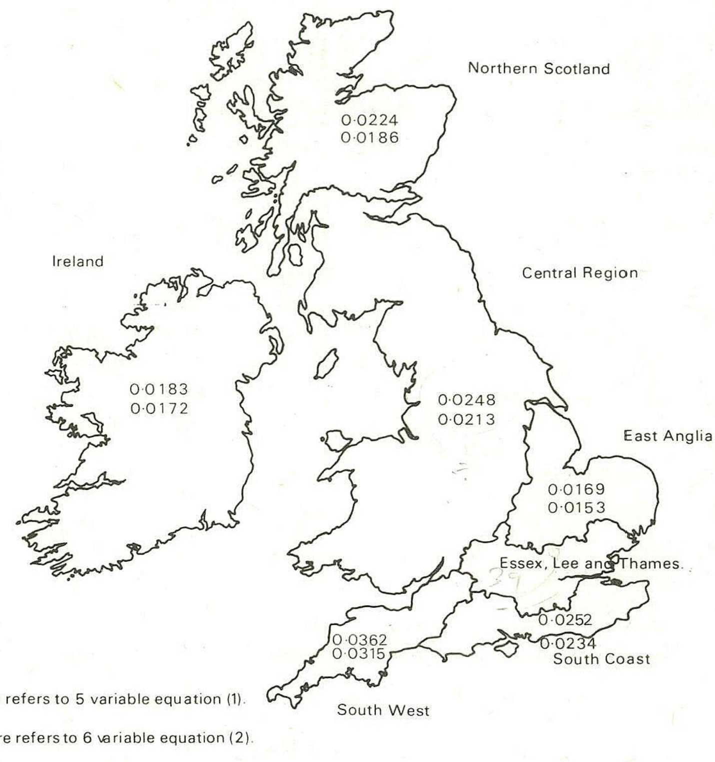
$$\bar{Q} = 0.0213\ (\text{AREA})^{0.94}(\text{STMFRQ})^{0.27}(\text{S1085})^{0.16}(\text{SOIL})^{1.23}(\text{RSMD})^{1.03}(1+\text{LAKE})^{-0.85} \qquad (4.14)$$

In this expression STMFRQ and S1085 both have small exponents, 0.27 and 0.16 respectively. SOIL has the largest exponent, 1.23; the RSMD, 1.03, is only a little greater than unity. LAKE has a negative exponent, −0.85; this implies that a catchment which drains wholly through a lake has a predicted mean annual flood about 55% of that of a completely similar

(1) $Q = \text{Const AREA}^{0.87}\ \text{STMFRQ}^{0.31}\ \text{SOIL}^{1.23}\ \text{RSMD}^{1.17}\ (1 + \text{LAKE})^{-0.97}$

(2) $Q = \text{Const AREA}^{0.94}\ \text{STMFRQ}^{0.27}\ \text{SOIL}^{1.23}\ \text{RSMD}^{1.03}\ (1 + \text{LAKE})^{-0.85}\text{S1085}^{0.16}$

(3) $Q = 0.302\ \text{AREA}^{0.70}\ \text{STMFRQ}^{0.52}\ (1 + \text{URBAN})^{2.50}$



1st figure refers to 5 variable equation (1).

2nd figure refers to 6 variable equation (2).

Equation (3) refers to Essex, Lee and Thames region.

Fig 4.15. Multipliers for regional equations.

catchment without the lake. The regional multipliers (the antilogarithms of the intercepts) are also given in the table and are illustrated in Figure 4.15. The multiplier for south west England is the highest, and is about double the lowest multiplier, that for East Anglia. Ireland is also low. The large central region is nearest to average as is to be expected from the number of stations in it. Northern Scotland is lower than average and the south coast is above average.

The standard logarithmic error of estimate for the recommended equation is 0.168 and the standard factorial error is 1.47. If the distribution of residuals were Normal 68 % of the catchments studied would have logarithmic residuals within the range ±0.168, and 95 % would be within ±0.336. The antilogarithms of these figures give the percentage error ranges for the mean annual flood. 68 % of actual floods are within the range +47 %, −32 % of the prediction and 95 % are within +117 %, −54 % of the prediction. This standard error for predictions of log mean annual flood can be compared with the error of estimate based on N years of record in order to find the value in terms of record length of this prediction. The standard error of estimate of the natural logarithm of the mean annual flood computed from N years of record is equal to CV/$\sqrt{N}$, where CV is the coefficient of variation of the annual floods. CV is hard to estimate for a single station and it is probably more reliable to use the average countrywide value of 0.357 in this case. Thus,

$$\text{se}\,(\log_e \bar{Q}) = \frac{\text{CV}}{\sqrt{N}}, \quad \text{or} \quad \text{se}\,(\log_{10} \bar{Q}) = 0.434\,\frac{\text{CV}}{\sqrt{N}}. \tag{4.15}$$

Substituting CV = 0.357 and the standard error of $\log_{10} \bar{Q}$ (0.168) gives the equivalent length of record as 0.9 years. In other words, the prediction from catchment characteristics gives a slightly more precise estimate of the mean annual flood than would be obtained from one year of record. It is obvious, therefore, that estimates from these equations must be used with extreme caution; even a few years of record at or near the site will be of greater value in estimating the mean annual flood. It is recommended that these equations should be used only for preliminary flood estimates during the earliest stages of the design of a project, and that as soon as a site is decided upon for the project a gauging station should be established to collect records from which more precise flood estimates may be made.

These equations might be useful in gauging station design. The predicted mean annual flood and the regional frequency curve (see Chapter 2) could be used to draw up a tentative flood frequency curve for the proposed site. This curve could then be used during the gauging station design to ensure that the station has adequate range to measure the floods required. Because of the errors in estimating this tentative curve any design should be conservative in case the errors are such as to underestimate the flood of a given return period.

In practice the designer often has available information which will allow him to correct these predictions. Take, for example, the case of a reservoir project. This will often involve a small catchment in the headwaters of the river system. The density of gauging stations in this country is such that gauging stations are commonly found on the same river as the development site or on nearby streams. The predicted mean annual floods at the gauged sites should be compared with the measured means. Often it will be found that there is a consistent pattern of over or underestimation in the locality. This could then be used to adjust the predicted flood at the site of interest.

No hard and fast rules can be given for this adjustment process; each case must be treated on its merits and the engineer must use his judgement to decide whether the prediction should be altered. Figure 4.23 (in Volume V) shows a map of residuals from the 6 variable equation of Table 4.20g with the average multiplier of 0.0201, and from equation 4.16 for Region 6; a number of areas with similar residuals can be seen. For example, the Spey valley shows mostly negative residuals, averaging about −0.30 which means the predicted mean annual flood is about twice that measured. Other regions showing mostly negative residuals are the Trent valley, the eastern half of Ireland and Peebles and Midlothian in Scotland. Positive residuals occur in the extreme south west peninsula and along the Pennine chain in England. However, it is not recommended that this map should be used as a guide in correcting estimates of the mean annual flood; the most recent available data on the mean annual flood should always be used.

Reservoir projects usually take several years to come to fruition and this period should be used to collect records at the site. If only a few years of record are available they could be used to derive a unit hydrograph. Chapter 6 describes a method of using this with rainfall statistics to derive flood estimates. If a longer record can be collected during the development phase of the project then the record of monthly maxima could be extended as described in Chapter 3 and the resulting annual maxima used to derive the mean annual flood.

As an example of a rather different problem consider the case of a flood relief project for a town farther down the river system. Here it is probable that there will be gauging stations within the catchment which could be used to correct the predictions as above. In addition, there will often be historical flood levels available (see Volume IV). Chapter 2.8 describes how these can be used.

The Thames, Lee and Essex region, that is hydrometric areas 36, 37, 38 and 39, were excluded from the above analysis and were considered separately. There were 50 gauging stations in this region. Optimal regression was used to relate BESMAF to subsets of variables chosen from AREA, STMFRQ, S1085, SOIL, URBAN, LAKE, RSMD, DVF and MSL. The dry valley factor (DVF) and mainstream length (MSL) had not been used in the rest of this study. DVF was included here because this region includes many chalk areas, where dry valleys are common. Mainstream length was included in the set as on the chalk the streams are often short in relation to catchment area. Thus, DVF and MSL can be regarded, in part, as complementary. The correlation matrix of the data, after transformation, is

Table 4.21 Correlation matrix for Region 6 (hydrometric areas 36–39).

	BESMAF	AREA	STMFRQ	S1085	SOIL	URBAN	LAKE	RSMD	SMDBAR	DVF	MSL
BESMAF	1.000	0.557	0.460	0.525	0.410	0.118	−0.029	−0.310	0.060	−0.585	0.632
AREA	0.557	1.000	−0.171	−0.898	0.073	−0.313	−0.226	−0.187	0.018	−0.410	0.844
STMFRQ	0.460	−0.171	1.000	0.041	0.537	−0.029	0.128	−0.446	0.343	−0.590	0.232
S1085	0.525	−0.898	0.041	1.000	−0.222	0.324	0.200	0.227	−0.063	0.454	−0.823
SOIL	0.410	0.073	0.537	−0.222	1.000	−0.113	0.158	−0.454	0.331	−0.402	0.205
URBAN	0.118	−0.313	−0.029	0.324	−0.113	1.000	−0.127	0.089	−0.043	0.201	−0.331
LAKE	−0.029	−0.226	0.128	0.200	0.158	−0.127	1.000	0.274	−0.356	0.138	−0.235
RSMD	−0.310	−0.187	−0.446	0.227	−0.454	0.089	0.274	1.000	−0.867	0.402	−0.383
SMDBAR	0.060	0.018	0.343	−0.063	0.331	−0.043	−0.356	−0.867	1.000	−0.286	0.184
DVF	−0.585	−0.410	−0.590	0.454	−0.402	0.201	0.138	0.402	−0.286	1.000	−0.693
MSL	0.632	0.844	0.232	−0.823	0.205	−0.331	−0.235	−0.383	0.184	−0.693	1.000

Calculated from 50 stations' data. Logarithmic transform has been applied. LAKE and URBAN have been transformed to the form log (1 + LAKE).

No. var.	Name	Coeff.	seb	t	R	R^2	see	Const.	Multiplier
1	MSL	0.80	0.142	5.6	0.632	0.399	0.391	−0.1122	0.772
2	AREA	0.59	0.082	7.2	0.792	0.627	0.311	−0.0926	0.808
	STMFRQ	0.49	0.078	6.3	—	—	—	—	—
3	AREA	0.70	0.070	10.0	0.870	0.757	0.254	−0.4285	0.373
	STMFRQ	0.52	0.064	8.2	—	—	—	—	—
	URBAN	2.52	0.509	5.0	—	—	—	—	—
4	AREA	0.74	0.071	10.3	0.879	0.773	0.248	−0.5208	0.301
	STMFRQ	0.51	0.062	8.2	—	—	—	—	—
	URBAN	2.71	0.509	5.3	—	—	—	—	—
	LAKE	1.65	0.941	1.8	—	—	—	—	—

Table 4.22 Regressions for Essex, Lee and Thames area (Region 6).

shown in Table 4.21, while Table 4.22 shows the best one to four variable subsets. MSL is the best single variable describing nearly 40% of the variance of the mean annual flood. From the correlation matrix we see that DVF is the second best choice with AREA lying third. The best two variable equation uses catchment area and the stream frequency STMFRQ.

Thereafter the variables are added in a regular fashion. URBAN is added as the third variable and LAKE as the fourth. However, the exponent of LAKE is not significantly different from zero and the same applies to all other variables added. It is interesting that no rainfall variables appear; it is not until the best 6 variable subset that RSMD is chosen. This is due to the small variation in rainfall across the region; for example, the annual average rainfall for the catchments in the region varies only from 577 to 839 mm. The recommended equation for the mean annual flood to be used in hydrometric areas 36, 37, 38 and 39 is

$$\bar{Q} = 0.302\ \text{AREA}^{0.70}\text{STMFRQ}^{0.52}(1+\text{URBAN})^{2.5}. \qquad (4.16)$$

For this equation, $R^2 = 0.773$ and the standard error of estimate is 0.248, which gives a factorial error of 1.77. This estimate is considerably less precise than that for the other regions. However, this region has a very diverse character; some catchments of the region are highly urbanised (up to 80% urban) while others are on very pervious Chalk or Oolite formations. This wide diversity of catchment type makes prediction difficult. Indeed it is largely because of the isolation in this region of many of the unusual catchments that the degree of success achieved in the rest of the country was possible. Figures 4.16 and 4.17 are nomograms for the prediction equations.

Regressions for CV

Table 4.23 attached shows the results of an optimal regression for CV. The best one to four variable subsets are shown. RSMD accounts for 9%

Table 4.23 Regressions of coefficient of variation (CV) (all stations).

No. var.	Name	Coeff.	seb	t	R	R^2	see	Const.	Multiplier
1	RSMD	−0.411	0.057	7.2	0.300	0.090	0.205	0.1456	1.398
2	S1085	−0.065	0.022	2.9	0.322	0.104	0.204	0.2753	1.885
	RSMD	−0.518	0.067	7.7	—	—	—	—	—
3	STMFRQ	−0.071	0.028	2.5	0.338	0.114	0.203	0.1341	1.362
	S1085	0.073	0.023	3.2	—	—	—	—	—
	RSMD	−0.434	0.075	5.8	—	—	—	—	—
4	STMFRQ	−0.087	0.032	2.8	0.342	0.117	0.203	0.1996	1.583
	S1085	0.072	0.023	3.2	—	—	—	—	—
	SOIL	0.107	0.093	1.2	—	—	—	—	—
	RSMD	0.447	0.076	5.9	—	—	—	—	—

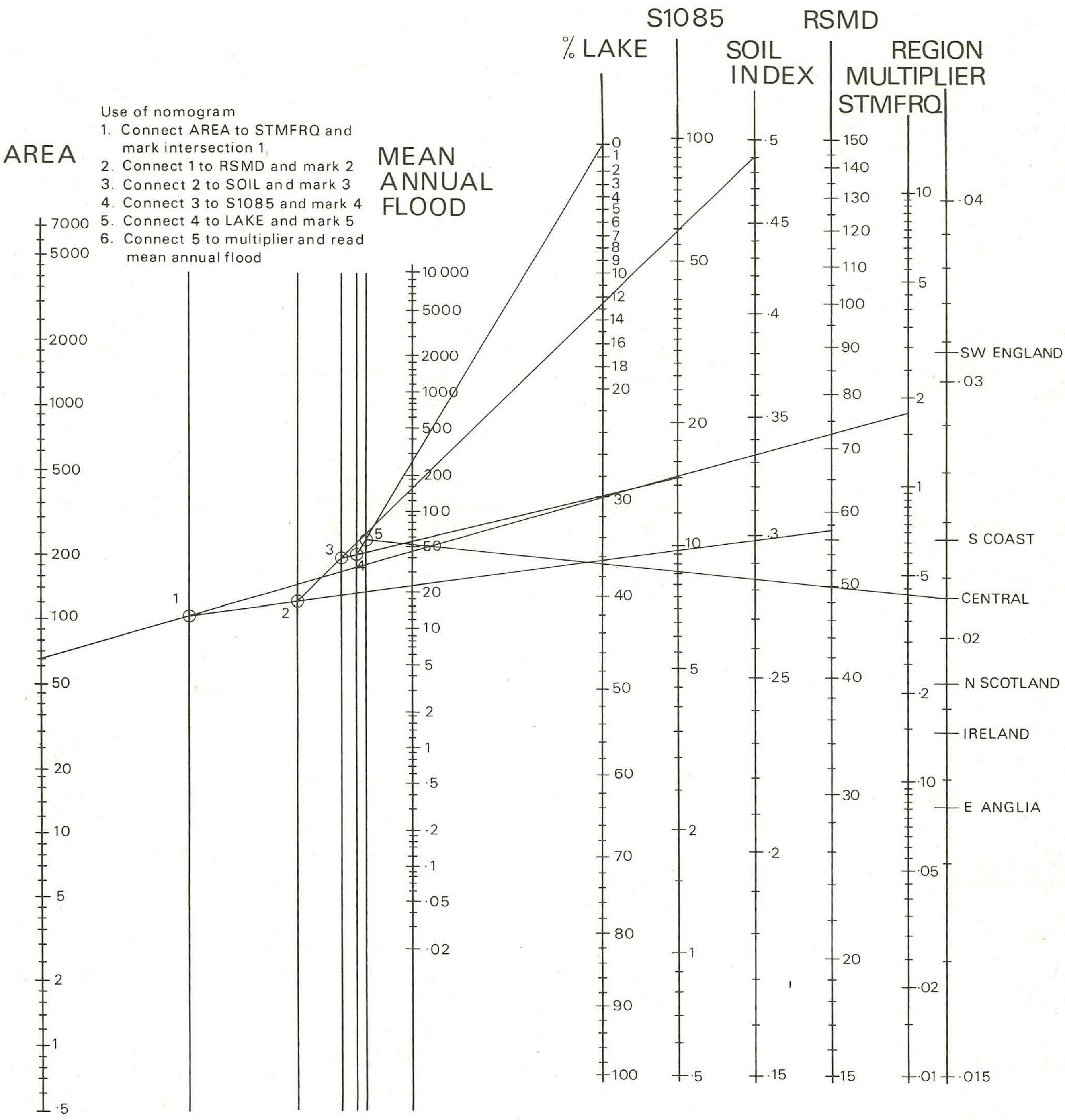


Fig 4.16 Nomograms for regional prediction equations.

of the variation in CV. The addition of S1085 and STMFRQ brings this figure up to 11.4%. SOIL is the next variable to be added, but its coefficient is not significantly different from zero. Much of the variation of observed CV values (see Figure 4.22 in Volume V) must be attributed to the sampling variance which can be shown to depend on higher moments of the flood distribution. A tendency was observed for CV to be better correlated with catchment characteristics as the length of record increased. The CV was still most closely related to climate characteristics; the variations between regional growth curves (Chapter 2), from steep curves in the dry south east to shallow curves in the north west, are consistent with such a relationship. The effect of steeper rainfall growth curves in the drier parts of the country is reinforced by the greater variation in percentage runoff between events in the south east. These poor results from regression are not surprising and it is recommended that the regional frequency curves of Chapter 2 be used for ungauged sites.

4.3.11 Comparison with other investigations

The other investigations considered here are those by Benson (1962), Nash & Shaw (1965) and Cole (1965). The last two studies were of catchments in Great Britain but Benson used catchments in New England. Benson and Nash & Shaw produce regression equations for flood statistics in terms of catchment characteristics. Some of the catchment characteristics used were not available in this study so a full comparison cannot be made. However, a partial comparison can be made using common characteristics.

Benson's study was of 164 catchments in the New England area of the north eastern United States. The catchments ranged in size from 4.25 to 25 000 km². This upper limit is considerably greater than the largest catchment in the present study. The variables in common with the present study were catchment area and the 10–85% slope which was proposed by Benson. When translated to metric units Benson's equations for the mean annual flood are

$$\bar{Q} = 0.561\ \text{AREA}^{0.85},\ (R^2 = 0.841,\ \text{standard error} = 0.194)$$

and

$$\bar{Q} = 0.0765\ \text{AREA}^{1.06}\ \text{S1085}^{0.52},\ (R^2 = 0.922,\ \text{standard error} = 0.142).$$

The data of the present study give the equations

$$\bar{Q} = 0.677\ \text{AREA}^{0.77},\ (R^2 = 0.563,\ \text{standard error} = 0.436),$$

and

$$\bar{Q} = 0.0236\ \text{AREA}^{1.19}\ \text{S1085}^{0.84},\ (R^2 = 0.759,\ \text{standard error} = 0.325).$$

The numerical values in the equations are roughly similar. Note the close fit that Benson obtained (high R^2 or small standard error). His catchments appear to be much more predictable than those used in this study.

Benson's two variable equation was applied to the catchments used in this study in order to see how closely it fitted the data. The average error (in logarithmic terms) was -0.022 corresponding to an average 5% overestimate of the mean annual flood on the catchment. Considering that the equation was developed for a different country the agreement achieved must be considered very good. However, this result is the average of over 500 values ranging from an 85% underestimate (predicted flood only 15% of recorded flood) to a 4100% overestimate.

Nash & Shaw studied 57 catchments in Great Britain, ranging in size from 7.8 to 9900 km². Many of these catchments are, of course, included in the present study. The equations for the mean annual flood in metric units are

$$\bar{Q} = 0.76\ \text{AREA}^{0.74},\ (R^2 = 0.60,\ \text{standard error} = 0.405),$$

and

$\bar{Q} = 9.65 \times 10^{-8}$ AREA$^{0.85}$ SAAR$^{2.2}$, ($R^2 = 0.92$, standard error $= 0.179$).

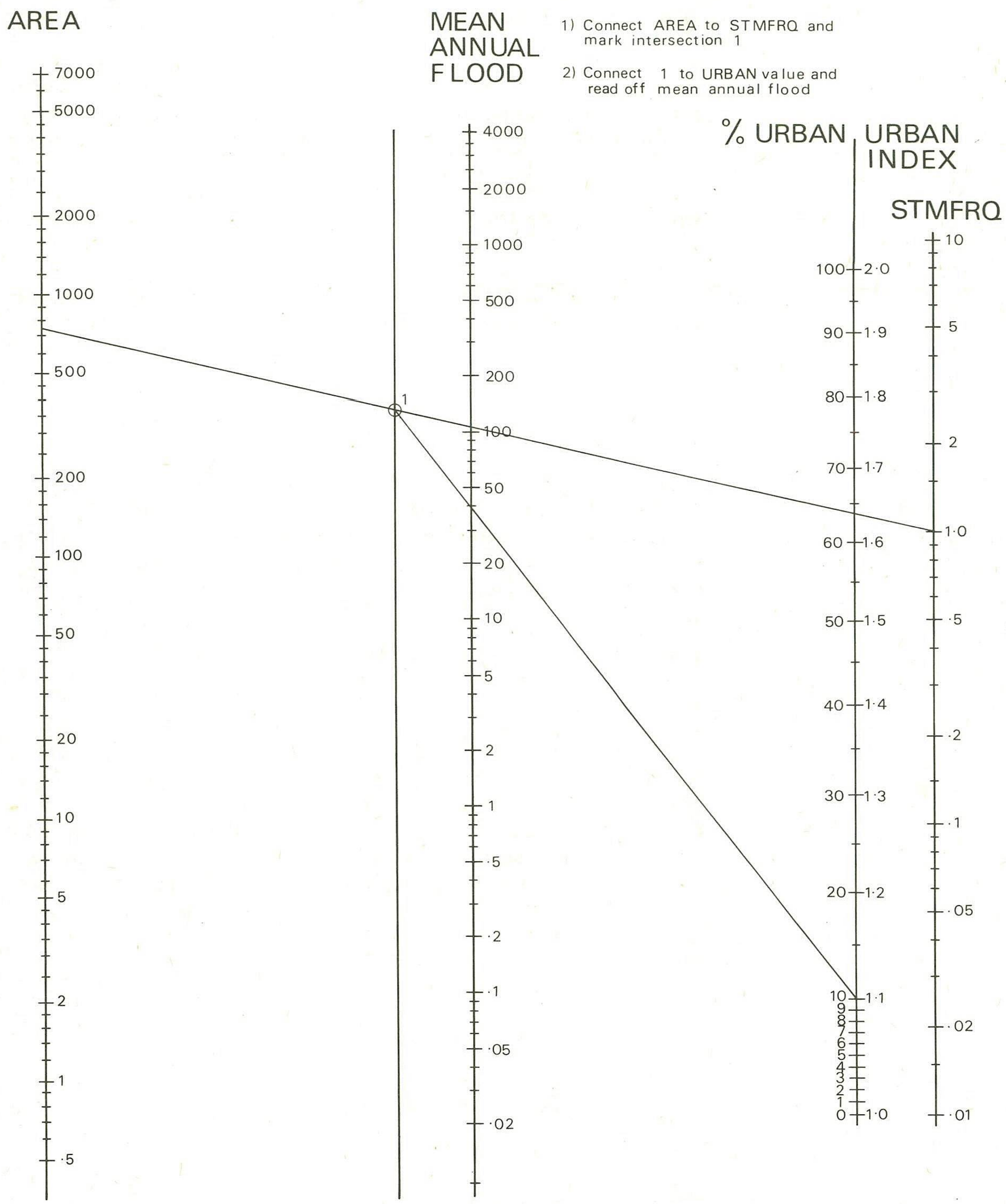
The corresponding equations in this investigation are

$\bar{Q} = 0.677$ AREA$^{0.77}$, ($R^2 = 0.563$, standard error $= 0.436$),

and

$\bar{Q} = 2.242 \times 10^{-7}$ AREA$^{0.84}$ SAAR$^{2.09}$, ($R^2 = 0.805$, standard error $= 0.292$).

Fig 4.17 Nomograms for Essex, Lee and Thames prediction equation.



Reasonably close agreement in the coefficients is evident. Note also the better fit obtained by Nash & Shaw. The 2 variable equation was applied to the data of the current study. The mean error in prediction was 0.024 (in logarithmic terms) which implies, on average, a 6% underestimate.

Cole (1965) studied 56 catchments in England and Wales. He plotted mean annual flood against catchment area on log–log paper. The points were found to lie close to one of six parallel lines with a slope of 0.85. That implies that the equation for the mean annual flood can be written as

$$\bar{Q} = C \times \text{AREA}^{0.85}$$

where C is a regional coefficient and takes the following values in the six regions: 0.028, 0.126, 0.277, 0.53, 1.01 and 2.27. A map of the six regions defined in this manner is given. There are some similarities with the regions of this study. The south west peninsula was divided off in a similar fashion; East Anglia and the south coast are joined as one region which extends into parts of the Thames and Essex region which was treated separately in the current regional study. However, the central region is divided into a large number of fragments which belong to one of four regions.

These other studies are of interest as they show rough agreement with the present study and hence help to increase confidence in its results.

4.3.12 Example

As an example of the use of these equations, consider the estimation of the flood frequency curve on the River Rhymney at Gilfach-Bargoed, in South Wales. A detailed description of this catchment is given in Chapter 6, where other methods are used to estimate floods. This catchment is in the central region so the equation from Table 4.20g for the mean annual flood is

$$\bar{Q} = 0.0213 \ \text{AREA}^{0.94} \ \text{STMFRQ}^{0.27} \ \text{S1085}^{0.16} \ \text{SOIL}^{1.23} \ \text{RSMD}^{1.03} (1+\text{LAKE})^{-0.85}. \tag{4.17}$$

The catchment characteristics are: AREA 63.2 km^2, STMFRQ 1.84 junctions /km^2, S1085 14.7 m/km, RSMD 57.3 mm; 20% of the catchment is soil Class 4 and 80% Class 5 giving SOIL = 0.49 and there are no lakes. The mean annual flood is thus 51 cumecs. The range for one standard error either side of this estimate is 35–75 cumecs. If the residual map (Figure 4.23) had shown that nearby sites were consistently over or underestimated, and if site visits confirmed the similarity of the catchments, some account could be taken of this. The mean annual flood estimate can be used to scale the region curve for Wales (Region 9) to give the flood frequency curve for this station as shown in the table.

Return period (years)	$Q(T)/\bar{Q}$	$Q(T)$ (cumecs)
2.33	1.0	51 (mean)
5	1.22	62
10	1.43	73
50	1.94	100
100	2.19	110
500	2.80	140

The error of estimate of these rare floods is of course greater than for the mean as the region curve has a standard error that should be added to the error of estimate of the mean (see Section 2.11.5).

4.4 References

ARMENTROUT C.L. & BISSELL R.B. (1970) Channel slope effect on peak discharge of natural streams. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, **96, (HY2),** 307–315.

BEALE E.M.L., KENDALL M.G. & MANN D.W. (1967) The discarding of variables in multivariate analysis. *Biometrika*, **54,** 357–366.

BELL F.C. & SONGTHARA OM KAR (1969) Characteristic response times in design flood estimation. *Journal of Hydrology*, **8,** 173–196.

BENSON M.A. (1959) Channel slope factor in flood frequency analysis. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, **85**, **(HY4)**, 1–19.

BENSON M.A. (1962) Factors influencing the occurrence of floods in a humid region of diverse terrain. *United States Geological Survey Water Supply Paper* 1580–B.

BERAN M.A. & SUTCLIFFE J.V. (1972) An index of flood producing rainfall based on rainfall and soil moisture deficit. *Journal of Hydrology*, **19,** 229–236.

BOULTON A.G. (1965) Morphometric analysis of river basin characteristics. *U.K. Water Resources Board, TN2*, 10.

CARLSTON C.W. & LANGBEIN W.B. (1960) Rapid approximation of drainage density: line intersection method. *U.S. Geological Survey, Water Resources Division Bulletin*, 10 February, 11.

CARLSTON C.W. (1963) Drainage density and streamflow. *U.S. Geological Survey Professional Paper*, 422–C.

CHIANG S.L. & PETERSEN G.W. (1970) Soil catena concept for hydrologic interpretations. *Journal of Soil and Water Conservation*, **25,** 225–227.

CHOW VEN T. (1962) Hydrologic determination of waterway areas for the design of drainage structures in small drainage basins. *University of Illinois Bulletin*, 59(65), *Engineering Experimental Station*, Bulletin No. 462.

COLE G. (1965) An application of the regional analysis of flood flows. *Institution of Civil Engineers, Symposium on River Flood Hydrology* (published 1966), 39–57.

CUNNANE C. (1973) A particular comparison of annual maxima and partial duration series methods of flood frequency analysis. *Journal of Hydrology*, **18**, 257–271, **19**, 377.

DRAPER N.R. & SMITH H. (1966) *Applied Regression Analysis*. Wiley, New York.

EFROYMSON M.A. (1960) Multiple regression analysis. *Mathematical Methods for Digital Computers*, Ed. by A. Ralston and H.S. Wilf. Wiley, New York.

GRAY D.M. (1964) Physiographic characteristics and the runoff pattern. *Proceedings of the Hydrological Symposium* 1964, No. 4, 147–166. National Research Council of Canada.

GRINDLEY J. (1970) Estimation and mapping of evaporation. *International Association of Scientific Hydrology Symposium, World Water Balance, Reading. IASH Publication* **92,** 200–213.

GUPTA S.N., BHATTACHARYA A.P. & JINDAL S.R. (1969) Statistical correlation of Himalayan and Bundelkhand basin characteristics with flood flows. *International Association of Scientific Hydrology, Symposium, Floods and their computation, Leningrad, IASH Publications* **85,** 874–880.

INSTITUTION OF CIVIL ENGINEERS (1967) Flood Studies for the United Kingdom.

INTERNATIONAL COMPUTERS LIMITED (1969) ICL Statistical analysis mark 2 1900 series. *2nd Edition Technical Publications 4162,* August 1969. ICL, London.

KENNEDY R.J. & WATT W.E. (1969) The relationship between lag time and the physical characteristics of drainage basins in Southern Ontario. *International Association of Scientific Hydrology, Symposium, Floods and their computation, Leningrad, IASH Publications* **85,** 866–874.

LYNN M.A. (1971) Flood estimation for ungauged catchments. *Transactions of the Institution of Engineers of Ireland,* **96,** 154–176.

MANDEVILLE A.N., O'CONNELL P.E., SUTCLIFFE J.V. & NASH J.E. (1970) River flow forecasting through conceptual models, III, the Ray catchment at Grendon Underwood. *Journal of Hydrology,* **11,** 109–128.

MELTON M.A. (1958) Correlation structure of morphometric properties of drainage systems and their controlling agents. *Journal of Geology,* **66,** 442–460.

MUSGRAVE G.W. (1955) How much of the rain enters the soil? *Water: The Yearbook of Agriculture,* United States Department of Agriculture, 151–159.

NASH J.E. (1960) A unit hydrograph study with particular reference to British catchments, *Proceedings of the Institution of Civil Engineers,* **17,** 249–282.

NASH J.E. & SHAW B.L. (1965) Flood frequency as a function of catchment characteristics. *Institution of Civil Engineers, Symposium on River Flood Hydrology,* (published 1966), 115–136.

NEWSON M.D. (1974) Mapwork for flood studies N.E.R.C. Institute of Hydrology, Report No. 25.

PAINTER R.B. (1971) A hydrological classification of the soils of England and Wales. *Proceedings of the Institution of Civil Engineers,* **48,** Technical Note 29, 93–95.

POTTER W.D. (1953) Rainfall and topographic factors that affect runoff. *Transactions of the American Geophysical Union,* **34,** 67–73.

REICH B.M. (1970) Land surface form in flood hydrology. *Symposium on Environmental Geomorphology, University of New York.*

RODDA J.C. (1967) The significance of characteristics of basin rainfall and morphometry in a study of floods in the United Kingdom. *International Association of Scientific Hydrology, Symposium, Floods and their computation,* Leningrad, *IASH Publications* **85,** 834–845.

SOIL CONSERVATION SERVICE, U.S. DEPARTMENT OF AGRICULTURE (1964) Hydrology: part 1, watershed planning.

STRAHLER A.N. (1950) Equilibrium theory of slopes approached by frequency distribution analysis. *American Journal of Science,* **248,** 800–814.

TAYLOR A. & SCHWARZ H. (1952) Unit hydrograph lag and peak flow related to basin characteristics. *Transactions of the American Geophysical Union,* **33,** 235–246.

WONG S.T. (1963) A multivariate statistical model for predicting mean annual flood in New England. *Annals of the Association of American Geographers*, **53**, 298–311.

5 Estimation of flood volumes over different durations

5.1 Introduction

Although instantaneous peak flow is usually needed in flood design, the total flood volume is often required. For reservoir routing and spillway design where flood volumes over short durations are needed, methods of hydrograph analysis and prediction are appropriate and these are described in Chapter 6.

However, where quick preliminary results are required for durations over a day and moderate return periods, an alternative method is proposed. Typical cases requiring estimates of flood volumes include design of flood control storage reservoirs or retarding basins, and flood damage studies needing information on depth, area and duration of flooding. Information on flood volumes becomes particularly important in studies of floods in the lower reaches of large rivers, but in such cases direct analysis of flow records may be essential. The method of estimating flood volumes presented here relates flows over different durations to either the arithmetic mean annual instantaneous flood (AMAF) or the mean annual calendar day flood (CALMAF), which can be derived either from flow records (Chapter 2) or by approximate estimates obtained from regressions on catchment characteristics.

The ratios of the mean flows over different durations to either AMAF or CALMAF can be plotted against duration, giving a reduction curve (Figure 5.1). The shape of these curves varies with catchment characteristics, the steeper catchments having steeper reduction curves with more pronounced curvature. The parameters of the curves based on AMAF were found not to be highly related to catchment characteristics, and as an alternative the two simple ratios of 3 and 10 day floods to CALMAF (AR3 and AR10 respectively) were also correlated with catchment characteristics. These two ratios, which give a reasonable indication of the reduction curve, can be estimated from the channel slope.

5.2 Flow duration reduction curves

5.2.1 Source of basic data

The mean annual flood discharges over different durations were derived from daily mean flow records, computed by either the Water Resources Board or the river authority; time did not permit a review of these standard hydrometric data as was possible for instantaneous flood events. For each water year of station record a program computed average flows over durations of 1–10 days, and the maximum average flows for each of the 10 durations were selected. These need not be derived from nesting periods, so that the 1 day flood was not necessarily included in the 10 day flood period.

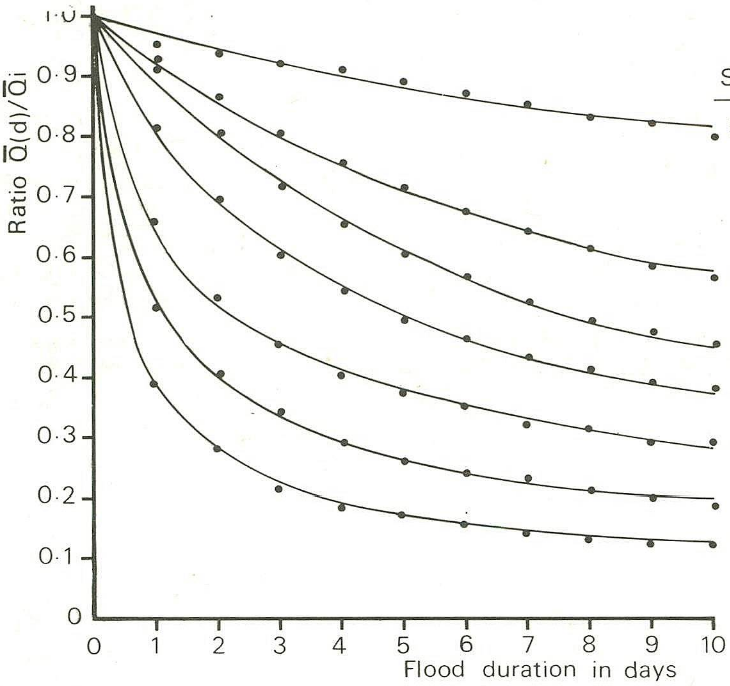
To reduce errors introduced by using calendar day flows, flood volumes were interpolated from a quadratic curve fitted to the daily mean flows. This increased the mean discharges by about 1–3 %.

The instantaneous flood peaks were derived by the Floods Study Team from the original records, being independently extracted from the original charts and processed using checked ratings, as described in Volume IV. Thus, the instantaneous flows were not necessarily directly comparable with the daily mean flows and those of longer duration, which were obtained from the hydrometric records of the Water Resources Board and river authorities.

5.2.2 *General form of reduction curves*

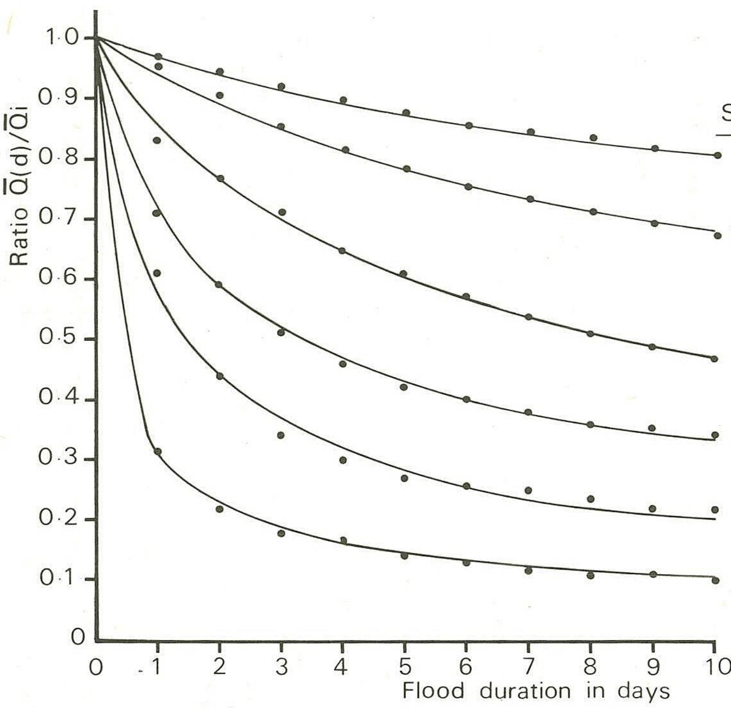
Average flood discharges over various durations up to 10 days were computed and compared with the instantaneous and daily mean discharges in

Fig 5.1 Typical reduction curves fitted to observed data.



Station	N	B	Area(km^2)	S1085
55/3	0·21	0·19	886	2·32
55/1	0·72	0·12	4040	3·04
21/31	0·96	0·14	680	3·39
45/1	0·56	0·48	601	4·40
55/7	0·45	1·63	1280	3·70
23/3	0·48	2·99	1010	3·17
55/10	0·54	5·08	27	43·30

— Fitted curve
• Observed data



Station	N	B	Area (km^2)	S1085
39/6	0·19	0·22	365	1·90
33/6	0·42	0·16	275	1·34
54/2	0·49	0·36	2210	0·80
47/1	0·42	1·26	917	1·60
72/1	0·57	1·61	995	2·86
25/2	0·51	8·96	217	12·88

the form of a ratio which reduces with increasing duration. Figure 5.1 shows typical examples of these curves, which can be described in the form

$$Q(d) = \frac{Q_i}{(1+Bd)^N} \quad \text{or} \quad r = \frac{1}{(1+Bd)^N} \tag{5.1}$$

where

$Q(d)$ is the average flood discharge over duration d days, for a given return period;

Q_i is the instantaneous peak discharge for the given return period;

r is $Q(d)/Q_i$, the reduction ratio;

N, B are parameters determined empirically.

The form of this equation was suggested by the rainfall intensity duration relationship used in the meteorological studies of Volume II, Chapter 3.

The parameters N and B were found for the 78 stations shown in Figure 5.2, and are listed in Table 5.1, together with the length of records varying from 5 to 49 years used to derive the instantaneous and longer duration floods. The data to which the reduction curves have been fitted were the ratios of mean annual maximum flood flow over durations of 1–10 days, to the mean annual instantaneous flood, derived from the same set of stations but not necessarily the same years or length of record. In some cases the length of the instantaneous and volume flood records differ significantly and the effects of this are examined below.

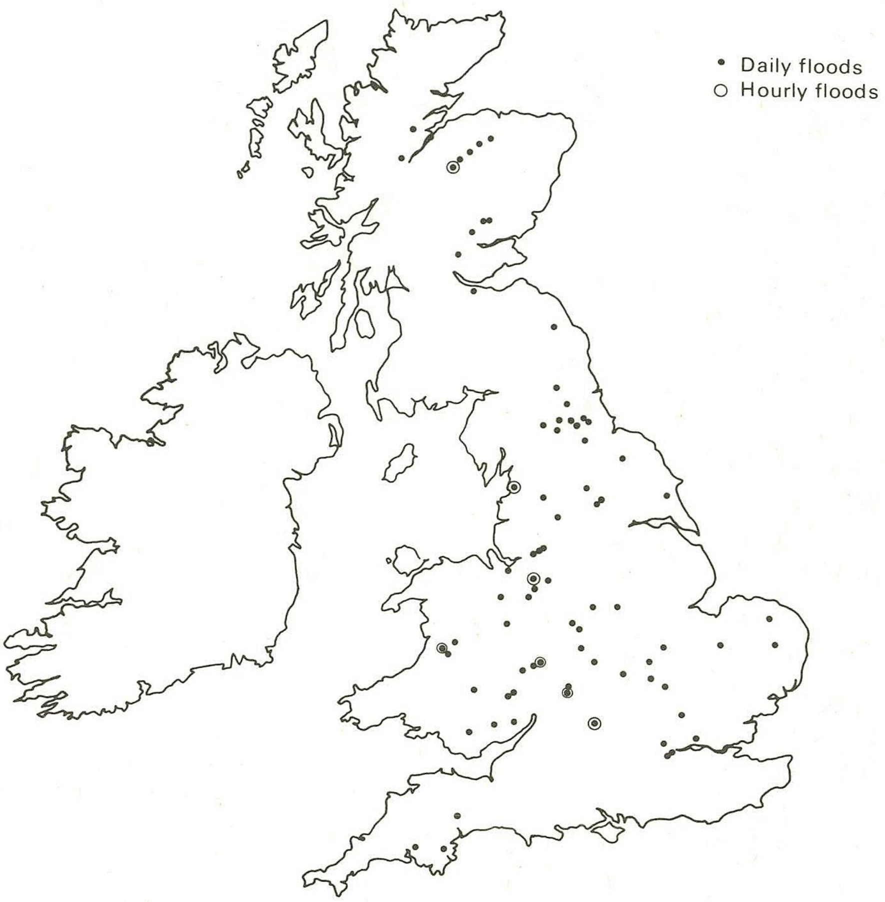


Fig 5.2 Location of stations used in the estimation of flood volumes.

Table 5.1 Basic data used.

Station no.	Name	Reduction curve parameters		SAAR (mm)	AREA (km²)	STMFRQ (nodes/km²)	S1085 (m/km)	Record length (years)	
		N	*B*					Vol. flood	Inst. flood
1. 34 station data set: All records concurrent to within a year									
4/1	Conon at Moy Bridge	0.26	5.32	1758	971	0.91	6.70	11	11
21/31	Till at Etal	0.96	0.14	838	648	1.45	3.39	12	13
23/2	Derwent at Eddys Bridge	0.45	2.46	932	118	2.14	10.67	10	10
23/3	North Tyne at Reaverhill	0.48	2.99	1062	1010	2.38	3.17	10	10
24/1	Wear at Sunderland Bridge	0.50	1.76	945	658	2.31	5.01	11	12
24/2	Gaunless at Bishop Auckland	0.46	2.30	752	93.0	0.78	9.82	11	11
24/3	Wear at Stanhope	0.58	3.25	1318	172	3.68	14.33	10	11
24/4	Bedburn Beck at Bedburn	0.48	2.35	854	74.9	2.14	21.16	10	10
25/2	Tees at Dent Bank	0.51	8.96	1717	217	3.08	12.88	10	10
25/4	Skerne at South Park	0.66	0.42	671	219	0.49	1.82	13	12
27/12	Hebden Water at High Greenwood	0.39	3.91	1415	36.0	4.92	32.90	15	16
32/2	Willow Brook at Fotheringhay	0.50	0.39	602	89.6	0.21	2.87	28	29
33/6	Wissey at Northwold	0.42	0.16	660	275	0.13	1.34	13	13
34/2	Tas at Shotesham	0.84	0.40	668	147	0.27	1.65	10	11
37/1	Roding at Redbridge	0.77	0.25	635	303	1.17	1.22	19	19
38/3	Mimram at Panshanger	0.44	0.55	665	134	0.03	2.59	16	16
39/6	Windrush at Newbridge	0.19	0.22	785	365	0.42	1.90	19	19
39/12	Hogsmill at Kingston	0.44	5.65	691	69.1	0.19	2.55	11	10
47/1	Tamar at Gunnislake	0.42	1.26	1229	917	1.79	1.60	13	13
54/2	Avon at Evesham	0.49	0.36	683	2210	0.66	0.80	32	32
54/4	Sowe at Stoneleigh	0.57	0.53	707	264	0.72	1.92	17	18
54/6	Stour at Kidderminster	0.44	1.03	721	324	0.42	2.38	16	17
54/7	Arrow at Broom	0.61	0.92	709	319	1.00	3.30	12	13
54/8	Teme at Tenbury Wells	0.98	0.17	871	1130	0.90	3.04	13	13
55/1	Wye at Cadora	0.72	0.12	1039	4040	1.04	1.10	32	32
55/3	Lugg at Lugwardine	0.21	0.19	836	886	0.60	2.32	29	29
55/7	Wye at Erwood	0.45	1.63	1425	1280	1.16	3.70	31	31
55/10	Wye at Pant Mawr	0.54	5.08	2461	27.2	3.01	45.30	14	14
56/1	Usk at Chain Bridge	0.48	1.34	1388	912	1.18	2.84	12	12
68/1	Weaver below Ashbrook	1.12	0.12	772	609	0.38	1.30	30	31
68/4	Valley Wistaston Brook at Marshfield Bridge	0.55	0.48	785	88.1	0.36	4.04	11	12
68/6	Dane at Hulme Walfield	0.54	2.98	1034	152	1.42	9.41	16	16
71/3	Croasdale Beck at Croasdale Flume	0.53	6.05	1839	10.4	3.36	35.60	12	12
72/1	Lune at Halton	0.57	1.61	1577	995	2.05	2.86	9	10
2. 52 station data set: Those above plus the following 18 stations whose records are concurrent to within 2–4 years									
6/906	Allt Bhlaraidh at Invermoriston	0.40	3.46	1689	27.5	0.76	33.27	9	7
8/1	Spey at Aberlour	0.44	1.21	1168	2640	0.98	1.74	28	30
8/4	Avon at Delnashaugh	0.48	4.18	1119	544	1.44	10.55	14	18
8/10	Spey at Grantown	1.08	0.12	1237	1750	0.86	2.17	14	18
15/5	Melgam at Loch of Lintrathen	0.35	4.54	1166	40.9	0.37	14.82	18	16
16/1	Earn at Kinkell Bridge	0.68	0.33	1608	591	0.95	2.19	18	20
19/1	Almond at Craigie Hall	0.38	3.74	914	369	0.75	4.87	10	14
24/5	Browney at Burn Hall	0.62	0.88	770	178	1.32	5.36	12	15
25/3	Trout Beck at Moor House	0.51	7.20	2182	11.4	2.89	35.79	11	7
27/7	Ure at Westwick Lock	0.56	0.64	1163	914	1.16	3.24	10	14
28/4	Tame at Lea Marston	0.73	0.14	734	795	0.43	1.34	9	13
28/5	Tame at Elford	0.16	9.15	714	1480	0.44	1.01	9	13
45/1	Exe at Thorverton	0.56	0.48	1280	601	0.86	4.40	11	13
54/13	Clywedog at Cribynau	0.56	2.88	1803	57	2.40	15.38	10	6
57/4	Cynon at Abercynon	0.48	1.73	1801	109	2.33	7.30	11	8
67/2	Dee at Erbistock Rectory	0.38	0.75	1455	1040	1.62	1.42	32	30
68/2	Gowy at Picton	0.55	0.85	754	156	0.35	1.89	18	20
69/1	Mersey at Irlam Weir	0.47	1.10	1118	679	1.19	2.52	34	30
3. 64 station set: The above plus 12 stations with record differences of 5 years and over									
15/3	Tay at Caputh	0.25	1.70	1648	3210	1.18	1.52	9	18
15/4	Inzion at Loch of Lintrathen	0.26	5.03	1107	24.7	0.61	13.55	16	39
27/1	Nidd at Hunsingore Weir	0.45	2.08	993	484	1.23	4.01	24	35
27/2	Wharfe at Flint Mill Weir	0.47	1.57	1161	759	1.31	2.67	13	32
27/10	Hodge Beck at Bransdale Weir	0.51	4.73	1057	18.9	2.38	30.64	13	32
28/2	Blithe at Hamstall Ridware	0.41	2.17	790	162	0.72	3.36	30	15
28/10	Derwent at Longbridge	0.48	0.86	996	1120	1.24	2.56	18	33
33/9	Great Ouse at Harrold Mill	0.44	0.38	655	1320	0.47	0.82	12	18

Station No.	Name	Reduction curve parameters		SAAR mm	AREA km²	STMFRQ nodes/km²	S1085 m/km	Record length (years)	
		N	*B*					Vol. flood	Inst. flood
3. 64 station set (continued)									
34/3	Bure at Ingworth	0.32	1.71	699	165	0.33	2.31	5	10
68/5	Weaver at Audlem	0.37	1.84	772	262	0.24	1.50	16	33
69/3	Irk at Scotland Weir	0.50	5.53	1064	74.3	0.58	5.26	19	14
69/6	Bollin at Dunham Massey	0.38	1.83	911	256	0.64	4.00	15	33
4. Deleted data: 14 stations									
6/901	Ness at Ness Castle Farm	21.17	0.002	1824	1790	1.23	2.17	25	33
8/2	Spey at Kinrara	2.57	0.04	1364	1010	1.18	2.36	16	19
8/5	Spey at Boat of Garten	6.66	0.02	1311	1270	1.02	2.25	16	19
26/3	Foston Beck at Foston Mill	0.04	2.00	719	57.2	0.12	1.79	10	9
28/9	Trent at Colwick	225.77	0.00	785	7490	0.68	0.63	10	11
32/8	Whilton Branch at Dodford	0.25	40.95	673	107	0.36	4.38	21	22
33/2	Great Ouse at Bedford	1507.54	0.00	650	1460	0.46	0.63	26	10
39/4	Wandle at Beddington	0.32	85.27	800	122	0.01	4.36	15	27
39/5	Beverley Brook at Wimbledon Common	0.45	16.66	64	43.5	0.20	2.28	9	7
39/10	Colne at Denham	350.59	0.00	726	743	0.15	1.73	12	16
46/2	Teign at Preston	0.23	89.76	1267	381	0.36	9.83	11	13
54/1	Severn at Bewdley	689.53	0.00	945	4330	0.94	0.60	49	30
54/5	Severn at Montford	1783.50	0.00	1179	2030	1.53	1.20	16	17
55/2	Wye at Belmont	391.26	0.00	1267	1900	1.05	2.00	31	58

Table 5.1 Basic data used (continued).

Table 5.2 Hourly flood volumes.
Arithmetic mean annual flood volumes over hourly durations in cumecs.

	Flood duration (hours)										
	Instantaneous	1	2	3	6	9	12	18	24	36	48
8/2 Spey at Kinrara (19 years)											
Mean	144.64	137.95	137.81	137.60	136.81	135.81	134.67	132.21	129.15	123.22	118.92
Mean (daily)									(140.93)		(124.11)
cv %	43.41	47.14	47.17	47.26	47.48	47.68	47.84	47.80	47.95	46.31	43.50
39/6 Windrush at Newbridge (19 years)											
Mean	13.31	13.14	13.14	13.13	13.26	13.33	13.37	13.39	13.36	13.27	13.18
Mean (daily)									(12.80)		(12.47)
cv %	26.41	26.10	26.14	26.17	25.59	25.30	25.08	24.81	24.74	24.93	25.31
54/2 Avon at Evesham (31 years)											
Mean	155.95	158.84	158.77	158.70	157.95	156.70	155.20	152.15	148.77	141.86	135.77
Mean (daily)									(129.93)		(120.79)
cv %	55.66	53.13	53.07	52.95	52.83	52.83	52.87	52.65	52.36	51.52	49.72
54/6 Stour at Kidderminster (16 years)											
Mean	21.76	22.27	22.23	22.17	21.81	21.25	20.57	18.58	17.42	14.87	13.02
Mean (daily)									(16.14)		(13.78)
cv %	95.33	91.12	91.00	90.82	89.51	87.24	84.38	77.74	71.90	65.06	60.12
55/8 Wye at Cefn Brwyn (19 years)											
Mean	16.91	16.40	14.60	13.61	11.26	9.89	8.89	7.62	6.74	5.39	4.74
Mean (daily)									(8.48)		(6.19)
cv %	50.55	50.59	43.37	40.36	32.40	31.76	32.26	33.63	34.40	34.08	33.08
68/1 Weaver below Ashbrook (31 years)											
Mean	61.11	60.22	60.14	60.03	59.50	58.53	57.21	54.58	52.13	47.74	44.01
Mean (daily)									(57.14)		(49.76)
cv %	57.45	55.86	55.74	55.71	55.56	53.64	49.78	43.78	39.93	35.49	33.51
72/1 Lune at Halton (9 years)											
Mean	671.22	671.85	667.74	661.30	628.43	586.97	547.28	476.38	423.53	345.65	296.87
Mean (daily)									(406.82)		(295.49)
cv %	21.20	21.08	21.15	21.38	22.18	23.37	25.08	29.17	31.41	30.75	28.36

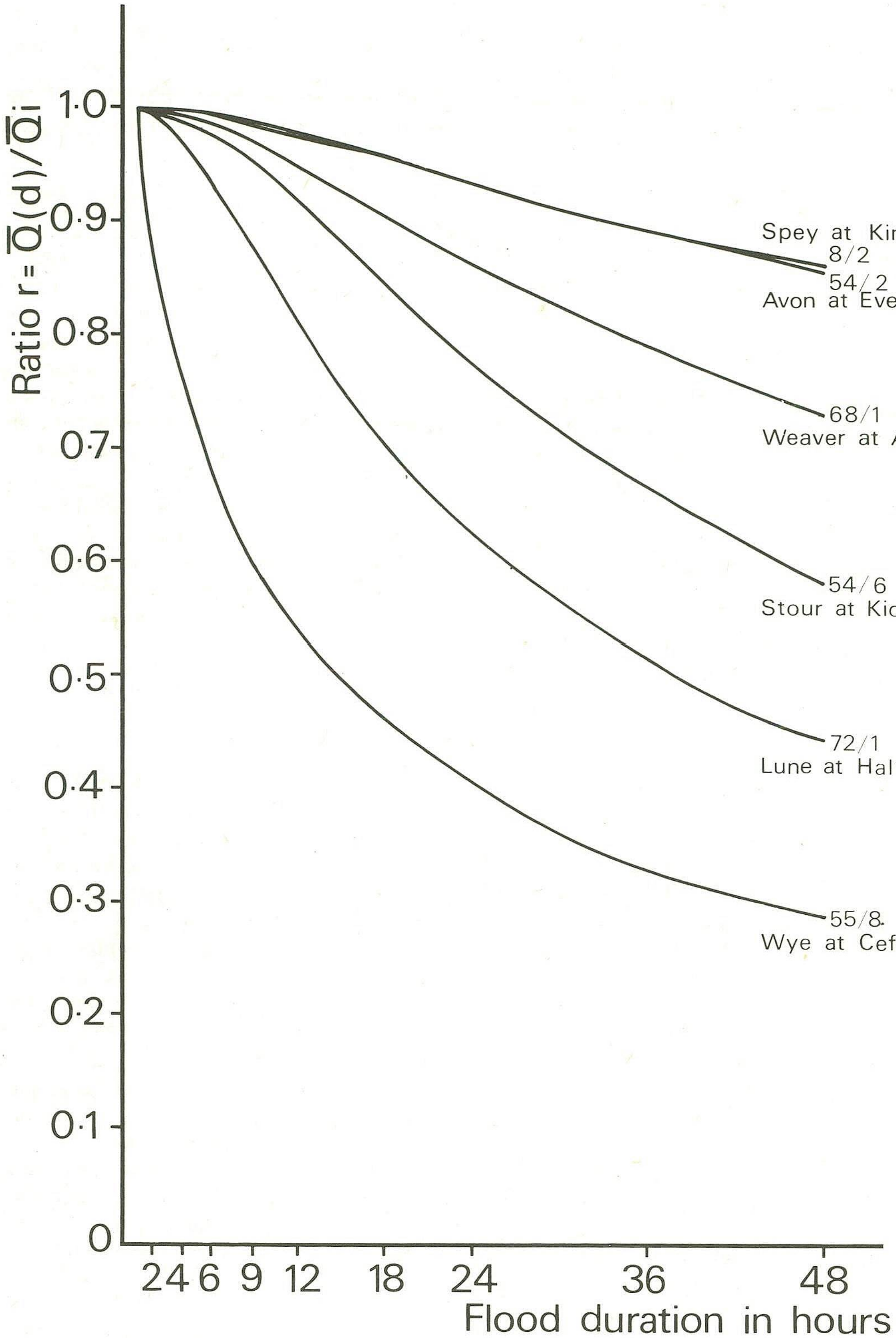


Fig 5.3 Reduction ratios for hourly flood volumes.

It was found that N and B have similar effects on the function of Equation 5.1, behaving as though they were not independent. Unfortunately N and B were poorly correlated with catchment characteristics, but as this function fitted the data well an alternative was not used. The observed reduction ratio data can be compared with the fitted reduction curves in Figure 5.1 for a selection of typical stations and in most cases the fitted ratio is within 2–3 % of the actual ratio. For stations with at least 10 years of record this function provides a simple tool for relating flood discharges over durations of up to 10 days to either AMAF or CALMAF.

5.2.3 Hourly reduction curves

In addition to computing daily flood volumes over 1–10 day durations, flood volumes were computed over periods of 1–48 hours to examine the behaviour of the reduction curves over shorter time periods. For this the basic records were obtained by digitising river authority charts, and as this involved considerable work seven stations only were selected. These data should be directly comparable with the data used for the instantaneous floods. Record length varied from 9 to 31 years, the average length being 16 years.

As with the daily floods, the annual maximum flood over each duration was found and ranked over the record length. The mean annual flood was computed for each duration with its standard deviation and coefficient of variation, as shown in Table 5.2. The 1 and 2 day flows, computed from daily mean flows, are shown in parenthesis below the values for data interpolated over 24 and 48 hours, not calendar days. The 24 hour volumes computed from the daily volumes program differ from the hourly program results with a range from a 12.7 % underestimate (54/2) to a 25.8 % overestimate (55/8). The reduction ratios for periods up to 48 hours have been plotted for the seven stations in Figure 5.3. Because the instantaneous and hourly floods have been computed in slightly different ways there are errors of up to 5 % in the hourly ratios, which at 1 hour should be close to unity; in some cases the 1 hour volume was used in place of the instantaneous flood. The Windrush at Newbridge (39/6) has been omitted from this graph as its record was anomalous in this respect. Figure 5.3 illustrates the range of reduction curves for different types of catchment and the possible differences between the 24 hour floods computed from records of daily mean flows and floods of shorter durations. As expected the small upland catchments show the greatest differences. For example at 55/8 the mean flows are 132 % at 12 hours, 167 % at 6 hours and 202 % at 3 hours, of the 24 hour mean flow.

Figure 5.4 shows the 12, 24, 36, 48 hourly ratios plotted on the graphs with the fitted curves for daily ratios. While the differences between the two sources of data can be seen in stations 39/6 and 54/2, the slopes of the curves are similar and the other stations compare favourably. This leads to the conclusion that, while the use of reduction curves derived from hourly data is preferable, the daily data reduction curves may be extrapolated or interpolated for durations of 1 to 48 hours without great risk.

5.2.4 Stability of reduction curves with return period

The annual maximum series of floods may be described by the general equation for frequency analysis:

$$Q_T = \bar{Q} + K_T . \sigma \qquad (5.2)$$

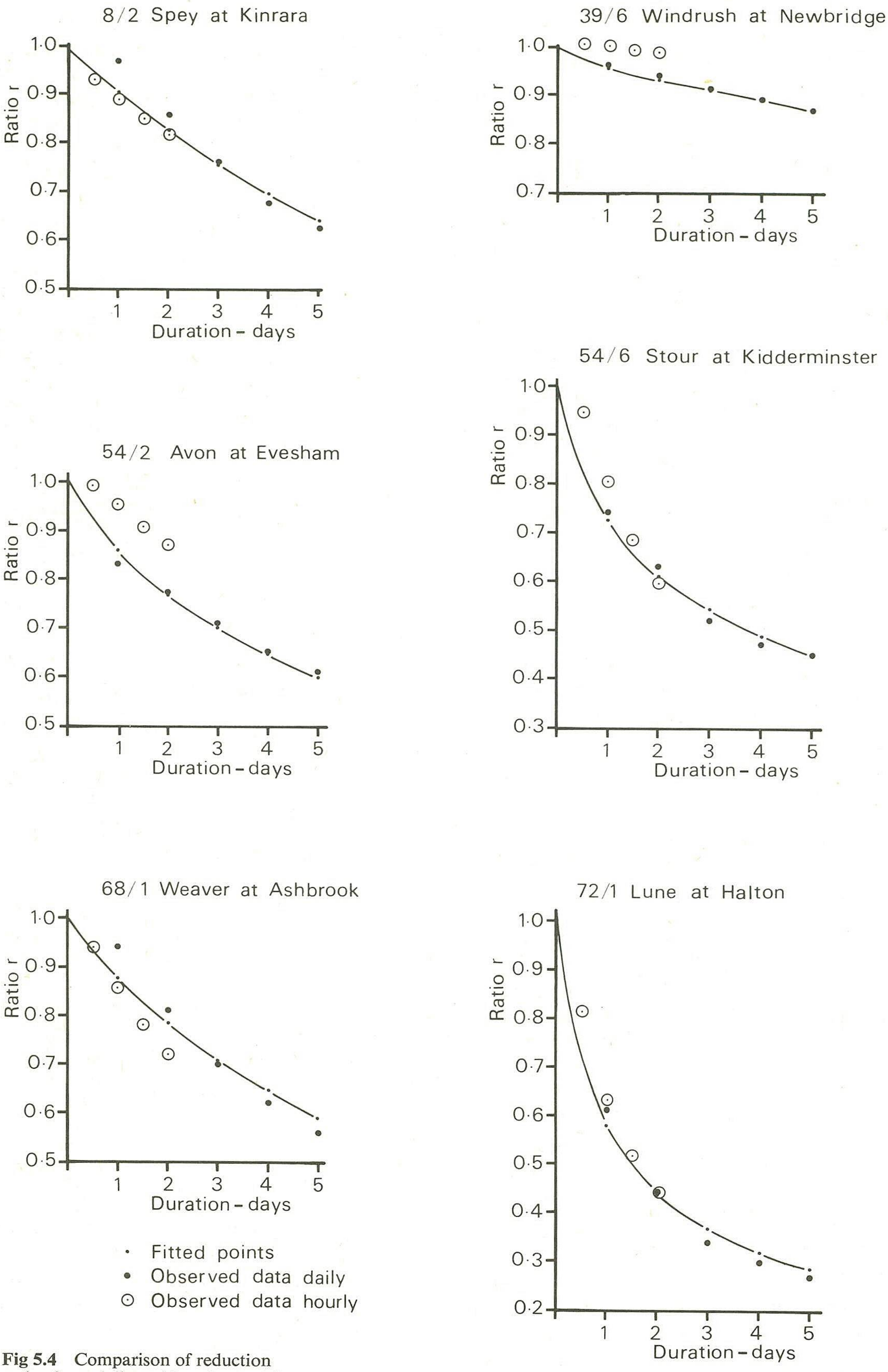


Fig 5.4 Comparison of reduction ratios from daily and hourly records.

which can be standardised to

$$\frac{Q_T}{\bar{Q}} = 1 + K_T.\text{cv} \tag{5.3}$$

where

Q_T is the flood of return period T years,

K_T is the frequency factor for the appropriate distribution,

$\bar{Q}$ is the mean annual flood,

σ is the standard deviation,

cv is $\sigma/\bar{Q}$.

As long as cv and the distribution remain constant at different durations, the ratios do not vary with return period and only the ratios for the mean annual flood need be computed.

The values of cv at selected durations are given in Table 5.3 for the 34 station data set (see Section 5.4.1). Although cv varies with duration at individual stations, and 38% of the stations have values of cv at 10 days which differ by over 25% from the instantaneous flood cv, the average values of cv at each duration have a smaller range. As the flood duration increases, the mean cv decreases, as illustrated in Table 5.3 by the averages of the cv values found at each duration for the 34 stations. The mean

Table 5.3 Variation in cv (%) for different duration floods.

Station	Duration (days)				
	Inst.	1	3	6	10
4/1	28.41	33.24	30.68	28.69	26.99
21/31	26.56	27.89	28.42	26.29	22.04
23/2	30.91	34.42	33.21	28.07	27.20
23/3	21.93	18.86	21.27	21.71	22.81
24/1	32.17	28.31	30.24	24.44	20.91
24/2	38.28	41.73	40.19	37.13	38.66
24/3	26.04	36.72	25.52	22.65	20.31
24/4	27.99	32.47	33.03	28.27	25.47
25/2	20.33	18.27	19.50	16.83	17.79
25/4	37.56	41.32	40.56	36.43	32.67
27/12	37.37	31.02	25.04	21.30	20.93
32/2	57.73	113.73	59.46	62.35	57.73
33/6	29.38	26.15	25.85	25.27	23.80
34/12	120.77	106.27	82.12	66.42	53.14
37/1	34.48	31.38	31.03	31.72	32.76
38/3	39.53	145.47	86.57	58.90	45.85
39/6	26.41	25.08	25.62	26.67	26.41
39/12	38.40	59.52	50.30	37.25	29.18
47/1	31.04	23.59	18.31	21.73	21.42
54/2	55.66	43.41	41.19	38.96	38.36
54/4	49.14	50.12	39.31	34.89	31.45
54/6	95.33	61.96	51.48	42.90	40.99
54/7	34.55	34.89	31.44	34.20	34.90
54/8	39.80	35.82	27.86	25.47	25.47
55/1	24.18	26.36	30.23	30.23	29.26
55/3	11.15	22.63	22.19	23.75	25.09
55/7	39.30	72.31	45.59	33.50	30.65
55/10	43.13	38.39	26.31	19.84	22.43
56/1	27.31	29.77	30.04	24.58	21.03
68/1	57.45	38.49	33.90	28.73	25.85
68/4	34.05	35.75	36.77	32.69	32.35
68/6	41.87	48.22	34.92	26.19	23.69
71/3	41.54	54.83	37.39	32.40	23.26
72/2	21.20	38.16	30.53	29.04	26.50
cv	38.85	44.30	35.97	31.76	29.33
σ	20.36	27.37	14.97	11.24	9.15

value of CV for an instantaneous flood (AMAF) was found to be 0.389 for this small sample, which is close to the value of 0.357 found for the full set of 532 stations (Section 4.3.10). As CV has a large sampling error, the assumption was made that CV remains constant over all durations, although floods of longer durations and longer return periods may be slightly overestimated.

5.3 Calendar day runoff prediction

5.3.1 Regression of CALMAF with catchment characteristics

The mean annual calendar day flood (CALMAF) is an initial alternative to AMAF. A prediction equation for CALMAF has been developed from catchment characteristics and may be used where no flow records exist.

The analysis was confined to the 236 stations for which both CALMAF and AMAF estimates were available from observed records, with CALMAF computed from the annual maximum calendar day flows extracted from the Water Resources Board archive. The average record length was 14 years and the geometric average of the 236 CALMAF estimates was 32 cumecs, or about 69 % of the corresponding AMAF value. The arithmetic average of the ratio RMAF (CALMAF/AMAF) was also 0.69 and varied between 0.24 and 1.20. The few cases of the ratio exceeding unity were caused by occasional differences in the ratings used to compute CALMAF and AMAF or by differences in record period. The similarity of the two flow statistics is emphasised by their correlation coefficient of 0.976 and their correlations with catchment characteristics (see Table 5.4). The RMAF correlations show its lower association with all characteristics except slope.

Table 5.4 Correlations with catchment characteristics.

Catchment characteristics	Correlation with		
	CALMAF	AMAF	RMAF
AREA	0.777	0.684	0.450
SAAR	0.469	0.538	−0.250
RSMD	0.442	0.520	−0.339
S1085	−0.161	−0.033	−0.593
SOIL	0.328	0.408	−0.353
URBAN	−0.100	−0.066	—
LAKE	0.268	0.255	—

5.3.2 Ridge regression analysis

Ridge regression was used to make an initial choice between catchment characteristics. This is a variant on conventional regression analysis. A small addition (θ) is made to the leading diagonal of the correlation matrix. This is numerically equivalent to increasing self correlation at the expense of intercorrelation among the variables (Hoerl & Kennard, 1970). The normal equations for multiple regression are then solved as usual, but the ridge regression coefficients so obtained are no longer optimal in the least squares sense. This adds to the significance test a second criterion for choosing variables, the stability of the regression coefficient under change to the correlation matrix. The ridge trace for a set of 13 variables

(Figure 5.5) shows the instability of several of the variables and the slow growth of the residual error with increasing θ.

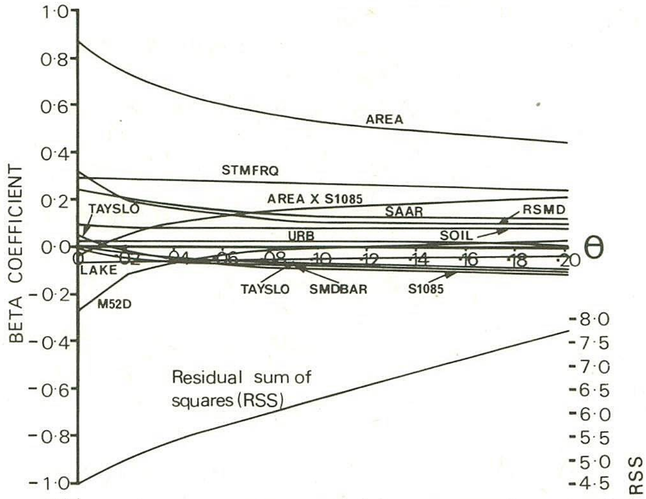


Fig 5.5 Ridge trace for 13 variables.

See note to Figure 5.6.

Where two or more variables measuring similar characteristics are included in a data set and their intercorrelation is high, for example M52D and SAAR which both measure catchment wetness, the effect is often to force unreasonable regression coefficients on them. In Figure 5.5 M52D has a negative exponent which becomes positive, the intuitively correct sign, as θ increases. A principal component analysis of the same data matrix showed that five or fewer components should be sufficient to account for the greater part of the variability. Basing judgement on the stability of the ridge trace and on the statistical significance of the regression coefficient, successive reductions were made to the number of variables left in the set. Figure 5.6 shows the ridge trace for a five variable set selected on this basis. Much of the instability has disappeared although some interaction between AREA and S1085 remains (the coefficient of S1085 is not significant). A parallel analysis using AMAF as dependent variable showed similar results but the regression coefficient of slope for $\theta = 0$ was significant though unstable.

Table 5.5 shows the CALMAF, AMAF and RMAF regression results. There is considerable similarity between the AREA, STMFRQ and RSMD coefficients in the CALMAF and AMAF equations. The SOIL coefficients differ by about two standard errors in the least squares set and by rather less in the more

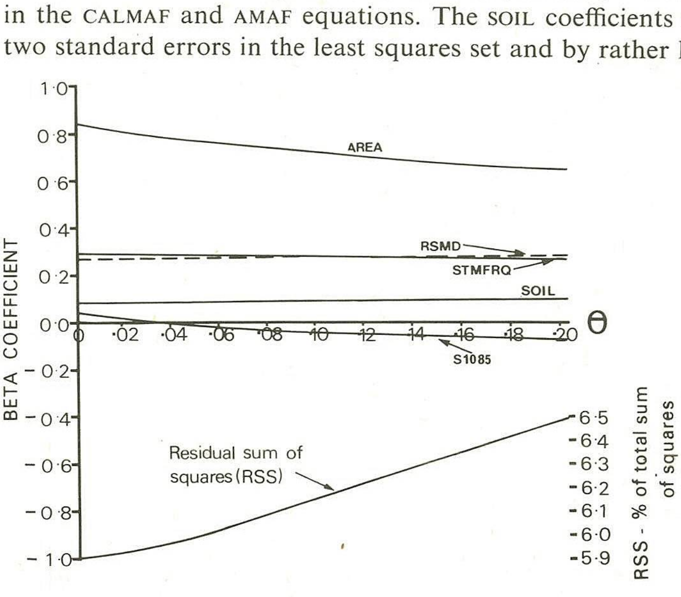


Fig 5.6 Ridge trace for 5 variables.

θ is added to leading diagonal of correlation matrix.
RSS is residual sum of squares expressed as a percentage of total sum of squares. It is related to coefficient of determination by $R^2 = 1 - \text{RSS}/100$.
BETA is the regression coefficient (b) standardised by $\beta = b/\sigma$.

stable ridge regression set which leaves the slope variable S1085 and the intercept terms as the main differences. The slope variable is the dominant term in the RMAF equation with SOIL as the only other significant variable. Slope appears in other expressions where response time is an important component, as in the prediction equations for reduction curve parameters and Equation 6.18 and Table 6.5. When S1085 was omitted, the prediction equation became CALMAF = 0.00560 $\text{AREA}^{0.967}$ $\text{STMFRQ}^{0.456}$ $\text{RSMD}^{1.086}$ $\text{SOIL}^{0.486}$ (R^2 = 0.941, see = 0.159).

Table 5.5 Comparison of regressions with catchment characteristics.

Variable name	Dependent variable					
	CALMAF		AMAF		RMAF	
	b	se(b)	b	se(b)	b	se(b)
AREA	0.997	0.030	0.967	0.038	0.030	0.021
STMFRQ	0.449	0.037	0.398	0.047	0.051	0.026
S1085	0.067	0.052	0.235	0.066	−0.169	0.037
RSMD	1.005	0.100	1.009	0.126	−0.003	0.070
SOIL	0.478	0.120	0.790	0.152	−0.312	0.084
Multiplier	0.0058	—	0.0106	—	0.542	—
R^2	0.941	—	0.904	—	0.397	—
see	0.159	—	0.200	—	0.111	—
Factorial error	1.44	—	1.58	—	1.29	—

b is regression coefficient; se(*b*) is the standard error of the coefficient.

5.3.3 Regional analysis

A map was prepared showing residuals from the CALMAF prediction. The resemblance to the AMAF residual map (Figure 4.23) was quite marked despite the fewer stations. Of particular note were areas of positive residual in the south west peninsula and through west Lancashire and Cheshire, and negative residuals in East Anglia and along the Spey. The implication of the similarity between residuals is that valley storage may not be an important factor in determining the flood peak.

S1085 was omitted from the variable set and an analysis similar to Section 4.3.9 was carried out. The assumption was made that for each of the 10 regions in Great Britain the regression coefficients remained constant and only the intercept varied. A significance test supported the impression given by the residual map that the intercepts were significantly different. Table 5.6 gives the values of the coefficients and the nomogram in Figure 5.7 enables CALMAF to be rapidly estimated.

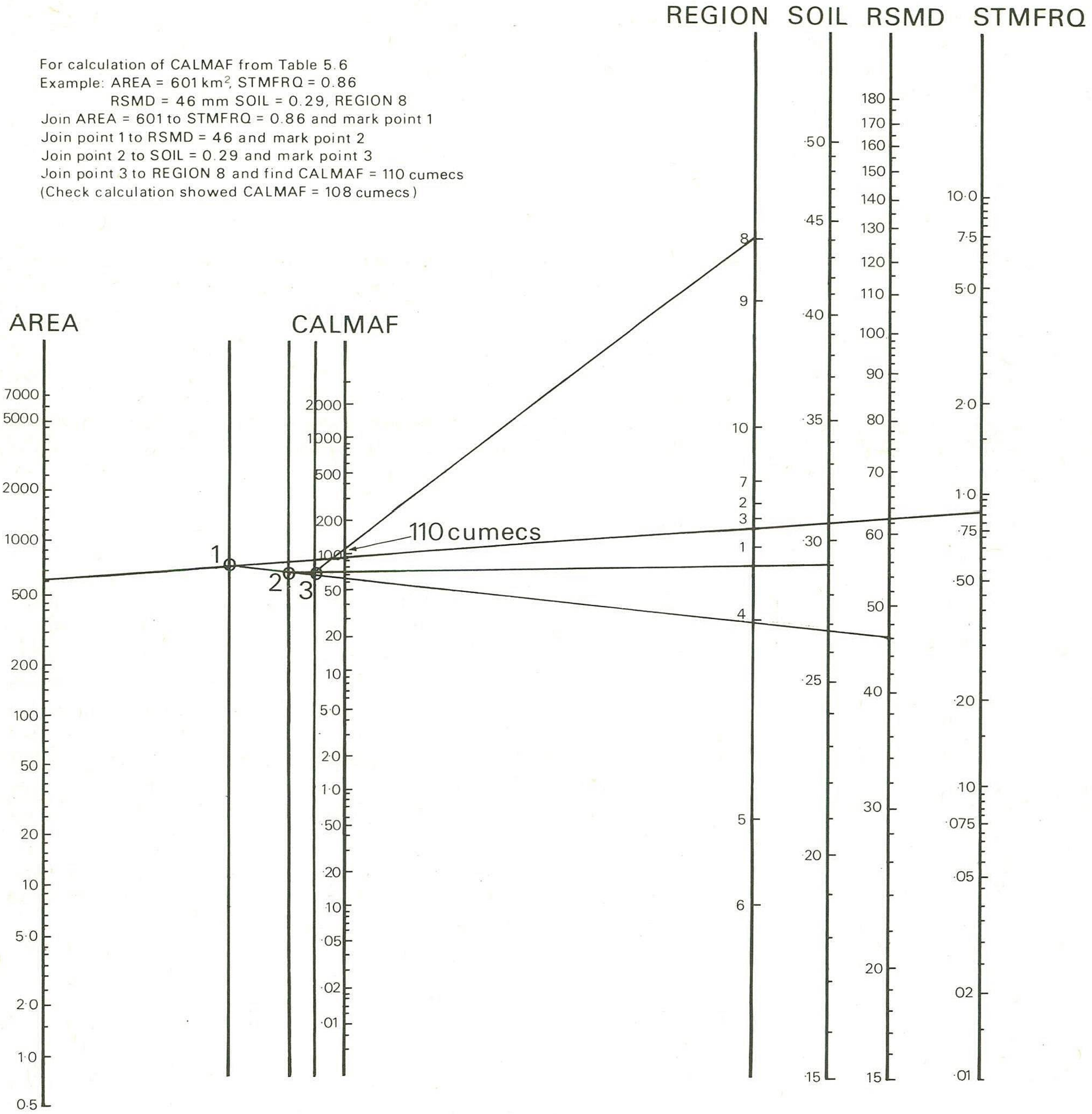
Table 5.6 Results of regional regression.

Variable	Coefficient	see	*t*	Multiplier	
				Region	Value
AREA	0.9475	0.019	50.5	1	0.0395
STMFRQ	0.4068	0.036	11.2	2	0.0417
RSMD	0.6280	0.110	5.72	3	0.0410
SOIL	0.7102	0.121	5.88	4	0.0360
				5	0.0279
R = 0.964				6	0.0250
R^2 = 0.929				7	0.0428
see = 0.142				8	0.0585
Factorial see = 1.39 (+39%, −28%)				9	0.0539
				10	0.0459

For example the equation for Region 1
CALMAF = 0.0395 $\text{AREA}^{0.9475}$ $\text{STMFRQ}^{0.4068}$ $\text{RSMD}^{0.6280}$ $\text{SOIL}^{0.7102}$.
Region boundaries can be seen on Figure 2.4.

The intercepts show a smooth geographical transition from near 0.06 in the south west through values around 0.04 in Scotland and the south coast to a minimum of near 0.025 in East Anglia. No daily flow records from Ireland were analysed.

Fig 5.7 Nomogram for estimating CALMAF.



5.4 Regression analysis of reduction curve parameters on catchment characteristics

5.4.1 Choice of data sets

The original data set comprised 78 stations for which mean daily flows were available and for which reduction curve parameters were found

(Table 5.1). 14 stations were deleted from this set on the grounds that the values of either *N* or *B* were extreme and therefore unrepresentative; for example, high values of *B* suggest a large instantaneous flood not included in the flood volume record period. The remaining set of 64 stations includes a number where the record length for the flood volumes is not the same as for the instantaneous floods. Where extreme events occur in one record and not in the other this is likely to affect the reduction ratios in an arbitrary way.

In order to test the effect of the varying record lengths three data sets were selected. The 64 station set has records which have differences of up to 23 years between the length of instantaneous and volume flow records. By selecting stations that have differences of not more than 4 years between the two records, the data set is reduced to 52. By further limiting the difference to not more than 1 year, the data set is reduced to 34 stations. Similar record length does not always ensure coincidence of years (see Volume IV) but makes it much more likely.

The reduction curve parameters were found to be sensitive to concurrent records because the regressions with catchment characteristics improved with the elimination of nonconcurrent station records. This is illustrated by Table 5.7, where for the logarithmic transform of *B* and *NB* the coefficient of determination and the standard error of estimate improve as nonconcurrent records are deleted. By comparison the results for ratios of 3 to 1 and 10 to 1 day floods, AR3 and AR10, based only on the concurrent daily flow records, change very little. This confirms that the effect of introducing nonconcurrent AMAF records is to add further noise to the data, which may already be affected by rating differences. It was concluded that the best regression equations for reduction curve parameters will be those derived from the 34 station record, but the full data set of 64 stations can be used with advantage for regressions with the ratios AR3 and AR10.

Table 5.7 Variation of coefficient of determination (R^2) and standard error of estimate with sample size for regression equations with three independent variables (after logarithmic transformation).

	Coefficient of determination (R^2 %)			Standard error of estimate		
No. of stations	64	52	34	64	52	34
N	5.3	6.5	10.8	0.159	0.164	0.161
B	43.8	40.3	65.6	0.408	0.443	0.377
NB	52.5	48.4	64.2	0.340	0.373	0.364
AR3	45.8	49.4	50.2	0.043	0.044	0.050
AR10	44.2	45.2	43.4	0.073	0.075	0.086

Table 5.8 gives a summary of the observation data for the 64 and 34 station data sets. The differences between data sets are small but the smaller set has slightly steeper, smaller catchments.

Table 5.8 Summary of observation data for two data sets.

	64 stations				34 stations			
	Mean	Minimum value	Maximum value	Standard deviation	Mean	Minimum value	Maximum value	Standard deviation
N	0.512	0.160	1.120	0.189	0.547	0.190	1.120	0.202
B	2.159	0.120	9.150	2.183	2.046	0.120	8.960	2.331
AREA	618.766	10.400	4040.000	774.000	551.903	10.400	4040.000	782.273
STMFRQ	1.231	0.030	4.920	0.982	1.390	0.030	4.920	1.194
TAYSLO	6.770	0.840	51.200	9.574	8.024	0.900	51.200	11.690
S1085	7.452	0.800	43.300	10.030	8.474	0.800	43.300	11.528
AR3	0.714	0.540	0.970	0.094	0.711	0.540	0.970	0.110
AR10	0.463	0.310	0.840	0.112	0.461	0.310	0.840	0.130
SAAR	1087.062	602.000	2461.000	417.037	1063.647	602.000	2461.000	481.978

5.4.2 *Choice of variables*

The choice and derivation of catchment characteristics used throughout the Floods Study are discussed in Chapter 4 and the parameters used in this chapter have been selected from the standard Master List computer file (Section 4.3.2). The flow duration reduction curves are essentially indicators of 'flashiness', and the independent variables likely to be associated with this are catchment or channel slope, catchment area, stream frequency and rainfall. In the regression of flood statistics on catchment characteristics (Section 4.3), it was found that catchment area, stream frequency and the climate index were always the first three to enter the regression and the most relevant variables considered were AREA, TAYSLO, S1085, STMFRQ, SAAR, RSMD. It was found in Chapter 4 that there was little difference between the alternative variables for slope (TAYSLO and S1085) and rainfall (RSMD and SAAR), so on the grounds of ease of measurement S1085 and SAAR were chosen for use here.

The four independent variables used here were therefore AREA, S1085, STMFRQ, and SAAR, and all were transformed logarithmically as were the dependent variables. The correlation matrix for all variables is given in Table 5.9. The variable which accounts for most of the variance is the channel slope, S1085, which enters first into most regression equations, and further variables account for very little additional variance (see Tables 5.10 and 5.11).

Table 5.9 Correlation matrix after logarithmic transformation.

	N	*B*	*NB*	AREA	STMFRQ	TAYSLO	S1085	AR3	AR10	SAAR
34 stations										
N	1.000	−0.268	0.012	0.046	0.066	−0.140	−0.132	−0.292	−0.479	−0.144
B	−0.268	1.000	0.960	−0.550	0.526	0.781	0.784	−0.748	−0.670	0.675
NB	0.012	0.960	1.000	−0.558	0.565	0.770	0.776	−0.861	−0.835	0.659
AREA	0.046	−0.550	−0.558	1.000	−0.136	−0.751	−0.725	0.568	0.492	−0.232
STMFRQ	0.066	0.526	0.565	−0.136	1.000	0.561	0.629	−0.448	−0.500	0.700
TAYSLO	−0.140	0.781	0.770	−0.751	0.561	1.000	0.964	−0.661	−0.574	0.681
S1085	−0.132	0.784	0.776	−0.725	0.629	0.964	1.000	−0.664	−0.588	0.689
AR3	−0.292	−0.748	−0.861	0.568	−0.448	−0.661	−0.664	1.000	0.944	−0.526
AR10	−0.479	−0.670	−0.835	0.492	−0.500	−0.574	−0.588	0.944	1.000	−0.427
SAAR	−0.144	0.675	0.659	−0.232	0.700	0.681	0.689	−0.526	−0.427	1.000
64 stations										
N	1.000	−0.449	−0.164	0.031	0.129	−0.055	−0.042	−0.327	−0.495	0.053
B	−0.449	1.000	0.955	−0.498	0.381	0.663	0.661	−0.592	−0.499	0.468
NB	−0.164	0.955	1.000	−0.539	0.464	0.714	0.716	−0.761	−0.714	0.499
AREA	0.031	−0.498	−0.539	1.000	−0.069	−0.794	−0.762	0.514	0.465	−0.162
STMFRQ	0.129	0.381	0.464	−0.069	1.000	0.455	0.533	−0.451	−0.504	0.665
TAYSLO	−0.055	0.663	0.714	−0.794	0.455	1.000	0.970	−0.637	−0.579	0.545
S1085	−0.042	0.661	0.716	−0.762	0.533	0.970	1.000	−0.645	−0.596	0.579
AR3	−0.327	−0.592	−0.761	0.514	−0.451	−0.637	−0.645	1.000	0.938	−0.447
AR10	−0.495	−0.499	−0.714	0.465	−0.504	−0.579	−0.596	0.938	1.000	−0.366
SAAR	0.053	0.468	0.499	−0.162	0.665	0.545	0.579	−0.447	−0.366	1.000

The two parameters describing the reduction curves, *N* and *B*, and their product *NB*, were first chosen as dependent variables. Poor correlation was found between *N* and catchment characteristics, the best R^2 being only 12%. However, although R^2 is low, the standard error of estimate of *N* is also relatively low at 0.16 (+45%, −31%) because the observed values of *N* do not vary greatly. As shown in Table 5.7, better values of R^2 were obtained for *B* and *NB* which had 65.6% and 64.2% respectively of the initial variance accounted for by the regression of the 34 station data set.

In order to use CALMAF, two additional dependent variables were chosen, AR3 and AR10, the ratios of the mean flood over 3 and 10 days

Table 5.10 Results of regressions of reduction curve parameters on catchment characteristics after logarithmic transformation (34 stations).

Dependent variable	Independent variables	Regr. coeff.	Standard error	Intercept	R	R^2	see
N	SAAR	−0.136	0.165	0.116	0.144	0.021	0.164
N	STMFRQ	0.112	0.083	0.771	0.275	0.076	0.162
	SAAR	−0.353	0.229	—	—	—	—
N	AREA	−0.112	0.084	0.142	0.329	0.108	0.161
	STMFRQ	0.160	0.094	—	—	—	—
	S1085	−0.246	0.134	—	—	—	—
N	AREA	−0.092	0.090	0.587	0.347	0.120	0.163
	STMFRQ	0.174	0.098	—	—	—	—
	S1085	−0.193	0.158	—	—	—	—
	SAAR	−0.176	0.276	—	—	—	—
B	S1085	0.941	0.132	−0.607	0.784	0.615	0.366
B	S1085	0.729	0.176	−3.057	0.806	0.650	0.355
	SAAR	0.864	0.494	—	—	—	—
B	AREA	−0.141	0.175	−3.358	0.810	0.656	0.357
	S1085	0.538	0.296	—	—	—	—
	SAAR	1.118	0.588	—	—	—	—
B	AREA	−0.130	0.200	−3.455	0.811	0.658	0.363
	STMFRQ	−0.027	0.210	—	—	—	—
	S1085	0.561	0.353	—	—	—	—
	SAAR	1.136	0.614	—	—	—	—
NB	S1085	0.897	0.129	−0.869	0.776	0.602	0.359
NB	S1085	0.709	0.174	−3.048	0.794	0.630	0.351
	SAAR	0.769	0.489	—	—	—	—
NB	AREA	−0.159	0.172	−3.387	0.801	0.642	0.352
	S1085	0.493	0.291	—	—	—	—
	SAAR	1.055	0.579	—	—	—	—
NB	AREA	−0.222	0.196	2.860	0.804	0.646	0.355
	STMFRQ	0.147	0.213	—	—	—	—
	S1085	0.368	0.345	—	—	—	—
	SAAR	0.960	0.600	—	—	—	—

Table 5.11 Results of regressions of 3 and 10 day flood volume ratios on catchment characteristics after logarithmic transformation (64 stations).

Dependent variable	Independent variables	Regr. coeff.	Standard error	Intercept	R	R^2	see
AR3	S1085	−0.081	0.012	−0.101	0.645	0.442	0.043
AR3	AREA	−0.045	0.009	−0.262	0.661	0.437	0.043
	STMFRQ	−0.060	0.014	—	—	—	—
AR3	AREA	0.026	0.017	−0.193	0.673	0.453	0.043
	STMFRQ	−0.040	0.021	—	—	—	—
	S1085	−0.035	0.027	—	—	—	—
AR3	AREA	0.030	0.018	−0.083	0.677	0.458	0.043
	STMFRQ	−0.035	0.021	—	—	—	—
	S1085	−0.025	0.031	—	—	—	—
	SAAR	−0.043	0.053	—	—	—	—
AR10	S1085	−0.127	0.022	−0.269	0.596	0.355	0.078
AR10	AREA	0.068	0.015	−0.518	0.664	0.441	0.073
	STMFRQ	−0.116	0.023	—	—	—	—
AR10	AREA	0.059	0.030	−0.489	0.665	0.442	0.073
	STMFRQ	−0.108	0.035	—	—	—	—
	S1085	0.015	0.047	—	—	—	—
AR10	AREA	0.055	0.031	−0.598	0.666	0.444	0.074
	STMFRQ	−0.113	0.037	—	—	—	—
	S1085	−0.026	0.053	—	—	—	—
	SAAR	0.042	0.092	—	—	—	—

respectively to the mean calendar day flood (CALMAF). These two dependent variables AR3 and AR10 were found to have poorer regression coefficients than did N, B and NB, but this is compensated by much lower standard errors of estimate (Table 5.7), and the better prediction of CALMAF from catchment characteristics.

5.4.3 Regression results

The regression analysis was carried out using the ICL Statistical Analysis Program, Mark 2 (1969) and the optimal regression option was used in all cases. The number of independent variables required is specified, in this case up to four, and the program finds the best set of independent variables to describe the variation in the dependent variable. The full results of the regressions are given for the 34 station data set in Table 5.10 and for the 64 station data set in Table 5.11. The tables set out the regression coefficients and their standard errors of estimate, intercept, multiple correlation coefficient R and the standard error of estimate for the equation. The t statistics for the equations relating N to catchment characteristics suggest doubtful validity. The regression coefficients in Equations 5.4 and 5.5 all have t statistics indicating highly significant regression equations.

The estimation of reduction curve parameters from catchment characteristics is clearly open to considerable error and could only be used as a very crude first approximation (see Table 5.10). Rather better results were obtained from the regression of the ratios of 3 to 1 and 10 to 1 day flood volumes, AR3 and AR10, and these results are given in Table 5.11. Here the errors of estimate are much smaller, $\pm 10\%$ for AR3 and $+19\%$, -16% for AR10, although rather less of the variance is accounted for by the equations. Once slope was included the addition of further variables produced no significant improvement to the prediction equations, and the best equations are:

$$\log(\text{AR3}) = -0.101 - 0.081 \log(\text{S1085}) \tag{5.4}$$

$$\log(\text{AR10}) = -0.269 - 0.127 \log(\text{S1085}). \tag{5.5}$$

Figure 5.8 shows the effect on the reduction ratio r of the standard error of estimate of B for three values of B, with N constant at 0.5. Figure 5.9 shows the corresponding effect of the standard error of estimate of N for the mean values of N and B. The direct estimate of the reduction ratio in the form AR3 or AR10 is clearly preferable, as it can be derived from catchment characteristics with errors in r of $\pm 10\%$ and less than $\pm 20\%$ respectively.

5.5 Conclusions

It was found that for the 78 stations studied, the relationship between the ratio of average to instantaneous flood flow and the duration d in days, closely fitted the equation

$$r = \frac{1}{(1+Bd)^N}.$$

For a reduced sample with concurrent instantaneous and volume flood records, N was found to have a mean of 0.55 and a standard deviation of

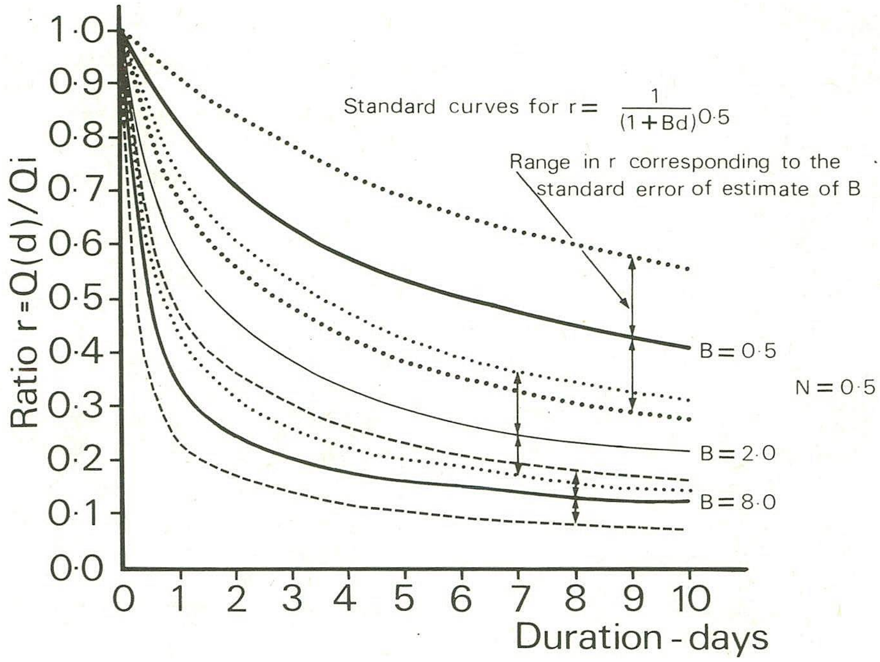


Fig 5.8 Error in r for standard error of estimate of B with N constant.

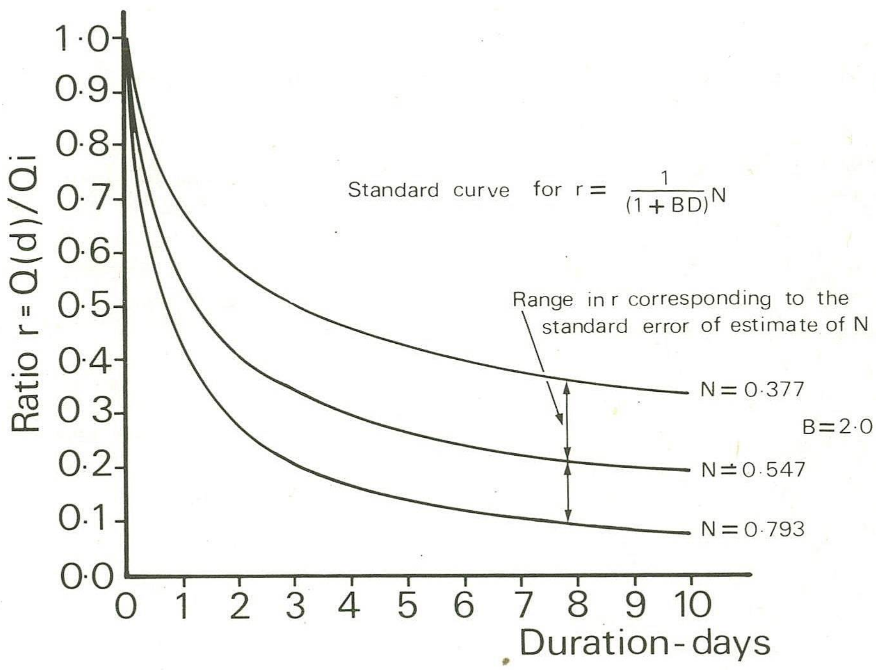


Fig 5.9 Error in r for standard error of estimate of N with B constant.

0.20; B was found to have a mean of 2.05 and a standard deviation of 2.33. B is particularly sensitive to nonhomogeneity of instantaneous and daily flow data due to different record lengths, methods of computation or ratings. N is not very sensitive and might be assumed to be constant at 0.50.

The reduction curves were found to be reasonably stationary over differing return periods, so the ratio between the instantaneous and d day duration flood was assumed constant for all return periods. This requires reduction curves to be computed for the mean annual floods only.

An attempt was made to provide equations relating the reduction curve parameters, N and B, to catchment characteristics. Partly because of the difficulty in obtaining truly homogeneous data the results were disappointing. Better results were obtained by relating the ratios of 3 and 10 day to 1 day flows to catchment characteristics. An important reason for success is thought to be that in this case the data are homogeneous.

This result is useful because mean daily flows are more readily available than peak instantaneous flood flows. It has also been found that the calendar day mean annual flood (CALMAF) can be estimated from catchment characteristics rather more accurately than can the instantaneous mean annual flood (AMAF). The ratio RMAF (CALMAF/AMAF) was found to have an average value of 0.69. From the regressions of RMAF with catchment characteristics it was found that slope was the dominant factor, as has been the case with other parameters (such as time to peak, T_p) that have catchment response time as a major component.

The instantaneous-hourly reduction curves were computed for only seven selected stations, but the data used were homogeneous and more reliable than for the instantaneous-daily reduction curves. It was shown that the hourly reduction curves are very similar to those computed at daily intervals and that interpolation in the instantaneous daily curves over durations of less than 24 hours may be made.

In Section 5.6 a simple method is given for deriving the reduction curve from the two ratios AR3 and AR10. Actual flow estimates can be made by reference to CALMAF, for which a prediction equation is also given.

5.6 Simple method of application

5.6.1 Gauged catchments

For stations with adequate record lengths of mean daily flow (see Introduction), computer programs have been developed for calculating average flood flows over various durations, so that flood statistics, $\bar{Q}(d)$ and CV, and actual reduction curves can be derived. These will enable estimates to be made of flood frequencies at selected durations.

5.6.2 Ungauged catchments

For ungauged catchments ratios of mean flood flows to CALMAF may be estimated from catchment characteristics. CALMAF can be estimated from a prediction equation from Table 5.6 or by use of the nomogram (Figure 5.7).

AR3, the ratio of 3 day flood flow to CALMAF, the 1 day flood flow, and AR10, the 10 day to 1 day ratio, can be estimated from the following prediction equations, using 10–85% channel slope (S1085).

$$\log(\text{AR3}) = -0.101 - 0.081 \log(\text{S1085})$$

$$\log(\text{AR10}) = -0.269 - 0.127 \log(\text{S1085}).$$

These two ratios can be plotted on Figure 5.10, which gives a family of typical observed reduction curves. The curve can then be sketched in by reference to the other curves.

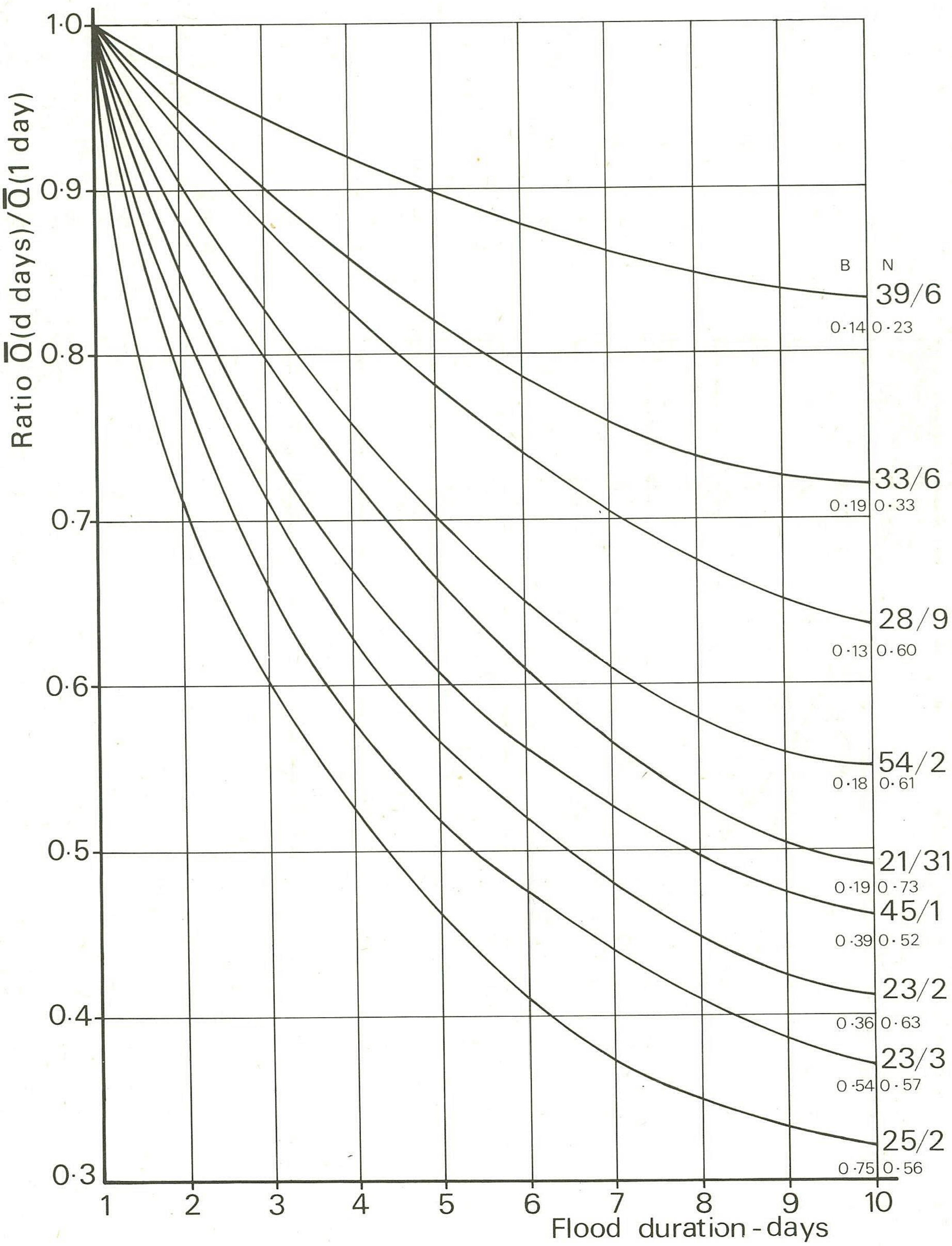


Fig 5.10 Typical reduction curves related to CALMAF.

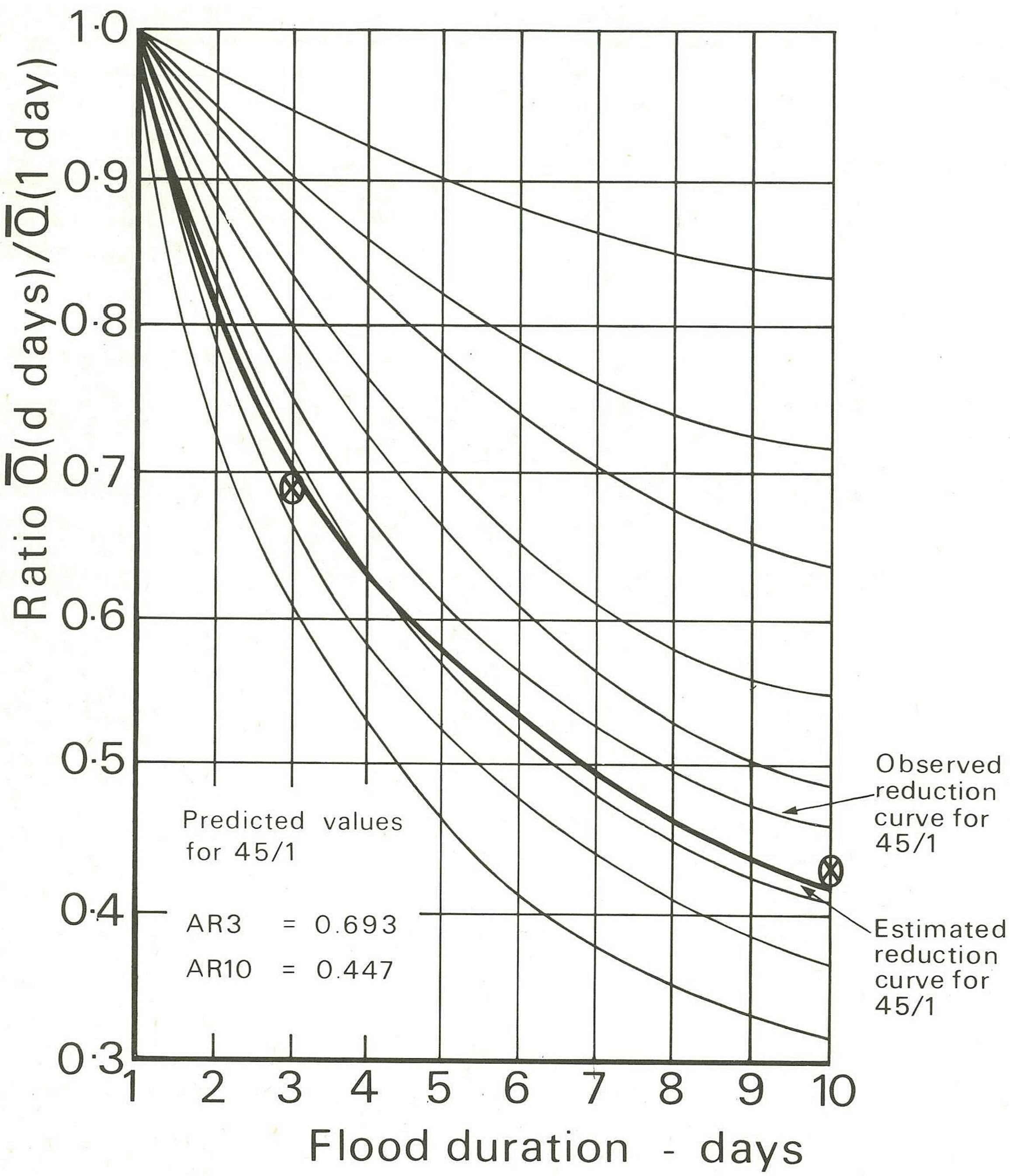
Any duration of flood flow from 1 to 10 days can have its reduction ratio from CALMAF estimated from this curve. On the assumption that the reduction curve is constant with return period, the ratio of $Q_T/\overline{Q}$ will not vary with duration and the region curves of Chapter 2 can be used to deduce the flood volume corresponding to a given duration and return period from the mean annual flood of the same duration.

5.6.3 Example

Estimate the 2 and 6 day flood flows with a return period of 100 years for station 45/1, Region 8, the Exe at Thorverton. For catchment 45/1 the characteristics are obtained as described in Chapter 4:

S1085 = 4.40, AREA = 601, STMFRQ = 0.86, RSMD = 45.62, SOIL = 0.293.

Fig 5.11 Example of use of prediction equations to sketch reduction curve.



CALMAF is estimated from the relevant equation for Region 8 in Table 5.6:

$$\log(\text{CALMAF}) = \log(0.0585)+0.9475\log(601)+0.4068\log(0.86) +0.6280\log(45.62)+0.7102\log(0.293)$$

$$\text{CALMAF} = 108 \text{ cumecs}$$

(using the nomogram in Figure 5.7 a value of 110 cumecs is obtained).

Using prediction equations 5.4 and 5.5:

$$\log(\text{AR3}) = -0.101-0.081\log(4.40) \qquad \text{AR3} = 0.693$$

$$\log(\text{AR10}) = -0.269-0.127\log(4.40) \qquad \text{AR10} = 0.447$$

These two points are plotted and a reduction curve drawn as in Figure 5.11. From this the required ratios are estimated to be: AR2 = 0.825, AR6 = 0.546.

Referring to Table 2.39, the $Q_T/\bar{Q}$ ratio for $T = 100$ years is found to be 2.42 for Region 8. Therefore

100 year flood over 2 days = CALMAF × AR2 × 2.42 = 215 cumecs

100 year flood over 6 days = CALMAF × AR6 × 2.42 = 143 cumecs.

These values have been obtained using only the data available at an ungauged catchment. For comparison the corresponding values obtained from 11 years of data at station 45/1 are given in Table 5.12 below. The differences illustrate the value of installing a gauge while a project is being investigated; even though the period of records may be short, CALMAF could be estimated more reliably than from catchment characteristics.

	Predicted from catchment characteristics alone	Predicted from 11 years of observed data
AR3	0.693	0.745
AR10	0.447	0.460
AR2	0.825	0.850
AR6	0.546	0.560
CALMAF	108 cumecs	155 cumecs
$\bar{Q}$, 2 days	89 cumecs	131 cumecs
$\bar{Q}$, 6 days	59 cumecs	87 cumecs
Q_{100}, 2 days	215 cumecs	309 cumecs
Q_{100}, 6 days	143 cumecs	171 cumecs

Table 5.12 Comparison of predicted and observed results for example.

5.7 Reference

HOERL A.E. & KENNARD R.W. (1970) Ridge regression: applications to nonorthogonal problems. *Technometrics*, **12**, 69–82.

6 Synthesis of the design flood hydrograph

Readers who wish to apply the techniques described in this chapter are recommended to work from Section 6.8. Much of the material in earlier sections is of an explanatory nature and is included in order to present a complete account of the investigation. However, it is suggested that these sections provide the background of data problems, confidence limits and method reliability which is essential to a proper use and understanding of the results.

6.1 Introduction

Contents: The introduction to the chapter explains that the method is complementary to the statistical approach (6.1.1) and describes the additional information it provides by way of hydrograph shape (6.1.2). The selection of a suitable response model is described in Section 6.1.3 and the fundamental problem of how to use the model in designing against a specified return period of *flow* is discussed in Section 6.1.4. Section 6.1.5 comprises a summary of the chapter's structure and content.

6.1.1 A technique of peak flow estimation

This chapter describes how data were collected and analysed for the purpose of providing a simple deterministic model to convert a design rainstorm into a flow hydrograph on a gauged or an ungauged catchment.

In applying the model the aim is to ensure that the peak of the resulting flow hydrograph has an exceedance probability related to that of the rainstorm. The method may therefore be considered as an alternative to the application of results from the statistical study of peak flows themselves.

Owing to the greater length of record and more widespread coverage, rainfall statistics are more reliable than peak flow statistics in the United Kingdom as a whole and on the large majority of individual gauged catchments. As the use of a simple deterministic model in design depends on the statistics of rainfall rather than flow, the method acquires increasing importance as the design probability of larger and rarer events decreases. Whereas the method might not even be considered in the design of a minor culvert it is likely to be the primary technique for specifying a spillway design flood of 'probable maximum' proportions. However, over the whole range of engineering design situations calling for estimates of peak flows of specific exceedance probability, it is hoped that the two basic techniques will be viewed as complementary. This topic is discussed in more detail in the Introduction to this volume.

6.1.2 The complete hydrograph

The peak flow is one design variable which results from the application of a simple deterministic model to a design rainstorm. However, in many design situations, the complete hydrograph may be needed and those models which provide it are particularly useful. Although an exceedance probability cannot be attached to a particular hydrograph, the fitting of a *characteristic* shape to the predicted runoff volume helps to ensure that the peak flow and the runoff volume are equally likely.

6.1.3 *Choice of a simple deterministic model*

Many techniques for determining either the peak flow alone or the complete hydrograph from a design rainstorm and catchment characteristics have been described and applied. Engineers in the United Kingdom will be familiar with the Lloyd–Davies or rational formula, the unit hydrograph, and the TRRL method (Watkins, 1972) of sewer design. Each of these may be described, in the broadest sense, as a simple model of flood wave formation.

The later models tend either to have a larger number of parameters or to involve more complicated equations. In either case, it is often not possible to solve the equations directly and optimisation techniques are employed; the 'best' values of parameters are found according to some predetermined search technique. In the calibration of the simple models and even simpler formulae, the parameters (or coefficients) are found directly by arithmetic.

Of these models, it is the *unit hydrograph/'losses'* model which, over many years and in many parts of the world, has proved to be easily understood and applied to both gauged and ungauged catchments. It is a simple model in the sense that optimisation need not be involved and it is a flexible model in that it can be adapted to meet particular requirements. The basic proposition of the model is that a total flow hydrograph may be separated into runoff which is a direct response to the preceding rain and runoff which is not. The difference between the rainfall and the response runoff is the 'loss'. The response runoff hydrograph is analysed to yield the unit hydrograph.

The unit hydrograph/'losses' model was used as the main tool for catchment response studies because it was the only technique for which the flood event data could be collected and analysed for application to ungauged catchments in the time available. At the same time, however, the development of a relatively simple conceptual model was initiated; this work is described in Chapter 7.

6.1.4 *The unit hydrograph/'losses' model and the design situation*

The usual design problem is to produce a hydrograph with a peak flow of specified annual exceedance probability or return period. It is sometimes assumed that the rainfall to be applied to the unit hydrograph should have the same return period as the peak flow; in other words, the 100 year flood should follow from the 100 year rainstorm. But the 100 year flood can arise from any one of many combinations of rain depth, storm duration, profile, and catchment antecedent condition. This is the fundamental problem associated with any attempt to base a design flow on the statistics of rainfall.

It may be useful, at this point, to summarise the steps that are necessary to apply the unit hydrograph/'losses' model to flood hydrograph specification on a catchment without flow records so that the need for the analyses described in this chapter may become clearer. Given that the return period of the peak flow has been specified, they are:

1 *Select* the duration of storm to be considered.

2 Estimate the rainfall volume (Volume II, Chapter 8) using the catchment area and location, selected duration (step 1), and *selected* return period of the rainfall.

3 Estimate the response runoff volume from a relationship between volume of response runoff and (a) volume of rain, (b) antecedent catchment condition, and (c) catchment characteristics. Note that (a) is determined from step 2, (c) are measurable but (b) is a design choice and calls for *selection.*

4 Distribute the total rainfall volume according to a *selected* time distribution (storm profile).

5 Reduce the total rainfall profile (step 4) so that the net rainfall volume equals the runoff volume (step 3).

6 Synthesise a unit hydrograph for the catchment from a relationship between unit hydrograph parameters and catchment characteristics.

7 Apply the synthetic unit hydrograph to the net rainfall profile (step 5) and hence obtain the response runoff hydrograph.

8 Determine nonresponse runoff from a relationship between nonresponse runoff and (a) antecedent catchment condition and (b) catchment characteristics, and add it to the response runoff hydrograph.

There are four selection stages (in italics) in the procedure where a suitable value of some variable is to be chosen; a rigorous solution to the problem that the design should correspond to a specific return period of *flow* requires that every combination of the variables should be applied to the unit hydrograph and the resulting population of peak flows analysed as if it was an annual maximum series. The chapter is therefore concerned with establishing the three relationships mentioned in steps 3, 6, and 8 and with providing guidance in the selection stages.

6.1.5 Section contents within Chapter 6

6.2. A basic introduction to the theory and practice of the unit hydrograph intended for nonspecialists.

6.3 A description of the data and the procedures adopted for their collection and processing.

6.4 A description of the analytical procedures applied to the prepared data; i.e. the methods of response runoff separation and unit hydrograph derivation.

6.5 The analysis of results leading to the establishment of the three relationships required for steps 3, 6, and 8. This is a long section describing the results, problems encountered, and alternative approaches. For some readers, perhaps engaged in research, it will be preferable to follow the contents quite closely. Others may wish to scan the section quickly at a first reading and thereafter to refer to it like an Appendix.

6.6 The results of testing the three relationships on recorded events not used in the analyses. The section includes attempts to reproduce several 'extreme' floods and readers who are particularly interested in such events might like to read Sections 6.2.3, 6.5.3, 6.6.3, and 6.8.3 as a 'set'.

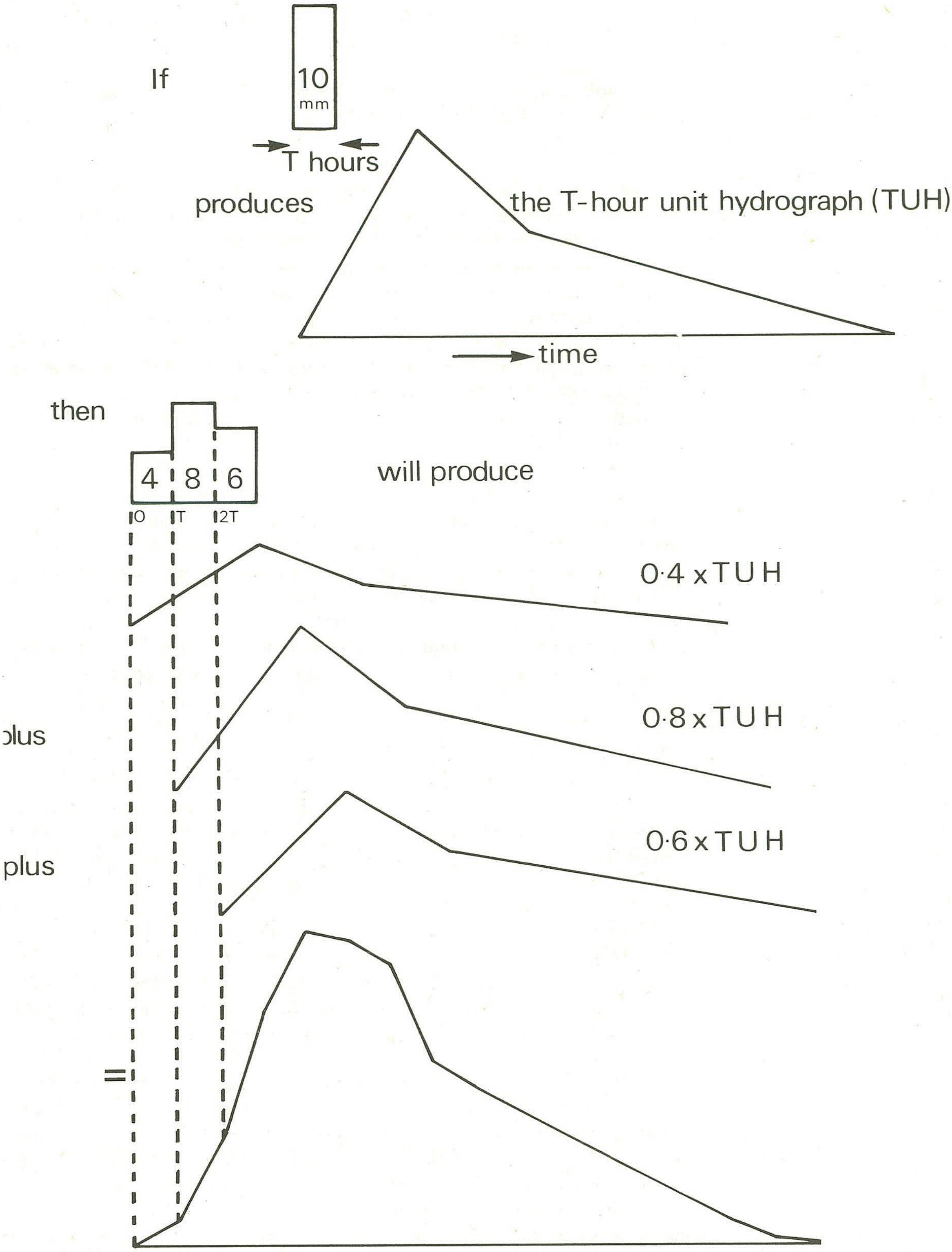
6.7 A description of an investigation into the effects on the resultant peak flow of different values of the four variables which need to be selected in the synthesis described in 6.1.4 above. Rules for the selection are proposed such that the required return period of peak flow is achieved.

6.8 The application of the results. A full description of the recommended procedures with worked examples.

6.2 Unit hydrograph; theory and practice

Fig 6.1 Unit hydrograph definition sketch.

Contents: The *T* hour unit hydrograph (TUH) is defined in Section 6.2.1; brief mention is made of the instantaneous unit hydrograph (IUH). Various



techniques for derivation of the TUH are mentioned in Section 6.2.2 and various aspects of its application are discussed in Section 6.2.3. The relationship between time of concentration, TUH, and IUH is given in Section 6.2.4.

6.2.1 Definition of the unit hydrograph

The runoff hydrograph of quick response to P mm of net (or effective) rainfall generated uniformly over the catchment area at a uniform rate for T hours is called the P mm T hour unit hydrograph. In this study, $P = 10$.

The theory of the unit hydrograph states that it will always be the same shape, that any greater or lesser volume of rain in the same T hours will produce only a proportional change in the flow values while the base-length remains the same, and that composite hydrographs can be produced by lag and addition, as shown in Figure 6.1.

The theory is capable of more elegant expression in terms of system linearity but this is not necessary here and the interested reader is referred to one of the many more mathematical treatments; two recent reviews are by O'Donnell (1966) and Nash (1966).

Brief mention should be made here of the instantaneous unit hydrograph (IUH) which is the unit hydrograph for $T = 0$ and can be conceived as the hydrograph resulting from 10 mm of rain deposited instantly on an impervious surface. Because there is no need to attach a duration label to it, the IUH is a neat summary of the catchment response. It has been attractive to mathematical hydrologists and the subject of many papers presented during the last two decades. In practice, however, the IUH is difficult to derive analytically from discrete interval data and it is not considered to have any practical advantages over unit hydrographs from some fixed duration of rainfall. In this study, one hour is used as the standard for comparing unit hydrographs between catchments.

6.2.2 Derivation of the unit hydrograph

The superposition principle illustrated in Figure 6.1 can also be expressed as a series of equations (Equation 6.1). In these equations q_t are the ordinates of the response runoff hydrograph, u_t are the ordinates of the T hour unit hydrograph (both at intervals of T hours), and p_t are the net rainfall volumes in T hourly blocks.

The apparently obvious way of finding the set of u values given the values of q and p is to start in the first equation and work forwards, or the last one and work back. But this is unsatisfactory because real data, particularly net rainfall, are imperfect and nature does not follow the unit hydrograph theory precisely. Oscillations of the u values soon start and magnify rapidly.

A more satisfactory and more direct technique is to study an event where the rainfall was suitably widespread and reasonably uniform in time and then divide the ordinates of the response hydrograph by the net rainfall in centimetres. This will yield the unit hydrograph for the same duration as the storm rainfall. S curve methods, well described by Wilson (1969) and elsewhere, can be used either to increase or decrease the duration of the unit hydrograph but they are rarely satisfactory. However, this

$$
\begin{array}{lr}
p_1u_1 & = q_1 \\
p_2u_1 + p_1u_2 & = q_2 \\
\vdots & \\
p_nu_1 + p_{n-1}u_2 + \ldots p_1u_n & = q_n \\
\quad p_nu_2 + \ldots \quad p_2u_n + p_1u_{n+1} & = q_{n+1} \\
\qquad\qquad \vdots & \\
\qquad\qquad p_nu_n + p_{n-1}u_{n+1} + \ldots & \\
\qquad\qquad\qquad\qquad \vdots & \\
\qquad\qquad\qquad\qquad \ldots p_2u_{m-n} + p_1u_{m-n+1} & = q_{m-n+1} \\
\qquad\qquad\qquad\qquad\qquad \vdots & \\
\qquad\qquad\qquad\qquad\qquad p_nu_{m-n} + p_{n-1}u_{m-n+1} & = q_{m-1} \\
\qquad\qquad\qquad\qquad\qquad\qquad p_nu_{m-n+1} & = q_m
\end{array}
\tag{6.1}
$$

simple technique is not suitable for large scale application to the types of heavy rainfall event and resulting hydrograph which are generally observed in this country. Rainfall intensity usually changes markedly during a storm and hydrographs are frequently multipeaked. It might be possible to find one apparently suitable large event but it is not usually enough to study only one event. In general, at least five unit hydrographs should be derived for each catchment. A more powerful technique of derivation must therefore be used to tackle the more typical data.

An illustration of this point is provided by Figure 6.2 which shows a complex event and the unit hydrograph derived from it by such a technique. This could not be done by a direct method.

The more complicated techniques are concerned essentially with a search for the dominant signal (unit hydrograph) in the noise ('imperfect' but real data) and involve either (a) trial and error or iterative solutions; (b) direct analytical solutions; or (c) solutions based on an initial assumption of the functional form of the signal.

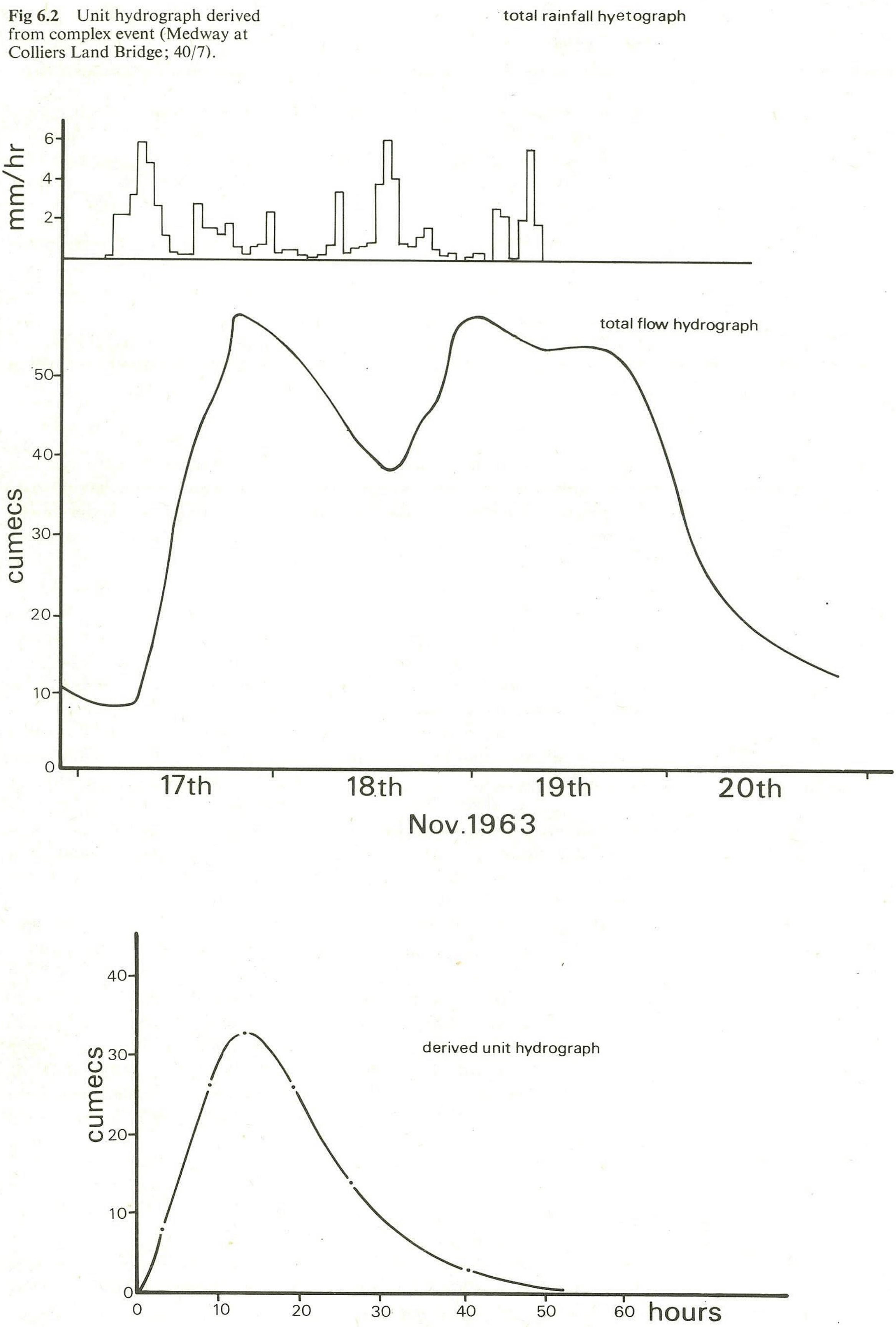
An example of type (a), suitable for desk calculator or even hand solution, is the Collins method as described by Wilson (1969). The best known type (c) is due to Nash (1960).

Although type (a) methods have been converted for use with modern computers, this is unnecessary when type (b) methods, which are intended for the computer, are more efficient. In the Floods Study, Nash's method was compared with two type (b) methods: the harmonic analysis technique due to O'Donnell (1966) and the matrix inversion technique due to Snyder (1955). A full discussion of the comparisons would be out of place in this report but will be published later. Briefly, it was found that the method which gave the most consistent results for a particular catchment was matrix inversion with smoothing. This method is described in detail in Section 6.4.6.

6.2.3 Limitations of the unit hydrograph

Criticisms of the unit hydrograph concept are aimed mainly at the assumption of linearity, i.e. that 20 mm of net rain produces double the flow expected from 10 mm of net rain. Hydraulic theory, applied to overland

Fig 6.2 Unit hydrograph derived from complex event (Medway at Colliers Land Bridge; 40/7).



and channel flow, has been invoked to show that greater depths of water move faster so the time distribution of runoff should be altered. This has led many investigators to postulate a nonlinear process which is either applied explicitly in the more complex catchment models referred to earlier or is implicit in recommendations to adjust the unit hydrograph according to the size of the design storm. For example, the U.S. Corps of Engineers (1959) suggest that slimmer and taller (25–50% higher) unit hydrographs might be used for large design floods than are found from recorded events.

As Wolf (1965) points out in a review paper, the unit hydrograph has been developed from Sherman's (1932) original proposition on the false assumptions that it is applicable in all situations and that the net data are capable of meaningful and exact specification. Sherman never made these claims and they would not be made today but it is likely that, in the intervening period, the unit hydrograph approach has sometimes been misunderstood.

The most common fault would appear to have been to use it for analysis in situations where assumption of nearly uniform net rainfall over the catchment could not be justified. In this study, an arbitrary limit of 500 km^2 was applied and individual storms were rejected if found to be very variable in space.

It was sometimes implied that, in deriving net rainfall and runoff, physical reality is being described. Although this claim is no longer made it is desirable to approach realism as far as is practicable particularly in the time distribution of losses. On the other hand, such descriptions as 'surface runoff', 'baseflow', and 'interflow' are not necessary to an appreciation of the unit hydrograph as an analytical tool. Such descriptions are not used in this report and the unit hydrograph is considered to apply to *quick response runoff* which is separated from the total runoff according to arbitrary rules outlined in Section 6.4.3. (The qualifier 'quick' is not always used; all references to response runoff refer to the short term response as defined by these arbitrary rules.) At any time, the proportion of total flow which is not so separated is described simply as 'nonseparated flow' and includes whatever flow would have occurred if the rain event had not taken place and the 'slow response' to the rain event.

6.2.4 *Time of concentration*

It may be helpful to mention the relationship between the unit hydrograph and the time of concentration which is generally defined as the time taken for the effect of rain falling on the most remote part of the catchment to reach the outlet. It is not capable of direct physical measurement but, in unit hydrograph theory, it is equivalent to the baselength of the instantaneous unit hydrograph (1 hour less than the baselength of the 1 hour unit hydrograph). Also, with uniform rainfall of duration greater than, or equal to, the time of concentration, the peak of the resulting hydrograph (or the attainment of a virtually constant rate of flow) should occur at the time of concentration. Therefore, in a small catchment with a time of concentration of 1 hour or less, the time to peak of the 1 hour unit hydrograph will be equal to the time of concentration.

6.3 Data for the unit hydrograph/loss study

This section is, in part, a summary of data collation procedures which are more fully described in Volume IV.

Contents: Criteria for the selection of suitable catchments for analysis are discussed in Section 6.3.1 and the types of data required in Section 6.3.2. The process of checking the data against snowmelt or large spatial variation in rainfall is described in Section 6.3.3 and further checks on the relative timing of rainfall and runoff in Section 6.3.4.

6.3.1 Catchments

There were four main criteria used in the selection of catchments for this study:

a Catchment area should not exceed 500 km^2.

b Gauging station rating curves should be classified A or, exceptionally, B (see Volume IV, Table 1.2 for description of classification scheme).

c There should be one or more autographic raingauges on or near the catchment.

d The catchment should display some evidence of a short term response to heavy rainfall.

These criteria were not always rigidly followed. The need for catchments in particular areas sometimes forced a relaxation of *a* and/or *b*. Unfortunately, no suitable catchments could be found for Northern Scotland or Northern Ireland. The list of catchments selected is shown with a map of their locations as Figure 6.3. One catchment in the Republic of Ireland was also used.

A number of catchments were rejected during analysis, namely:

Catchment	Reason for omission
21/17	Poor position of recording raingauge in relation to catchment
28/2	The effect of a reservoir
35/4	Recording raingauge records found to be unreliable
48/2	Hydrograph analysis confirmed doubts about the rating for the station
84/16	No suitable events unaffected by snowmelt

Other catchments were difficult to deal with and some of the more general problems are discussed here; catchments with specific problems are discussed in Section 6.5.12.

The requirement that net rainfall and, by implication, losses should be uniform over the catchment is relaxed when unit hydrographs are desired for use on the same catchment; then the spatial pattern need only be *characteristic* of the catchment and nonuniformity will be incorporated in the unit hydrograph. However, when comparing unit hydrographs between catchments, departure from a spatially uniform pattern can be crucial. The important factor is catchment heterogeneity and in this country the obvious example is chalk. If a catchment is 50% chalk and 50% clay, the derived unit hydrograph will in general be for the clay half, unless an extreme event is being studied (Section 6.5.3). (Another example is when the required short term response originates on an urban area only rather than a catchment as a whole.) Although attempts were made to find a rule for subtracting chalk areas from the topographic catchment area, it was not possible to generalise in regions where the extent of drift or derived deposits (boulder clay, clay with flints) is highly variable.

T S	19/1	ALMOND at CRAIGIE HALL
S	19/2	ALMOND at ALMOND WEIR
S	19/5	ALMOND at ALMONDELL
S	20/1	TYNE at EAST LINTON
S	21/5	TWEED at LYNE FORD
	21/17	ETTRICK at BROCKHOPERIG
S	23/2	DERWENT at EDDYS BRIDGE
S	23/5	NORTH TYNE at TARSET
S	24/3	WEAR at STANHOPE
S	24/5	BROWNEY at BURN HALL
	24/7	BROWNEY at LANCHESTER
	25/3	TROUT BECK at MOORHOUSE
S	25/4	SKERNE at SOUTH PARK
	25/810	SIKE WEIR at MOORHOUSE
S	27/1	NIDD at HUNSINGORE
S	27/10	HODGE BECK at BRANSDALE
S	27/26	ROTHER at WHITTINGTON
S	27/27	WHARFE at ILKLEY
	27/34	URE at KILGRAM BRIDGE
	28/2	BLITHE at HAMSTALL RIDWARE
S	28/16	RYTON at SERLBY
	28/23	WYE at ASHFORD
	28/26	ANKER at POLESWORTH
	28/33	DOVE at HOLLINSCLOUGH
	28/801	BURBAGE BROOK at BURBAGE
	28/802	MAUN at MANSFIELD
S	29/1	WAITHE BECK at BRIGSLEY
T S	29/3	LUDD at LOUTH
S	30/1	WITHAM at CLAYPOLE MILL
	31/6	GWASH at BELMESTHORPE
S	32/801	FLORE RESEARCH CATCHMENT (R.R.L.)
S	33/14	LARK at TEMPLE WEIR
S	33/15	OUZEL at WILLEN WEIR
S	33/24	CAM at DERNFORD
S	33/29	STRINGSIDE at WHITE BRIDGE
	33/45	WITTLE at QUIDENHAM
	33/809	BURY BROOK at BURY WEIR
S	34/3	BURE at INGWORTH
S	34/5	TUD at COSTESSEY PARK
	34/7	DOVE at OAKLEY PARK
	34/11	WENSUM at FAKENHAM
	35/4	ORE at BEVERSHAM BRIDGE
S	35/8	GIPPING at STOWMARKET
S	36/8	STOUR at WEST MILL
S	37/1	RODING at REDBRIDGE
S	37/3	TER at CRABBS MILL
S	37/7	WID at WRITTLE
	37/8	CHELMER at SPRINGFIELD
	37/807	CROUCH at WICKFORD
	38/3	MIMRAM at PANSHANGER
	38/7	CANONS BROOK at HARLOW
	39/4	WANDLE at BEDDINGTON
	39/5	BEVERLEY BROOK at WIMBLEDON
T S	39/12	HOGSMILL at KINGSTON
S	39/17	RAY at GRENDON UNDERWOOD
	39/22	LODDON at SHEEPBRIDGE
	39/25	ENBORN at BRIMPTON
	39/26	CHERWELL at BANBURY
S	39/813	MOLE at IFIELD WEIR
	39/814	CRAWTERS at HAZELWICK
S	39/820	DOLLIS BROOK at HENDON LANE
	39/830	BECK at RECTORY ROAD
	39/831	CHAFFINCH at BECKENHAM
S	40/6	BOURNE at HADLOW
S	40/7	MEDWAY at COLLIERS LAND BRIDGE
S	40/8	STOUR at WYE
S	40/9	TEISE at STONE BRIDGE
	40/10	EDEN at VEXOUR BRIDGE
S	41/5	OUSE at GOLD BRIDGE
S	41/6	UCK at ISFIELD
S	41/7	ARUN at PARK MOUND
	41/801	HOLLINGTON at HOLLINGTON
S	41/811	CHESS STREAM at CHESS BRIDGE
S	45/2	EXE at STOODLEIGH
S	45/3	CULM at WOODMILL
T S	45/4	AXE at WHITFORD
	45/804	BARLE at BRUSHFORD
	45/805	EXE at PIXTON
S	46/3	DART at AUSTINS BRIDGE
	46/5	EAST DART at BELLEVER
	46/802	SWINCOMBE at INTAKE
	46/805	BALA BROOK at RESERVOIR
S	47/7	YEALM at PUSLINCH
	48/2	FOWEY at RESTORMEL
	49/3	DE LANK at DE LANK
S	52/4	ISLE at ASHFORD MILL
S	52/5	TONE at BISHOPS HULL
S	52/6	YEO at PEN MILL
S	52/10	BRUE at LOVINGTON
	52/805	GALLICA BROOK at RHYME INTRINSICA
S	53/5	MIDFORD at MIDFORD
S	53/7	FROME at TELLISFORD
	53/8	AVON at GREAT SOMERFORD
	53/9	WELLOW at WELLOW
S	54/4	SOWE at STONELEIGH
S	54/6	STOUR at KIDDERMINSTER
S	54/7	ARROW at BROOM
S	54/10	STOUR at ALSCOT PARK
S	54/11	SALWARPE at HARFORD
T S	54/16	RODEN at RODINGTON
S	54/19	AVON at STARETON
S	54/20	PERRY at YEATON
	54/22	SEVERN at PLYNLIMON
	55/8	WYE at CEFN BRWYN
S	56/3	HONDDU at FORGE BRECON
	56/5	AFON LLWYD at PONTHIR
S	56/6	USK at TRALLONG
S	57/4	CYNON at ABERCYNON
	57/5	TAFF at PONTYPRIDD
S	58/1	OGMORE at BRIDGEND
S	58/2	NEATH at RESOLVEN
S	58/3	EWENNY at EWENNY PRIORY
S	60/2	COTHI at FELIN MYNACHDY
	60/7	TOWY at DOLAU HIRION
S	61/1	WESTERN CLEDDAU at PRENDERGAST MILL
S	64/1	DOVEY at DOVEY
S	65/1	GLASLYN at BEDDGELERT
	66/1	CONWAY at CWM LLANARCH
	66/2	ELWY at PONT YR ONEN
S	67/5	CEIRIOG at BRYNKINALT
	67/8	ALYN at PONT Y CAPEL
T	67/10	GELYN at CYNEFAIL
S	68/6	DANE at HULME WALFIELD
S	68/802	SANDERSON at SANDBACH (R.R.L.)
	69/8	DEAN at STANNEYLANDS
	69/11	MICKER BROOK at CHEADLE
S	71/3	CROASDALE BECK at CROASDALE FLUME
	71/802	RIBBLE at HALTON WEST
	71/804	DUNSOP at FOOTHOLME
S	72/2	WYRE at ST MICHAELS
	73/804	BRATHAY at BRATHAY HALL
	74/1	DUDDON at DUDDON HALL
	76/5	EDEN at TEMPLE SOWERBY
	76/8	IRTHING at GREENHOLME
	76/1	COAL BURN at COALBURN
S	77/11	ESK at CANONBIE
	83/2	GARNOCK at DALTY
	84/2	CALDER at MUIRSHIEL
	84/8	ROTTEN CALDER at RED LEES
S	84/12	WHITE CART at HAWKHEAD
	84/16	LUGGIE at CONDORRAT
S	85/2	ENDRICK at GAIDREW
	65/801	NANT PERIS at TAN-YR-ALLT

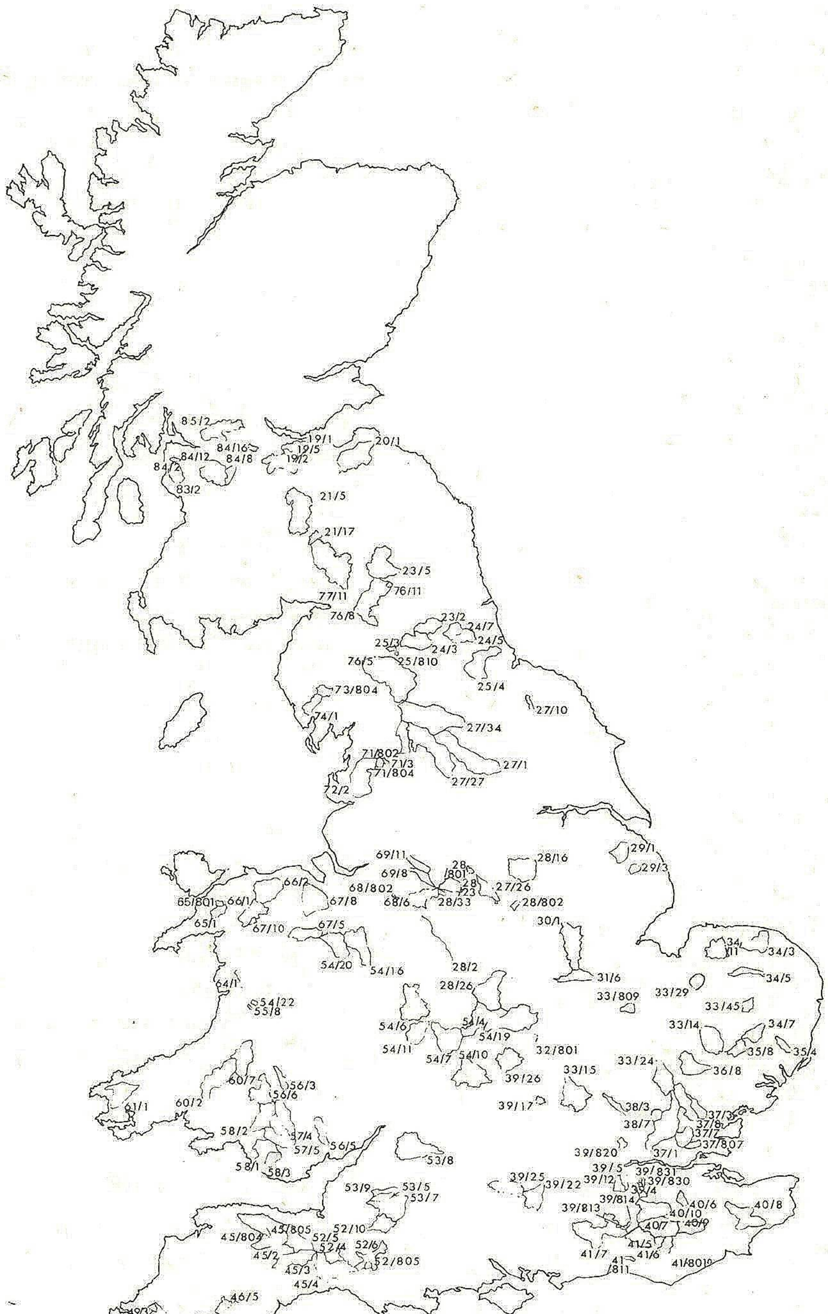


Fig 6.3 Catchments selected for hydrograph analysis. In the list 'T' indicates catchments used for testing (Section 6.6) and 'S' those used in the simulation study (Section 6.7).

Another type of catchment which posed problems is the small one with a very short time to peak. The importance of relative timing error between rainfall and flow data is greatly increased. In addition, the response of those small catchments which are partially urban is often determined by factors which, in a general study, are not known. For example, information on artificial drainage methods or extent was not collected. The determination of sewer design hydrographs for urban catchments is the subject of the TRRL method (Watkins, 1962).

In other catchments flood plain storage becomes effective only during large floods; accordingly, derived unit hydrographs from these events tend to be longer and flatter than those from minor events.

6.3.2 *Types of data collected for each event*

Events were selected mainly according to the size of the peak flow. Preference was given to events which were separated from others on either side by a reasonable period of recession. The number of events chosen ranged from 2 to 25 per catchment with an average of 10. The total number of events processed was 1631; 143 of these were rejected before analysis.

The hydrograph of flow at hourly intervals (sometimes 0.5 or 0.2 hours where the catchment is very flashy—see Table 6.1) was obtained from digitisation of microfilm copies of the original level or flow charts. Where necessary a rating curve was obtained from or agreed with the gauging authority (usually a River Authority).

Daily rainfall totals for all standard gauges within the catchment or just outside it were obtained from copies of the Meteorological Office's magnetic tapes (1961–71) or from their archived records (pre1961). Hourly rainfall totals were extracted directly from the original chart record or a microfilm copy; where official tabulations existed these were used instead. The rain data were combined to produce a hyetograph of hourly average rainfall over the catchment.

Estimates of soil moisture deficit (SMD) on the day of the event at the nearest climate stations were obtained from the Meteorological Office records. The published maps of SMD were also consulted and the mean SMD for the catchment on the day of the map was estimated. Daily rainfall totals for 28 days before the event were collected from a raingauge near the centre of the catchment.

The data collation procedure is illustrated in Figure 6.5.

6.3.3 *Data screening*

The reader is reminded that data collection and processing are described in detail in Volume IV. Three points are stressed here which have a bearing on the analysis:

a As is made clear in Section 7.2 and in Chapter 7, Volume II, there is very little quantitative snowmelt data available in the United Kingdom. From snow lying records obtained from the Meteorological Office and also from their weather summaries (published in various sources including *Water and Water Engineering*), it was possible to check on suspicions that snowmelt was involved in particular events. If it proved to be a significant factor, the event was not analysed.

b Another difficulty which is often discussed is that of storm movement. Storm cells moving down a catchment could be expected to produce hydrographs somewhat earlier and peakier than those moving up. The unit hydrograph theory could be extended to cope with this situation by allowing the time to peak to be predicted in terms of some measure of this storm movement. But this would require:

i that the representation of a rain event as a single cell moving with constant velocity in a straight line is reasonable;
ii that the network of autographic gauges is sufficient to quantify the storm velocity (speed and direction);
iii that the frequency distributions of storm velocity can be defined.
At present none of these is generally true (although radar measurements may provide answers in the future) and, in analysis, the conventional assumption was made that the storm was stationary. Some events were rejected when the rain patterns from two or more autographic gauges were seen to be very different.
c A problem was encountered with the estimation of SMD for the catchment on the day of the event. The network of stations for which daily estimates are available (within the Meteorological Office) from 1941 is shown in Figure 4.8. It was hoped that estimates from the nearest station to the catchment would be adequate but a study of the published maps of SMD showed that large variations in short distances could occur. Either the maps, which are produced at fortnightly intervals, or the end of month values for all stations (readily available) can be used to adjust the nearest station value to something more appropriate to the catchment. All these sources were used to provide a reasonable estimate of the average SMD over the catchment on the day of the event.

6.3.4 Inspection prior to analysis

Despite all objective checking procedures it was considered vital that the rainfall and runoff should be inspected visually before the analysis. The flow hydrograph was plotted by the computer as part of the routine processing and the hyetograph was added afterwards. As a result of checking the plots, final adjustments could be made to ensure the deletion of unrelated rainfall data before or after the main event, crude data errors could be investigated and, in some cases, new evidence for snowmelt or instrument failure was observed.

Sometimes, however, the relation of rainfall to runoff remained inexplicable despite rigorous checks on the data. Figure 6.4 provides an example of this; the event had to be rejected as no reason could be found for the apparent timing discrepancy.

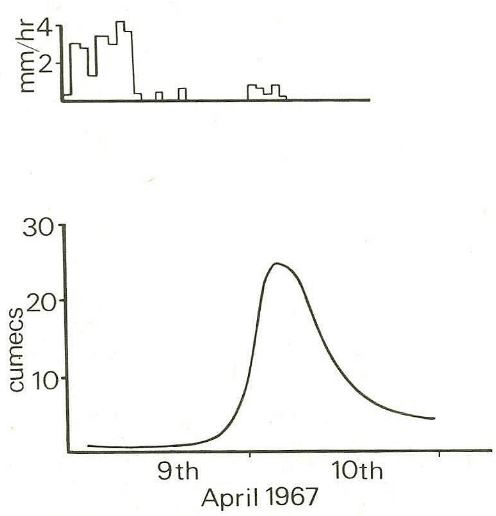
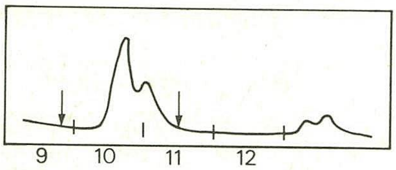
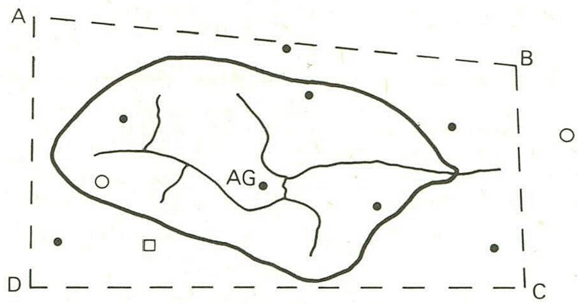


Fig 6.4 Examples of rejected event—inexplicable timing (Teise at Stone Bridge; 40/9).

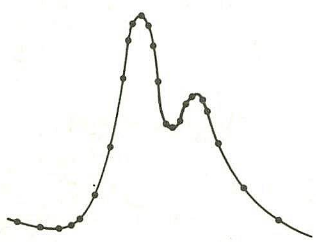


Event specified (arrows). Preliminary examination of rain records shows that some rain occurred end of 9th.



Rain recorder (○) charts collected for 9th, 10th, 11th. Hourly rainfalls extracted. Input together with definition of enclosing quadrilateral ABCD to program which obtains (from magnetic tape) daily falls for all gauges (●) inside ABCD, weights hourly patterns according to distance from centre of catchment, calculates average fall on each of the three days and distributes; also extracts 28 days of antecedent rain for gauge nearest centre (AG).

Microfilm copy of stage hydrograph projected onto table of d-mac pencil follower. Calibration, rating, and coding data punched to paper tape via keyboard. Hydrograph digitised at as many points as necessary for adequate definition.

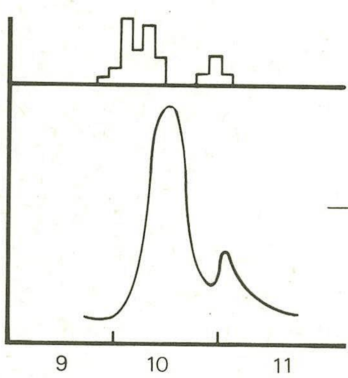


SMD data obtained (□)

Paper tape to computer which produces flow hydrograph at specified intervals (usually hourly) and plots it.

Rain program produces catchment average pattern of hourly rainfall totals. Trivial amounts at either end of event are removed.

Added to plot



Plot checked for 'sense'. Edits made if necessary.

Data types:

1. Flow hydrograph
2. Hourly rainfall
3. SMD
4. Antecedent rainfall
5. Comment.

Checked, brought together and stored on magnetic tape.

To Figure 6.6

Fig 6.5 Summary of data collation procedure.

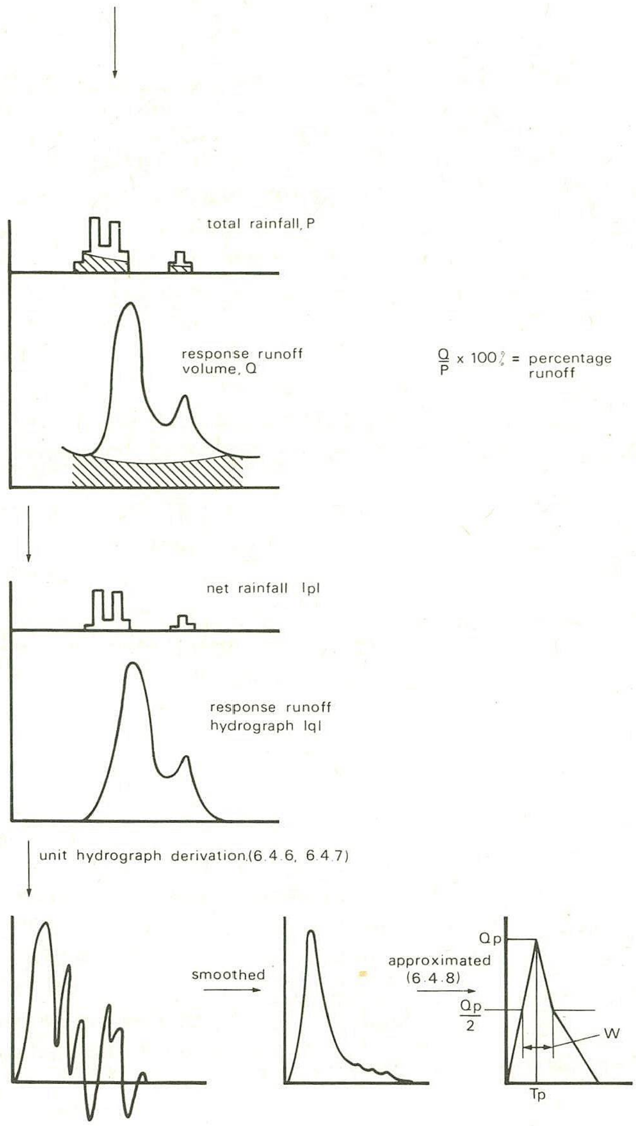


Fig 6.6 Summary of hydrograph analysis.

6.4 Hydrograph analyses

Contents: Section 6.4.1 reiterates the requirements of the study (and the need to do hydrograph analysis). An objective measure of lag time is described in Section 6.4.2 and used as a basis for separating quick response runoff (6.4.3). A catchment wetness index (CWI) is described (6.4.4) and used in the establishment of a profile of net or effective rainfall (6.4.5). The method of unit hydrograph derivation is given, in some detail, in Section 6.4.6 and relative timing problems which affect it are discussed in Section 6.4.7. Finally, the way in which the shape of the TUH was summarised for application in regressions on catchment characteristics is described in Section 6.4.8.

6.4.1 *Introduction*

The main aim of the study described in this chapter is to synthesise a design flood hydrograph on an ungauged catchment. This is to be done by the prediction of losses to deduct from rainfall and by the prediction of a unit hydrograph with which to convert the net rainfall profile into a response runoff hydrograph. The hydrograph analyses described in this section are therefore required to provide the predictive relationships from the data collected as described in Section 6.3. The main requirements are to separate the response runoff hydrograph (this gives the losses), to separate the net rain profile to derive a unit hydrograph from the separated data, and to represent the unit hydrograph by a number of parameters. Because the method of response runoff separation is linked to the measured lag time the latter is described first. Likewise, the index of catchment wetness is introduced before its use in the net rainfall separation is described.

Two conflicting factors affected the choice of techniques in hydrograph analysis. On the one hand, there were 1500 events to be analysed automatically in the space of a few months. On the other, the techniques should be capable of application in a design office without necessarily using a computer. It must be admitted that the techniques to be described in this section are not suitable for manual application but, at each stage, some discussion is included as to the likely effect on the overall results of using a simpler technique.

The more complicated techniques were adopted in the study because of the need to cope with data of varying quality and complexity. Simple methods would tend to give similar answers when the data are particularly suitable. The method of response runoff separation, for example, becomes less important as the event increases in size relative to the preceding flow. Similarly, separation of the net rainfall profile is less dependent on technique with storms that consist of nearly steady rainfall.

The data preparation system and computer program for hydrograph analysis will be available, on request, for application to further events.

Figure 6.6 illustrates the analytical procedures.

6.4.2 *A measure of lag time*

The time taken for the effect of rainfall to reach the outlet of the catchment is called the lag time. It varies between events according to the spatial and temporal distribution of the rainfall and to the prior condition of the catchment. It has been more closely defined (and renamed) in many different ways of which the following are examples:

a Time from start of rainfall to peak flow.
b Time from centroid of net rainfall to centroid of response runoff.
c Time from end of net rainfall to inflexion point on recession limb.
d Time from centroid of total rainfall to peak flow.

In this study, the adopted definition is based on *d*. Its main advantage over *b* for example is that it can be measured from data without any assumptions regarding losses. In order to avoid possible anomalies in multipeaked events, the definition has been widened to be the time from centroid of gross rainfall to 'centroid of peaks'. Figure 6.7 is the definition sketch which shows a numerical example of what is meant by centroid of peaks.

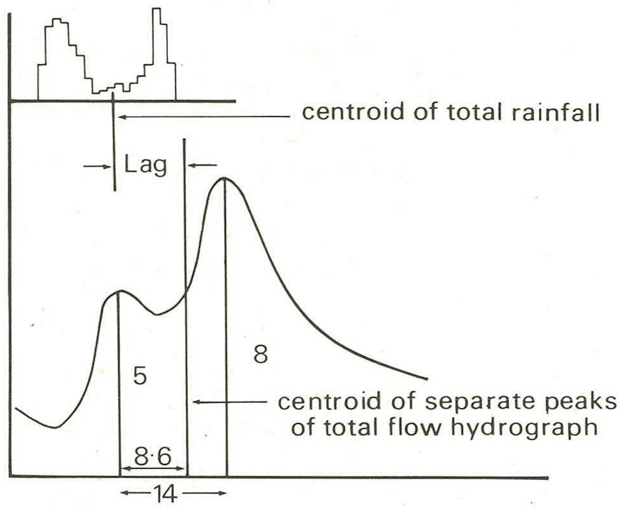


Fig 6.7 Definition of 'LAG'.

6.4.3 Hydrograph separation

For the purposes of this study, quick response runoff is defined as shown in Figure 6.8. It starts when the flow begins to increase and it ends at point **B**, where the time from end of rainfall is 4 × LAG.

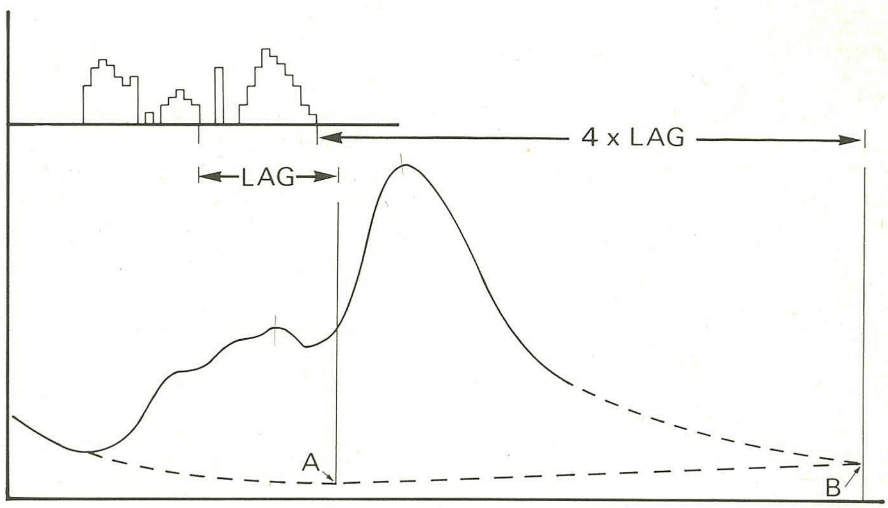


Fig 6.8 Definition of quick response runoff.

Figure 6.8 shows the preceding and succeeding recessions extended as necessary; the former to the centroid of peaks and the latter to the response runoff endpoint. A pilot study was concerned with establishing the most appropriate method of extending recession curves. Five lengthy recessions from each of five catchments were obtained. Different models of the recession curve were fitted to the first half of each curve and the predicted second half compared with a simple linear recession, with a nonlinear recession and with a model which can be interpreted as a set of linear reservoirs in parallel. The third model was found to be best at matching the extended recessions.

A full description of the pilot study is not presented here because, compared with the other assumptions involved in hydrograph analysis, the method of recession extension is not very important. However, a consistent technique was required and the most realistic model was chosen.

If hydrograph analysis is being performed manually, a simple sketched extension of the recessions would produce results very little different from the computer technique in the majority of cases and particularly when the preceding flow is low or almost constant.

The final step in separation of response runoff is to join, by a straight line, the points A and B in Figure 6.8.

6.4.4 An index of catchment wetness

The difference between total rainfall and response runoff is called 'loss'. This is an unfortunate term as it refers, in part, to water which is beneficial to crops and groundwater levels; it is however well established in hydrograph analysis as referring to water lost to the stream network. One of the factors affecting the amount of loss is the antecedent condition of the catchment. Previous studies (Kohler & Linsley, 1951) have indicated this condition by an antecedent precipitation index (API) and a seasonal index (e.g. week number) to represent the annual cycle of the net radiation component of potential evaporation. The API is an exponentially decaying index (k commonly 0.9 per day) with rainfall (P) added directly.

$$\text{API}_d = P_{d-1}+k.P_{d-2}+k^2.P_{d-3}+\ldots = P_{d-1}+k.\text{API}_{d-1} \quad \text{(truncating at 30 days)} \qquad (6.2)$$

In the United Kingdom, the Meteorological Office collect data from a network (Figure 4.8) of stations where potential evaporation is calculated from net radiation and wind run data. By making certain assumptions about the typical distribution of vegetation with different root constants (Grindley, 1970) actual evaporation is estimated. Recorded rainfall is subtracted from and estimated evaporation added to the soil moisture deficit (SMD). Although it may be a rather oversimplified measure, as an index of catchment condition during dry weather it is better than a simple seasonal index. An SMD of zero indicates field capacity which is a condition applying to most of the country between November and April. However, the extent to which a catchment will produce response runoff will vary during this period as a result of recent rainfall which could raise the soil moisture above field capacity. As an index to this increased soil moisture, a short term API is proposed:

$$\text{API5}_d = 0.5^{\frac{1}{2}}\,(P_{d-1}+0.5\,P_{d-2}+(0.5)^2\,P_{d-3}+(0.5)^3\,P_{d-4}+(0.5)^4\,P_{d-5}) \qquad (6.3)$$

where the constant outside the brackets ensures that, given a 0.5 daily decay factor, the assumption of uniform distribution of rain through the day is consistent with the value of API5 at the end of the day. SMD and API5 may be combined to give a full range index of catchment wetness. A simple arithmetic combination is proposed:

$$\text{CWI} = 125+\text{API5}-\text{SMD}. \qquad (6.4)$$

The introduction of the constant, 125, is intended to ensure that, in the majority of flood situations, CWI remains positive because SMD rarely exceeds 125 mm. The advantages of a variable which is always positive will become more apparent when the method of loss separation is discussed in the next section.

The CWI may be calculated on a daily basis as implied above or at smaller time intervals. It will be seen that, where SMD is initially nonzero, the CWI increases more rapidly with rainfall because the rain affects both components but, when SMD is zero, the increase is due to API5 alone.

6.4.5 Definition of net rainfall profile

A full discussion of the several ways of determining a net rainfall profile is given in Section 6.5.5 because it was not until the analysis had been completed that evidence for or against different techniques could be studied. Prior to the analysis it was possible only to draw on previous experience of hydrograph analysis. It appeared that the use of a loss *rate* was in general favour. This could be assumed to be constant throughout the storm (Φ index) as illustrated in Figure 6.9a, to consist of an initial loss and a continuing loss rate (Figure 6.9b), or to change in the form of a curve (Figure 6.9c). It seemed that the latter was more appropriate if the curve could be defined. The concept of the loss rate curve is the applied hydrologist's extension of the infiltration curve (Figure 6.9d), originally due to Horton (1940).

$$f_t = f_c + (f_o - f_c)e^{-kt} \qquad (6.5)$$

where f_t = infiltration rate at time t, f_o = initial infiltration rate, f_c = final constant infiltration rate.

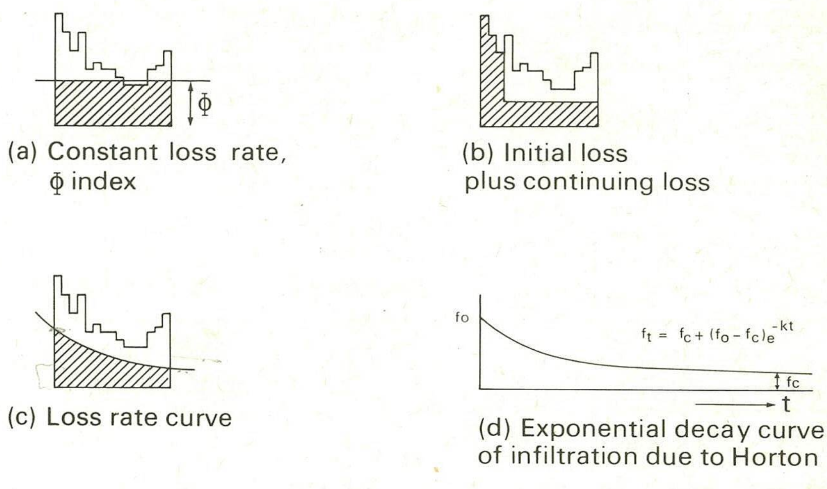


Fig 6.9 Variation of the loss rate concept.

The loss rate curve is assumed to include the effects of all forms of loss in addition to infiltration. Nowadays, hydrologists would recognise the possibility that water apparently lost at a constant rate of infiltration could continue travelling sideways as interflow and be included in the response runoff. Infiltration curves, when applied to areas much larger than experimental plots, have lost much of their physical justification for this and other reasons. Nevertheless, a decaying form for the loss abstraction curve is still, intuitively, a realistic proposition. It would be even more realistic if the loss rate could recover during dry periods within the storm and to have higher initial values and decay faster in dry antecedent conditions. Such a behaviour is observed in the reciprocal of the CWI as it increases through the storm (Figure 6.10), and a simple model of loss rate can be constructed if loss rate at time t is taken to be K/CWI_t. The value of K is the necessary proportional increase in all ordinates of the $1/\text{CWI}$ curve such that the volume of net rain equals the response runoff.

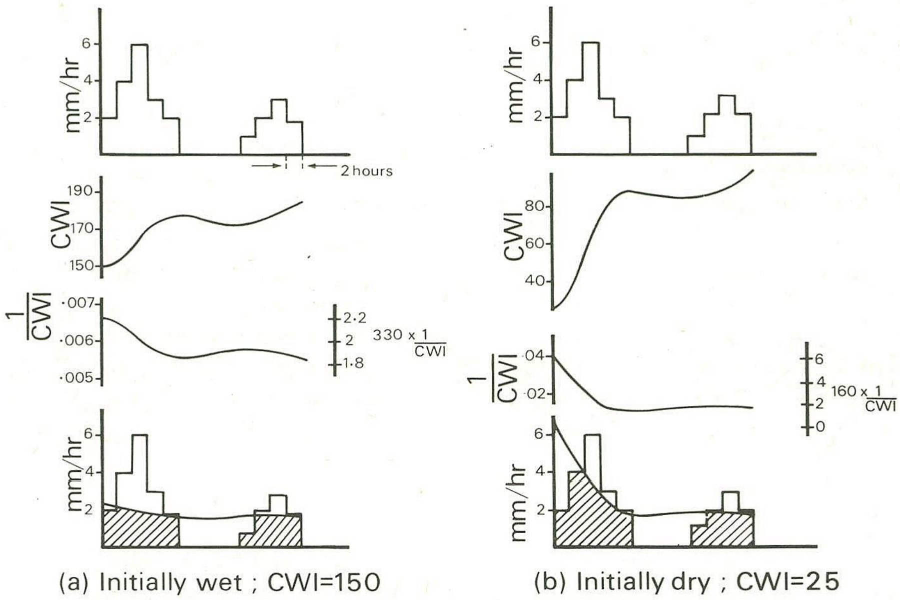


Fig 6.10 Establishing the loss rate curve.

It must be stressed that this particular model of loss rate distribution is not intended to describe physical process but objectively to distribute the losses during the storm with some recognition of the changing state of the catchment.

Figure 6.11 typifies a summer event where the loss rate curve can be seen to be reasonable by a comparison of the relative volumes involved in the two major blocks of the storm.

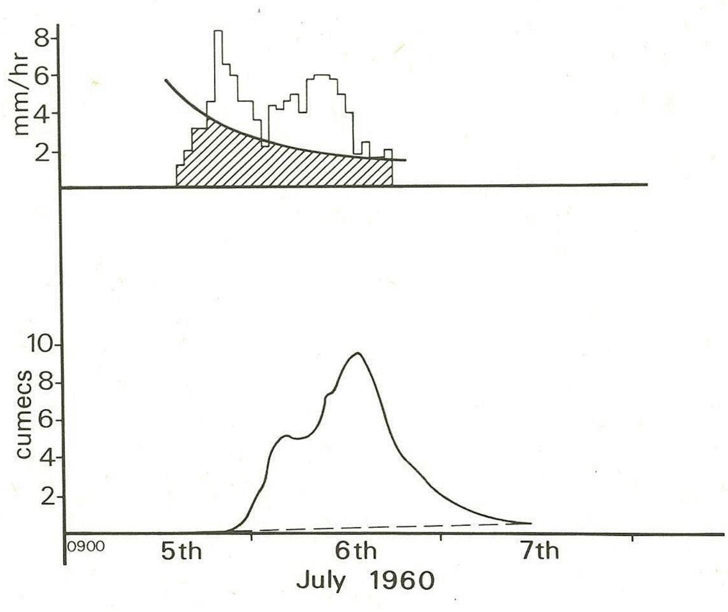


Fig 6.11 Example of net rain separation (Croasdale Beck; 71/3).

To ensure that net rainfall always started first it was assumed that 100 % of rainfall from at least c % of the catchment area always contributed to response runoff. The remaining $(100-c)$ % was then eligible for reduction by the loss rate curve. The combined technique is illustrated in Figure 6.12. The standard method of analysis used $c = 1$.

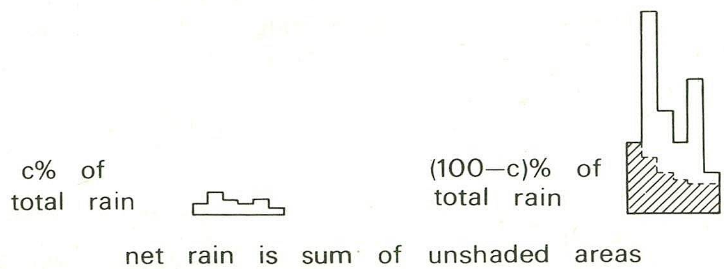


Fig 6.12 Net rainfall separation technique used in analysis.

Although the main reason for introducing this modification was computational convenience, there was also an impression, which grew as more data were examined, that the concept of contributing area (see Section 6.5.5) was highly relevant to the type of catchment and the type of rainfall event which are typical of this country. On some catchments trials were made with fixed percentage contributions greater than 1 % and improvements in the stability of the derived unit hydrograph were observed. The RMS difference between ordinates of the observed and reproduced (using the derived unit hydrograph) response runoff hydrographs was used as a measure of the improvement. Figure 6.13 shows the unit hydrographs resulting from fixed percentage assumptions of 1 and 20 % and the event from which they were derived.

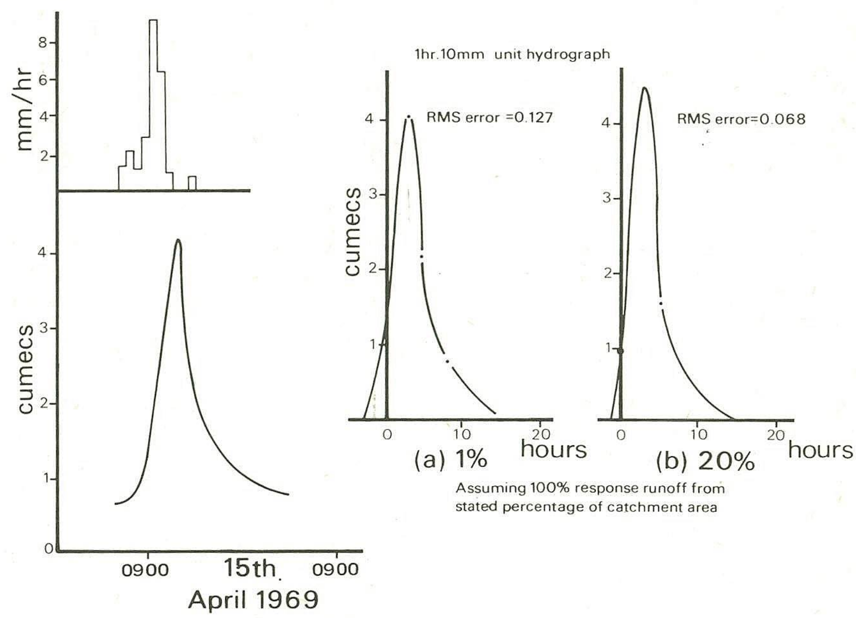


Fig 6.13 Effect of contributing area assumptions on derived unit hydrograph (Severn at Plynlimon; 54/22).

The statistical analysis of results of the loss studies is given in Section 6.5.5 but it may be helpful to mention the main conclusion here. This is that the total loss to be expected from a given rainfall is best described as a *percentage* of that rainfall rather than as a *rate*. It was not possible to repeat all the hydrograph analyses using an entirely percentage based method of separation but it was done on a few catchments to study the effects on the derived unit hydrograph.

The results given in Table 6.13 and in Table 3.2 of Volume IV are those based on loss rate separation but the finally recommended design method and prediction equations are based on percentage runoff. The use of different techniques in analysis and design can cause difficulties. The way in which these were minimised is fully discussed in the sections to follow. The intention here is to warn the reader of this problem so that repeated reference to different methods will not be too confusing.

6.4.6 Method of unit hydrograph derivation

Once the response runoff hydrograph (Section 6.4.3) and the net rainfall sequence (Section 6.4.5) have been determined, the unit hydrograph may be derived. A general and very brief account of the alternative methods of derivation was given in Section 6.2.2. The method adopted in this study was matrix inversion with smoothing. Without smoothing, the method is also known as 'least squares', i.e. the sum of squares of differences between ordinates of the observed and reconstituted hydrographs is minimised.

Equations 6.1 (Section 6.2.2) may be filled out with zero terms until there are $m-n+1$ terms on the left hand side of each equation. Such a set of equations may be represented by the product of two matrices

$$|\mathbf{p}|.|\mathbf{u}| = |\mathbf{q}| \tag{6.6}$$

The solution for $|\mathbf{u}|$ requires that matrix $|\mathbf{p}|$ be made square (number of rows made equal to number of columns), inverted, and then used as a multiplier on $|\mathbf{q}|$. The squaring is done by multiplying by the transpose (the 'transposed' matrix is the result of interchanging columns and rows). Thus

$$|\mathbf{p}^{\mathrm{T}}|.|\mathbf{p}|.|\mathbf{u}| = |\mathbf{p}^{\mathrm{T}}|.|\mathbf{q}| \tag{6.7}$$

and

$$|\mathbf{u}| = \{|\mathbf{p}^{\mathrm{T}}|.|\mathbf{p}|\}^{-1}.|\mathbf{p}^{\mathrm{T}}|.|\mathbf{q}|. \tag{6.8}$$

To demonstrate that this matrix manipulation provides the least squares estimate of $|\mathbf{u}|$ it may be helpful to use a simple numerical example.

Consider the equation set with $n = 3$ and $m = 6$

$$\begin{aligned}
p_1u_1 &= q_1\\
p_2u_1+p_1u_2 &= q_2\\
p_3u_1+p_2u_2+p_1u_3 &= q_3\\
p_3u_2+p_2u_3+p_1u_4 &= q_4\\
p_3u_3+p_2u_4 &= q_5\\
p_3u_4 &= q_6
\end{aligned}$$

For a least squares solution it is required that the sum of the squares of the differences between observed (q) and predicted (sum of $p.u$ terms) should be minimised, i.e.

$$(q_1-p_1u_1)^2+(q_2-p_2u_1-p_1u_2)^2+(q_3-p_3u_1-p_2u_2-p_1u_3)^2$$
$$+(q_4-p_3u_2-p_2u_3-p_1u_4)^2+(q_5-p_3u_3-p_2u_4)^2+(q_6-p_3u_4)^2$$

should be a minimum.

Differentiating with respect to each u value in turn and equating to zero gives the $m-n+1$ (in this case 4) normal equations. For example, for u_1:

$$-2q_1p_1+2p_1^2u_1-2q_2p_2+2p_2^2u_1+2p_2p_1u_2-2q_3p_3+2p_3^2u_1+2p_3p_2u_2+2p_3p_1u_3 = 0$$

which may be written:

$$(p_1^2+p_2^2+p_3^2)u_1+(p_1p_2+p_2p_3)u_2+(p_1p_3)u_3 = p_1q_1+p_2q_2+p_3q_3.$$

The four normal equations (of which the above is the first) may be written as a matrix product:

$$\begin{vmatrix} p_1^2+p_2^2+p_3^2 & p_1p_2+p_2p_3 & p_1p_3 & 0 \\ p_1p_2+p_2p_3 & p_1^2+p_2^2+p_3^2 & p_1p_2+p_2p_3 & p_1p_3 \\ p_1p_3 & p_1p_2+p_2p_3 & p_1^2+p_2^2+p_3^2 & p_1p_2+p_2p_3 \\ 0 & p_1p_3 & p_1p_2+p_2p_3 & p_1^2+p_2^2+p_3^2 \end{vmatrix} \times \begin{vmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{vmatrix}$$

$$= \begin{vmatrix} p_1q_1+p_2q_2+p_3q_3 \\ p_1q_2+p_2q_3+p_3q_4 \\ p_1q_3+p_2q_4+p_3q_5 \\ p_1q_4+p_2q_5+p_3q_6 \end{vmatrix}$$

and this can be expanded to:

$$\begin{vmatrix} p_1 & p_2 & p_3 & 0 & 0 & 0 \\ 0 & p_1 & p_2 & p_3 & 0 & 0 \\ 0 & 0 & p_1 & p_2 & p_3 & 0 \\ 0 & 0 & 0 & p_1 & p_2 & p_3 \end{vmatrix} \times \begin{vmatrix} p_1 & 0 & 0 & 0 \\ p_2 & p_1 & 0 & 0 \\ p_3 & p_2 & p_1 & 0 \\ 0 & p_3 & p_2 & p_1 \\ 0 & 0 & p_3 & p_2 \\ 0 & 0 & 0 & p_3 \end{vmatrix} \times \begin{vmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{vmatrix}$$

$$= \begin{vmatrix} p_1 & p_2 & p_3 & 0 & 0 & 0 \\ 0 & p_1 & p_2 & p_3 & 0 & 0 \\ 0 & 0 & p_1 & p_2 & p_3 & 0 \\ 0 & 0 & 0 & p_1 & p_2 & p_3 \end{vmatrix} \times \begin{vmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{vmatrix}$$

which is a specific example of Equation 6.7 thus demonstrating that the solution of $|\mathbf{u}|$ in Equation 6.6 automatically provides least squares optimisation.

The matrix manipulations of Equation 6.8 can be handled by standard computer library subroutines.

The u values are the ordinates of the 'least squares' unit hydrograph. They form the series of numbers which, when recombined with the original net rain pattern, produce a response runoff hydrograph with minimum sum of squares deviations from the observed runoff hydrograph. However, they do not necessarily form themselves into the shape of a hydrograph; although the position of the dominant signal is usually quite clear, the actual ordinate values are often affected by oscillations and some kind of smoothing scheme is needed to reduce these.

The most suitable form of smoothing was found to be a simple moving average method. Each ordinate was replaced by the average of itself and its two neighbours; this was done twice in succession. The smoothed ordinates were adjusted so that the product of their sum and the data interval was 277.8 cumec hours (equivalent of 10 mm on 100 km^2). Final ordinates are therefore in units of cumecs/100 km^2. The need for smoothing presents problems, mainly because the choice of a suitable method is arbitrary. The only reason for smoothing is to enable the peak of the unit hydrograph to be assessed (see Section 6.4.8) with less subjectivity. Its use might be criticised in the case of a good original unit hydrograph where the effect is to reduce peaks and increase troughs just as it does with an oscillating unit hydrograph. However, as long as the data interval is small enough to produce three values within 20% of the peak and five within 50%, the reduction in peak by smoothing cannot be more than 20%. Moreover, the reduction can be compensated for and this will be described in Section 6.5.9. Figure 6.14 shows the effect of this smoothing scheme on the unit hydrograph. Two events from each of two catchments are used to illustrate the greatest and the least observed effect on a particular catchment. Smoothing has a more marked effect on the peak in the case of 39/12 where it might be argued that the data interval of 1 hour is too long when the time to peak is only 3 or 4 hours.

6.4.7 Timing problems with the data

Every effort was made, in collation of the rainfall and runoff data, to ensure that the timing was correct. However, there are many sources of

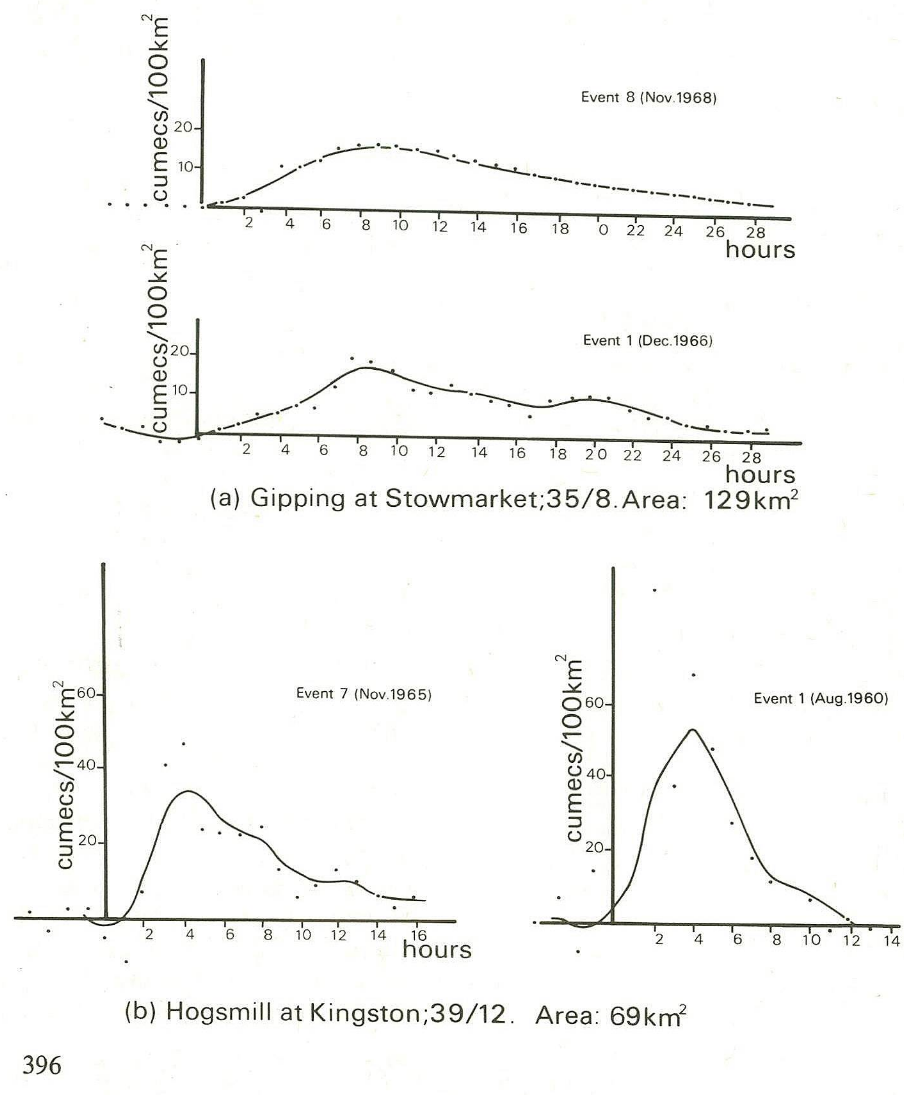


Fig 6.14 Effect of smoothing the unit hydrograph. Unsmoothed values are shown as dots; smoothed values are joined by continuous curve.

error not the least of which is the difficulty of deciding whether the chart annotations of times on and off refer to GMT or BST. Sometimes there were no annotations at all and the weekly chart scale had to be taken at its face value. In short, it was found that relative timing could not be determined better than plus or minus 1 hour in most cases.

As a result of the timing uncertainties, it was decided to introduce a facility for relative timing corrections. Coarse errors could be detected from the graphical plots (Figure 6.4) before analysis but errors up to ± 3 hours were inferred from the results of initial analysis and the correction applied to a second run. To avoid the danger that subjective corrections of this type might be influenced by individual expectations, a team of checkers was used with some ground rules for the correction. The plotted unit hydrograph was used as the basis for the small corrections.

On most catchments, relative timing corrections were highly variable between events suggesting that no systematic bias was being introduced. On the two catchments where positive corrections of 3 hours or more were required in most events, attempts were made to look for other explanations. These are discussed in 6.5.12.

6.4.8 Defining the unit hydrograph

A principal aim of the study is to produce a method of unit hydrograph synthesis on an ungauged catchment. For this purpose the derived unit hydrograph is best represented by a few parameters which can be used as dependent variables in regressions on catchment and storm characteristics. Three parameters were extracted:

a peak flow in cumecs/100 km² Q_p
b time to peak in hours T_p
c width at half the peak (hours) W

When synthesising a unit hydrograph in terms of these three parameters it would be possible to draw in a smooth curve around the specified points. However, in the context of the design problem as a whole, it is thought that a simple linear construction should be adequate. It is therefore assumed that the rising limb is a straight line and that the recession limb consists of two straight lines meeting at a flow value of half the peak flow. The time base of the unit hydrograph (TB) is controlled by the other parameters and the need to produce the 'unit' volume. Thus:

$$\text{TB} = 2\left(\frac{556}{Q_p} - W\right). \qquad (6.9)$$

The approximation of the derived unit hydrograph by three straight lines requires that the three parameters be extracted with this in mind. In Figure 6.15b, for example, it would be incorrect to record the three parameters as measured directly from the 'observed' unit hydrograph because the reconstruction, while matching exactly at the peak, would be a poor approximation overall. The aim is to record the three parameters of that set of three lines which are a 'best fit' to the observed unit hydrograph. Figure 6.15a illustrates the correct approach. The subjectivity of the technique was reduced by adopting the following procedure:

1 The rising limb and the upper half of the recession limb of the observed unit hydrograph were replaced by straight lines fitted by eye.
2 The intersection of these lines was taken as the approximating peak.

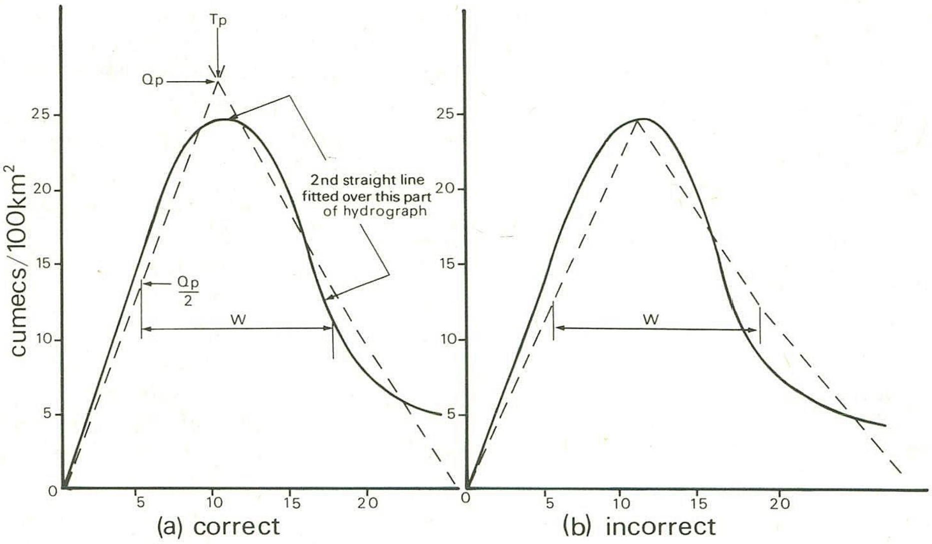


Fig 6.15 Straight line approximation to the unit hydrograph.

3 The recession side line was extended to the base line and the intercept there was taken as $2W$.

It might be thought that this could be done by the derivation program, but in practice it was preferred to incorporate it as part of the checking procedure. Even with smoothing and other refinements, there remained a significant minority of cases where additional adjustment was required before an acceptable unit hydrograph could be determined. These adjustments usually comprised further graphical smoothing of the computer result. Figure 6.16 shows a unit hydrograph which has been derived from a two part storm. Due to a combination of inadequate definition of the time pattern of catchment rainfall and the limitations of the method of data separation, the signal has been subdued; a secondary peak reflects the apparent link between the first block of rain and the second peak of the hydrograph. The checker, being aware of this, can adjust the shape before superimposing the three straight lines. It is necessary to ensure that further manual smoothing does not alter the area beneath the unit hydrograph.

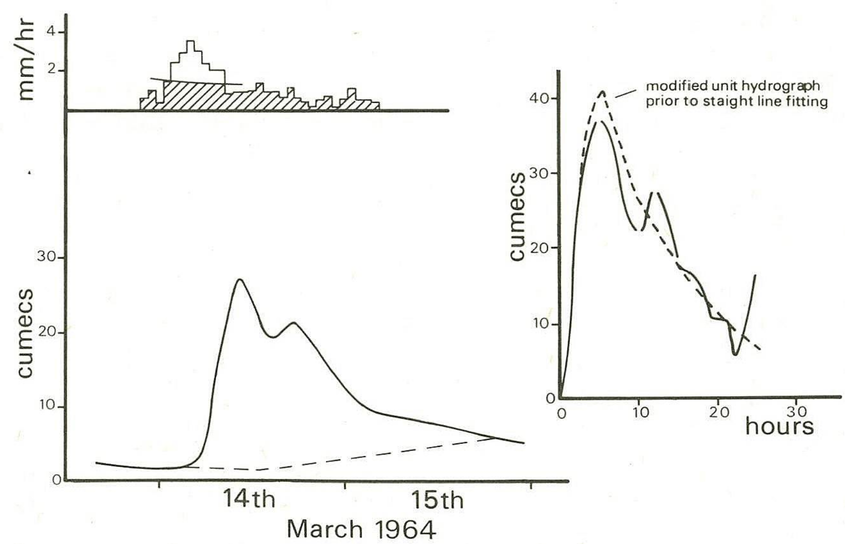


Fig 6.16 Unit hydrograph from a two peak hydrograph—subjective adjustment (Browney at Burn Hall; 24/5).

This aspect of subjective correction concerned only a minority of events. However, it is important to recognise that hydrograph analysis cannot be performed entirely automatically. A judicious blend of computer efficiency and human judgement was required to get the best out of the data.

Despite all the objective and subjective attempts to identify the unit hydrograph, there were 137 events (out of 1488) where this could not be done. In the main, it was assumed that the cause was incorrect definition of the time pattern of rainfall and the events were not necessarily rejected from the 'losses' part of the study.

For most catchments the unit hydrographs were derived from data given at 1 hour intervals; they are 1 hour unit hydrographs. Where a different interval had been used in the data extraction, the unit hydrograph had to be converted. A very simple method was used; the time to peak was adjusted up or down by half the difference between the original data interval and 1 hour. Q_p and W were then recalculated by assuming that the product Q_pT_p and the ratio W/T_p would remain the same. For a 0.5 hour catchment with $Q_p = 100$, $T_p = 2$, and $W = 3$, conversion to 1 hour would cause T_p to increase to 2.25, Q_p to decrease to $200/2.25 = 89.0$, and W to increase to $3/2 \times 2.25 = 3.38$. The same computation can, of course, be applied in reverse to give unit hydrographs of smaller durations.

Bearing in mind (a) the nature of straight line approximation to the unit hydrograph and (b) the variation of unit hydrographs on any one catchment, this simple approach gave results which were as accurate as necessary.

Those catchments which were analysed with a data interval other than 1 hour are listed in Table 6.1. With one exception, a smaller interval was required because the hydrographs would otherwise have been poorly defined. The 2 hour interval was required for 339/1 where events lasted for several days. (This catchment in Ireland is also referred to as 99001 in Table 6.13 and in Volume IV, Table 3.2.)

Table 6.1 Catchments with data interval other than 1 hour.

Catchment	Interval (hours)	Catchment	Interval (hours)
25/810	0.5	46/802	0.5
28/810	0.5	46/805	0.5
28/802	0.2	54/22	0.5
32/801	0.5	55/8	0.5
38/7	0.5	68/802	0.5
39/4	0.5	71/804	0.5
39/5	0.5	76/11	0.5
39/814	0.5	84/8	0.5
39/830	0.5	85/2	0.5
39/831	0.2	339/1	2.0

6.5 Analysis of results

Contents: Alternative approaches to the statistical analysis of the results of the hydrograph studies are discussed in Section 6.5.1. The relationship between the three extracted parameters of the TUH is given in Section 6.5.2; it is suggested that only one, the time to peak (T_p) need be retained. The variation of T_p on a particular catchment is discussed (6.5.3) with reference to ideas of linear or nonlinear catchment behaviour and to observations of 'catastrophic' floods. The prediction of an average T_p in terms of catchment characteristics is described in Section 6.5.4.

Section 6.5.5 compares and describes methods expressing storm 'losses'; it is suggested that a percentage loss (or runoff) is appropriate in this country. The variation of percentage runoff between events is described in Section 6.5.6, different models for predicting it in Section 6.5.7 and regressions on all events in Section 6.5.8. The effect of runoff separation technique on unit hydrograph parameters is discussed in Section 6.5.9. In Section 6.5.10, the prediction equations are compared with the U.S. Soil Conservation Service technique and the possible effects of land use are mentioned.

The prediction of 'base flow' in terms of CWI is described in Section 6.5.11. Some problems which arose on some individual catchments are discussed in Section 6.5.12.

6.5.1 *Alternative approaches to analysis*

Nearly 1500 events from 138 catchments produced unit hydrograph and loss information (the results from all events are summarised as Table 3.2 in Volume IV). In order that a design flood hydrograph could be synthesised on an ungauged catchment, a number of questions had to be answered, e.g.

a Which was the best way of expressing loss? As a total, a percentage, or a loss rate?

b Do unit hydrographs vary systematically between events?

c Which catchment characteristics (20 were available) are useful in predicting unit hydrograph parameters and losses?

It was concluded that multiple regression was the only feasible tool. Graphical techniques could be employed in parallel—perhaps to indicate suitable transformations before regression—but the main effort had to be computer oriented.

For both the 'loss' study and the unit hydrograph study, the variation between events on a single catchment could be examined either separately or together with the variation between catchments. In the loss study, where variation between events on a single catchment was expected, plotting showed the data to be inadequate for the separate approach; there were not enough events on each catchment. In the unit hydrograph study, plotting T_p against storm characteristics suggested that the variation between events was not systematic. The combined (all events together) approach was adopted for both studies at first but, when the lack of significant within-catchment variation of T_p was confirmed, the unit hydrograph studies continued on a between-catchment basis only, using average values (Section 6.5.3).

It was decided to withdraw six catchments (see Figure 6.3), selected at random, from the data set so that the prediction equations derived from the remaining 132 could be tested rigorously. These prediction equations would also be tested on certain very large floods on which there were only limited data and on some overseas events. The testing is described in Section 6.6.

6.5.2 *Unit hydrograph parameters*

As described in Section 6.4.8, three parameters of the unit hydrograph were extracted; Q_p, the peak flow in cumecs/100 km^2, T_p, the time to peak, and W, the width (time) at half the peak. There is clearly an inverse relationship between Q_p and W as their product is likely to be nearly constant (it would be constant if the unit hydrograph were being described by a simple triangle). A linear relationship between T_p and W would be expected.

Because of this obvious interdependence, it was decided to transform the unit hydrograph by dividing the abscissae by T_p and multiplying the ordinates by T_p. The three new parameters are T_p, Q_pT_p and W/T_p.

The choice of T_p as the key parameter in defining the unit hydrograph follows a long tradition in investigations of this type. Moreover, it is the least affected by smoothing and the net rain separation method.

There were three regressions to be made:

a T_p on catchment and storm characteristics.
b Q_pT_p on T_p, catchment and storm characteristics.
c W/T_p, on T_p, catchment and storm characteristics.

The subject of the following two paragraphs is *a* wherein the lack of significant association between T_p and storm characteristics is fully discussed. This led to the adoption of catchment average values for all three parameters so that regressions of Q_pT_p and W/T_p were on T_p and catchment characteristics only. Table 6.2 gives the regression results.

Table 6.2 Regressions of secondary UH parameters.

No. var.	Variable name	Coeff.	Stnd error of coeff.	*t* statis.	*R*	R^2	Stnd error of est.	Const.
Dependent variable: Q_pT_p								
No. of observations: 129								
1	T_p	2.587	0.407	6.35	0.491	0.241	33.434	162.20
2	T_p	2.439	0.409	5.96	0.515	0.270	33.034	171.04
	SAAR	−0.0062	0.0030	2.02	—	—	—	—
3	T_p	2.612	0.423	6.18	0.527	0.284	32.869	162.80
	SAAR	−0.0074	0.0031	2.35	—	—	—	—
	DD	5.856	3.889	1.51	—	—	—	—
Dependent variable: W/T_p								
No. of observations: 129								
1	T_p	−0.00834	0.00325	2.57	0.222	0.049	0.267	1.399
2	T_p	−0.00946	0.00341	2.77	0.241	0.058	0.267	1.454
	DD	−0.032	0.031	1.07	—	—	—	—
3	T_p	−0.0127	0.00412	3.08	0.271	0.073	0.265	1.800
	RSMD	−0.0022	0.0015	1.49	—	—	—	—
	URBAN	−25.7	17.2	1.49	—	—	—	—

SAAR, standard annual average rainfall; DD, drainage density; URBAN, urban fraction; RSMD, see Section 4.2.4.

T_p alone explains 24% of the variance of Q_pT_p with catchment characteristics explaining small additional amounts. The only coefficient significant at the 1% level, however, related to T_p and the prediction is

$$Q_pT_p = 2.6\,T_p + 162. \tag{6.10}$$

Figure 6.17 shows this weak positive association between Q_pT_p and T_p and the fitted line. There is, of course, the difficulty that, if Q_p and T_p were independent, there is bound to be a significant correlation between their product and T_p.

The effect of unit hydrograph smoothing in reducing Q_p values has been mentioned in Section 6.4.6. The result is to make the line in Figure 6.17 steeper than it should be. The question of correcting Equation 6.10 is pursued in Section 6.5.9 where the effect on Q_p of changing the method of net rain separation is also discussed. It is, however, convenient to present the revised prediction here:

$$Q_pT_p = 220. \quad (6.11)$$

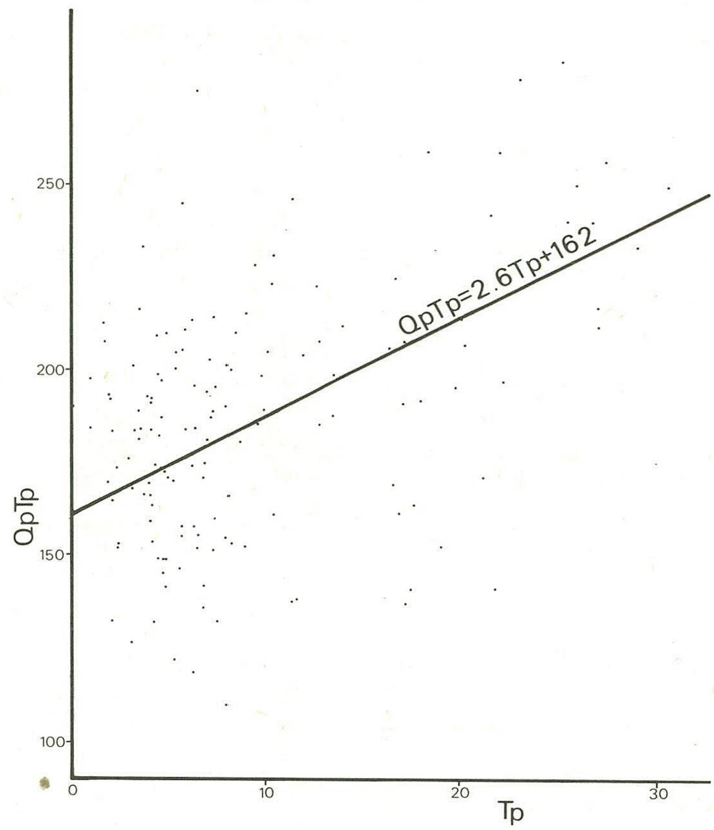


Fig 6.17 Q_pT_p *versus* T_p (catchment average values).

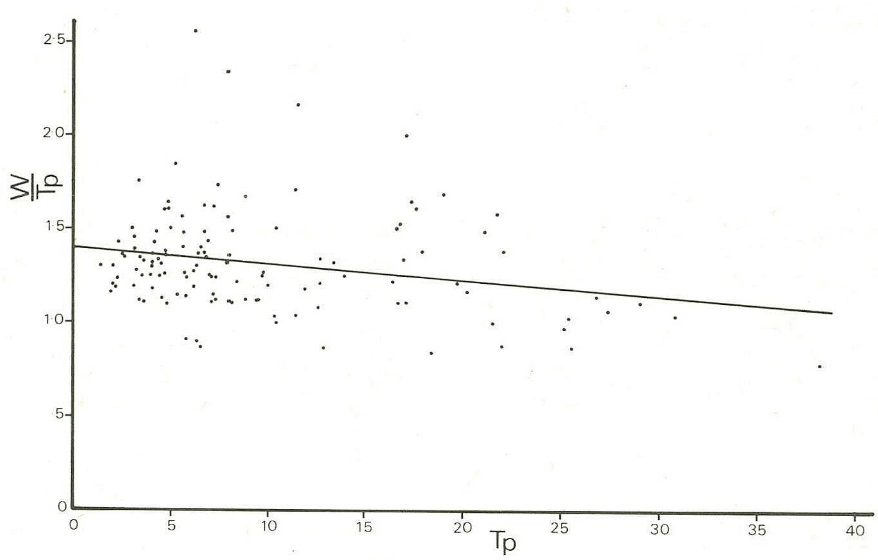


Fig 6.18 W/T_p *versus* T_p—average values.

Figure 6.18 shows the association between W/T_p and T_p; also the fitted prediction line. In this case, T_p explains only 5% of the variance of W/T_p but, again, it is the only variable significant at the 1% level.

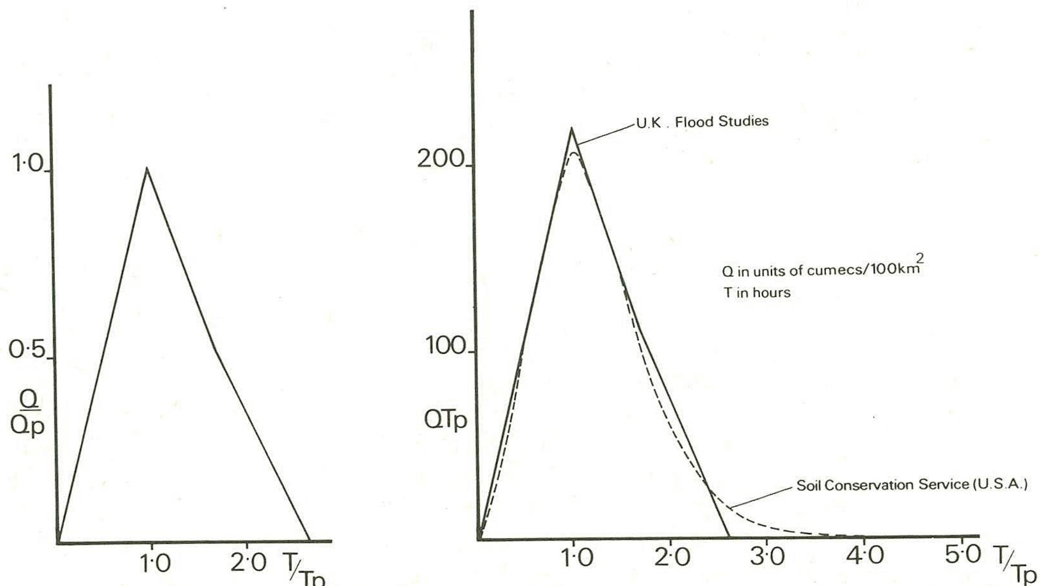
It is thought that it is largely the effect of unit hydrograph smoothing that has introduced the slope in Figure 6.18. As the peak is reduced by smoothing, it follows that the signal is made broader; the level at which it is measured is also lower. To compensate for the undesirable effects of smoothing, therefore, it is proposed that W/T_p should be regarded as constant, e.g.

$$W/T_p = 1.2. \tag{6.12}$$

Given the replacement of Equation 6.10 by 6.11 (described in Section 6.5.9), it is possible to produce a dimensionless unit hydrograph (Figure 6.19(a)). If, on Figure 6.19(a) the curvilinear dimensionless unit hydrograph produced by the U.S. Soil Conservation Service (Mockus, 1957) were to be plotted, the two lines would be indistinguishable over much of the range. They may more easily be compared in Figure 6.19(b) in the form of $Q.T_p$ v. T/T_p; in this case they are constrained to the same enclosed area. It will be seen that the representation of the recession limb by two straight lines is a refinement which can now be abandoned. The simple triangle would be achieved exactly by the equation

$$W/T_p = 1.26. \tag{6.13}$$

Fig 6.19. Dimensionless unit hydrographs.



(a) Dimensionless unit hydrograph (b) Semi-dimensionless unit hydrographs

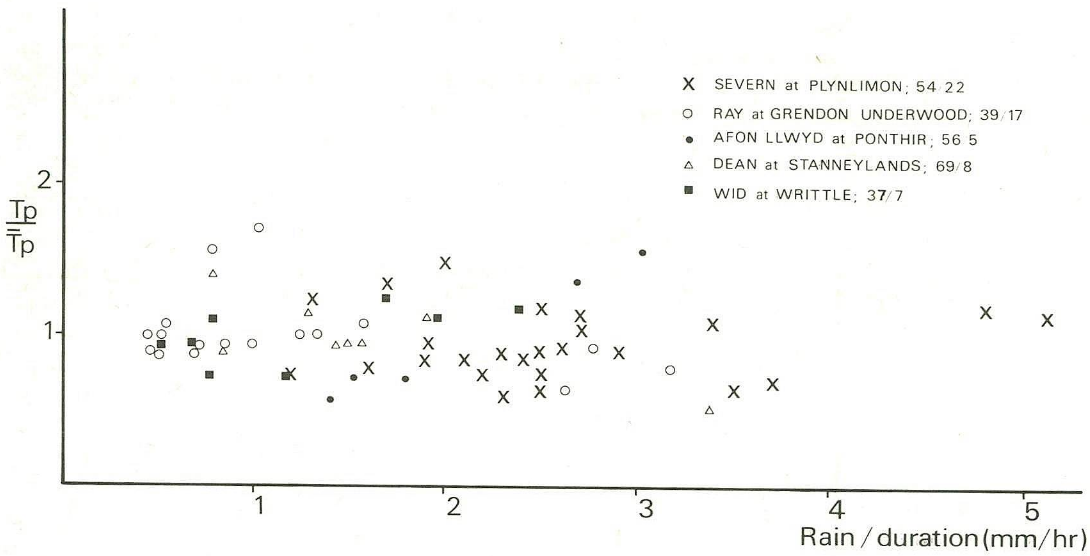
The reduction of the unit hydrograph to a triangle is a simplifying measure in accordance with the accuracy of estimate associated with T_p (Section 6.5.4). It will also be shown, in Section 6.6, that in reproducing recorded events it is rather more important to estimate losses correctly than the unit hydrograph. However, on a gauged catchment, the correct procedure would be to derive several unit hydrographs and, after aligning the peaks in time, to average all the ordinates. In such a situation there is no need to represent the unit hydrograph by a reduced number of parameters.

6.5.3 Variation of T_p with storm characteristics

Theoretical reasons why recorded T_p values might be expected to decrease with some measure of storm intensity or size were given in Section 6.2.3.

This was investigated initially by plotting T_p against mean rainfall intensity. In Figure 6.20, values of T_p have been divided by their mean and the variation shown for five catchments. (The values of *mean* intensity are, of course, much less than intensities observed in the core of an event but the two are related.) One catchment (69/8) shows a strong tendency for T_p to decrease as the intensity increases; another (39/17) shows a slight trend. The others either show no trend at all or suggest the opposite effect. This variation in trend is typical of that found in the full set of catchments; whether or not it can be related to catchment characteristics will be the subject of further research.

Fig 6.20 $T_p/\overline{T}_p$ versus average rain intensity for five catchments.



The regression on all events showed that T_p was not dependent on storm characteristics to any significant extent. (There was a statistically insignificant tendency for *higher* T_p values to be associated with increased volumes of runoff.) This regression masks the possibility of individual catchments displaying different trends but, as their direction cannot be predicted at present, they must be ignored.

In view of the assumptions made and the arbitrary methods of data separation used in analysis, the within catchment variation in T_p (average cv = 23%) is not unexpected. Typically, the ratio of the highest observation to the lowest is 2.5 to 1.

A similar range of results is reported from a large scale investigation carried out in South Africa (Pullen, 1969). This variation in T_p illustrates the importance of deriving unit hydrographs from more than one event on a catchment.

It should be emphasised that these results do not disprove ideas about nonlinearity which have been put forward in the past. Minshall (1960) for example, who was one of the first to write about nonlinearity in the context of unit hydrograph parameters in particular, was concerned with very small experimental watersheds only. Moreover, on his 27 acre catchment, the case for T_p to be inversely related with rainfall intensity rested on five events where the observed rainfall, had it occurred in this country, would have had a return period of 200 years or more.

In very small areas where the infiltration process is dominant and surface runoff is the result of overland flow, the hydraulic arguments (Section 6.2.3) become intuitively more acceptable. However, over larger areas it is probable that the dominant process is subsurface flow. This is particularly likely in the United Kingdom with its relatively low rainfall intensities and only occasional observation of widespread surface (overland) flow.

There is one more possible explanation of why this study failed to produce any clear evidence for the nonlinear ideas. If the concept of contributing area is accepted then it follows that, as the catchment gets wetter, the size of the contributing area extends back from the channels and up into the hills. The wetter the catchment, therefore, the longer is the average travel time. This would act against the nonlinear storage effect.

Although there was no evidence, from the data as a whole, for lagtimes to decrease with rainfall or runoff intensities, there were indications for some catchments that, in the most extreme events, response runoff does concentrate more quickly than usual. In this context, the phrase 'most extreme events' refers to such floods as Louth 1920, Lynmouth 1952, Bowland 1967. The catchments which display the effect particularly strongly tend to be those where it is possible to speculate that the physical nature of response runoff changes in the case of extreme rainfall. The Lud at Louth is probably the most impressive example. The geology is chalk with some clay drift deposits and it seems likely that response runoff normally originates on the latter. Since 1966, when formal records started, the highest recorded discharge is about 7 cumecs and the average lag for the catchment about 5 or 6 hours. But in 1920, the peak flow was estimated at 150 cumecs and the lag at 1.5 hours; it seemed certain to those who investigated the flood at the time that response runoff was being generated on the chalk areas of the catchment.

The conclusion of this aspect of the investigation was that the average T_p of all events on the catchment should be used as the independent variable in regressions on catchment characteristics. Before discussing these regressions, however, it is useful to present the prediction of T_p in terms of the lag measure (Section 6.4.2).

$$T_p = 0.92\,\text{LAG}^{0.99} \qquad (6.14)$$

(not significantly different from $T_p = 0.9$ LAG).

It is most useful in situations where a flow recorder (or even a stage recorder) has been installed for long enough to record a few large events. They may not be suitable for a full unit hydrograph derivation but a simple measure of LAG will allow synthesis of a unit hydrograph which will be as useful as the real thing in many design problems. Details of this regression are given in Table 6.3.

Table 6.3 Regression of T_p on LAG and other catchment characteristics. (S1085 = S).

No. var.	Variable name	Coeff.	Stnd error of coeff.	t statis.	R	R^2	Stnd error of est.	Const.	Antilog (const.)
Dependent variable: log (T_p)									
No. of observations: 129									
1	Log (LAG)	0.992	0.174	57.01	0.981	0.962	0.061	−0.034	0.92
2	Log (LAG)	0.932	0.028	33.30	0.982	0.964	0.060	0.062	1.15
	Log (S1085)	−0.054	0.020	2.70	—	—	—	—	—
3	Log (LAG)	0.940	0.029	32.59	0.982	0.964	0.060	0.084	1.21
	Log (S1085)	−0.061	0.021	2.91	—	—	—	—	—
	Log (AREA)	−0.012	0.011	1.11	—	—	—	—	—

6.5.4 *Prediction of T_p on an ungauged catchment*

There have been many investigations of the time delay parameter of a catchment; Table 6.4 lists some of them. Stream length and slope have been the most popular catchment characteristics and have often been combined in the form $L/\sqrt{S}$ which, from the open channel formulae of the form velocity $= k\sqrt{\text{channel slope}}$, can be expected to be a measure of travel times. The regression of T_p on $L/\sqrt{S}$ resulted in

$$T_p = 2.8\left(\frac{L}{\sqrt{S}}\right)^{0.47} \quad (6.15)$$

where L is in km, S in m/km, T_p in hours.

Table 6.4 Some investigations into the relationship between catchment characteristics and a measure of response time.

Author	Date	Time parameter	Catchment characteristics
Bransby-Williams	1922	Time of concentration	L (straight line to outfall), circularity, A, S
Kirpich	1940	Time from lowest to highest stage at outfall	S, L (straight line to outfall)
Ramser	1927		
Bernard	1935	The 'U' factor of his 'distribution graph'	L, S, shape/network, conveyance factor
McCarthy	1938	Time to peak from beginning of rain	A, stream pattern, average elevation increase per unit area
Snyder	1938	Time to peak from centre of mass of rainfall excess	L, L_{ca}
Johnstone & Cross (textbook which also gives details of three American studies of the 1930s listed above)	1949	Base-length of area *versus* time diagram	L, S, stream pattern
Taylor & Schwarz	1952	As Snyder	L, L_{ca}, S
Hickok, Keppel & Rafferty	1959	As Snyder	L, Dd, S (average land slope)
Nash	1960	Time from centre of mass of excess rainfall to centre of mass response runoff	L, S
Chow	1962	As Synder	L, S
Gray	1964	Time to peak	L, S
Espey, Morgan & Masch	1965	Rise time	L, S, percentage impermeable
Kennedy & Watt	1967	As Nash	L, S, storage, population
Bell & Om Kar (a review of many methods)	1969	Time from centre of mass of excess rain to time when 50% of response runoff has occurred	L, S also A and vegetation cover
Painter	1971	Time to peak	L, S, infiltration index
Mitchell	1972	As Snyder	L, S, surface storage

L = stream length, L_{ca} = length to centroid of catchment, S = stream slope, A = area, Dd = drainage density.

However, with a multiple regression package available, it seemed preferable not to combine variables in any particular way until they had been tried separately.

Apart from L and S the complete set of catchment characteristics was available (see Chapter 4) including some which had been extracted specifically for the unit hydrograph analysis. All variables were subjected to logarithmic transformation. The variable expressing fraction of urban area (URBAN) was transformed

$$\text{URBT} = 1 + \text{URBAN} \quad (6.16)$$

so that, when there was no urban area, the transformed URBAN would be 1.0 and there would be no effect on a prediction of the type:

$$T_p = k \cdot v_1^a \, v_2^b \, v_3^c \ldots \quad (6.17)$$

The best equation with four variables allowed into the regression was

$$T_p = 46.6S^{-0.38}\ \text{URBT}^{-1.99}\ \text{RSMD}^{-0.4}\ L^{0.14}. \qquad (6.18)$$

RSMD (fully defined in Section 4.2.4) is a climatic index of flood runoff potential. Full details of the regression are given in Table 6.5, from which it can be seen that a slope measure was the most important variable in explaining the variance of T_p. The most surprising result is that stream-length is not more important; this was unexpected in view of results from elsewhere. The most plausible explanation lies in the significant inverse correlation ($R = -0.58$) of log (stream length) with log (stream slope) which is observed in the 139 catchments used in hydrograph analysis (Figure 6.21).

Fig 6.21 Association between stream slope and stream length and slope prediction equation.

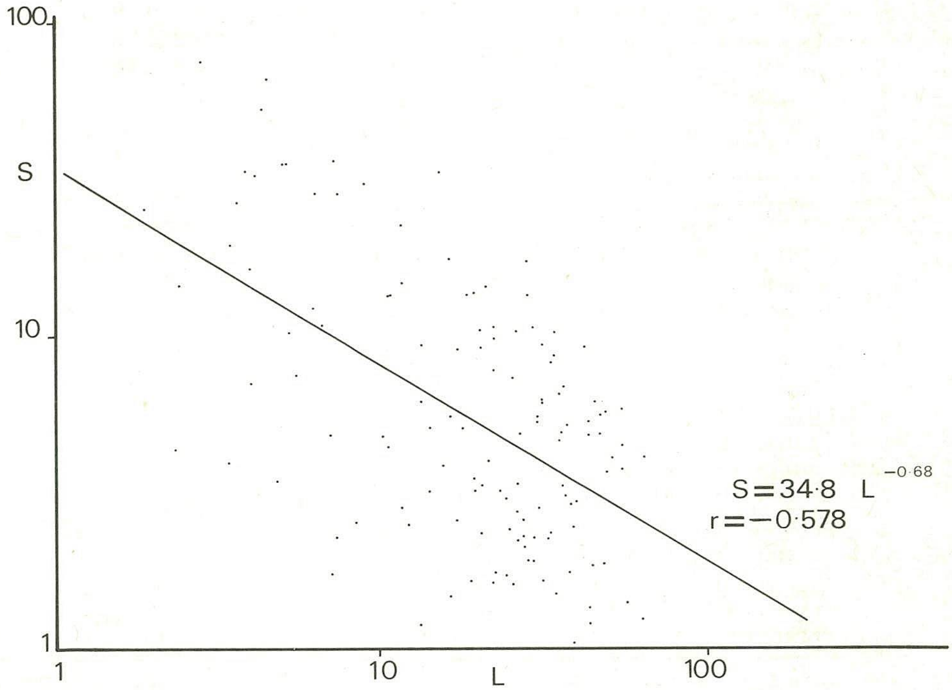


Table 6.5 Regression of T_p on catchment characteristics (MSL forced into four variable equation).

No. var.	Variable name	Coeff.	Stnd error of coeff.	t statis.	R	R^2	Stnd error of est.	Const.	Antilog (const.)
Dependent variable: log (T_p)									
No. of observations: 130									
1	Log(S1085)	−0.598	0.038	15.41	0.807	0.651	0.186	1.311	20.46
2	Log(S1085)	−0.597	0.032	18.65	0.870	0.757	0.156	1.355	22.65
	Log(URBT)	−2.028	0.273	7.42	—	—	—	—	—
3	Log(S1085)	−0.516	0.048	10.68	0.876	0.767	0.154	1.673	47.10
	Log(URBT)	−2.205	0.281	7.85	—	—	—	—	—
	Log(RSMD)	−0.240	0.108	2.22	—	—	—	—	—
4	Log(S1085)	−0.381	0.066	5.77	0.883	0.780	0.150	1.669	46.67
	Log(URBT)	−1.995	0.284	7.02	—	—	—	—	—
	Log(RSMD)	−0.417	0.118	3.52	—	—	—	—	—
	Log(MSL)	0.139	0.050	2.77	—	—	—	—	—
5	Log(S1085)	−0.188	0.108	1.73	0.888	0.789	0.147	1.785	60.95
	Log(URBT)	−1.927	0.281	6.85	—	—	—	—	—
	Log(RSMD)	−0.451	0.118	3.84	—	—	—	—	—
	Log(MSL)	0.096	0.053	1.81	—	—	—	—	—
	Log(TAYSLO)	−0.241	0.108	2.23	—	—	—	—	—

S1085 = S, MSL = L, TAYSLO = alternative measure of channel slope (Section 4.2.2), URBT = 1 + URBAN.

In order to examine a different aspect of this rather surprising result, the catchments were divided into three classes according to size and the regressions repeated. Table 6.6 gives the results. It can be seen that, although the stratified samples produce better individual predictions, the improvement over the overall prediction is only slight. There is thought to be no advantage in using separate predictions.

Table 6.6 T_p regressions; separating catchments by area.

No. var.	Variable name	Coeff.	Stnd error of coeff.	t statis.	R	R^2	Stnd error of est.	Const.	Antilog (const.)
Dependent variable: log (T_p) No. of observations: 33 Catchments ≤ 50 km²									
1	Log(TAYSLO)	−0.422	0.090	4.70	0.645	0.419	0.184	1.056	11.38
2	Log(S1085)	−0.480	0.066	7.33	0.811	0.658	0.144	1.204	16.00
	Log(URBT)	−1.810	0.353	5.12	—	—	—	—	—
3	Log(S1085)	−0.477	0.065	5.96	0.820	0.672	0.143	1.162	14.52
	Log(URBT)	−1.815	0.351	5.17	—	—	—	—	—
	Log(SHAPE)	0.152	0.130	1.17	—	—	—	—	—
Dependent variable: log (T_p) No. of observations: 53 Catchments 51–200 km²									
1	Log(TAYSLO)	−0.378	0.078	9.46	0.798	0.637	0.161	1.393	24.72
2	Log(TAYSLO)	−0.692	0.073	9.50	0.838	0.707	0.148	1.387	24.38
	Log(URBT)	1.843	0.561	3.28	—	—	—	—	—
3	Log(TAYSLO)	−0.499	0.089	5.58	0.868	0.753	0.135	2.600	398.1
	Log(URBT)	−2.379	0.541	4.40	—	—	—	—	—
	Log(SAAR)	−0.446	0.138	3.23	—	—	—	—	—
Dependent variable: log (T_p) No. of observations: 44 Catchments > 200 km²									
1	Log(RSMD)	−1.345	0.131	10.26	0.848	0.719	0.143	3.113	1287.0
2	Log(RSMD)	−0.754	0.211	3.58	0.884	0.781	0.128	2.428	267.9
	Log(S1085)	−0.426	0.127	3.37	—	—	—	—	—
3	Log(RSMD)	−0.677	0.204	3.31	0.898	0.806	0.122	2.307	202.8
	Log(S1085)	−0.391	0.122	3.21	—	—	—	—	—
	Log(DD)	−0.269	0.122	2.21	—	—	—	—	—

The variables are defined in preceding tables, except for SHAPE which is defined in Volume IV, Chapter 3.3.

The emergence of urban area fraction as an important independent variable is naturally limited to the small and medium catchments but it is still the second most significant variable overall. This significance is attributable to a handful of catchments with a large urban area and the fraction may be estimated quite roughly without reducing significance or altering coefficients. It was found that expressing the fraction to the nearest tenth was adequate.

A physical explanation for the importance of the urban area may be made in terms of the more efficient drainage network and the higher velocities associated with paved areas and pipes as compared with the predominantly subsurface nature of response runoff from natural catchments. A similar decrease in lag time with increased urban area is reported by Hall (1973), and, previously, by Espey, Morgan & Masch (1965). Such an effect might be reversed with large storms if the drainage capacity is inadequate resulting in ponding.

Figure 6.22 shows the residual differences between 'observed' T_p values (average of all events on a catchment) and those predicted by Equation 6.18. The residuals are plotted against the observed values (Figure 6.22a) to indicate the remaining variance and also against the predicted values

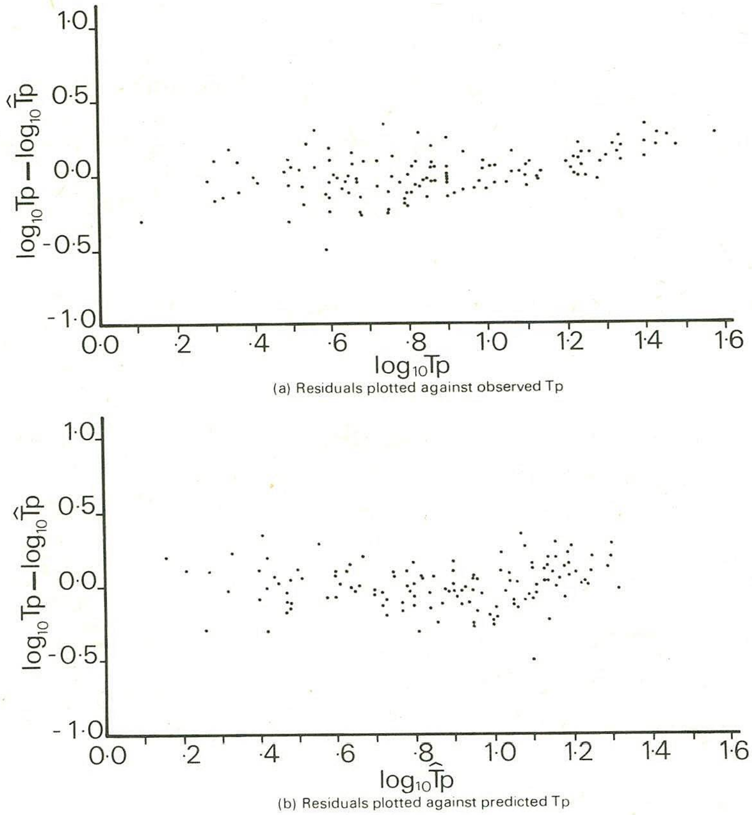


Fig 6.22 Unexplained variation of T_p between catchments.

(Figure 6.22b). Some of the residual variance may be associated with the error of estimating the *mean* value of T_p on each catchment, some in terms of systematic data error and some remains unexplained. Figure 6.23 shows the same residuals plotted on a map. Also, to test whether untried variables would help, six pairs of neighbouring catchments with opposite residuals were compared; the results are given in Table 6.7. Although the data set would have to be broken down into regions to prove the point statistically, it was concluded that there was no geographical pattern of residuals and indeed there is no rational reason why there should be.

Table 6.7 Comparison of nearby pairs of catchments with opposite errors of predicting T_p.

Catchment no.	Residual/ standard error (logarithms)	Geological difference	Land use difference	Storage difference
71/804 71/3	−1.07 +1.21	No real difference	No difference	No difference
74/1 73/804	−0.47 +1.88	No difference	No difference	'Slower' catchment has more lakes
52/805 52/6	−1.36 +0.64	No real difference	No difference	'Slower' catchment has reservoir
33/29 33/14	−1.61 +1.40	'Faster' catchment has no drift cover and responds from near gauge	No difference	'Slower' catchment has ponds
55/8 54/22	−0.64 +0.64	No difference	'Slower' catchment is forested	No difference
46/802 46/5	−0.15 +0.34	No difference	'Slower' catchment has some forest	No difference

NB 'Faster' catchment has the negative residual.

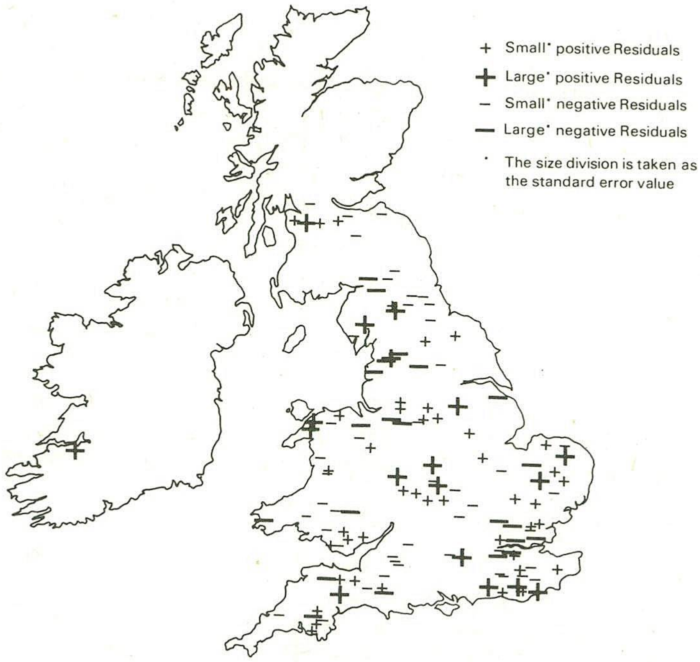


Fig 6.23 Map of T_p residuals.

From the paired catchments only one of the four new factors might be described as significant and rational, namely the presence of lakes or reservoirs. However, there are very few catchments in the set which are affected by such storages and the inclusion of a 'lakes' index could make little improvement overall. The prediction must therefore be regarded as applying to catchments with no storages. If T_p is required for a catchment with lakes, the T_p estimate must be modified according to their size and location.

The conclusion of the study of T_p and catchment characteristics is that it has produced quite good results. Where the published information allows comparison, it would seem that the prediction equations presented here can be used with a confidence equal to that implied in the best of other large scale investigations.

However, the standard error associated with the prediction of log T_p by this regression on catchment characteristics is 0.15 compared with 0.06 by the regression on measured lag. In many design situations it will be possible to install a water level recorder at an early stage in the investigation. (A recording raingauge would also be required if none was already on the catchment.) A reliable estimate of average catchment lag could be made as soon as a few large events had occurred. If the interest in the catchment were to develop further, the gauging site would be calibrated and the 'proper' unit hydrograph could be derived.

6.5.5 Comparison of ways of expressing storm 'losses'

This section considers the alternative approaches to loss analysis. General readers may prefer simply to note that it was decided to express loss and response runoff as percentages of rainfall and to omit this section at a first reading.

The analyses have produced, for each event, figures for total loss (mm), percentage runoff, and two values of loss rate (mm/hour) applying to the beginning and end of the event. It had to be decided which of these was most suitable for regression on catchment and storm characteristics. The method of effective rainfall separation used in analysis, described in Section 6.4.5, assumed a model based largely on the loss rate curve. A decision had to be made and it was not until all the results were available that its suitability could be questioned. However, as it was used in analysis, it may be argued that it should be used in design whether or not it still appears to be suitable. The loss rate method has the advantage that the total value of loss and its distribution are specified at the same time. If either absolute loss or percentage loss are predicted the distribution remains to be specified separately.

Despite the case which could be made for using loss rates in design it was decided to investigate the different techniques more closely. There were three points to investigate; in order of importance they were,

a regressions to determine the best form of the loss prediction model;
b the influence of the hyetograph on loss rates;
c trials of alternative techniques in analysis.

Before discussing the findings, the alternative methods of describing losses should be defined.

1 Simple water balance. The antecedent catchment condition is expressed in the form of an initial moisture deficit. Rain which falls is assumed to reduce the deficit to zero whereafter further rain becomes runoff. Such a model has proved reasonably successful when applied to rainfall and total runoff volumes measured in time periods of a day or so. It is less relevant to quick response runoff.

2 Infiltration. Response runoff takes place whenever the rate of rainfall exceeds the rate at which infiltration can occur (Figure 6.9d). The shape of the infiltration curve is determined by the initial state of the soil and soil properties. The method is usually replaced, in applied hydrology, by the concept of loss rate. The loss rate is considered to include surface retention, interception, and other losses in addition to infiltration; it is specified as a single index (Figure 6.9a), as a two part loss (Figure 6.9b) or as a loss rate curve such as was used in analysis. This type of model is based, essentially, on the belief that what is observed at a point may be considered to apply over the catchment as a whole.

3 Contributing area. Catchments comprise many different zones which are in various states of readiness to transmit response runoff. An extreme example would be a catchment where the high ground is on the chalk downland (70%) and there are clay areas or perhaps a conurbation (30%) lower down; the concept of contributing area would suggest 100% runoff from 30% of the area and none from the rest. More generally, the contributing area can expand as the catchment gets wetter and percentage runoff can increase through the storm. It is a concept which is currently receiving much attention as hydrologists recognise the importance of subsurface flow and variable source areas of runoff.

Regressions to determine the best form of the model

The three main concepts may be expressed:

1 $Q = P - \text{SMD}$ (6.20)

2 (a) constant loss rate $\dfrac{P-Q}{De} = k_1$ (6.21)

(b) two part loss rate $\dfrac{P-Q-k_2.\text{SMD}}{De} = k_3$ (6.22)

3 $\dfrac{Q}{P} = f(\text{SMD})$ e.g. $\dfrac{Q}{P} = k_4 - k_5.\text{SMD}$ (6.23)

where SMD is the initial deficit and *De* the duration of net rain.

Rearranging to have the loss volume on the left:

1 $P-Q = \text{SMD}$ (6.24)

2 (a) $P-Q = k_1\ De$ (6.25)

(b) $P-Q = k_3\ De + k_2\ \text{SMD}$ (6.26)

3 $P-Q = P(1-k_4+k_5\ \text{SMD})$. (6.27)

In the regression, CWI (defined in Section 6.4.4) was used as the deficit index in place of SMD, and the duration of net rain was replaced by the duration of total rain (*D*). With loss volume as the dependent variable, the independent variables allowed were CWI, *D*, *P* and catchment characteristics including the soils index, SOIL (Section 6.5.7). All variables were transformed by taking logarithms.

It was reasoned that the coefficients of the variables provided by the regression procedure would indicate the most appropriate model. If Equation 6.27 applied, the exponent of *P* should approach 1, while the loss rate model would be supported by the exponent of *D* approaching 1. The results from 1450 events are shown in Table 6.8. Loss is related to rainfall with an exponent near one but, significantly, not exactly one. It is related to antecedent condition and to the soil index but there is no significant relationship with storm duration. From this it was concluded that the percentage model was more appropriate than the loss rate model.

Table 6.8 Regressions to predict loss volume.

No. var.	Variable name	Coeff.	Stnd error of coeff.	*t* statis.	*R*	R^2	Stnd error of est.	Const.	Antilog (const.)
Dependent variable: log (LOSS)									
No. of observations: 1450									
1	Log RAIN	0.807	0.020	41.3	0.735	0.540	0.179	0.0418	1.10
2	Log RAIN	0.909	0.018	49.2	0.792	0.627	0.162	−0.354	0.44
	Log SOIL	−0.550	0.030	18.3	—	—	—	—	—
3	Log RAIN	0.903	0.018	49.2	0.797	0.635	0.160	−0.0292	0.93
	Log SOIL	−0.480	0.032	14.8	—	—	—	—	—
	Log CWI	−0.138	0.025	5.5	—	—	—	—	—
4	Log RAIN	0.902	0.021	43.3	0.797	0.635	0.160	−0.0288	0.94
	Log SOIL	−0.480	0.032	14.8	—	—	—	—	—
	Log CWI	−0.139	0.026	5.4	—	—	—	—	—
	Log IDURN	0.002	0.018	0.1	—	—	—	—	—

RAIN = *P*, IDURN = *D*.

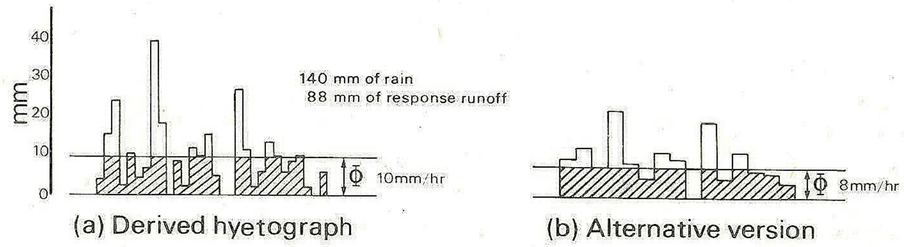


Fig 6.24 'Error' in loss rate determination.

Influence of the hyetograph on loss rates

Consider the sequence shown in Figure 6.24(a). The pattern in Figure 6.24(b) could have been obtained by

a a slightly different interpretation of the chart record;

b a different combination of recording raingauges; or

c extraction at 2 hour intervals rather than 1 hour.

Whatever the reason, the effect is to lower the derived loss rate. This is seen in practice as well as in theory; values of loss rate are markedly dependent on the pattern of rainfall intensity. Figure 6.25 illustrates an actual event where the loss rate curve has been forced up to an unreal level because of one single burst of very intense rainfall. Consequently, the only net rainfall resulting from the second shower is the nominal 1% (Section 6.4.5) and this is not enough to account for the response runoff which followed. It was examples like this which suggested strongly that the contributing area concept was more realistic than the threshold concept in determining the pattern of net rainfall. This in turn led to the regression tests described above.

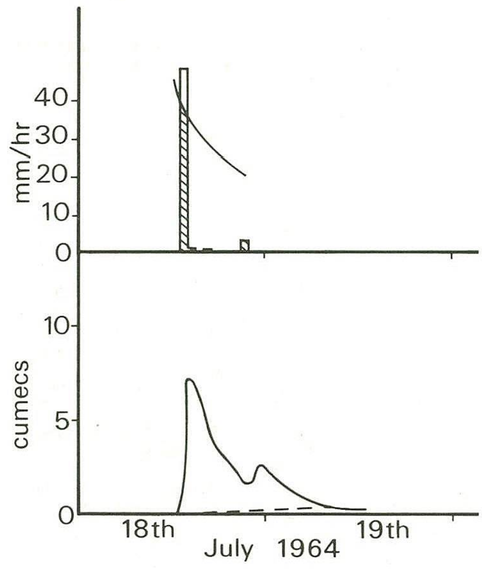


Fig 6.25 Example of poor loss rate separation (Burbage Brook; 28/801).

Trials of alternative techniques in analysis

The aim of this part of the investigation was to see if a net rainfall separation based solely on this percentage runoff concept would produce greater or less consistency between derived unit hydrographs. The effect on the average values of the three parameters was also examined.

Five catchments were chosen for testing; the results are shown in Table 6.9. The technique of percentage based separation was taken to its logical conclusion and the percentage runoff allowed to vary with CWI as it increases through the event. A simple linear dependence was used;

$$\frac{q_t}{p_t} = k\,\text{CWI}_t \qquad (6.28)$$

(This refinement was shown, on three of the five catchments, to be little different from a constant percentage assumption. But it is a better technique when the CWI is initially low.)

		1%+loss rate —most analyses			Percentage varying with CWI			Percentage constant		
Catchment	Event	Q_p	T_p	W	Q_p	T_p	W	Q_p	T_p	W
19001	7	21.9	7.8	10.0	24.5	8.2	8.5	—	—	—
	8	25.5	5.7	7.4	32.5	6.4	6.1	—	—	—
	9	30.5	3.2	6.6	35.0	3.5	5.7	—	—	—
	10	25.3	5.6	7.1	32.0	6.0	7.1	—	—	—
	11	24.0	8.9	7.8	24.0	9.2	7.8	—	—	—
	Mean	25.4	6.2	7.8	29.6	6.6	7.0	—	—	—
	SD	3.2	2.1	1.0	4.5	2.2	1.3	—	—	—
35008	1	15.6	10.6	14.0	17.0	9.0	11.7	17.2	9.2	11.7
	3	15.0	13.0	16.0	16.2	11.8	13.2	16.6	11.6	12.5
	4	18.0	8.4	14.5	18.8	8.3	13.7	18.6	8.6	13.6
	7	11.4	17.0	20.8	13.2	15.8	19.0	12.0	16.2	19.7
	8	17.6	8.4	14.3	17.7	8.6	14.4	17.6	8.7	14.2
	9	15.4	8.2	15.0	15.3	9.0	14.2	15.4	9.2	14.4
	12	14.3	8.2	15.9	14.0	9.0	16.8	13.9	9.1	17.0
	13	16.4	9.0	15.0	17.0	9.7	14.7	16.8	10.5	14.5
	14	16.0	11.1	14.5	15.4	10.6	15.0	15.7	11.5	14.0
	Mean	15.5	10.4	15.6	16.1	10.2	14.7	16.0	10.5	14.6
	SD	1.9	2.8	1.9	1.7	2.2	2.0	1.9	2.3	2.4
39012	1	57.6	3.1	4.5	55.0	3.9	4.4	—	—	—
	3	32.8	5.4	7.2	40.0	5.0	5.4	—	—	—
	4	48.4	4.2	5.3	52.0	4.0	5.0	—	—	—
	5	57.2	3.5	4.5	56.0	3.9	4.0	—	—	—
	6	34.0	4.3	6.8	56.0	4.0	4.9	—	—	—
	8	51.6	3.9	4.5	66.0	3.0	3.7	—	—	—
	9	36.9	4.0	6.0	36.0	4.9	5.2	—	—	—
	11	37.0	4.5	6.3	48.0	5.0	5.2	—	—	—
	12	47.0	3.0	4.9	54.0	4.0	4.0	—	—	—
	Mean	44.7	4.0	5.6	51.4	4.2	4.6	—	—	—
	SD	9.3	0.6	0.7	8.8	0.5	0.9	—	—	—
41007	2	8.1	14.5	30.5	8.3	15.0	31.0	8.0	15.3	30.6
	4	11.3	25.6	24.0	11.3	24.8	24.0	11.3	25.3	24.8
	5	8.2	32.7	30.2	8.5	33.5	29.7	8.4	33.2	31.0
	7	10.0	34.0	28.5	9.8	33.6	23.9	10.0	35.0	26.5
	8	7.0	27.8	39.0	7.2	25.7	38.5	8.0	30.5	27.0
	9	8.8	31.5	32.0	8.8	29.6	33.8	8.8	29.8	32.8
	10	7.8	32.0	35.6	8.1	28.8	36.3	8.1	28.7	36.5
	11	13.3	20.8	15.4	14.6	19.8	14.4	14.2	20.0	14.8
	Mean	9.3	27.4	29.4	9.6	26.3	28.8	9.6	27.2	28.0
	SD	2.0	6.4	6.8	2.2	6.1	7.4	2.1	6.2	6.1
68006	2	56.6	3.2	3.9	68.0	3.0	3.6	68.0	3.0	3.4
	3	49.6	3.6	4.1	61.2	3.1	4.2	60.0	3.3	4.2
	4	47.4	4.3	6.4	49.0	4.0	6.4	48.4	4.2	6.2
	5	45.8	3.9	4.5	60.6	4.5	4.6	60.0	4.5	4.6
	6	45.0	5.0	4.7	49.2	5.0	4.5	50.0	5.0	4.6
	7	42.2	4.3	4.5	64.0	4.0	4.6	63.6	4.1	4.1
	Mean	47.8	4.1	4.7	58.7	3.9	4.7	58.3	4.0	4.5
	SD	4.5	0.6	0.8	7.2	0.7	0.9	7.0	0.7	0.8

Table 6.9 Comparison of unit hydrograph parameters following varying loss separation assumptions.

It will be seen that mean values of T_p and W are hardly affected by the choice of technique. Q_p values are affected on those catchments with times to peak less than 10 hours; on 68/6 (T_p = 4 hours), Q_p increases from 48 to 58 cumecs/100 km^2 when the percentage based method is substituted for the loss rate curve method (see Section 6.5.9).

The standard deviations about the mean values of the three parameters were not significantly affected so there is no argument for or against a particular method on the grounds of consistency between unit hydrographs.

Although the conclusion to this section is that, in typical flood producing situations in the United Kingdom, the contributing area concept is the most useful for the purpose of relating runoff to rainfall, the occurrence of *widespread* surface runoff during extreme ('catastrophic') floods on small catchments is indisputable. In these circumstances, where a large part of the catchment is behaving like a lake and therefore producing 100% runoff, the normal concept of contributing area is inadequate. Although there were no data from an event of this type which were suitable for hydrograph analysis, all available information was collected and four of these events are discussed in Section 6.6.3.

6.5.6 Variation of percentage runoff between events on a single catchment

Figure 6.26 shows three typical plots of percentage runoff against catchment wetness index. There is a definite trend for wetter catchments to give more runoff but the scatter is large. Errors of estimation of SMD, rainfall, and runoff all contribute to this scatter. Without reopening the issue of whether or not percentage runoff is the most suitable dependent variable it is valid to explore the possibility of other event variables explaining some of the residual variance—assuming that straight lines have been fitted to the data of the form

$$\frac{Q}{P} = a.\text{CWI}+b. \qquad (6.29)$$

There are three possible variables; rainfall, duration, and rainfall/duration. In the previous section (6.5.5) the logarithmic transform regressions showed that absolute loss was dependent on rainfall to an exponent of nearly but not exactly unity,

$$P-Q=\text{c}.\,P^{0.9}. \qquad (6.30)$$

Thus,

$$\frac{Q}{P}=1-\text{c}.\,P^{-0.1}. \qquad (6.31)$$

The annotations beside the data in Figure 6.26 are the rainfall totals. It is clear, from a visual inspection of such plots for all catchments, that there are too few events from any one catchment to draw conclusions about the effects of rainfall. In particular, there is a shortage of events occurring on a dry catchment. Formal regressions for individual catchments confirmed this initial impression and the other two event variables, duration and average intensity, were also unhelpful as percentage runoff predictors. It was decided that dependence of percentage runoff on event variables other than CWI could only be studied by regression studies on all events together.

6.5.7 Catchment characteristics and the form of the percentage runoff prediction model

The catchment characteristics available for regression purposes are fully described in Chapter 4. However, the two which were found to be most important in this study of percentage runoff are the soils index (SOIL) and the percentage of urban area (URB). (Note that URB = 100 × URBAN where URBAN is the name used for the *fraction* of urban area).

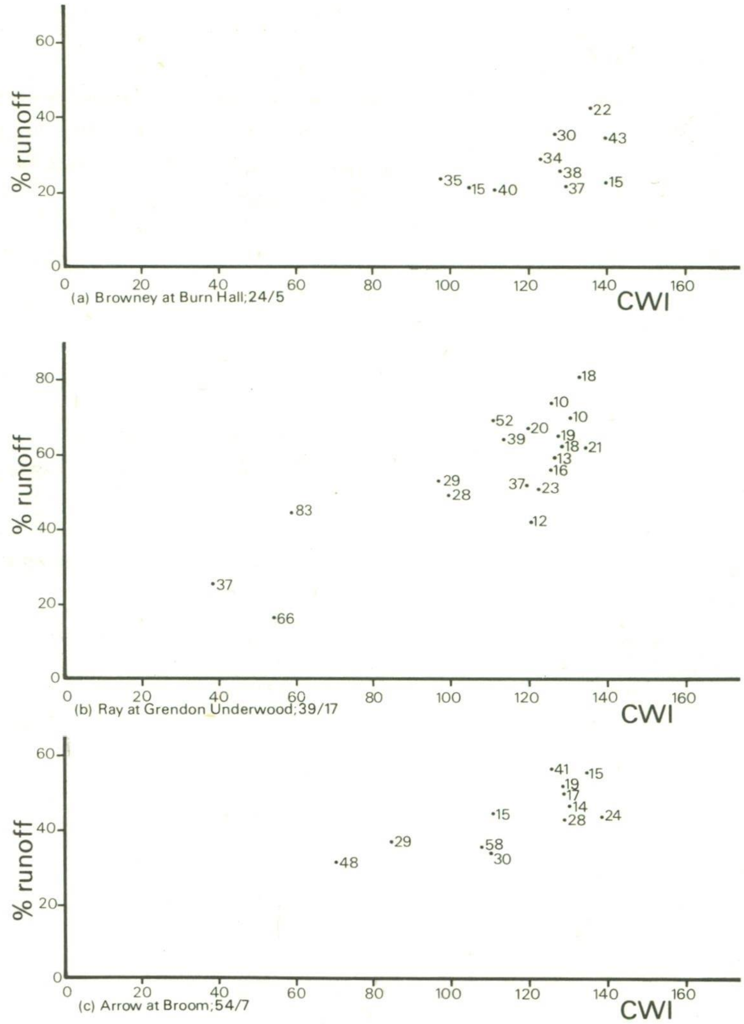


Fig 6.26 Variation of percentage runoff with catchment wetness index (CWI).

The soils index was derived from the map produced by the Soil Surveys (Figure 4.18) wherein all soils are allocated to one of five classes according to their physical properties and topographic factors. The classes are ranked according to the capacity of the soil to accept rain in winter conditions. Thus, class 1 includes light dry soils derived from chalk and class 5 includes thin soils over impermeable rock. The unclassified areas on the map are covered by water or urban development. The classification is described in detail in Section 4.2.3.

When the fraction (S_n) of each soil class within a catchment had been determined, regressions on the five fractions showed that average percentage runoff (adjusted to a CWI of 125) was predicted best by:

$$\frac{Q}{P} = 0.21\ S_1 + 0.40\ S_2 + 0.51\ S_3 + 0.54\ S_4 + 0.58\ S_5 - 7.5. \qquad (6.32)$$

Thus, if a catchment consists entirely of soil class 1, the expected percentage runoff is 13.5%.

From these results a soil index was defined as

$$\text{SOIL} = (0.15\ S_1 + 0.30\ S_2 + 0.40\ S_3 + 0.45\ S_4 + 0.50\ S_5)/(1 - S_u) \qquad (6.33)$$

where S_u is the unclassified fraction of the catchment.

The percentage urban area was measured from existing maps as described in Chapter 4.

The individual catchment plots of percentage runoff *v.* CWI were compared in the hope that the slope and/or intercept of the 'best straight line' would be seen to vary systematically with these or other catchment characteristics. It was hoped to choose, *a priori*, the most suitable form of model for percentage runoff prediction. The main alternatives were

a the additive model, illustrated in principle in Figure 6.27(a)

$$\frac{Q}{P} = a.\text{CWI} + b.\text{SOIL} + c.P + d.\text{URB} \ldots + \text{constant} \quad (6.34)$$

b the partial multiplicative model (Figure 6.27(b)), e.g.

$$\frac{Q}{P} = a.\text{CWI}.\text{SOIL} + b.P + c.\text{URB} \ldots + \text{constant} \quad (6.35)$$

c the exponent model (Figure 6.27(c))

$$\frac{Q}{P} = a.\text{CWI}^{b}.\text{SOIL}^{c}.P^{d}.\ \text{URB}^{e} \ldots \quad (6.36)$$

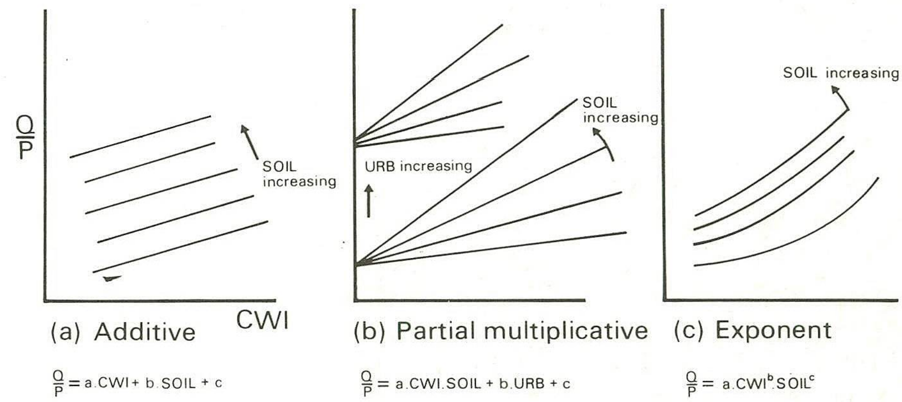


Fig 6.27 Alternative concepts of percentage runoff prediction.

There are many variations on *b* but the interaction of SOIL and CWI was the only one which visual inspection suggested was worth investigating. In fact, examination of the plots (Figure 6.26) was inconclusive. The slopes and intercepts did not appear to vary systematically. From this point, all comparisons of models were made by regression analysis on all events.

6.5.8 Percentage runoff regressions on all events from all catchments

Table 6.10 gives the regression results for the three alternative models. The best equations are

a $$\frac{Q}{P} \times 100 = 0.22\text{CWI} + 95.5\text{SOIL} + 0.10P + 0.12\text{URB} - 28.4 \quad (6.37)$$

b $$\frac{Q}{P} \times 100 = 0.72\text{CWI}.\text{SOIL} + 0.10P + 0.05\text{URB} + 1.4 \quad (6.38)$$

c $$\frac{Q}{P} \times 100 = 10.48\text{CWI}^{0.47}\text{SOIL}^{1.39}P^{0.07}\text{URB}^{0.64}. \quad (6.39)$$

Other catchment characteristics give minor improvements in the proportion of explained variance but these improvements are not significant at the 1% level.

The choice between the models cannot be made on the basis of these regressions and these data. It must be made by considering such factors as ease of application, relation to the percentage runoff (contributing area) concept, and behaviour in the extreme design case.

Equation 6.37, involving neither cross product nor exponent, is easier for rapid calculation. It is also a logical way of expressing the contributing area hypothesis. This is that a certain percentage of the catchment is ready to transmit 100% response runoff when CWI = 125. With CWI greater than 125, more of the catchment is ready and *vice versa*. As the rain continues, contributing area increases and so the average over the event as a whole is dependent on the rain total. The effect of urban areas is a constant additive at all times.

It is felt that equally rational hypotheses relating to other equation forms require more assumptions about the physical behaviour of the catchment than the data warrant.

Finally, as is illustrated in Figure 6.28 and Table 6.11, the predictions of runoff are larger and therefore more conservative with version *a* than with *c*. Version *b* is not plotted; as Table 6.10 shows, it is similar to *a*.

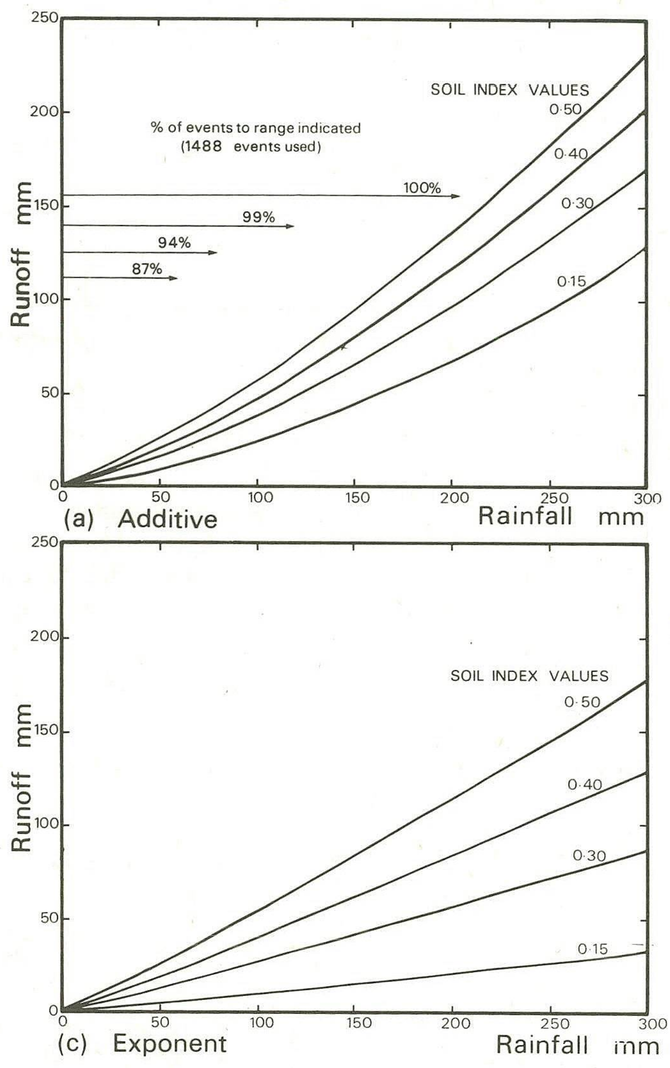


Fig 6.28 Rainfall–runoff curves for alternative forms of percentage runoff prediction equation (CWI = 125; URB = 0.0).

Table 6.10 Regression results for 3 alternative forms of percentage runoff prediction.

No. var.	Variable name	Coeff.	Stnd error of coeff.	*t* statis.	*R*	R^2	Stnd error of est.	Const.	Antilog (const.)
a Dependent variable: PERC									
No. of observations 1447									
1	SOIL	114.913	4.099	28.0	0.594	0.353	16.09	−4.2	—
2	SOIL	94.243	4.254	22.2	0.643	0.413	15.32	−18.7	—
	CWI	0.184	0.015	12.2	—	—	—	—	—
3	SOIL	87.673	4.408	19.9	0.651	0.424	15.19	−20.3	—
	CWI	0.191	0.015	12.7	—	—	—	—	—
	RAIN	0.089	0.017	5.1	—	—	—	—	—
4	SOIL	95.485	4.700	20.3	0.657	0.432	15.09	−28.4	—
	CWI	0.222	0.016	13.5	—	—	—	—	—
	RAIN	0.097	0.017	5.6	—	—	—	—	—
	URB	0.117	0.026	4.6	—	—	—	—	—
b Dependent variable: PERC									
No. of observations 1447									
1	CWI.SOIL	0.712	0.022	32.0	0.643	0.413	15.31	5.9	—
2	CWI.SOIL	0.685	0.023	30.3	0.653	0.426	15.15	3.6	—
	RAIN	0.093	0.017	5.5	—	—	—	—	—
3	CWI.SOIL	0.718	0.027	26.7	0.654	0.428	15.13	1.4	—
	RAIN	0.097	0.017	5.7	—	—	—	—	—
	URB	0.054	0.024	2.2	—	—	—	—	—
c Dependent variable: log (PERC)									
No. of observations 1447									
1	Log(SOIL)	1.476	0.045	32.9	0.655	0.429	0.25	2.161	145
2	Log(SOIL)	1.290	0.047	27.4	0.683	0.466	0.24	1.287	19.4
	Log(CWI)	0.385	0.038	10.1	—	—	—	—	—
3	Log(SOIL)	1.413	0.055	25.8	0.688	0.473	0.24	1.169	14.75
	Log(CWI)	0.460	0.042	11.0	—	—	—	—	—
	Log(URB)	0.592	0.137	4.3	—	—	—	—	—
4	Log(SOIL)	1.386	0.056	24.9	0.689	0.475	0.24	1.020	10.48
	Log(CWI)	0.472	0.042	11.3	—	—	—	—	—
	Log(URB)	0.644	0.138	4.7	—	—	—	—	—
	Log(RAIN)	0.071	0.028	2.5	—	—	—	—	—

PERC = percentage runoff = $Q/P \times 100$; RAIN = P.

Table 6.11 Predicted values of percentage runoff and runoff from alternative forms of equation (CWI = 125, URB = 0.0).

	Rain	Soil class 1 %	Runoff	2 %	Runoff	3 %	Runoff	4 %	Runoff	5 %	Runoff
a	50	18.6	9.3	32.9	16.5	42.5	21.3	47.3	23.7	52.0	26.0
	100	23.4	23.4	37.7	37.7	47.3	47.3	52.1	52.1	56.8	56.8
	150	29.2	43.7	42.5	63.6	52.1	78.0	56.9	85.2	62.6	93.9
	200	33.1	66.2	47.4	94.8	57.0	114.0	61.8	123.6	66.5	133.0
	250	38.0	95.0	52.3	131.0	61.9	155.0	66.7	167.0	71.4	178.5
	300	42.8	128.4	57.1	171.3	66.7	200.1	71.5	214.5	77.0	231.0
b	50	21.1	10.6	33.9	17.0	42.5	21.3	46.8	23.4	51.1	25.6
	100	25.7	25.7	38.5	38.5	47.1	47.1	51.4	51.4	55.7	55.7
	150	30.4	45.5	43.2	64.6	51.8	72.6	56.1	84.1	60.4	90.5
	200	35.0	70.0	47.8	95.4	56.4	112.8	60.7	121.4	65.0	130.0
	250	39.6	99.0	52.4	131.0	61.0	152.6	65.3	163.1	69.6	174.0
	300	44.3	132.9	57.1	171.3	65.7	197.1	70.0	210.0	74.3	222.9
c	50	9.8	4.7	25.7	12.9	38.3	19.2	45.0	22.5	52.1	26.1
	100	10.3	10.3	26.8	26.8	40.0	40.0	47.1	47.1	54.5	54.5
	150	10.6	15.9	27.6	41.4	41.2	61.8	48.5	72.8	56.0	84.0
	200	10.8	21.6	28.2	56.4	42.1	84.2	49.5	99.0	57.2	114.4
	250	10.9	27.7	28.5	71.3	42.6	106.7	50.1	125.3	58.0	145.0
	300	11.1	33.3	29.0	87.0	43.2	129.6	50.9	152.7	58.8	176.4

For the several reasons presented above, the recommended prediction for percentage runoff is

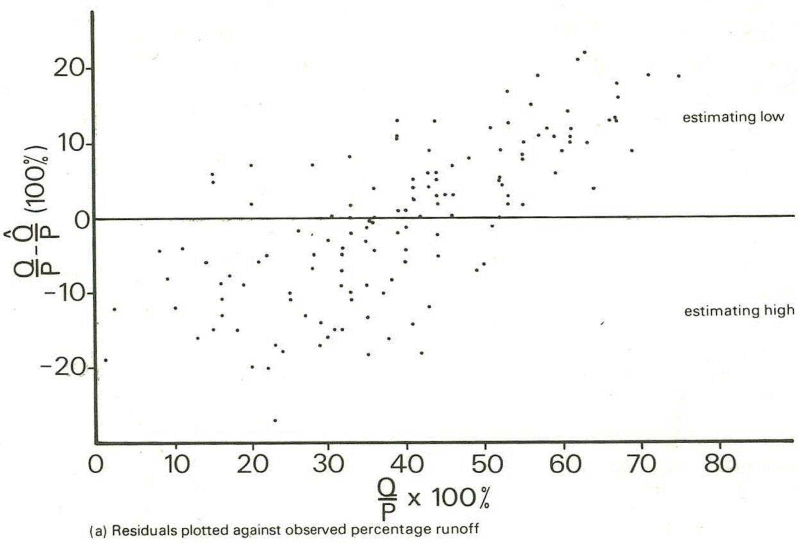
$$\frac{Q}{P} \times 100\,\% = 0.22\,(\text{CWI} - 125) + 0.10\,(P - 10) + 95.5\,\text{SOIL} + 0.12\,\text{URB}. \quad (6.40)$$

With CWI = 125 and 10 mm rain, the 'standard' percentage runoff for a catchment is given by the last two terms.

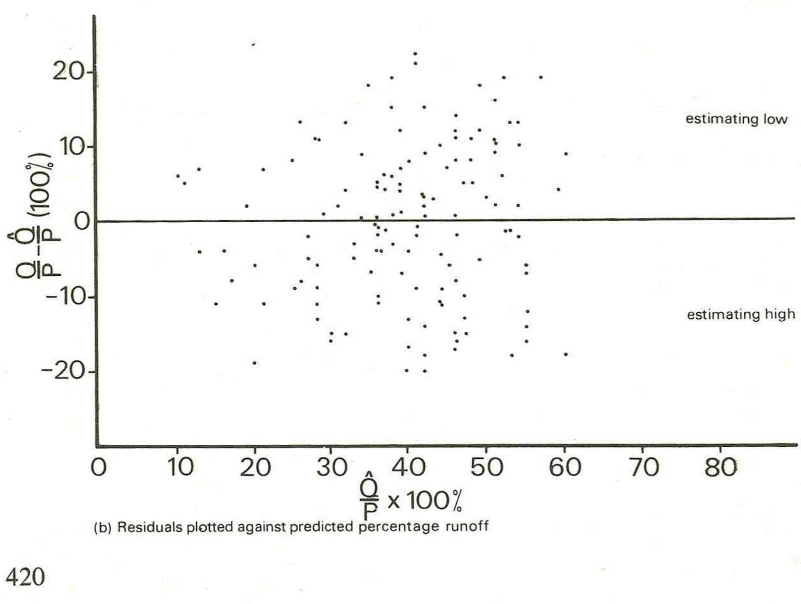
The regressions described here were not the only attempts. Space does not permit a full discussion of all the different combinations that were tried or the attempts that were made to perform regressions with an adjusted average figure of percentage runoff for each catchment.

The residuals from the final regression are shown plotted against observed percentage runoff in Figure 6.29(a) and against predicted percentage runoff in Figure 6.29(b). Each point is the average of all events on a catchment.

The main observation from Figure 6.29(a) is that percentage runoff tends to be underestimated on catchments where it should be high and



(a) Residuals plotted against observed percentage runoff



(b) Residuals plotted against predicted percentage runoff

Fig 6.29 Unexplained variation of percentage runoff between catchments.

overestimated when it should be small. In other words, the range of predicted percentage runoff is too small and this is the inevitable result of the model's inability to explain more than 45 % of the variance. Of the remaining 55 %, it is suggested that about 20 % is due to errors of measurement and the difficulties of using an arbitrary definition of response runoff; that leaves about 35 % which is unexplained.

The fact that little would be gained by trying other forms of the prediction equation is illustrated by the lack of trend in Figure 6.29(b).

Apart from trying regressions on all the catchment characteristics available in the study, an attempt was made to look at the possible effect of land usage and vegetation. This was prompted by the large influence that these variables have in an American prediction procedure (discussed in Section 6.5.9). The 10 mile to 1 inch maps of land classification and vegetation which are currently available (Ordnance Survey, 1945) are both old and approximate but it was thought that, if there was a significant influence, it would be obvious from a side by side inspection of the maps with the map of residuals (Figure 6.30). In fact, no such influence could be seen.

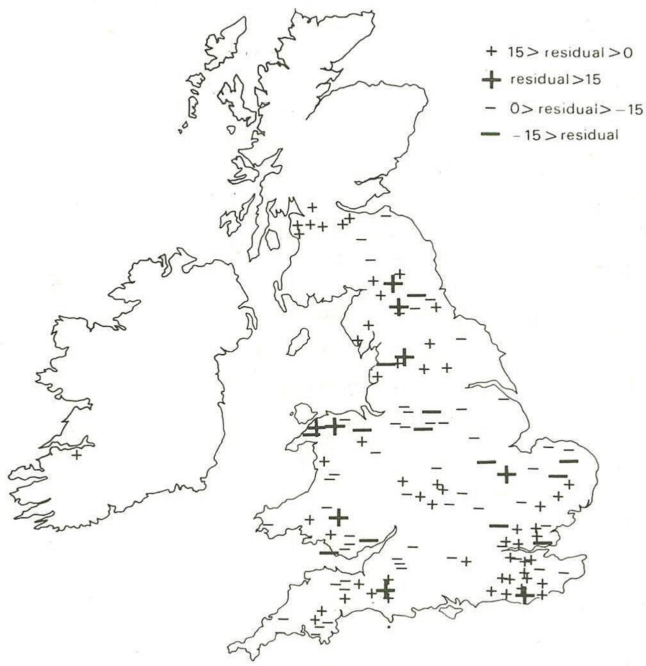


Fig 6.30 Map of percentage runoff residuals.

Discussions were held with those responsible for producing the map of soil classes and it was concluded that, although it was encouraging to find that the classes were ranked in the right order as regards average standard percentage runoff over all catchments, the range of possible values of standard percentage runoff within any one class was rather wide particularly within classes 4 and 5 which include all the high runoff catchments. It was stressed that the map was based on subjective judgement rather than measurements. The properties by which soils had been judged in these classes may not be the right ones and further research should be undertaken. In the meantime, the recommended prediction equation (6.40) must stand as the best available in the light of current knowledge.

6.5.9 Loss prediction in a design case—interaction with Q_p

Section 6.5.5 was concerned with establishing the case for predicting losses, and hence response runoff, as a percentage of total rainfall. In Section 6.5.7 it was concluded that, for a particular catchment, the percentage runoff from an event as a whole would vary with antecedent condition (CWI) and with total rainfall.

It is believed that this is a rational hypothesis and therefore there is a need to study the effect of employing a contrary hypothesis—that of loss rates—in the analyses. This was done in Section 6.5.5 and it was found that the only important difference in the unit hydrograph was that a larger Q_p value resulted when a percentage based separation was used.

If percentage runoff is to be presented as the best prediction device for the event as a whole, then it is logical to recommend a percentage based separation for the temporal sequence of net rainfall within the event. Therefore, it is necessary to modify Equation 6.10 so as to correct for the 'low' values of Q_p on which it was based. At the same time, a correction for the effect of unit hydrograph smoothing may be made (Section 6.4.6). In both cases, Q_p values require to be increased more when T_p is small.

A pilot study showed that increases should be applied as follows:

T_p	:	5	10	15	20	>20
% increase in Q_p	:	25	15–20	10	0–5	insignificant

The effect on Equation 6.10 is to remove the dependence on T_p and the much simpler

$$Q_pT_p = 220 \tag{6.41}$$

is the result.

6.5.10 Comparison of prediction equation with rainfall–runoff curves

It is interesting to compare the form of the prediction equation with the only widely available source of information on rainfall relations for ungauged catchments, namely the set of curves produced by the U.S. Soil Conservation Service. These curves may be found in a number of sources; they are particularly well presented by Miller & Clark (1965). Figure 6.31 shows the curves for 'condition II' which may be equated roughly to

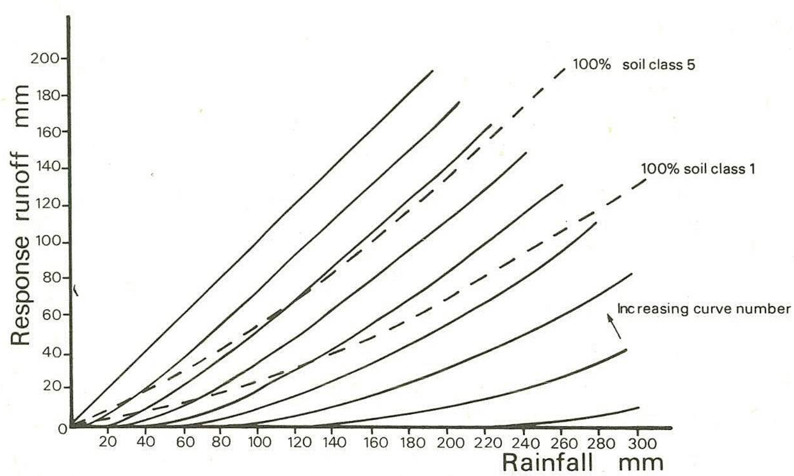


Fig 6.31 Rainfall-runoff curves compared with U.S. Soil Conservation Service results.

CWI = 125. The curve number increases as the effects of soil type, vegetal cover, and land use combine to produce greater amounts of response runoff. The soil class 1 and 5 lines from Figure 6.28a are superimposed.

There are two important points of difference between the two sets of curves. The first is the much greater influence of storm rainfall in the American curves. This can be seen in the greater curvature; if rainfall did not appear at all on the right hand side of the prediction equation the curves would be straight lines. Now the data for the American curves were, presumably, not confined to events which produced high flows. This restriction on the selection of data for the present study means that the predictions, while particularly relevant to high flows, do not comprise a general purpose model of rainfall and runoff.

The second major difference in the sets of curves is the difference in range. The small range of predicted percentage runoff in the British data was discussed in Section 6.5.8 and attributed, in part, to a still inadequate understanding of the effect of soil properties on response runoff. However it is interesting to note that the curve number in the American set is as dependent on land use or cover as it is on the soil type. This is illustrated in Table 6.12 which suggests, also, that the American curves were developed from, and primarily for, small catchments under agriculture. It is clear that the smaller the area the more likely it is to be homogeneous and, consequently, the greater the chance of its producing particularly high or low percentage runoff values. As the catchment area increases, different land types are involved and the percentage runoff tends towards some middle value. Table 6.13 shows that the larger catchments are generally well predicted while the worst predicted include many of the smallest.

This comparison with another set of curves has brought three useful conclusions:

1 The prediction equations produced by the current study are for *flood* prediction purposes only.

2 The percentage runoff predictions are likely to be better for large catchments than for smaller ones.

3 The predictions for small catchments should be supplemented by a detailed examination of nearby gauged catchments which have a similar land use and soil type.

Table 6.12 Effect of land use on curve number in U.S. Soil Conservation Service rainfall–runoff curves (condition II), (abridged from Miller & Clark, 1965).

Land use	Condition for infiltration	Soil type			
		A	B	C	D
Fallow		77	86	91	94
Row crops	Poor	72	81	88	91
(ploughed straight)	Good	67	78	85	89
Small grain	Poor	65	76	84	88
(ploughed straight)	Good	63	75	83	87
Permanent meadow		30	58	71	78
Woods	Poor	45	66	77	83
	Good	25	55	70	77
Farmsteads		59	74	82	86
Juniper forest in Western USA	Poor	—	70	79	—
	Good	—	40	58	—

CATCHMENT PARAMETER VALUES - OBSERVED AND PREDICTED

CATCH NO.	AREA (SQ.KM)	SOIL INDEX	PERC. URBAN	PREDICTED %ROFF (MEAN)	PREDICTED TP	OBSERVED %ROFF (MAX)	OBSERVED %ROFF (MIN)	OBSERVED %ROFF (MEAN)	OBSERVED TP (MAX)	OBSERVED TP (MIN)	OBSERVED TP (MEAN)	OBSERVED CWI (MEAN)
19001	369.00	0.46	1.7	47.8	9.3	58.6	44.4	52.4	8.9	3.2	6.2	127.1
19002	43.80	0.45	5.7	46.4	7.1	89.6	46.5	60.6	11.1	5.0	7.4	125.0
19005	229.00	0.46	10.0	47.3	6.9	72.0	44.7	56.8	13.2	4.0	6.1	122.5
20001	307.00	0.42	1.1	39.3	8.3	40.4	18.4	31.9	12.7	5.1	8.1	102.4
21005	373.00	0.42	0.1	44.9	8.7	57.6	25.7	39.5	8.9	7.5	8.1	136.1
23002	118.00	0.50	0.1	47.5	6.1	50.0	12.6	32.1	8.4	4.0	5.7	117.4
23005	285.00	0.50	0.0	51.2	8.5	82.9	44.0	59.8	12.0	4.0	6.5	130.6
24003	172.00	0.50	0.4	52.3	5.0	76.5	36.9	51.4	8.4	2.0	4.4	133.2
24005	178.00	0.45	5.8	45.4	8.1	43.1	13.3	28.3	9.8	4.2	7.1	121.8
24007	44.60	0.45	0.4	43.9	5.2	57.6	22.3	34.6	5.2	3.6	4.7	120.6
25003	11.40	0.50	0.0	51.5	2.8	84.1	49.9	67.0	4.8	2.5	3.2	129.7
25004	210.00	0.40	9.2	43.5	13.0	42.8	26.0	33.0	15.2	7.7	11.4	128.2
25810	0.94	0.50	0.0	46.1	1.4	30.0	29.2	29.6	2.3	1.5	1.9	99.5
27001	484.00	0.33	3.4	34.4	10.4	75.2	19.1	42.7	12.0	7.3	8.8	122.5
27010	18.90	0.49	1.7	41.8	3.8	70.0	13.4	40.2	7.3	3.4	4.6	84.1
27026	165.00	0.34	10.4	36.0	7.9	53.6	21.3	35.2	17.4	4.2	8.0	125.8
27027	443.00	0.42	0.3	44.5	8.8	91.3	29.1	54.7	7.4	2.6	4.8	135.2
27034	510.00	0.47	4.2	51.0	8.1	75.1	45.1	60.5	12.0	7.2	10.4	127.9
28016	231.00	0.37	6.9	34.8	12.2	35.0	23.6	28.2	27.5	16.6	21.1	127.6
28023	154.00	0.21	2.7	24.1	5.9	33.7	5.9	16.0	12.3	5.7	8.8	131.6
28026	351.00	0.45	6.9	45.7	15.2	48.8	37.8	43.8	31.0	19.4	26.8	121.9
28033	8.10	0.50	0.0	50.8	3.0	31.8	10.6	23.4	2.9	1.8	2.3	131.7
28801	13.0	0.47	0.0	46.0	3.0	53.9	9.5	29.0	6.6	1.2	3.0	114.6
28802	31.00	0.15	33.1	8.2	3.8	19.3	6.2	8.8	1.6	0.5	0.9	73.4
29001	108.00	0.16	1.0	14.6	11.2	3.4	1.0	2.4	12.0	5.0	7.9	108.4
29003	55.10	0.15	1.6	16.5	7.6	21.2	4.7	9.9	7.8	3.5	5.2	115.5
30001	298.00	0.39	5.9	40.5	14.4	37.5	17.4	27.5	21.2	18.8	19.7	127.1
31006	150.00	0.43	2.5	41.6	13.1	23.9	16.0	21.8	26.5	14.0	18.4	114.0
32801	7.00	0.45	0.0	44.2	5.1	66.5	15.8	40.2	6.5	2.5	4.5	122.4
33014	272.0	0.28	2.8	27.5	14.7	21.9	7.9	15.7	39.0	14.0	25.4	122.6
33015	277.00	0.38	3.8	35.5	19.5	44.2	18.3	31.7	33.2	13.0	19.0	110.0
33024	198.00	0.28	2.9	29.0	14.6	22.4	18.0	19.4	25.0	16.6	21.5	127.1
33029	93.50	0.19	0.4	16.5	13.5	13.8	3.1	9.0	10.3	5.8	7.9	110.2
33045	28.30	0.40	0.7	39.4	12.5	27.6	4.6	19.7	24.5	11.3	17.4	118.5
33809	65.30	0.35	1.0	34.7	17.1	76.5	31.6	52.9	19.0	13.6	17.0	121.1
34003	165.00	0.15	0.9	15.1	13.7	13.3	6.3	11.9	32.4	6.7	13.5	120.1
34005	73.30	0.38	3.4	39.5	14.7	23.4	18.5	23.0	45.2	20.8	29.0	127.0
34007	133.80	0.40	0.9	37.2	16.4	55.0	23.3	41.0	21.0	11.4	16.8	111.8
34011	127.10	0.18	1.9	20.1	16.1	27.5	8.5	14.2	42.6	9.0	17.9	127.6
35008	129.00	0.40	2.4	37.2	11.8	74.8	23.5	40.9	17.0	8.2	10.4	113.4
36008	224.00	0.39	2.2	36.3	17.0	59.6	19.7	41.5	31.0	12.5	22.1	112.4
37001	303.00	0.41	3.6	39.1	20.5	57.6	9.0	44.4	35.0	24.0	30.6	113.9
37003	77.70	0.36	1.0	34.2	15.0	52.7	7.7	33.8	23.0	11.2	17.6	111.3
37007	136.00	0.41	3.1	39.5	14.8	70.2	9.9	43.5	17.0	9.5	13.4	114.2
37008	190.1	0.39	2.6	38.0	18.6	62.9	24.9	43.8	27.0	22.7	25.2	118.7
37802	71.80	0.45	17.5	42.4	10.7	32.0	12.6	24.0	8.8	4.5	6.3	110.7
38003	134.0	0.29	1.5	19.5	12.7	2.7	0.7	1.3	5.0	1.5	3.9	79.2
38007	21.37	0.30	15.6	25.6	5.0	75.0	7.9	33.6	5.5	1.6	3.1	97.6
39004	122.0	0.15	[illegible]	11.9	3.0	2.0	0.5	1.2	2.6	1.2	1.7	71.7
39005	43.5	0.15	34.0	13.1	3.7	34.5	12.5	20.0	4.7	1.7	3.1	69.8
39012	69.10	0.30	61.0	27.9	4.6	27.6	8.5	15.8	5.4	3.0	4.0	77.0
39017	18.57	0.45	[illegible]	42.1	8.9	81.0	15.7	56.9	17.2	6.5	10.3	113.5
39022	164.50	0.35	5.2	35.4	13.0	49.1	34.0	36.4	25.6	20.0	22.0	127.3
39025	147.60	0.27	2.0	26.8	10.9	49.6	6.5	25.6	16.5	7.5	11.9	124.7
39026	109.4	0.43	1.5	42.7	14.8	59.6	21.6	28.6	28.8	14.2	20.2	122.0
39813	13.25	0.41	0.0	42.0	6.0	52.1	31.6	44.4	12.6	4.7	7.2	123.4
39814	4.50	0.45	92.7	42.5	2.0	86.3	21.0	42.3	2.4	1.0	1.6	66.8
39820	25.10	0.45	43.0	46.1	4.5	69.2	26.9	45.9	10.0	4.0	5.8	105.6
39830	10.00	0.15	53.0	10.4	2.7	34.8	8.6	15.3	3.8	1.8	2.9	70.7
39831	7.90	0.15	40.0	8.4	2.7	37.1	8.7	15.6	1.5	0.5	0.9	71.9
40006	49.70	0.28	2.3	28.3	7.6	36.7	7.0	21.8	8.4	5.3	6.7	120.4
40007	257.20	0.29	4.5	32.4	11.7	62.5	28.8	44.8	18.2	8.0	12.6	132.1
40008	230.00	0.35	4.2	36.4	13.5	55.0	25.5	36.3	23.2	12.0	16.4	130.6
40009	136.00	0.36	2.4	36.5	10.6	62.1	14.7	42.8	11.1	6.1	8.4	121.9
40010	210.00	0.38	3.3	38.6	11.8	75.5	21.4	50.7	25.5	9.6	16.6	126.9
41005	182.00	0.38	4.0	38.2	9.7	61.4	26.2	45.9	24.3	12.6	17.1	126.8
41006	88.70	0.38	1.9	37.8	9.0	77.5	14.8	56.6	15.5	10.6	12.9	122.6
41007	403.30	0.47	2.7	41.9	17.9	90.2	66.5	75.9	34.0	14.5	27.4	115.7
41801	3.52	0.46	40.0	35.0	2.2	82.5	13.7	35.7	5.6	2.0	3.5	79.7
41811	23.91	0.44	13.2	46.8	6.6	61.5	41.0	52.1	10.0	6.3	7.9	132.6
45002	422.00	0.33	0.4	35.5	7.0	60.6	8.6	33.8	10.6	3.6	7.2	127.6
45003	226.00	0.27	1.4	26.2	7.7	59.0	10.3	39.3	18.0	4.8	11.4	113.7
45004	298.00	0.28	1.6	27.9	9.2	60.9	13.9	39.1	12.5	4.8	8.5	117.9
45804	128.00	0.37	3.3	40.4	6.7	61.8	8.2	39.1	6.5	3.2	4.8	127.7
45805	160.00	0.32	0.4	35.9	6.2	37.2	15.4	25.2	9.0	4.4	6.4	135.5
46003	248.00	0.36	0.5	39.8	6.2	42.4	17.1	31.5	7.8	2.8	4.6	133.7
46005	21.50	0.50	0.1	53.9	3.2	91.5	38.8	66.5	5.0	2.5	3.6	134.9
46802	14.20	0.50	0.2	54.3	2.6	80.1	50.5	66.8	3.1	1.2	2.3	137.9
46805	5.67	0.50	0.0	55.3	1.6	53.9	44.9	49.6	3.1	1.6	2.0	165.3
47007	54.90	0.35	0.6	39.1	4.2	59.7	17.0	31.6	6.2	4.2	5.3	137.4
49003	21.70	0.50	0.0	52.7	4.6	74.5	15.8	51.9	6.5	3.3	4.5	129.5
52004	90.10	0.41	3.5	38.0	7.2	50.9	15.6	39.1	9.0	4.2	6.9	105.6
52005	202.00	0.31	2.1	30.7	8.7	50.5	14.3	33.5	10.8	8.0	9.7	116.9
52006	132.40	0.36	4.3	37.7	6.6	79.5	29.3	53.1	12.7	7.1	9.8	124.1
52010	135.0	0.47	3.3	42.9	11.1	67.6	34.4	46.0	11.0	6.0	9.4	125.8
52805	15.8	0.42	1.1	42.2	5.3	86.8	24.9	51.6	4.8	2.1	3.4	123.5
53005	147.0	0.25	4.0	25.0	9.8	29.8	3.5	17.1	15.5	4.9	9.5	119.6
53007	262.00	0.33	2.5	33.3	11.9	70.5	9.2	30.3	14.0	7.5	10.0	123.3
53008	303.00	0.32	1.1	33.3	14.7	58.3	11.7	28.5	16.0	9.5	12.7	125.4
53009	73.00	0.26	5.5	27.2	8.4	23.9	7.5	16.9	12.4	5.0	8.0	125.0
54004	264.00	0.41	21.8	42.2	10.6	63.4	13.7	45.4	18.0	4.5	11.5	117.5
54006	324.00	0.17	10.0	17.7	11.7	29.2	15.7	20.7	31.7	21.0	25.9	115.2
54007	319.00	0.43	2.9	41.7	12.6	56.3	31.1	44.4	16.0	12.0	14.0	118.1
54010	319.00	0.38	1.8	38.5	13.5	46.0	24.5	40.0	21.2	9.6	16.7	122.7
54011	184.00	0.45	7.2	43.9	9.5	65.1	12.1	32.9	17.5	8.8	12.7	119.2
54016	259.00	0.31	1.2	28.7	20.5	47.4	11.1	30.2	34.0	21.2	27.0	112.2
54019	347.00	0.36	4.0	35.7	18.2	68.3	17.0	41.2	44.0	32.0	38.2	118.8
54020	181.00	0.27	0.6	27.2	13.4	28.9	16.2	21.9	22.2	13.8	17.1	118.6
54022	8.05	0.50	0.0	54.0	1.8	76.5	10.9	43.8	4.5	1.2	2.0	140.3
55006	10.40	0.50	0.0	55.6	2.5	69.4	28.0	48.9	3.5	1.0	2.0	142.0
56003	62.20	0.34	0.3	36.1	5.8	74.9	18.9	35.5	5.4	3.1	4.0	129.2
56005	98.40	0.42	8.9	46.2	4.1	41.3	23.6	31.1	6.5	2.4	4.1	131.2
56006	184.00	0.39	0.2	47.7	5.8	73.1	56.2	63.5	6.0	2.5	4.5	138.9
57004	109.00	0.50	7.4	54.9	4.8	58.7	24.9	41.1	9.4	4.6	6.7	135.3
57005	454.80	0.49	11.1	55.8	4.5	60.9	28.0	43.4	10.5	4.0	6.5	139.5
58001	158.00	0.47	2.1	54.9	5.1	75.2	18.6	38.6	6.5	2.7	4.3	147.2
58002	190.90	0.50	0.2	59.7	4.4	86.8	48.5	68.5	5.0	2.9	4.0	146.0
58003	62.90	0.34	5.6	38.3	5.3	50.8	24.0	35.3	7.0	3.8	5.8	133.5
60002	297.80	0.38	0.3	45.0	7.9	69.9	37.1	51.7	10.0	3.8	6.7	141.2
60007	231.80	0.49	0.1	53.8	5.1	61.1	43.9	52.2	5.6	3.0	3.9	144.7
61001	197.60	0.33	0.3	35.8	10.0	39.7	11.7	25.9	9.8	5.2	7.1	135.1
64001	471.00	0.40	0.2	47.5	7.2	69.6	35.0	55.2	5.9	5.2	5.6	145.9
65001	68.60	0.50	0.1	60.6	2.5	56.7	30.1	42.2	11.1	3.5	5.5	139.9
65801	11.40	0.50	0.0	56.7	1.8	98.8	44.9	75.4	5.2	2.4	3.6	137.9
66002	220.00	0.33	0.2	32.0	8.2	19.3	16.4	17.7	7.8	4.8	6.3	112.5
66011	344.00	0.32	0.4	40.3	4.3	81.0	26.8	62.2	5.5	3.2	4.2	140.0
67005	114.00	0.41	0.5	45.8	6.1	74.9	27.3	38.9	6.0	3.5	5.2	137.1
67008	227.00	0.30	2.4	29.4	9.3	19.0	7.1	13.7	8.0	4.2	6.2	117.2
67010	13.10	0.45	0.0	50.9	3.2	81.8	49.6	61.4	6.7	2.5	5.3	136.9
68006	152.00	0.43	3.7	46.1	6.7	45.4	32.0	37.6	5.0	3.2	4.0	133.1
68802	5.40	0.45	0.9	46.8	8.6	44.3	21.6	37.0	5.8	3.2	4.6	131.6
69008	51.80	0.42	4.2	39.2	6.0	89.7	8.9	36.4	10.0	4.0	7.3	108.7
69011	67.30	0.44	30.0	47.7	3.9	40.6	29.3	34.8	5.2	4.6	4.9	127.8
71003	10.41	0.50	0.0	53.0	2.6	79.8	27.3	55.5	8.0	2.0	4.0	130.5
71802	206.70	0.48	0.3	52.9	8.1	87.8	60.0	71.1	9.0	4.0	7.6	136.2
71804	24.90	0.50	0.0	52.8	3.0	59.5	19.6	35.0	2.3	1.5	1.8	120.2
72002	275.00	0.46	0.6	49.2	9.8	85.5	44.2	61.1	7.0	4.2	5.6	136.1
73804	57.50	0.50	0.3	59.4	3.5	98.5	48.2	63.8	8.5	5.4	6.8	145.4
74001	85.20	0.45	0.1	49.6	4.0	80.1	29.0	52.6	5.5	2.3	3.4	133.9
76008	334.60	0.48	1.8	52.3	9.0	64.9	52.3	58.6	6.9	5.5	6.2	168.3
76011	1.50	0.50	0.0	51.1	3.0	84.6	31.4	67.5	3.5	1.4	2.3	126.6
77015	495.00	0.47	0.2	50.7	9.7	60.7	35.2	53.4	7.6	4.2	5.7	132.9
83002	98.80	0.47	0.2	53.1	4.3	79.2	53.5	66.3				142.4
84002	12.40	0.50	0.0	53.7	2.8	72.2	56.7	63.7	3.4	2.5	3.0	134.8
84008	51.30	0.45	30.0	49.5	3.2	87.0	33.5	58.8	6.6	2.8	4.0	128.3
84012	235.00	0.44	31.0	52.3	4.6	74.1	47.1	59.0	13.7	5.2	7.3	136.3
85002	220.00	0.44	0.2	46.6	6.2	66.4	50.5	54.9	5.2	3.9	4.6	135.1
90003	771.00	0.35	0.4	39.0	11.0	60.2	39.5	48.1	28.0	16.6	21.7	132.3

Table 6.13 Catchment parameter values—observed and predicted.

6.5.11 Base flow

The final step in the specification of a design hydrograph from storm rainfall is to add on a certain flow to represent the flow in the river before the event started and, to a lesser extent, the beginnings of slow response runoff from the event itself. This additional flow has been indicated in analysis by the average 'nonseparated' flow (ANSF) over the time base of the response runoff hydrograph. A regression of ANSF on the catchment wetness index and catchment characteristics gave the result:

$$\text{ANSF} = 0.000326\,(\text{CWI} - 125) + 0.00074\,\text{RSMD} + 0.003 \qquad (6.42)$$

where ANSF is the additional flow in cumecs/km^2 and RSMD the climatic index of flood runoff potential (described in Section 4.2.4).

It is recommended that this flow simply be added to all ordinates of the design hydrograph. In view of the almost trivial effect on the size of any design flood, nothing more complex could be justified.

6.5.12 Problem catchments

Several individual catchments are now discussed in detail although some general problems have already been mentioned in Section 6.3.1.

Maun at Mansfield; 28/802. A small catchment with a large urban contribution (33 %). The problem was that the typical response hydrograph is two peaked (Figure 6.32(a)) with a flashy response from the urban area and a later peak from other parts of the catchment. It was decided to analyse only the first peak but, despite a study of detailed drainage maps, it was not possible to define the effective catchment contributing to it. Both time to peak and percentage runoff are consistently overestimated by the prediction equations. Although URB, the percentage of urban area, is an important variable in both equations, it is unrealistic to expect its

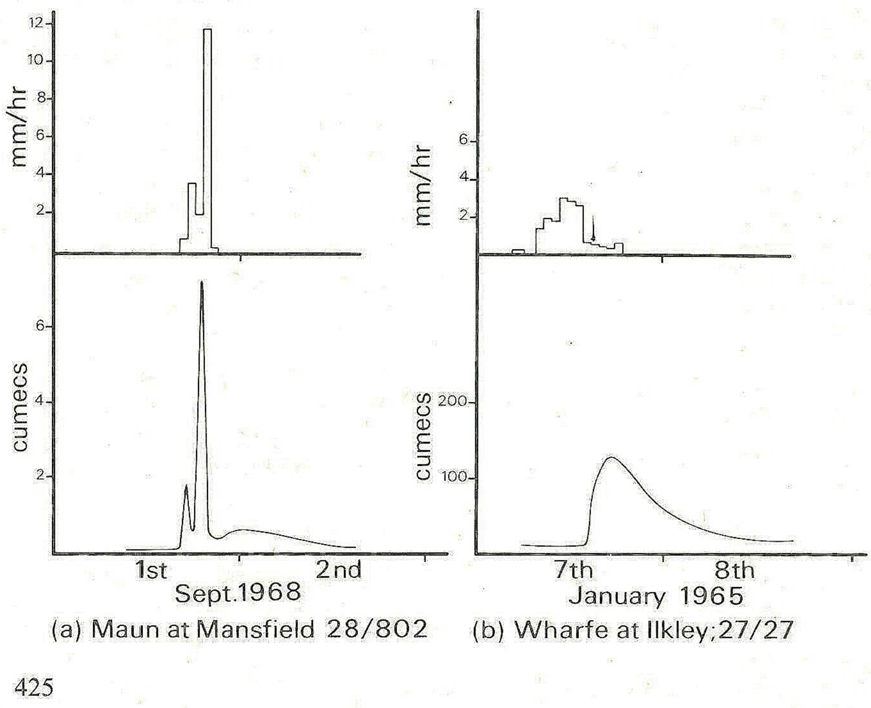


Fig 6.32 Typical response of two difficult catchments.

effect to be the same from one catchment to another. Such a coarse treatment cannot hope to model the complexities of different drainage systems, sewer sizes etc. and methods such as the TRRL technique (Watkins, 1962) must be preferred. The catchment was excluded from the T_p regressions.

Wandle at Beddington; 39/4. This illustrates the effect of a basically permeable catchment (chalk without any drift cover) which produces runoff solely because of urban development and a small area of clay close to the outlet. The area, as defined by contours, is 122 km^2 but it has little relevance to the river which effectively starts in springs only a few km upstream of the gauging station. As it was felt desirable to avoid, as far as possible, arbitrary rules designed to cope with particular catchments but which then become awkward to apply to borderline cases, the topographic area was retained in all cases. However, both time to peak and percentage runoff are overestimated and the effect of considering only the small area (15.2 km^2) below the chalk outcrop was examined. The time to peak prediction was improved because the urban area increases as a proportion of the whole; the prediction of percentage runoff worsened. The latter is predicted in terms of the soils index and the urban proportion; changes in these more than offset the increase in 'observed' percentage runoff due to the smaller nominal area.

The catchment response of 39/4 is largely urban runoff but this is not the main problem. Two other catchments, *Mimram at Panshanger, 38/3* and *Stringside Beck at Whitebridge, 33/29* show the same overestimation of T_p. The former includes a new town but the Stringside Beck catchment has no effective urban areas. What all three catchments do have in common are springs fed from the chalk and because, over all catchments, the effect of streamlength is explained largely by the streamslope variable, it may be that short spring fed streams have smaller slope than length alone would suggest.

Wharfe at Ilkley; 27/27. The problems of relative timing between rainfall and runoff were discussed in Section 6.4.7. This catchment is one where a timing correction was considered in two parts; a fixed correction of +3 hours and a further random correction as necessary. Figure 6.32b shows the typical response. It illustrates the main restriction of a linear approximation to the unit hydrograph; if, for any reason, the effective start of the rising limb comes significantly and consistently later than the start of net rainfall then a blind application of the straight line approximation to the rising limb would have a serious effect on the peak. That is why the systematic correction was applied. In explanation of the phenomenon it is possible to hypothesise about the shape of the drainage network —a long length of river without any tributaries immediately before the gauging station would tend to produce this effect. But this does not occur on this catchment. Alternatively, the presence of limestone in the upper half of the catchment could be the cause of a typically delayed response. Whatever the reason, it is thought to be better to introduce a deliberate timing error in this way than to allow such a gross distortion of the unit hydrograph. Lag times for this catchment will be, on average, 3 hours too short but predicted peaks should be of the right order.

Gwash at Belmesthorpe; 31/6. The symptoms here were exactly as outlined above except that the systematic timing correction was 6 hours. In this case, a possible explanation may be that clay in the top half of the catchment is responsible for the response runoff whereas water entering the limestone in the bottom half does not reach the stream again above the gauging station. Despite the 6 hour adjustment, it is interesting to note that the T_p value predicted from catchment characteristics is still 7 hours *less* than the 'observed'. If the 6 hours had not been subtracted, it would have been 13 hours less.

This catchment is also very badly predicted as regards percentage runoff. 22% is observed and 42% predicted (average figures) which makes it one of the worst predictions whether the error is considered in the absolute or percentage sense. It is believed that an explanation for some of this difference may lie in the soils index; the revised version of the soils map includes alterations in this region.

6.6 Testing of results

The procedure for hydrograph synthesis is summarised in Section 6.6.1 and is applied in 6.6.2 for the purpose of testing the prediction equations; observed and predicted hydrographs are compared for 64 events from six catchments. In 6.6.3 the testing concentrates on six very large floods and the results are discussed in detail.

6.6.1 Summary of procedure for hydrograph synthesis

The procedure summarised here is based on systematic analysis of a large number of high flow events resulting from rainfall. The final summary of the recommended design procedure is given in Section 6.8; prediction equations are not repeated here.

The steps required in design hydrograph specification were given in Section 6.1.4. In testing the results of this chapter by attempting to reproduce observed events the steps are the same, except that 1, 2 and 4 are unnecessary because the rainfall input is known from available data. The volume of runoff is estimated (step 3) by Equation 6.40.

Step 5 requires the loss $(P-Q)$ to be distributed through the storm. This may be based on the concept of a percentage runoff which varies with CWI within the event; a detailed description of the calculations required is given in Section 6.8. This method is recommended for reproducing recorded events and in applying the unit hydrograph/losses model to short term forecasting. In predicting hydrographs resulting from idealised design rainstorms, the simpler assumption of a percentage runoff which is constant during each event is considered adequate (see also Section 6.7.5).

Step 6 requires the synthesis of a unit hydrograph on the assumption that the catchment is ungauged. A simple triangular form is considered adequate for most purposes; the time to peak is given by prediction Equation 6.18 and the peak (in units of cumecs/100 km^2) by Equation 6.11 (6.41). Step 7, the convolution of the synthetic unit hydrograph with the net rainfall, is a simple desk calculator operation. It is shown in Figure 6.1 and numerical examples are given in Section 6.8. The prediction of nonresponse runoff (step 8) is given by Equation 6.42.

6.6.2 Testing on catchments not used in development of prediction equations

Six catchments (64 events) were omitted from the analysis. They are indicated in Figure 6.3. The prediction equations 6.18, 6.40 and 6.42 were applied to the 64 events with results summarised in Table 6.14. Synthetic unit hydrographs were applied to the 'observed' rainfall sequence, the predicted 'average nonseparated flow' was added, and the resulting hydrograph was compared with the original. The magnitude and timing of the observed and reproduced hydrograph peaks are compared in the first two columns of Table 6.14. The complete hydrographs of one event from each of the six catchments are compared in Figures 6.33–6.38. The examples are typical, being neither the best nor the worst reproductions of the several events on each catchment. In the case of 45/4 (Figure 6.36), however, the worst event (event 14) has been chosen, and Table 6.14 should be used to compare it with a more typical result.

These results provide a test of the prediction equations, though with the large data sets available the standard errors of estimate shown in the tabulated regression results are as useful for this purpose; they also show how the unit hydrograph and the 'losses' vary between catchments in relative importance in reproducing the peak flow. Figure 6.39 shows that error in hydrograph peak is much more closely related to error in percentage runoff than to error in unit hydrograph peak. This helps to place in perspective the approximations and assumptions made in unit hydrograph synthesis. Although the shape of the unit hydrograph can be very important in reproducing extreme events (Section 6.6.3) it is generally more important for peak flow prediction to predict percentage runoff correctly.

The test results may be summarised: of the 64 events, 32 peaks (50%) are predicted to within 25%, and 45 (70%) to within 50%; 22 predictions are low; of these, 41 are better than −25%, 18 better than $-33\frac{1}{3}$%; in only four events out of 64 is the predicted peak less than two thirds of the observed peak.

These results are capable of improvement because they refer to an *ungauged situation*. This is a severe test of any technique and one for which

Table 6.14 Comparison of observed and predicted features of events on six test catchments.

Event no.	Time to peak		Peak (cumecs)		Unit hydrograph T_p		Unit hydrograph Q_p		% runoff	
	Obs.	Pre.	Obs.	Pre.	Obs.	Pre.	Obs.	Pre.	Obs.	Pre.
Catchment No. 19/1										
7	19	20	149	138	7.8	9.6	21.9	22.9	57	49
8	14	17	106	93	5.7	—	25.5	—	44	47
9	12	18	114	65	3.2	—	30.5	—	53	46
10	22	24	130	136	5.6	—	25.3	—	50	51
11	31	30	170	136	8.9	—	24.0	—	59	46
Catchment No. 29/3										
2	12	16	2.5	5.3	4.5	7.6	29.0	28.9	7.3	16.9
3	12	15	1.8	4.5	5.0	—	41.2	—	4.7	16.8
4	25	27	5.3	10.6	7.2	—	30.5	—	4.9	10.3
5	19	19	7.3	9.1	7.8	—	27.0	—	16.5	19.9
6	13	17	3.9	5.4	3.5	—	32.0	—	8.7	15.3
7	15	18	3.7	7.9	3.5	—	36.0	—	6.1	17.1
Catchment No. 39/12										
1	5	6	16	16	3.1	4.6	57.6	47.8	13.2	18.6
3	20	19	15	18	5.4	—	32.8	—	27.6	39.0
4	6	7	11	25	4.2	—	48.4	—	13.2	37.6
5	5	7	10	19	3.5	—	57.2	—	8.5	25.6
6	8	8	13	16	4.3	—	34.0	—	13.6	22.4
7	10	11	12	23	—	—	—	—	23.9	37.8
8	10	11	11	17	3.9	—	51.6	—	9.7	23.9
10	22	19	23	55	—	—	—	—	20.3	32.0
11	15	15	10	10	4.5	—	37.0	—	9.6	12.6
12	9	11	12	15	3.0	—	47.0	—	12.4	21.9

Event no.	Time to peak		Peak (cumecs)		Unit hydrograph T_p		Unit hydrograph Q_p		% runoff	
	Obs.	Pre.	Obs.	Pre.	Obs.	Pre.	Obs.	Pre.	Obs.	Pre.
Catchment No. 45/4										
1	17	19	63	64	9.8	9.2	24.7	23.9	33.1	32.0
2	24	22	42	60	10.1	—	20.3	—	16.7	22.5
3	24	21	61	40	12.5	—	17.7	—	42.2	23.4
4	19	21	70	67	6.0	—	20.0	—	35.7	30.7
5	14	13	72	50	10.6	—	18.5	—	49.1	24.6
6	39	38	44	51	10.0	—	14.0	—	33.7	28.5
7	30	31	48	42	8.2	—	18.0	—	38.1	29.7
8	17	19	65	53	7.2	—	22.7	—	44.4	31.8
9	12	16	73	46	6.1	—	28.2	—	60.9	34.6
10	9	13	71	55	4.8	—	23.0	—	44.2	30.7
11	28	26	61	49	11.4	—	20.0	—	40.8	30.7
12	17	19	78	61	7.8	—	24.2	—	42.8	33.0
13	32	31	73	38	10.4	—	24.4	—	13.9	8.8
14	24	29	218	70	6.3	—	26.3	—	36.7	23.7
16	22	23	89	71	6.3	—	15.8	—	54.5	36.7
Catchment No. 54/16										
1	29	29	10.2	17.0	23.8	21.3	10.0	10.3	20.9	30.0
2	46	42	7.4	15.1	25.5	—	7.6	—	23.4	28.6
3	70	56	11.3	16.3	33.0	—	6.7	—	20.2	20.7
4	49	33	14.9	18.6	—	—	—	—	37.9	27.3
5	46	44	7.0	15.5	22.6	—	8.8	—	11.1	22.1
6	48	36	14.8	23.0	—	—	—	—	37.5	31.5
7	42	30	15.8	21.4	34.0	—	6.5	—	41.4	31.2
8	47	36	10.0	18.2	21.2	—	8.6	—	25.8	30.0
9	62	53	10.3	19.6	29.2	—	5.8	—	18.3	22.5
10	51	30	15.8	18.3	—	—	—	—	40.9	25.4
12	35	27	11.3	21.6	25.9	—	9.0	—	28.7	32.1
13	56	50	31.5	41.0	—	—	—	—	42.0	32.0
17	49	42	21.5	24.8	30.6	—	6.4	—	47.4	30.3
18	37	29	14.6	25.6	29.0	—	9.0	—	17.2	26.3
19	25	27	24.8	40.0	21.5	—	9.2	—	31.8	35.9
20	64	62	24.8	29.8	28.0	—	5.7	—	38.6	34.1
Catchment No. 67/10										
1	15	13	11.6	8.9	6.0	3.1	35.0	71.0	74.6	47.8
2	17	15	13.0	15.0	5.0	—	39.5	—	58.0	49.0
3	16	13	13.0	8.3	5.7	—	49.0	—	63.2	47.6
4	14	11	11.3	13.4	6.7	—	33.7	—	49.6	51.0
5	10	11	18.5	13.8	2.5	—	53.5	—	51.8	50.8
6	21	18	11.5	10.1	6.6	—	34.5	—	55.5	54.1
7	11	10	13.7	16.3	5.3	—	43.0	—	50.4	51.3
8	16	13	12.2	15.4	4.2	—	28.5	—	61.8	55.5
9	12	11	14.8	10.0	6.0	—	38.7	—	81.8	48.0
11	29	27	15.3	14.7	5.5	—	28.5	—	59.2	54.1
13	30	28	10.8	11.6	5.5	—	27.0	—	50.7	50.1
14	9	7	11.5	9.7	5.0	—	41.0	—	80.7	52.1

Table 6.14—continued

results are not normally presented. They support the argument for the establishment of a gauging station during the investigation of a project. If a catchment can be gauged for a year or so, the standard percentage runoff (SPR) may be assessed with much greater confidence than is possible by use of the 'soil' and 'urban' indices in Equation 6.40; it has been suggested above and in Section 6.5.8 that a better estimate of standard percentage runoff is the most significant single improvement that can be made. The improved estimates that could result from a short period of record are illustrated in three cases (catchments 29/3, 45/4, 54/6). The first five events on each catchment (excluding the illustrated test event) have been used to make new estimates of T_p and standard percentage runoff. The improved reproduction of the example events is seen in

Figures 6.34, 6.36, and 6.37. Further small improvements would be expected if average catchment values of Q_p and average nonseparated flow were to be used. Finally, average ordinates of actual unit hydrographs could be used if it were necessary to reproduce the recession of the hydrograph with more realism.

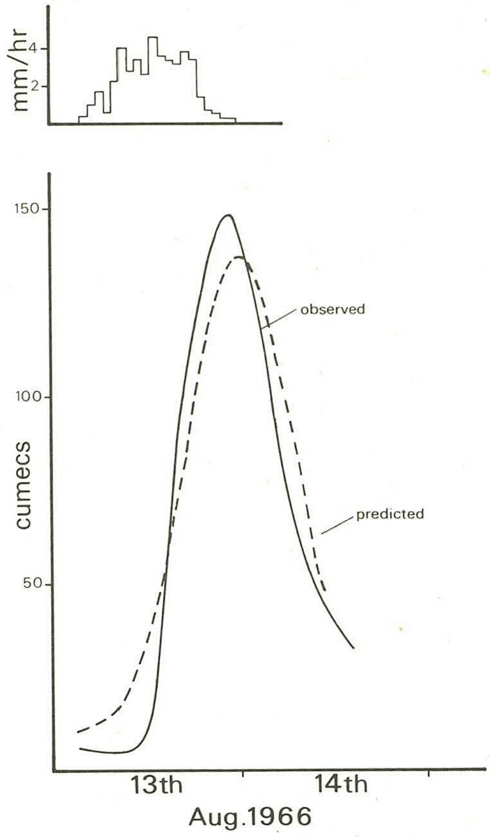


Fig 6.33 Observed and predicted hydrograph. Event 7 from the Almond at Craigie Hall; 19/1 (369 km²).

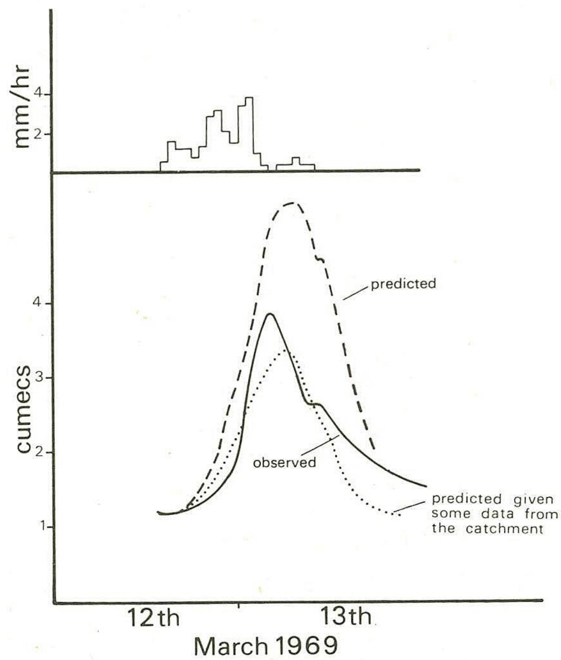


Fig 6.34 Observed and predicted hydrograph. Event 6 from the Lud at Louth; 29/3 (55.1 km²).

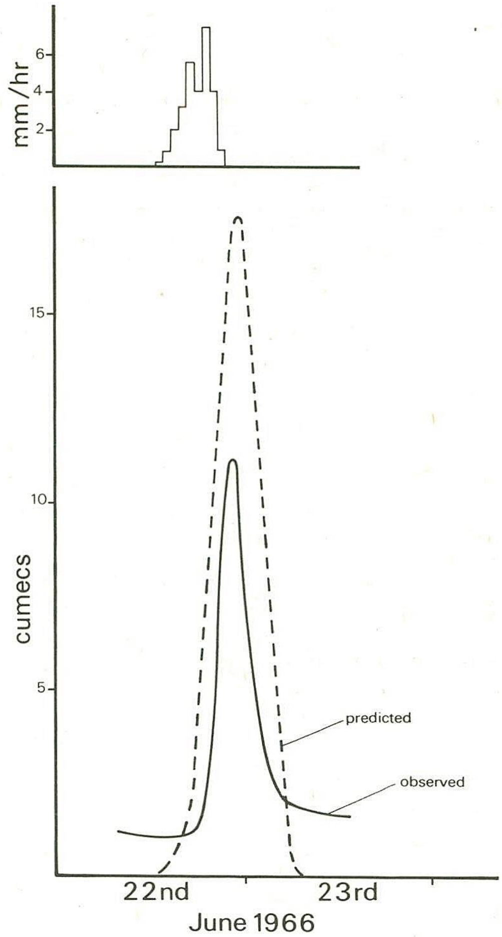


Fig 6.35 Observed and predicted hydrograph. Event 8 from the Hogsmill at Kingston; 39/12 (69.1 km²).

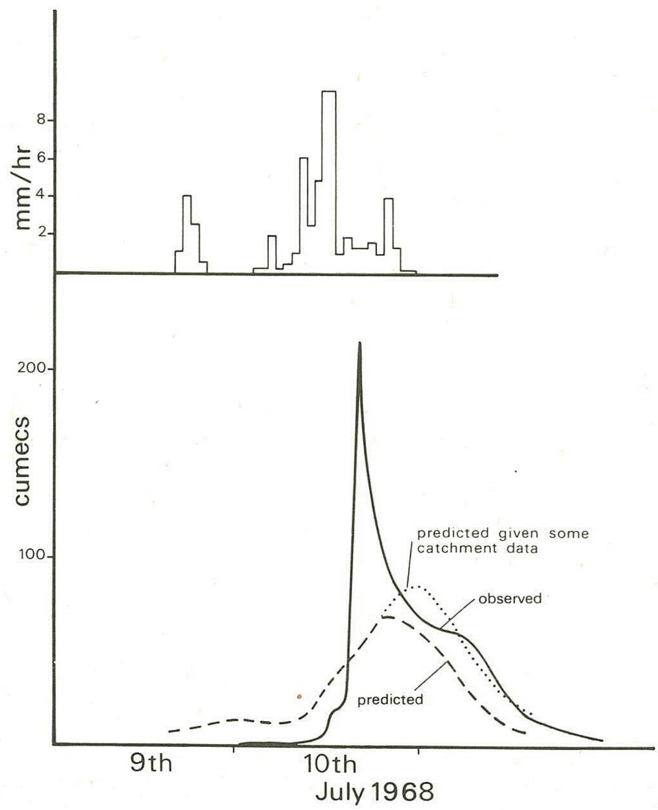


Fig 6.36 Observed and predicted hydrograph. Event 14 from the Axe at Whitford; 45/4 (298 km²).

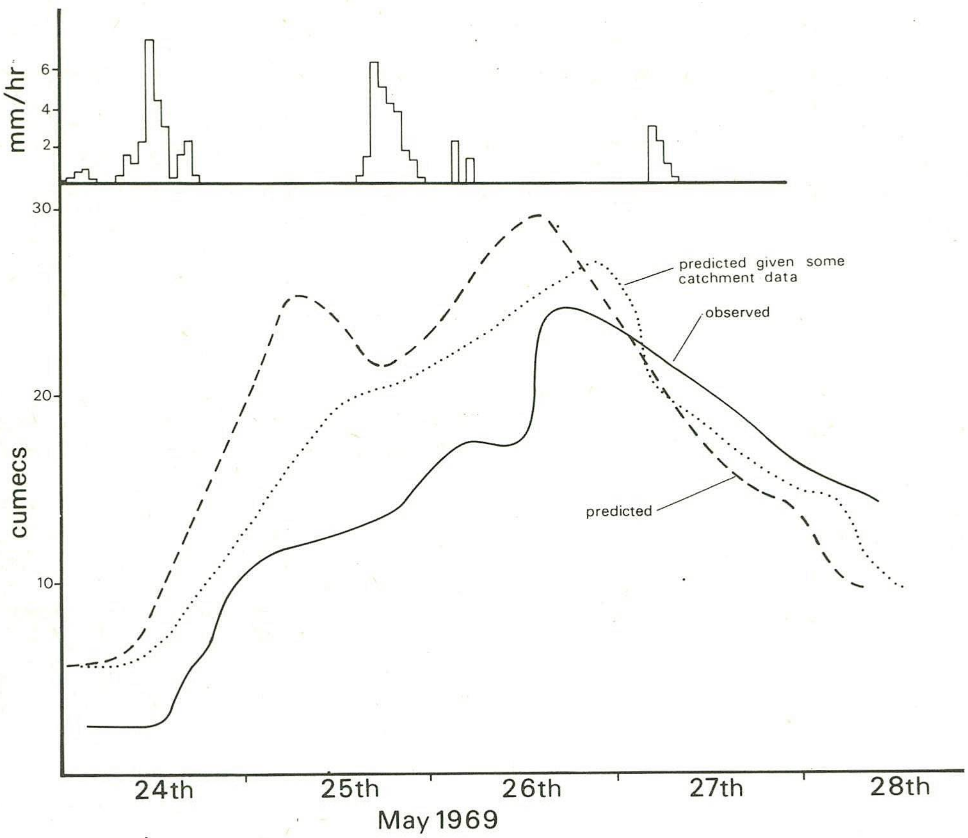


Fig 6.37 Observed and predicted hydrograph. Event 20 from the Roden at Rodington; 54/16 (259 km²).

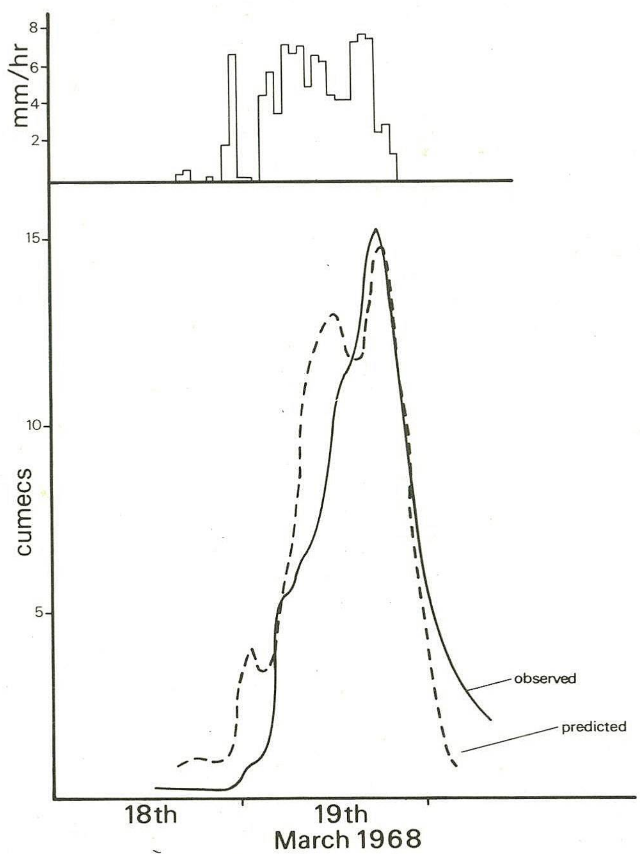


Fig 6.38 Observed and predicted hydrograph. Event 11 from the Gelyn at Cynefail; 67/10 (13.1 km²).

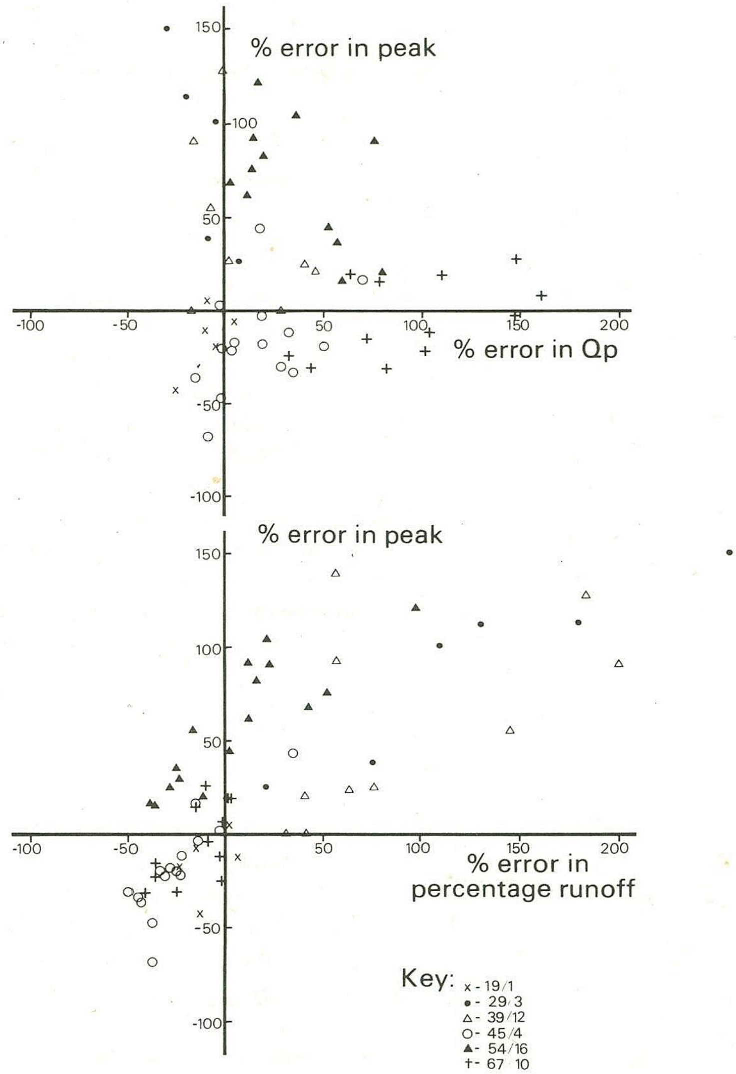


Fig 6.39 Main cause of error—test catchments.

6.6.3 Testing on notable events

Although the analyses included many large events—up to three or four times the mean annual flood—these did not approach the highest events listed in the 1933 ICE Report. Indirect estimates of instantaneous peak flow were available for these extreme events but the data were not suitable for unit hydrograph or loss analysis. In two cases there was enough information for a reasonable estimate of the basin average rainfall and its profile. The present methods were tested on these two events, on two other recent extreme events, and on two events from overseas. These six events are listed below. The ratio of estimated peak flow to the mean annual flood is included as an index of its rarity.

1 Lynmouth (W. Lyn), 15 August 1952. 34 people killed. Peak flow in River West Lyn estimated at 9.4 cumecs/km^2, about 15 times estimated mean annual flood. Area: 23.5 km^2. Rainfall investigated by Bleasdale & Douglas (1952); runoff by Dobbie & Wolf (1953).

2 Louth (29/3), 29 May 1920. 23 people killed. Peak flow in River Lud (chalk catchment) estimated at 2.7 cumecs/km^2, about 32 times mean annual flood. Area: 55.1 km^2. Rainfall investigated by Newnham (1921); rainfall and runoff by Crosthwaite (1921). Seven typical high flow events from this catchment (29/3) were also used for testing purposes (Section 6.6.2).

3 Dunsop Bridge (71/804) 8 August 1967. Peak flow in River Dunsop close to gauging station at Footholme estimated at 10.9 cumecs/km^2, 9 times estimated mean annual flood. Area: 24.9 km^2. Reported by Law (1968) and Duckworth (1969). Flows estimated by Hydraulics Research Station (1968).

4 Cefn Brwyn (Plynlimon) (55/8), 5 August 1973. Peak flow in River Wye estimated at 6.1 cumecs/km^2, about 4 times mean annual flood. Area: 10.4 km^2. 18 events from this experimental catchment (55/8) run by the Institute of Hydrology were included in the analysis. It was particularly useful to record such a large event (2.5 times the previous maximum) at the end of the Flood Study.

Following a request for flood event data which was sent to several countries in broadly similar climatic zones, the Floods Team received much interesting information. Data received from Tasmania and New Zealand were particularly useful.

5 Mersey-Forth area of Northern Tasmania, 23 August 1970. Peak flow in River Wilmot (above Forth River junction) estimated at 2.0 cumecs/km^2. Area: 293 km^2. Record rainfalls (up to 300 mm point falls) over large area; reported by Hydro-Electric Commission (1970).

6 Northland, New Zealand, 2 February 1967. Peak flow in Mangakahia River estimated at 3.1 cumecs/km^2, but described by New Zealand Ministry of Works as typical of 'flash floods' from tropical cyclones. Area: 240 km^2.

The hydrographs were predicted as described in the previous section. In each case the predicted peak flow (highest of the ordinates at hourly intervals; half-hour intervals for the Wye) was smaller than the field estimate. The worst prediction, for No. *2*, was only 27% of the field estimate; the best, for No. *5*, was 98%.

As illustrated by the earlier test results, there is a finite risk of such underpredictions resulting from the application of such a model to an ungauged catchment, even if the model were valid in such extreme events. The prediction equations for T_p and percentage runoff each have a standard error associated with them. Log T_p is predicted by Equation 6.18 with a standard error of ± 0.15, while percentage runoff is predicted by Equation 6.40 with a standard error of ± 15. The prediction errors may be summarised:

No. of standard errors	Probability that actual value lies within indicated range of the predicted value		
	Probability %	Range	
		T_p%	% runoff
1	68	71–141	± 15
2	95	50–200	± 30

To test whether the errors encountered are likely to arise by chance, the six events were reproduced using (a) the predicted percentage runoff plus one standard error, (b) the predicted T_p reduced by one standard error, and (c) both of these. Table 6.15 summarises these results. The prediction for Dunsop Bridge is still only 46% of the field estimate when both changes are made. The prediction for the New Zealand event increases from 51 to 81% of the field estimate, and for the Tasmanian event from 98 to 130%.

This underestimation is a disappointing test of the prediction method, but it should be remembered that these examples were chosen because the observed values of the dependent variable, the peak flow, were exceptionally high. In other words, far from being a random sample like the earlier set, they were those where the deviations from the predicted runoff, or in hydrological terms the ratio of peak discharge to rainfall intensity, would be expected to be high. The likelihood of such deviations can be taken into account in design in the same way as the risk of a given rainfall being exceeded in a design life. Nevertheless, the low predictions on six separate catchments made it necessary to investigate whether either the response model or the prediction equations were at fault in these extreme conditions. It was therefore investigated whether the low prediction was due to an underestimation of the percentage runoff or an overestimation of T_p, and whether data from other events from the catchment would have helped.

In the three cases where the data permitted full hydrograph analysis (both overseas events and the recent Cefn Brwyn event) the actual unit hydrograph derived from the event itself reproduced the peak flow little better than either the simple triangular approximation based on the event or the predicted triangle based on catchment characteristics. In the Tasmanian case it was worse.

Table 6.15 Observations and predictions of six notable flood events.

	1	1a	2	3	4	5	6
Catchment No.	(W.Lyn)		29/3	(71/804)	55/8	(Tasmania)	(New Zealand)
Area (km²)	23.5	23.5	55.1	24.9	10.4	292.7	240.0
Data interval (hours)	1.0	1.0	1.0	1.0	0.5	1.0	1.0
Rain (mm)	149	290	117	105	104	260	126
Duration (hours)	16	16	5	5	17	50	20
Peaks (cumecs) from:							
1. Straight application of prediction equations	72	183	40	79	32	562	384
2. %+15	91	222	67	99	40	678	504
3. $T_p \times 0.71$	84	209	55	100	36	625	463
4. (2) and (3)	106	254	92	126	44	753	608
5. Wetness variant model	111	349	50	93	41	849	481
6. (5) and (2)	140	424	84	117	51	1023	631
7. Indirect estimate	252	—	150	272	67	—	—
8. Rating curve extension	—	—	—	—	60	576	750†
9. Laboratory model	221	—	—	—	—	—	—
10. Reapplying UH from event itself	—	—	—	—	42	440	576
% runoff:							
Observed	—	—	28 app.	—	78	65	57
Predicted by equation	57	71	23	58	65	73	49
Predicted by catchment data	—	—	11	40	58	—	—
T_p (hours):							
Observed	—	—	1.5 app.	—	1.9	7.0	3.5
Predicted by equation	3.1	—	7.3	3.4	2.2	9.6	5.9
Predicted by catchment data	—	—	5.3	1.8	2.1	—	—
Main cause of error	Not known		T_p over estimated	T_p over estimated	?	Good prediction	T_p over estimated
Would catchment data have helped?	—	—	No	Yes‡	Probably not		Yes, possibly

†This estimate based on an assumed peak stage derived from wrack marks.

‡But catchment data suggest a standard percentage runoff *lower* than predicted.

Besides the probable underestimation of percentage runoff at Lynmouth and Cefn Brwyn, which is understandable in view of the residual plot of Figure 6.29 and discussion in Section 6.5.8, and a serious over-estimate of T_p at Louth, there is some indication that either the data or the response model were faulty in five out of the six events.

Data. Some of the difference is due to the field estimates of peak flow referring to instantaneous maxima whereas the reproduced peaks are maxima on the clock hour. Indirect flow estimates rely about equally on correct values of coefficients such as Manning's 'n' and proper interpretation of level records or evidence. Such estimates may be very much in error but the error is usually minimised by calculating flows at a number of sites and ensuring that they are consistent. A further serious source of error lies in the definition of the profile of basin average rainfall. Despite the most careful assessment of rainfall totals and distribution, accurate data from events of this magnitude are notoriously difficult to obtain. The assumptions of a stationary storm and a uniform spatial distribution may also be seriously at fault.

Response model. The relation between assumptions made in the model and real catchment response to extreme rainfall was discussed in Sections 6.5.3 and 6.5.5. The basic proposition is that subsurface flow is the usual source of response runoff and that overland flow occurs on rare occasions. However, eye witness reports of the Lyn catchment during the Lynmouth flood leave the reader in no doubt that overland flow was taking place over the major part of the catchment. In such an event it is not difficult to imagine 100% runoff applying to rain falling at the peak of the storm but the prediction equation allows no more than 80% maximum with an overall average of 69%. An explanation for such a low prediction is provided by Figure 6.29(a); extremes of percentage runoff are predicted badly because of the small proportion of variance explained. In other words, it is the regression model which is faulty and not necessarily the contributing area basis of the catchment model. The same argument could apply to T_p prediction but, while the contributing area concept allows changing behaviour through the storm and 100% runoff, the linear unit hydrograph is inflexible. Its validity has been confirmed by testing on minor events but, for want of any suitable data, there is no guarantee that it applies to Lynmouth type events which may involve a fundamental change in process.

It would be possible to modify the conventional unit hydrograph to allow for such discontinuities in the runoff generation process. Suppose that a fundamental change in response takes place within the event when CWI passes 200 and that thereafter T_p is reduced as the CWI increases; the unit hydrograph applied to successive increments of net rain becomes peakier. For instance, assume that when CWI is greater than 200, $T_p(\text{CWI}) = T_p \times (200-125)/(\text{CWI}-125)$. The results of this model are shown in Table 6.15 to indicate the effect on synthesised peak flows. This is a non-linear model but, if catchment wetness depended only on time, it would be a time variant linear model for which methods of analysis have been suggested by Mandeville & O'Donnell (1973).

The instantaneous flood peak on the West Lyn was estimated at 252 cumecs by slope area methods and 221 cumecs by laboratory model. The lower figure is approached if predicted percentage runoff is increased by one standard error, predicted T_p reduced by 1 standard error, and the wetness variant model is assumed. The normal model does as well if the rainfall rates in the most intense part of the storm are assumed to be 50% greater than estimated.

These events, and the test results, could be discussed further but it is suggested that failure to reproduce field estimates of peak flows could be due to any one or more of the following:

1 Inadequate definition of rainfall.
2 Gross departure from the assumption that rainfall is spatially uniform.
3 Inadequacy of response model.
4 Poor field estimates of flow.
5 T_p and/or standard percentage runoff badly predicted for the catchment.

The results of Table 6.15 suggest that the low predictions are caused by different combinations of these factors in each case but, until top quality data can be obtained for events of this rarity, it is not possible to be more definite. It cannot be said with certainty, for example, that the simple triangular unit hydrograph is inadequate for these conditions. It would also be incorrect to deduce that all upland catchments on soil class 5 produce more runoff than the prediction equation suggests. The most significant factor may be the smoothing, and therefore peak reducing, effect of assuming uniform rainfall over the catchment.

These results present a dilemma of whether or not to adapt the results of analysing a large number of events to take account of the underprediction of a small number of exceptional floods. Although it is outside the terms of reference of this report to suggest procedures which incorporate safety factors into design, these tests on extreme events point to the need for design to take account not only of the risks of exceedance of design rainfall but also of those associated with the variance of percentage runoff and T_p. This should apply whether the percentage runoff and T_p had been predicted from the regression equations or from other events studied on the design catchment.

In order to provide a comparison for Lynmouth, maximum rainfall (Volume II.4) was estimated as 290 mm in the same 16 hour duration that was considered for the 1952 event. The summer median profile (Figure II.6.1) was applied and the resulting peak flows (clock hour maxima) are shown in column 1a of Table 6.15. Higher flows would be expected from estimated maximum rainfalls in the shorter durations. An example of such a calculation is presented in detail in Section 6.8.3 where the 'probable maximum' flood is computed for the West Lyn catchment.

6.7 Choice of design specifications—a simulation exercise

Contents: The main purpose of the section—to examine the way in which the return period of peak flow is affected by design conditions—is introduced in Section 6.7.1. Section 6.7.2 describes the simulation procedure, and the data required to establish the probability distribution of each variable (duration, profile, volume, antecedent condition) are discussed in Section 6.7.3; these two sections may be omitted at a first reading.

In Section 6.7.4 the simulation procedure is used to deduce the probability distribution of peak discharge from the rainfall depth duration frequency relationship. The effect on the peak discharge of controlled variation of each variable in turn is examined in Section 6.7.5. Finally, in Section 6.7.6, experience gained from the sensitivity analysis is used to identify the choice of design conditions needed to preserve the desired return period through to the design flood.

6.7.1 The purpose of simulation

In the Introduction to this volume and Section 6.1.1 the conditions were outlined where the use of a catchment response model in conjunction with rainfall frequency information would be an appropriate flood design technique. The eight steps of the technique were described in Section 6.1.4. To summarise; the designer specifies the *return period* and *duration* of a rain storm, a temporal pattern or *profile*, and finally the antecedent condition or *CWI*. The choice of return period and storm duration determines the gross rainfall, from which the net or effective rainfall can be deduced using the chosen profile and CWI. The net rainfall is transformed to a design flood by means of a unit hydrograph, either observed or synthetic. Although conventionally the design flood is associated with the same return period as the storm which caused it, it is clear that many sources of variation, such as those due to the specifications in italics above, are in fact ignored. The design engineer, in recognition of this fact, uses stringent design specifications in order not to reduce the return period of the flood inadvertently.

The problem of preserving the desired return period by a judicious choice of design conditions is the primary topic of this section. It was investigated by generalising the design procedure to sample not from a single combination of conditions but from a large variety of different conditions. A computer program was written which made it possible to investigate the average outcome of an exhaustive sampling of different storm depths, durations, profiles and CWI values. This sampling procedure is referred to as 'simulation', although it operates not in the time domain as is usual with simulation but in the probability domain.

6.7.2 The simulation procedure

Generalisation of the technique for design hydrograph synthesis. The simulation procedure follows the steps used to estimate the design flood in common engineering practice, where an arbitrary single choice is made at each step (Section 6.1.4); in the simulation procedure a selection of possible values is used. Figure 6.40 illustrates the procedure as a tree diagram on which the 'single choice' method is represented by a single path.

In order to reproduce the flood variability of natural catchments, storm duration, profile, and antecedent condition were sampled in proportion to their frequency of occurrence. In common with the design procedure spatial uniformity was assumed in the simulation, which restricts its application to catchments suitable for unit hydrograph analysis (Section 6.2.3). To simplify analysis discrete distributions were used so that, for example, a continuous distribution of storm duration was approximated by a histogram of relative frequency at six or twelve particular durations chosen to represent the entire range. The derivation of all the distributions required by the simulation is given in Section 6.7.3.

Algebraic formulation. The procedure for simulating floods following a particular rainfall depth may be summarised algebraically by letting p_i be the probability of the *i*th duration, D_i; q_j be the probability of the *j*th storm profile, H_j; and r_k be the probability of the *k*th CWI, C_k. If Q_{ijk} is the flood magnitude resulting from the combination of D_i, H_j and C_k then, if independence may be assumed, the probability to be associated with Q_{ijk} is $W_{ijk} = p_i q_j r_k$. The average or expected flood magnitude is calculated from $\Sigma_i \Sigma_j \Sigma_k W_{ijk} Q_{ijk}$ while the average flood magnitude following all storms of, say, the fourth duration is calculated from $\Sigma_j \Sigma_k W_{4jk} Q_{4jk}$. Figure 6.41 illustrates this process in the simplified case where only duration and antecedent condition affect the discharge.

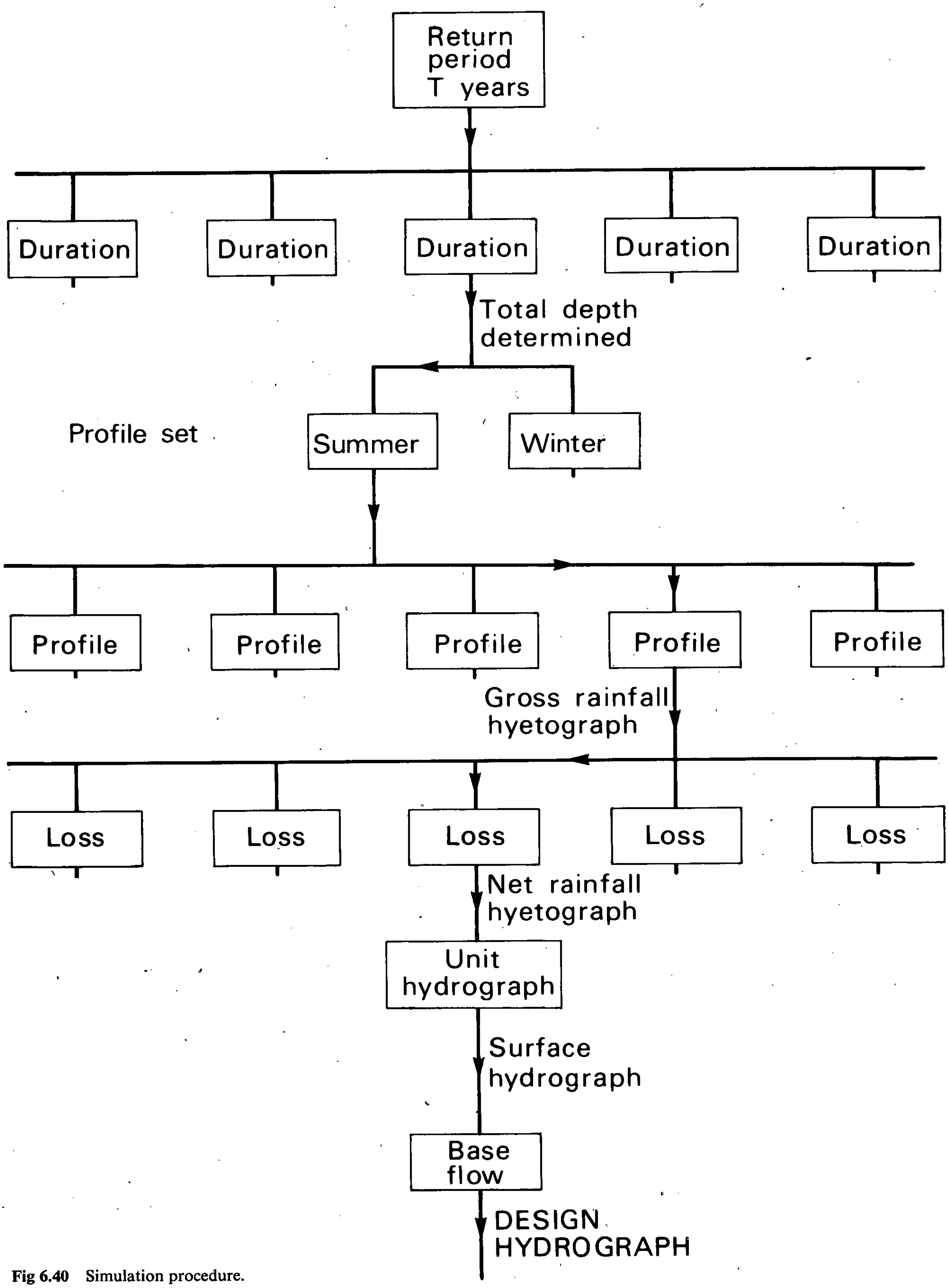
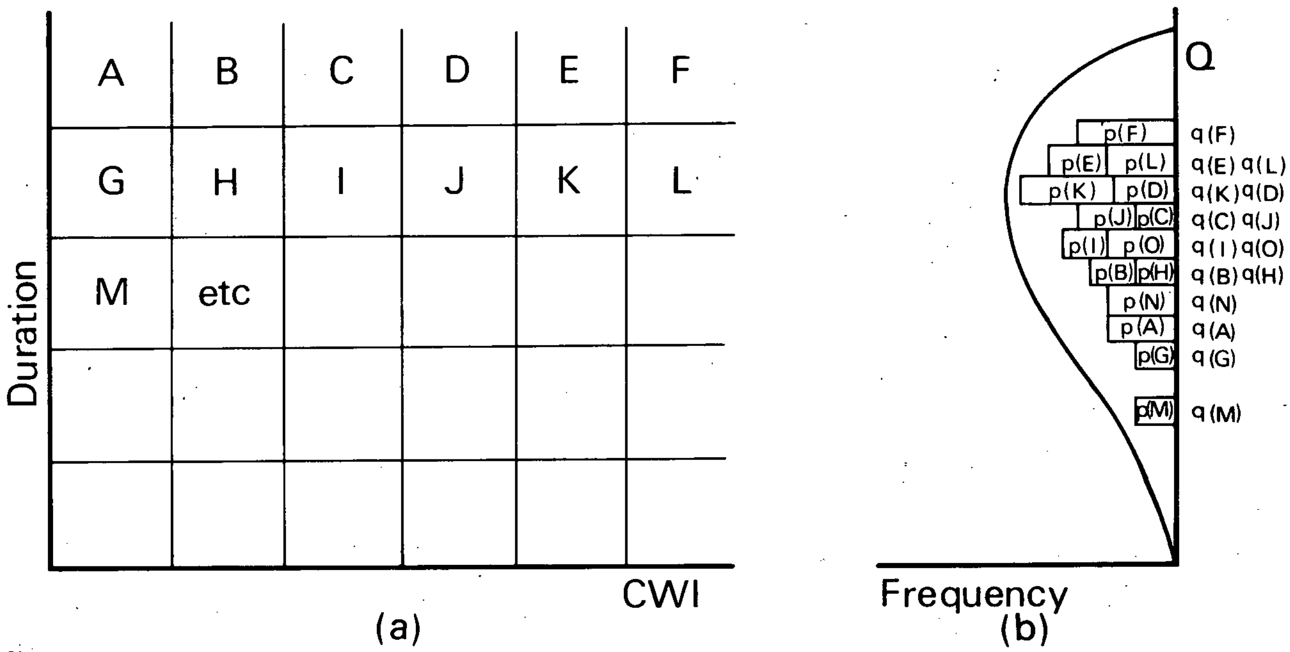
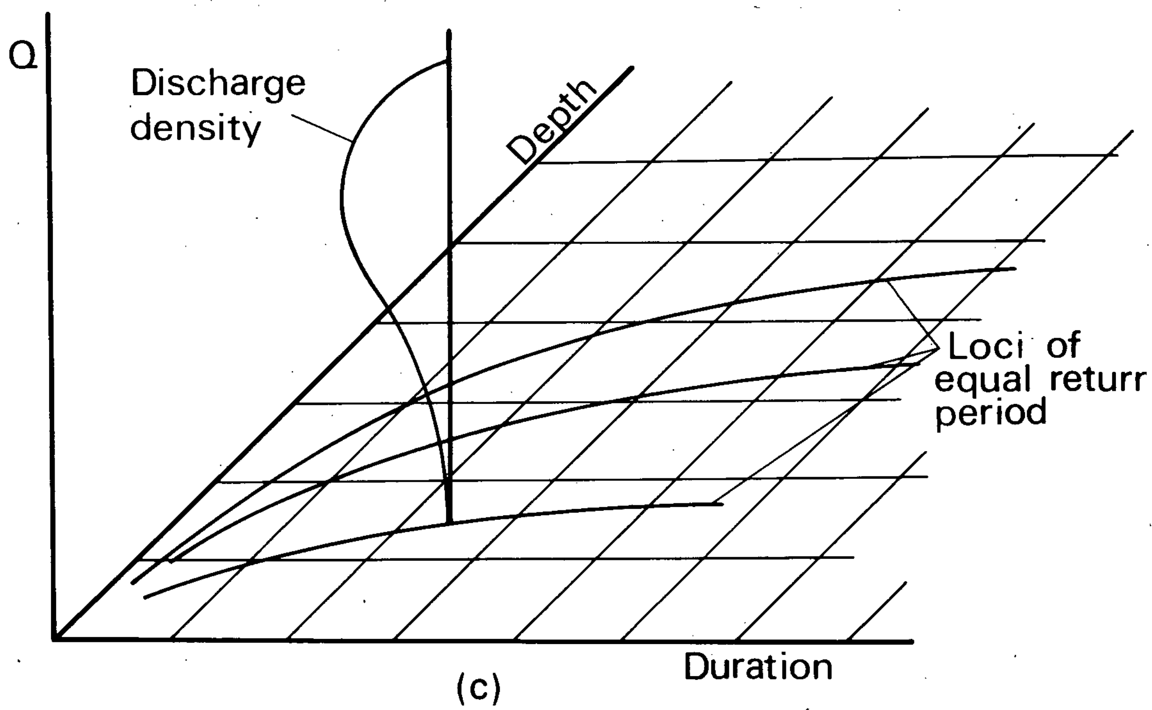


Fig 6.40 Simulation procedure.



Notes

(a) Consider case where two variables only affect discharge Q, for example, storm duration and CWI (Figure a).
(b) For each combination of duration and CWI a value of Q and a probability of occurrence can be calculated. For example, combining the duration in the fourth interval, D_4, with the CWI in the second interval, C_2, a discharge $q(H)$ and a probability $p(H) = p(D_4) \times p(C_2)$ are found.
(c) Summing all the probabilities in each discharge interval a discharge distribution (Figure b) may be constructed.
(d) This concept can be generalised to sample from further variables.



Notes

(a) Depth and duration are plotted on the base plane.
(b) Each combination is associated with a probability of occurrence as given by the depth duration frequency diagram.
(c) Contingent on each depth duration combination a distribution of discharges like Figure b can be visualised on the vertical discharge axis.
(d) Integrating such densities above all points on the base plane on a locus of equal return period yields the results of Section 6.7.5.
(e) Integrating over the entire base plane yields the distribution of discharge of Section 6.7.4.

Fig 6.41 Discharge distribution from CWI and duration.

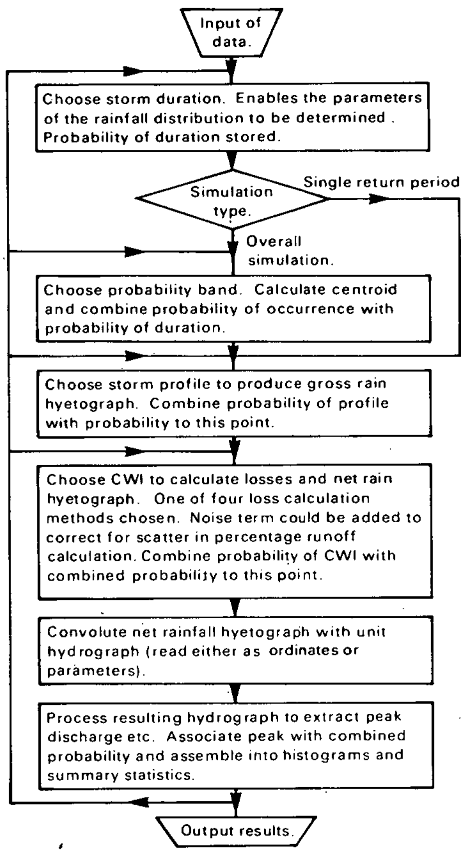


Fig 6.42 Flow chart of simulation program.

The computer program. A FORTRAN computer program was written to carry out the exhaustive sampling over all possible combinations. It was written generally in order to allow the choice of any number and value of storm durations, profiles, and CWI. Options included the input of the unit hydrograph as hourly ordinates or as parameters (Section 6.4.8); four loss subtraction procedures; and the addition of 'random noise' to reproduce the statistical nature of the loss relation (Section 6.5.8, Equation 6.40). It was possible to sample from the entire storm distribution or alternatively from storms of any one return period. An outline flow chart of the program is shown in Figure 6.42.

The program provided a line-printer histogram of all the simulated flood peaks and the histograms of floods following storms of specified durations, rainfall return periods, profile types and CWI values. Tables summarising the mean, standard deviation, maximum and minimum values of each of the histograms were also produced.

6.7.3 Probability distributions of the variables

Storm duration. The storm used for depth duration frequency relations is defined as the largest rainfall of a predetermined duration occurring in a year. Any one wet spell can contribute to the statistics of a number of different durations on this definition even though it may produce only a single flood peak. For this reason and also because a frequency distribution of storm durations was required it was felt that such a definition was not appropriate to the simulation's requirements..

Automatic rainfall records from five sites were analysed using a definition that allowed any wet spell to contribute only a single depth and duration to the statistics. The definition used (Beran, 1973) could be adjusted to take account of different catchment responses. It was found that the average storm duration appropriate to unresponsive catchments was longer than that appropriate to flashy ones but a central distribution (Table 6.16) could be assumed with little error. The distribution seemed very stable for different locations, rainfall regimes and return periods.

Table 6.16 Distribution of storm duration.

	Storm duration (hours)											
	1	3	6	9	12	15	18	21	24	30	36	48
Relative frequency (%)	5	12	26	20	13	10	8	1	1	2	1	1

Storm profiles. The profile sets of Volume II, Chapter 6, were used, with equal probability of summer and winter storm types.

Storm depth. The depth duration frequency information of Volume II, Chapter 3, was used in the simulation. This depth distribution was divided into 6 or 12 bands; Figure 6.43 shows the scheme and gives the necessary formulae.

CWI distribution. In Equation 6.40 the percentage runoff is expressed as the sum of two parts. The second and dominant part is constant for a given catchment (the standard percentage runoff) and is determined by the soil index and urban fraction. The first part allows for the effects of CWI and of storm rainfall. A distribution of CWI values was therefore required for each catchment.

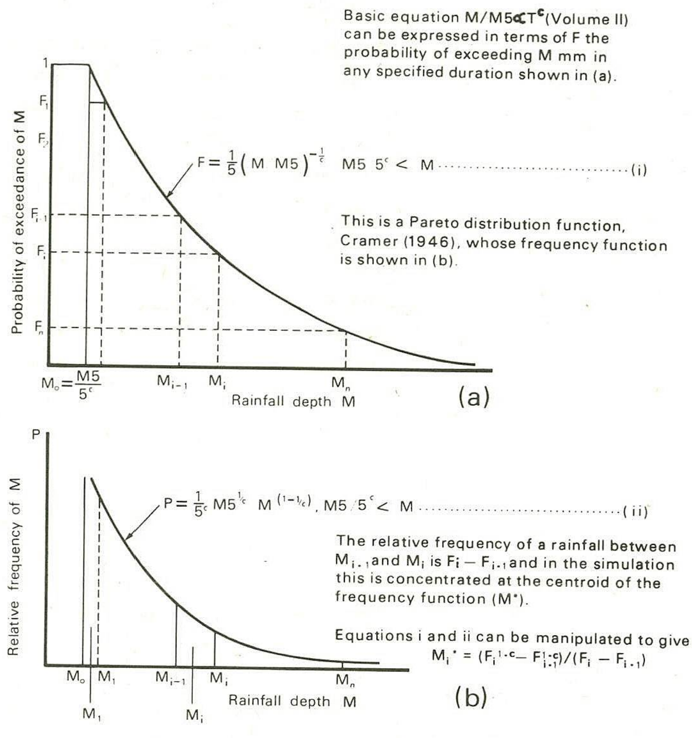


Fig 6.43 Depth distribution treatment.

The two components of CWI are rainfall and soil moisture deficit (SMD) which have been shown to be statistically independent within a season (Section 4.2.3). As explained there, this enables end-of-month CWI values to represent the total CWI distribution. The data from 15 SMD stations were used to estimate the median CWI and interquartile range at each station and the median was then related to annual average rainfall in order to predict values at ungauged sites. Figure 6.44 shows the relationship between median CWI and rainfall. The median CWI increases rapidly for low rainfall but varies only between 120 and 130 mm for annual rainfall greater than 800 mm.

A close straight line correspondence between the interquartile range (IQR) and the median CWI ($\mathrm{CWI_{med}}$) was found—the range reducing as the wetness increased according to the formula

$$\mathrm{IQR} = 258.5 - 1.85\ \mathrm{CWI_{med}}. \tag{6.43}$$

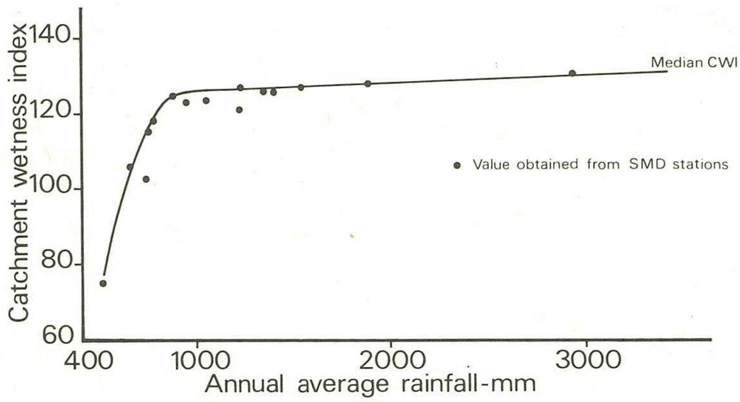


Fig 6.44 CWI–average annual rainfall relationship.

The median and interquartile range were used to standardise the CWI data and two characteristic distributions were found; a long-tailed distribution for wet areas (CWI > 117, annual rainfall > 770 mm), and a short-tailed one for dry areas.

Thus, a CWI distribution at any site can be obtained as follows:

1 estimate the catchment annual average rainfall;
2 interpolate in Figure 6.44 to obtain the median CWI;
3 calculate the interquartile range from Equation 6.43;
4 distribute according to the appropriate standardised frequency distribution (Table 6.17).

(a) Wet areas												
CWI interval												
from	−4.0	−3.5	−3.0	−2.5	−2.0	−1.5	−1.0	−0.5	0.0	0.5	1.0	1.5
to	−3.5	−3.0	−2.5	−2.0	−1.5	−1.0	−0.5	0.0	0.5	1.0	1.5	2.0
Relative freq. (%)	1	1	1	3	5	8	12	22	39	6	1	1
(b) Dry areas												
CWI interval												
from	−1.6	−1.4	−1.2	−1.0	−0.8	−0.6	−0.4	−0.2	0.0	0.2	0.4	0.6
to	−1.4	−1.2	−1.0	−0.8	−0.6	−0.4	−0.2	0.0	0.2	0.4	0.6	0.8
Relative freq. (%)	2	3	5	6	8	8	9	10	14	29	5	1

For example, in wet areas 8% of CWI values lie between $\mathrm{CWI_{med}} - 1.5\ \mathrm{IQR}$ and $\mathrm{CWI_{med}} - 1.0\ \mathrm{IQR}$.

Table 6.17 Standardised CWI frequency distributions.

Because of the importance of CWI in predicting the percentage runoff 12 values of CWI were necessary to represent its variation adequately. It was also necessary to incorporate into the calculation a 'noise' term representing the deviation of percentage runoff from the predicted value. The standard error of estimate of that prediction was 15.1 % and so a random normal deviate with that standard deviation was added to the predicted percentage. This produced occasional very high percentages and a limited distribution might have been more realistic.

Catchment constants. The program required other information to simulate flood hydrographs: the parameters or ordinates of the unit hydrograph; the data for calculating base flow (Equation 6.42); parameters for the rainfall areal reduction factor (Volume II, Chapter 5).

6.7.4 Prediction of the peak discharge distribution

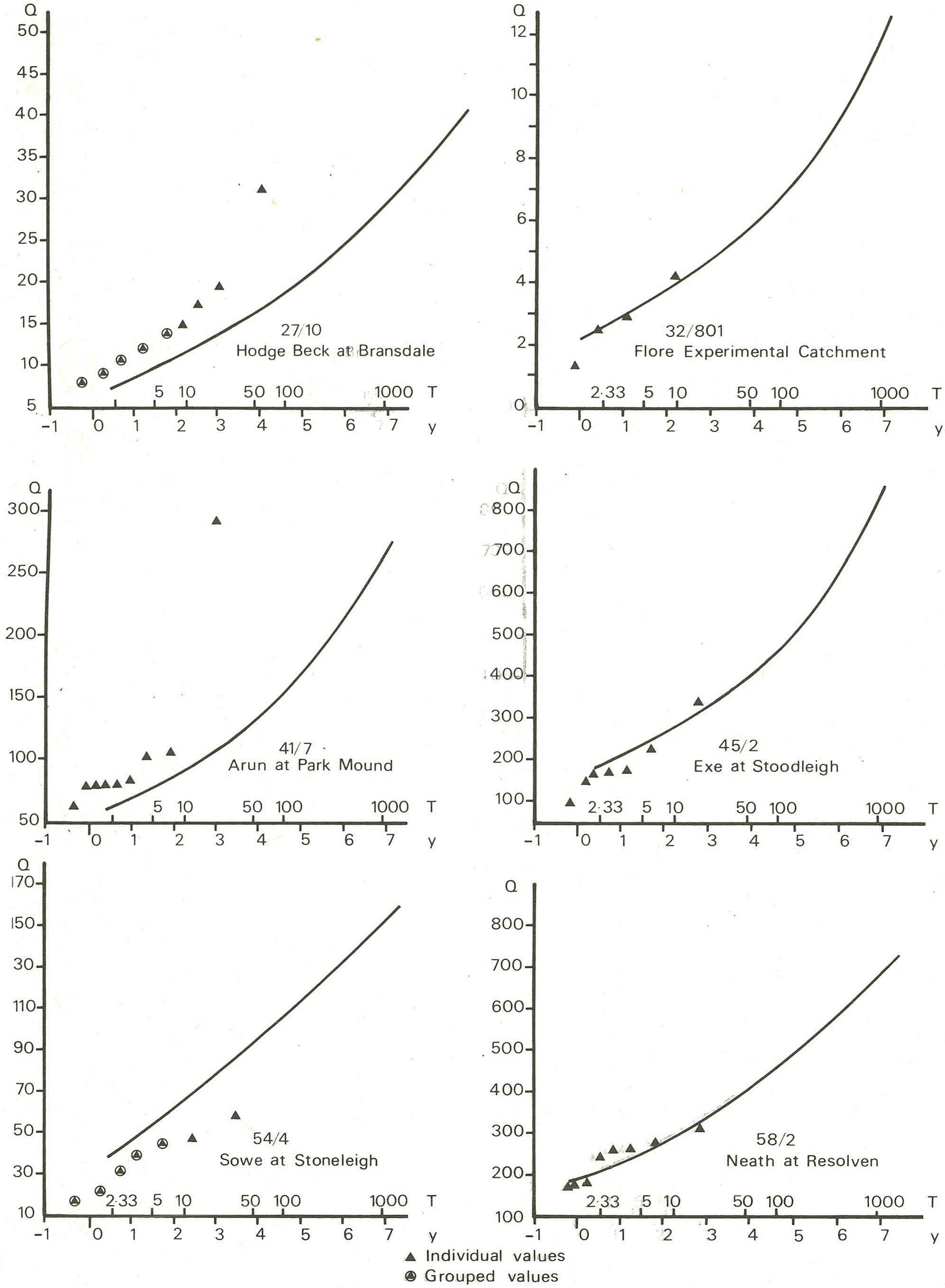
The two problems of a single choice of design parameters to preserve the desired return period, and the sensitivity of the design to change in those parameters, could be answered through a generalisation of the design procedure without involving the behaviour of real catchments. However, to construct a design flood which is unbiassed with respect to design rainfall is academic unless the design flood is realistic in terms of real catchment behaviour. The experiments described in this section show that the probability distribution of floods from real catchments can be adequately predicted by the simulation technique. The simulation was therefore carried out on 98 catchments for which unit hydrograph and loss predictions, and also a useful length of annual maximum flows, were available; 17 catchments were later rejected because their response was too flashy for successful simulation based on hourly rainfall. The location of the 80 catchments in the United Kingdom is shown in Figure 6.3; the 81st was the Maigue at Croom in Ireland.

Regional estimates of the depth, profile, duration and CWI distributions were used (Section 6.7.3). The only recorded values used were unit hydrograph parameters; the effect of substituting parameter values estimated from catchment characteristics is described in Section 6.7.5.

Growth curve comparison. Nine representative catchments were chosen for a full simulation enabling the $Q{:}T$ growth curve to be plotted to high return period. Figure 6.45 shows six typical graphs, comparing the simulated growth curves with the recorded annual maxima. The steepening growth curves are in accordance with the regional growth curves of Chapter 2 and in fair agreement with the trend of the observed points.

Comparisons at two return periods. Figure 6.45 shows that in some cases the simulated estimates are higher or lower than recorded floods; comparisons were made at two return periods. The mean annual flood (BESMAF) and the 10 year return period flood (RECQ10) were estimated from the recorded data of each of the 81 catchments. These are tabulated in Table 6.18 together with the simulated mean annual flood (SIMMAF) and 10 year return period flood (SIMQ10). A summary of the means and correlations between pairs is given in Table 6.19. Overall there was a 10% underestimate of the mean annual flood and a 3% overestimate of the 10 year flood. After taking logarithms the 10% underestimate became a 1 % overestimate indicating that the higher values are underestimated; the 10 year floods supported this. The correlation coefficients between recorded and simulated floods were very high which suggests that the

Fig 6.45 Simulated growth curves.



STATION NUMBER	MEAN ANNUAL FLOOD RECORDED	MEAN ANNUAL FLOOD SIMULATED	10 YEAR RP FLOOD RECORDED	10 YEAR RP FLOOD SIMULATED	COEFFICIENT OF VARIATION RECORDED	COEFFICIENT OF VARIATION SIMULATED
19001	132,700	158.0	166.7	227.0	26.46	40.9
19002	15,020	12.1	21.2	19.0	25.76	42.9
19005	97,820	104.2	107.1	160.0	18.62	41.5
20001	60,250	85.0	93.5	133.0	40.84	45.7
21005	162,040	140.8	196.1	219.0	32.74	39.7
23002	45,030	66.3	63.1	100.0	30.91	44.6
23003	249,430	184.3.	326.5	273.0	19.44	40.4
24003	126,700	116.5	186.1	177.0	26.04	40.8
24005	36,290	65.4	55.5	100.0	39.99	43.8
25004	24,720	31.4	37.1	53.0	37.56	53.9
27001	136,660	142.0	241.14	238.0	44.45	52.0
27010	10,440	9.1	16.7	14.0	46.97	42.3
27026	35,800	33.83	56.2	59.0	35.40	58.2
27027	263,710	200.5	374.05	320.0	20.73	46.4
28016	12,970	17.7	16.79	33.0	22.91	69.0
29001	1,520	9.2	3.16	21.0	48.20	94.7
29003	2,930	5.4	7.47	13.5	44.42	108.0
30001	17,230	33.0	28.76	58.0	50.36	61.1
32801	1,910	2.5	4.3	4.2	59.01	52.8
33014	9,017	18.2	15.4	33.0	59.42	81.8
33015	16,750	25.6	22.6	45.0	27.79	58.7
33024	8,187	15.2	11.5	32.0	31.37	85.2
33029	2,520	5.8	4.3	13.5	22.95	98.9
34003	4,770	9.7	13.5	23.0	43.43	99.5
34005	2,818	6.5	4.7	11.5	46.90	56.8
35008	10,590	19.2	24.7	34.0	45.71	63.8
36008	18,440	21.6	36.5	39.0	107.75	61.7
37001	22,400	26.1	34.7	45.0	34.48	58.6
37003	4,470	5.8	8.6	11.0	42.77	75.8
37007	14,020	17.6	28.7	31.0	46.60	72.2
39012	12,950	15.7	19.2	30.0	38.40	71.7
39017	7,879	4.2	14.0	6.9	59.00	51.8
39813	4,590	3.5	7.5	·5.7	110.16	48.1
39820	7,364	8.6	11.3	14.0	44.49	53.4
40006	10,600	9.3	15.0	17.0	143.72	66.6
40007	56,960	46.3	90.9	80.0	48.41	58.1
40008	19,610	32.7	34.8	55.0	29.45	54.0
40009	33,760	34.6	42.4	59.0	19.08	57.5
41005	38,500	28.5	65.0	47.0	53.56	49.1
41006	31,120	23.3	47.7	40.0	28.39	51.3
41007	94,890	59.2	124.0	93.0	72.24	45.4
41811	7,170	7.0	13.0	11.0	24.89	46.4
45002	146,490	169.3	270.4	280.0	50.82	51.1
45003	76,410	53.9	153.0	96.0	63.68	58.9
45004	106,070	75.0	233.4	133.0	50.52	57.0
46003	210,600	174.3	353.0	280.0	15.37	46.8
47007	27,980	25.7	25.7	42.0	9.00	50.3
52004	29,350	35.2	29.2	55.0	11.34	45.4
52005	50,250	48.1	97.8	81.0	49.48	54.4
52006	47,850	33.20	58.9	82.00	23.59	51.1
52010	47,770	42.4	85.1	67.0	43.52	46.2
53005	27,700	26.9	50.5	49.0	45.28	61.4
53007	66,790	61.2	103.3	103.0	42.05	53.3
54004	28,630	37.5	47.8	54.0	49.14	34.1
54006	19,010	19.0	31.0	40.0	95.33	84.5
54007	44,410	58.8	60.2	95.0	34.55	49.9
54010	35,380	47.5	66.6	80.0	52.03	54.4
54011	17,180	37.0	45.7	59.0	60.18	48.4
54016	20,960	20.2	25.5	36.0	48.74	60.2
54019	30,950	32.0	72.8	52.5	74.05	59.1
54020	9,360	13.3	12.3	24.5	11.76	64.5
56003	23,450	27.1	39.7	45.0	20.40	51.3
56006	162,310	131.1	294.9	207.0	36.24	45.5
57004	67,490	68.1	113.4	98.0	41.39	37.0
58001	103,990	90.4	162.1	138.0	33.40	40.9
58002	215,620	194.7	307.0	290.0	32.03	36.6
58003	19,340	19.6	21.9	32.0	10.54	52.6
60002	139,030	108.1	191.8	172.0	29.81	45.2
61001	44,440	59.00	62.00	145.70	30.59	52.8
64001	292,310	256.4	374.2	395.0	26.29	43.0
65001	62,310	71.3	73.6	105.0	17.66	36.8
66011	450,390	356.6	615.4	530.0	24.39	36.7
67005	36,120	38.1	54.5	60.0	28.50	45.0
68006	57,600	76.3	96.7	121.0	41.57	46.7
68802	1,110	1.7	1.9	2.6	17.77	46.1
71003	13,510	8.4	22.9	13.0	41.54	39.3
72002	152,350	130.2	172.9	198.0	11.05	40.6
77011	379,480	344.6	573.4	520.0	23.69	41.9
84012	106,490	95.5	162.0	150.0	33.76	44.0
85002	109,470	154.4	123.4	235.0	10.05	41.5
33903	93,390	77.90	113.6	125.0	20.35	45.1

Table 6.18 Results of simulation.

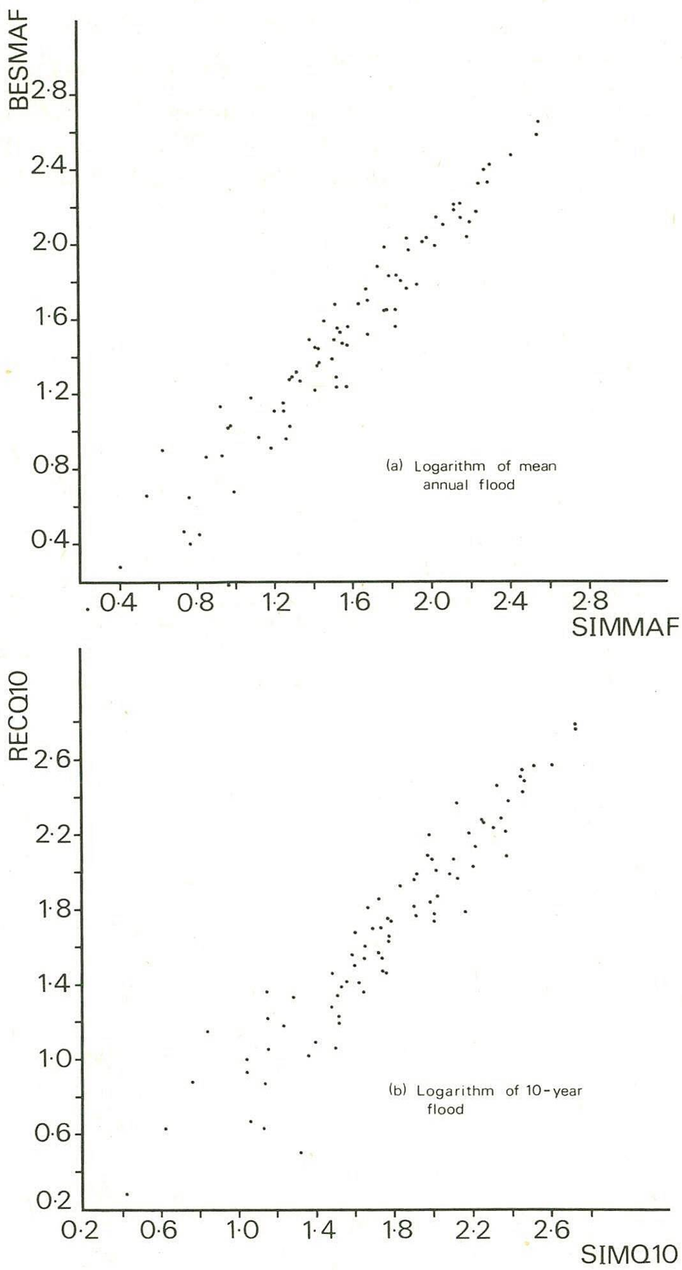


Fig 6.46 Comparisons between recorded and simulated floods.

technique could be used as an alternative method of flood prediction at ungauged sites. Moreover, the result was obtained by using regional parameter values for all input data except the unit hydrograph; further improvement might be expected if specific values were substituted after analysis of catchment data.

	Arithmetic mean (cumecs)	Geometric mean (cumecs)	Correlation and determination coeffs.			
			Arithmetic data		Logarithmic data	
BESMAF	70.3	33.0	0.980	0.960	0.964	0.929
SIMMAF	64.0	35.0	—	—	—	—
RECQ10	100.1	49.1	0.966	0.933	0.946	0.895
SIMQ10	102.3	59.5	—	—	—	—

Table 6.19 Summary of overall simulation results from 81 catchments.

Figures 6.46(a) and (b) show the same data in the form of a scatter diagram. These graphs confirm that the main source of underestimate is in the higher floods although the most serious individual departures are with smaller and medium flood catchments. The three largest deviations arise from 29/1 (Waithe Beck at Brigsley), 39/17 (Ray at Grendon Underwood), and 71/3 (Croasdale Beck at Croasdale Flume), which together account for over one third of the total error sum of squares.

Figure 6.47 shows the pattern of residuals from the ratio regression of BESMAF on SIMMAF

$$\text{BESMAF} = \text{SIMMAF}^{0.98}. \qquad (6.44)$$

The simulation tends to underestimate floods in the south and south west and to overestimate floods in East Anglia and the east coast.

A direct comparison between the prediction of the mean annual flood by simulation and prediction by regression on catchment characteristics can be gauged from Tables 6.19 and 6.20. 93% of the variability of BESMAF is accounted for by simulation and 91% by regression. The difference is not large, but was obtained with regional values of most of the input variables. The simulated prediction at any catchment could be materially improved by using information from a short period of records.

Figure 4.23 shows the residuals from the BESMAF regression of observed mean annual flood (BESMAF) on catchment characteristics. Their pattern resembles that of Figure 6.47 indicating that some areas are more or less flood prone than their measured catchment characteristics would predict either in regression or in simulation.

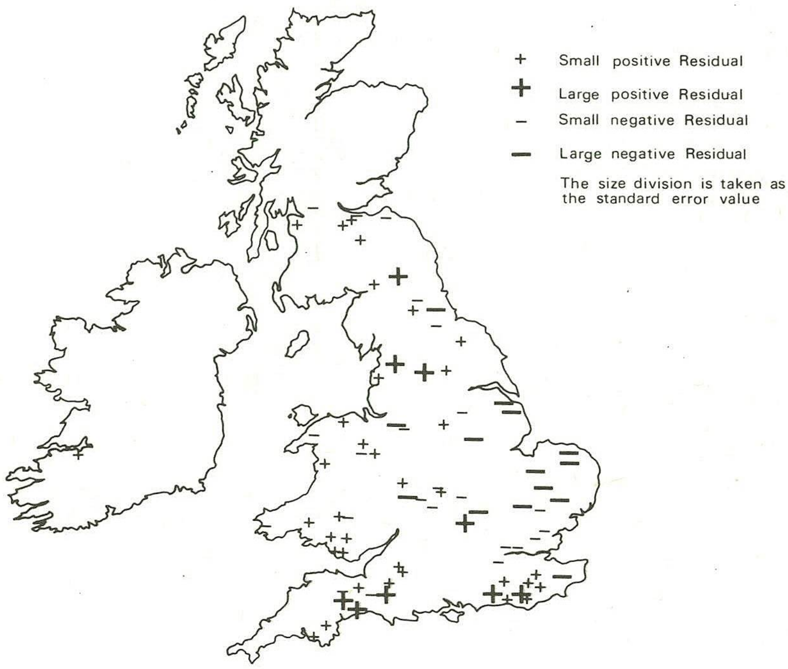


Fig 6.47 Map of residuals from the regression of observed mean annual flood (BESMAF) on simulated mean annual flood (SIMMAF).

The regressions of simulated floods on catchment characteristics are of some interest (Table 6.20). Although the independent variables are all used directly or implicitly in the simulation, only 96% of the variability in SIMMAF is explained. This demonstrates the effect of lack of fit (inappropriate form of model) and goes some way to quantifying the imperfect

Dependent variable	Regression coefficients				Intercept	R	R^2	see (%)	
	AREA	RSMD	SOIL	TAYSLO					
BESMAF	1.006	1.472	1.390	0.275	−2.500	0.955	0.912	+51,	−34
SIMMAF	0.993	1.152	1.018	0.257	−2.082	0.978	0.956	+29,	−22
RECQ10	0.985	1.290	1.238	0.289	−2.053	0.939	0.882	+58,	−37
SIMQ10	0.977	1.102	0.683	0.247	−1.882	0.975	0.951	+29,	−22

In all cases an optimal selection procedure produced the same four variable set as shown above.

Table 6.20 Regressions on catchment characteristics from 81 catchments.

predictions from such equations (Section 4.3.10). Table 6.20 shows the similarity between the coefficients obtained in regressions of floods simulated through the unit hydrograph approach and in direct regressions of recorded floods on catchment characteristics. The equation predicting simulated floods is broadly of the form

$$Q = c\ \mathrm{AREA.RSMD.SOIL.SLOPE}^{\frac{1}{4}}. \qquad (6.45)$$

The exponents derive from equations predicting losses and unit hydrograph dimensions, thus establishing a link between the unit hydrograph and statistical methods of flood prediction.

6.7.5 *Sensitivity analyses*

The sensitivity of the design flood to legitimate changes in the assumptions about the storm and catchment conditions is one of the fundamental problems of flood design. The importance of the problem is rooted in its relation to risk evaluation, and to the identification of areas where the design might be improved by refinements or by the acquisition of further data.

The first problem to be addressed is the effect of different return periods of design storm on the average outcome. Subsequent sensitivity analyses were carried out primarily on hydrographs resulting from 10 year storms. In turn selected values of either storm duration, hyetograph profile, or CWI were held constant while integrating the effect of the other two. For example, in investigating the sensitivity of the design flood to changes in storm duration all the floods following 10 year 3 hour storms were simulated; then all floods following 10 year 6 hour storms and so on through a representative range of durations. The variation of the mean flood with storm duration provides information on the sensitivity to storm duration.

Floods following storms of specified return period. The relationship between flood peaks following T year storms and the actual T year flood was investigated. Figure 6.48 shows how flood distributions from different rainfall probability bands are built up to form the total distribution. It is clear from this that the range of floods is such that almost any return period of flood can be obtained from any storm return period. The diagram also shows that the overlap is greatest between distributions from low return period storms.

The simulation program was written to allow the storm return period to be specified and distributions of floods following only such storms to be generated. The mean and mode of the flood distributions for $T = 2.33$, 10, 100 and 1000 have been compared with the simulated growth curve. There is good agreement at low and high return periods but the mean flood following storms of 5–100 year return period underestimates the

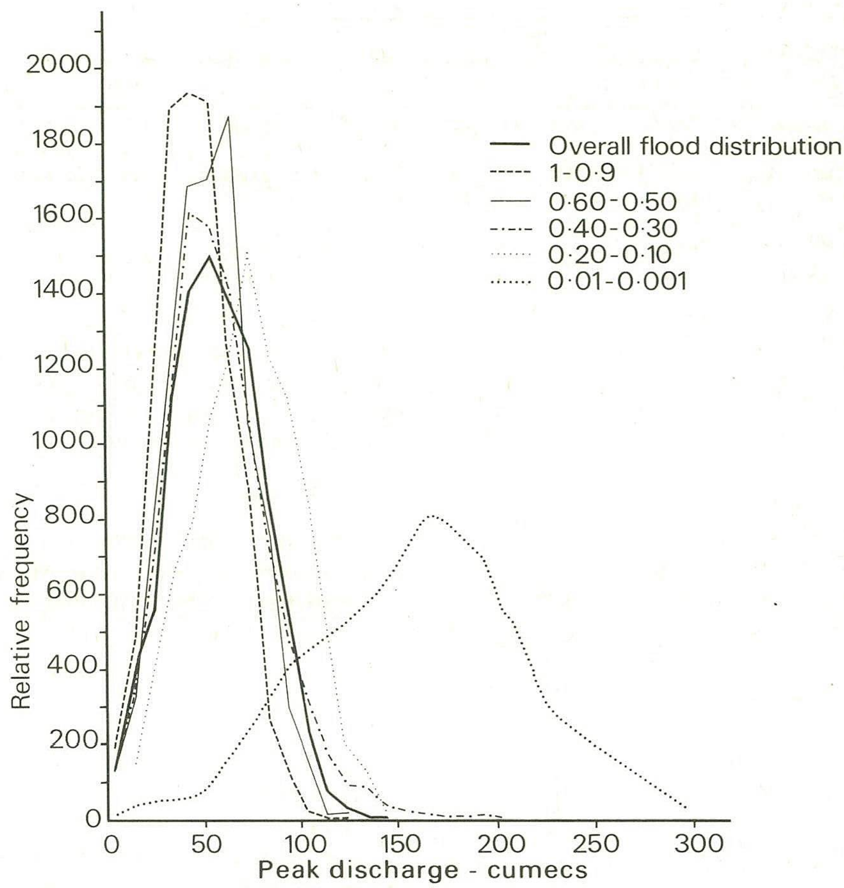


Fig 6.48 Histograms of flood peaks following rainfalls of specified probability.

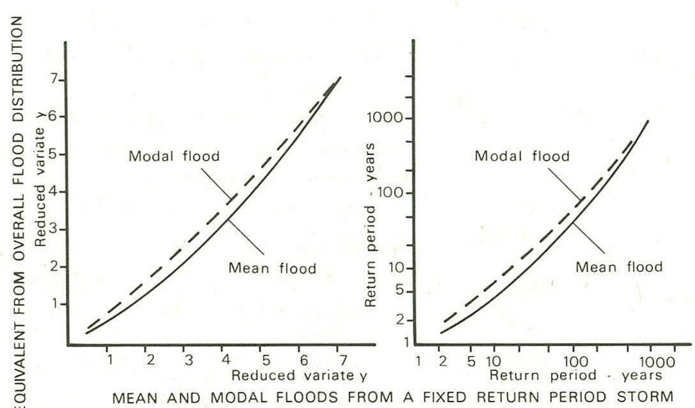
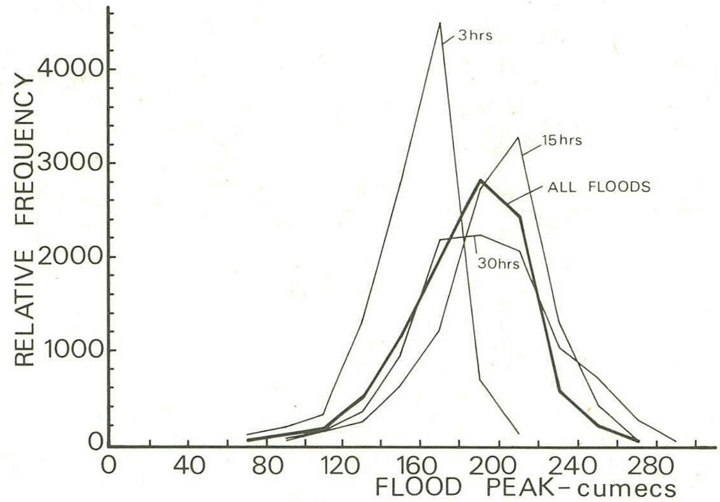


Fig 6.49 Reduced variate and return period equivalences; rainfall and peak flow.

flood of that return period. The average relationship is shown in Figure 6.49; a typical result is that the mean flood following a 100 year storm has a return period of 50 years.

Sensitivity to catchment wetness index change. CWI is an important factor controlling the percentage runoff and so has a considerable effect on both peak and volume floods. Figure 6.50 shows the simulated distribution of floods following 10 year return period storms which encounter specific CWI antecedent conditions. Two features should be noted; the rapid increase in the location of the distribution as the antecedent conditions



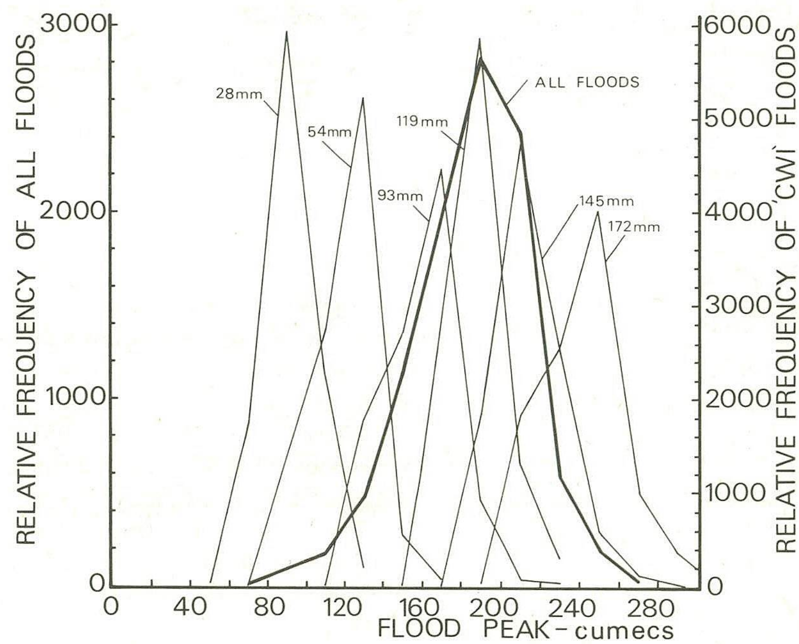


Fig. 6.50 Histograms of flood peaks following specified conditions.

become wetter, and the narrow range of values once the CWI is determined. Both reflect the importance of CWI, the first being a measure of the sensitivity of design to a choice of value, and the second indicating the relative unimportance of the other design specifications.

Figures 6.51(a) and (b) show the sensitivity of the average flood peak to CWI. In both the discharge has been standardised by dividing by the mean of all floods following 10 year return period storms. In (a) the CWI has been standardised in terms of the median and interquartile range of the CWI distributions at each site; the figure shows a stable point through which the lines appear to pass. The CWI axis was subsequently standardised by expressing CWI as a percentage of the CWI at this stable point (1.185 $\mathrm{CWI}_{med} - 25.9$). Figure 6.51(b) shows the resulting graph in which the slope of the line indicates sensitivity to CWI change. This slope was found to correlate very well with the catchment soil index, catchments with low soil index being more sensitive to CWI change than catchments with high

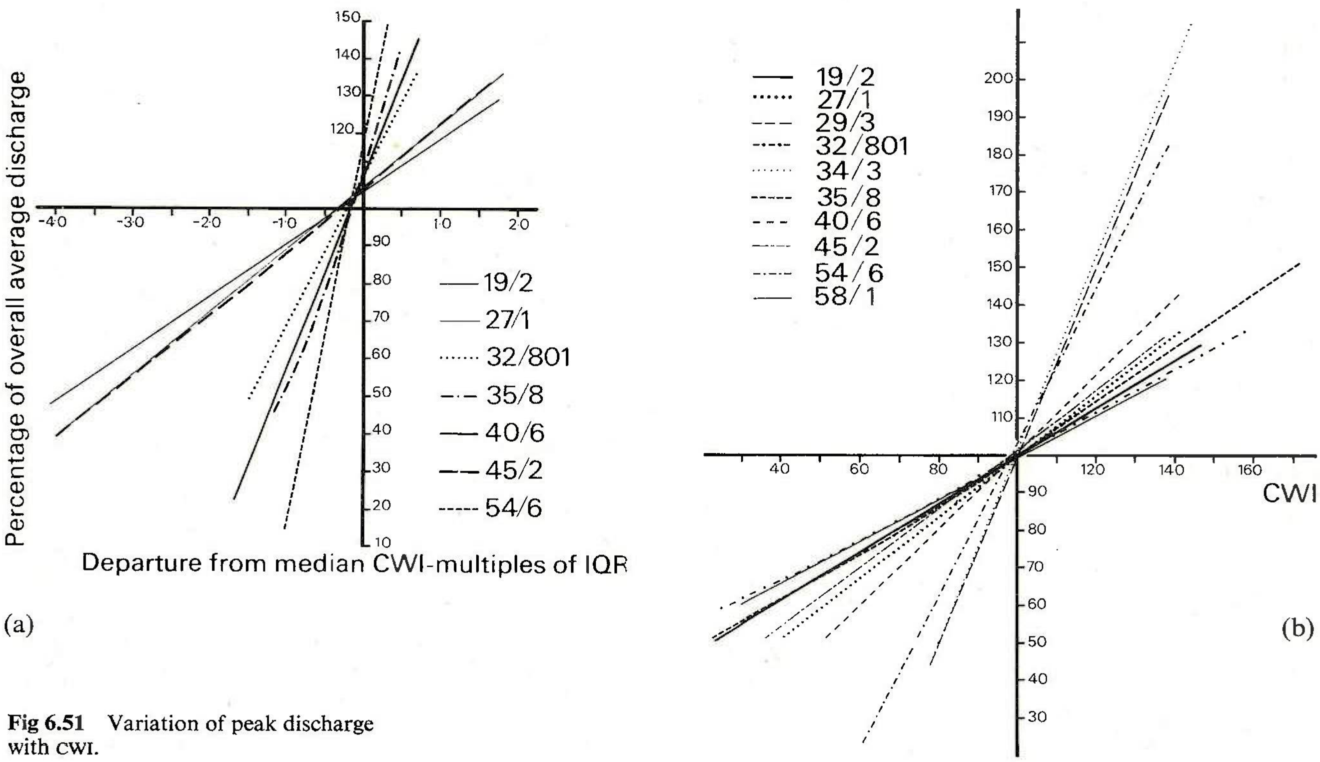


Fig 6.51 Variation of peak discharge with CWI.

soil index. This observation results from the form of the percentage runoff equation used, Equation 6.40. Because catchments with low soil index have low standard percentage runoff, the percentage increase in runoff for given CWI change is greater for such catchments than for catchments with a high soil index. For each 10% CWI increase the design flood peak increases by 20% for a catchment with soil index of 0.18, but by 5% for a catchment with soil index 0.50 (all soil class 5).

These results show the importance in flood design of correct choice of CWI in order to predict losses. They also have a bearing on the variability of floods from year to year.

Sensitivity to storm duration change. The effect of storm duration has historically held a prominent position in flood design. This was because storm duration is the only degree of freedom allowed in the rational formula and so over or under design will follow from under or over estimate of the time of concentration. In unit hydrograph methods, on the other hand, a balance is achieved between increasing storm duration and decreasing intensity such that the peak discharge varies over a restricted range for a very considerable range of storm durations. Figure 6.50 shows histograms of floods following 10 year return period storms of different durations.

With uniform rainfall and a simple triangular unit hydrograph, it can be shown algebraically that the duration leading to the highest peak depends on the time base of the unit hydrograph and the continentality factor n in Table II.3.8. Figure 6.52 shows the variation of discharge with storm duration for nine selected catchments. The critical duration producing the highest peak was found to increase with decreasing n and thus with increasing annual average rainfall. The sensitivity of simulated peaks to

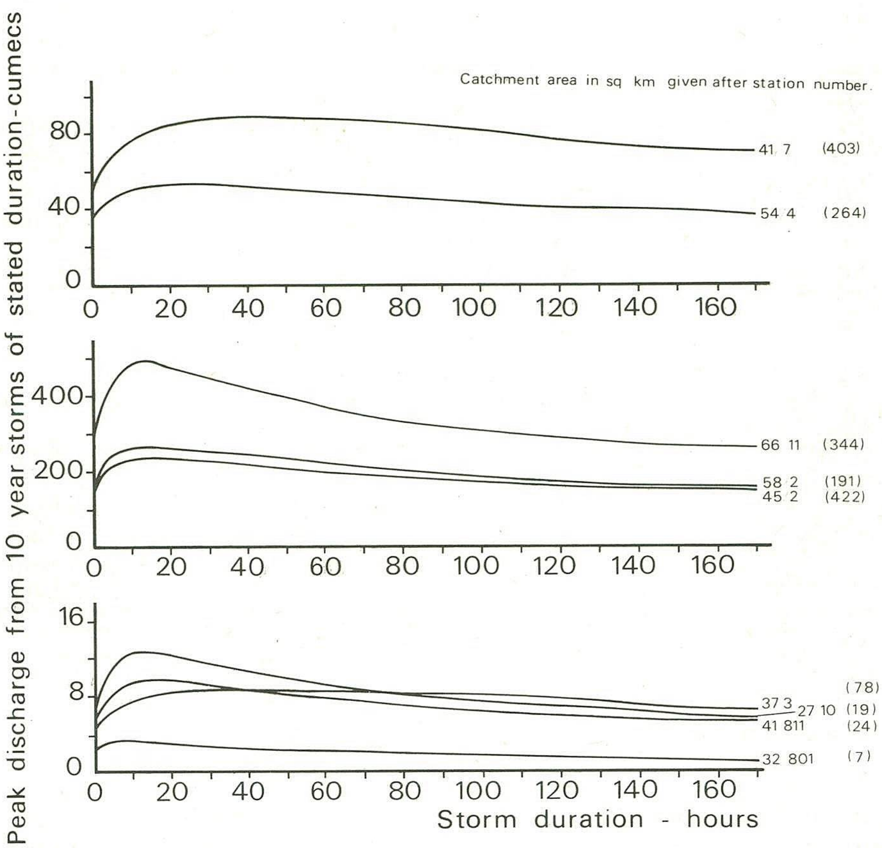


Fig 6.52 Variation of peak discharge with storm duration.

changes in duration is greater for durations below the critical value than for larger durations.

Sensitivity to hyetograph profile change. Some theoretical work was undertaken to investigate this source of variation. The hyetograph shape that maximises the peak from any given unit hydrograph is one with equal and opposite skewness to the unit hydrograph, provided that no portion of the hyetograph is of greater return period than the whole.

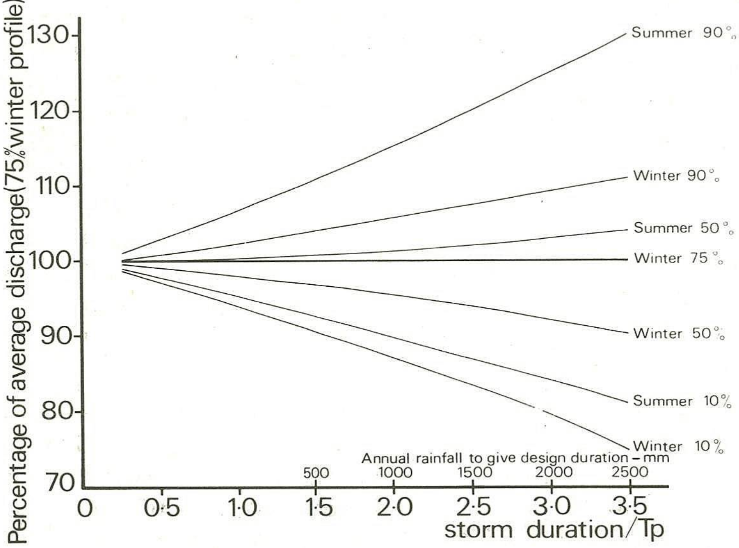


Fig 6.53 Variation of peak discharge with profile.

The distributions of floods following the two basic profile types (Chapter 6 in Volume II) were very similar. The mean peak following summer profiles was slightly higher than that following winter profiles, neglecting seasonal changes in CWI or storm duration. The 75% winter profile was found to yield a flood peak close to the average from all profiles. Figure 6.53 shows the effect on the peak of the profile subtypes within the broad summer and winter types. The flood increases with hyetograph peakiness; more markedly as the ratio duration/time to peak increases. This can be explained by the fact that the difference between the return periods of a storm and that of, say, its central third also increases with duration. For most catchments the range (of this ratio) inspected was from 0 to 2 over which profile has little effect on the outcome. However, for the design storms of Section 6.7.6 the effect can become important.

Sensitivity to unit hydrograph change. Observed values of T_p, Q_p, and W were used in the simulation. The effect of alternative values was studied by substituting estimates obtained from catchment characteristics (Equation 6.18) and was found to be the same as observed in trials reported in Section 6.6.3. If T_p is increased by 41% (1 standard error upwards), the peak is decreased by about 20% (average over all catchments); if T_p is decreased by 29% (1 standard error downwards), the peak is increased by about 17%.

When the unit hydrograph parameters are estimated from catchment characteristics, Q_p is deduced from T_p by assuming a constant Q_pT_p of 220. However, Figure 6.17 displays a wide scatter of recorded values of Q_pT_p (Section 6.5.2). The effect on the peak flow of changes in Q_pT_p was simulated; it was found that a change of 10 in Q_pT_p causes a cnange in the same direction of about 3% in the peak discharge.

Sensitivity to method of separating net rain. The method of separating net rain in hydrograph analysis was discussed at length in earlier sections. Table 6.9 shows the effect on the derived unit hydrograph of three different methods. The simulation program also enables the effect on synthesised peak discharges to be assessed. Four different methods were compared: A1, uniform loss rate; A2, loss rate curve—as used in analysis; B1, constant loss percentage—recommended for prediction of hydrograph from design storm; B2, loss percentage varying with CWI—recommended for forecasting of hydrographs from recorded storms.

It was found that on average A1 gave results similar to A2 and B1 gave results similar to B2. Type A methods gave results typically 5–10% higher than type B methods. The difference in peak discharge between B1 and B2 methods is small because storm profiles used in the simulation, as in design, are symmetrical. The rain intensity reaches its maximum in the middle of the storm, where the percentage runoff predicted by method B2 will approximate the overall average predicted by method B1. Though the prediction equations in this chapter are intended for use with method B2, the use of B1 in design will not introduce serious error.

6.7.6 Single choice of variables

The simulation study described in this section was intended to provide the basis for recommending the particular values of CWI, duration, profile, and rainfall depth that would yield a peak discharge of the desired return period. In past practice, the duration variable has been set to a critical value which leads, other things being equal, to the highest peak. It has usually been assumed that, if other variables are set to their most likely values, the return period of the peak discharge will equal that of the rainfall. In this report, the recommendation for a single choice of variables is based directly on the overall simulation results which have been shown to be in good agreement with records, although this agreement could only be assessed with the mean annual and 10 year floods. The approach has been to select 'appropriate' values for three of the four variables and to see what value is forced on the fourth variable so that the required peak is produced.

Storm profile. It was shown in Section 6.7.5 that the effect of profile can become quite important especially for responsive catchments. However, there will be many cases where the desired peak cannot be achieved by manipulating profile alone. It is therefore proposed that profile should be one of the 'fixed' variables. The '75% winter' profile is recommended for use in the single choice exercise. It is the profile which typically produces a peak of the same size as the average from sampling across the full set of summer and winter profiles.

Storm duration. The use of storm duration as the free variable was tested. Although it proved more flexible than profile, there were occasions when the required peak could not be achieved given reasonable specifications of the three other variables (T year rainfall to give T year flood; profile as above; a middle value of CWI). It was therefore decided that duration would be specified and a suitably stable value is that which gives the highest peak—the peak of the curves in Figure 6.52. Because these curves are flat topped, error in estimating duration is not critical. This duration d_m varies with average annual rainfall and unit hydrograph time to peak:

$$d_m = (1.0+\text{SAAR}/1000).T_p. \qquad (6.46)$$

Rainfall (SAAR) enters this equation because of its association with 'n', the continentality factor of II.3.3.3 (see Section 6.7.5, sensitivity to storm duration change).

CWI and storm depth. Either of the remaining variables, CWI or storm depth, is capable of acting as the free variable. If the storm depth were specified (T year rainfall to give T year flood) then the necessary value of CWI could be found. Similarly, if the CWI were specified then the required return period of rainfall could be determined. Both methods were tested and it was found that the graph comparing flood return period with required return period of rain (Figure 6.54) was similar between catchments. On the other hand, the necessary values of CWI when used as the free variable, were found to vary between catchments (see Figure 6.51 and Section 6.7.5).

The recommended value of CWI is the intersection point established by Figure 6.51(a). This value (one tenth of the interquartile range less than the median) is almost independent of catchment type. Figure 6.44 relates the median CWI to average rainfall.

Summary of recommended selection of variables. To achieve a peak discharge of T year return period, Figure 6.61 (an average of the curves in Figure 6.54) enables the appropriate storm return period to be chosen.

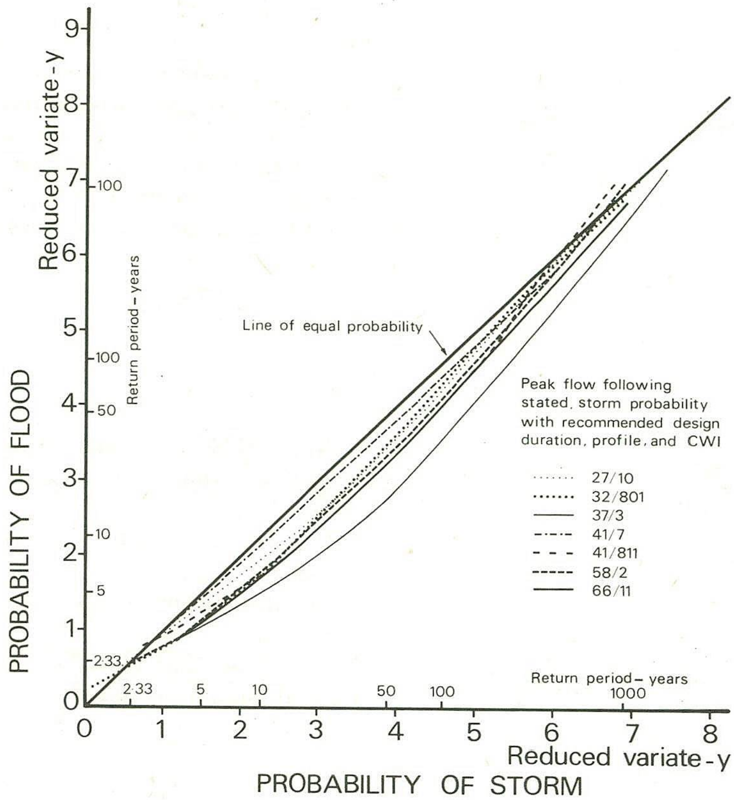


Fig 6.54 Specification of storm return period to obtain peak discharge of required return period using the single choice of variables.

The design CWI can be calculated from Equation 6.43 and Figure 6.44. The storm duration is derived from Equation 6.46. The '75% winter' storm profile is used.

6.8 Application of results

Contents: After a cautionary introduction (6.8.1), the full procedure for synthesising a design hydrograph with peak of specified return period is given in the form of a worked example (6.8.2). In Section 6.8.3, factors affecting 'probable maximum' flood design are discussed and a further worked example is presented.

The procedure for unit hydrograph analysis is summarised in Section 6.8.4 and the application to short term forecasting is briefly mentioned in Section 6.8.5. A computer program performing the basic steps of hydrograph synthesis is described in Section 6.8.6.

6.8.1 Introduction

The results of this chapter enable a design hydrograph, with a peak of specified return period, to be provided for an ungauged catchment. The data used were gathered from catchments under 500 km^2 in the United Kingdom and were mostly from events less than twice the mean annual flood. Application to larger catchments, to overseas catchments, or to the prediction of floods with higher return periods must be made with caution. The area limit was of course, a product of requirements for *analysis*. In design, it is quite normal to consider spatially uniform rainfall over much larger areas than 500 km². Indeed, any consideration of different rain amounts falling on subcatchments raises the question of assessing the probability of various combinations of such different amounts. It is thought that the methods of this chapter can probably be used for catchments up to 1000 km² but that for larger catchments it would be advisable to subdivide and route as discussed in the Introduction to this volume.

As regards overseas catchments, the main difficulty is in extracting catchment characteristics to the same specification. However, if soil index and stream slope can be assessed with reasonable confidence and the rainfall statistics are available there is every reason to suppose that the method can be applied to catchments in other cool temperature zones.

Specification of peak flows with return periods much greater than 1000 years can only be done with a large error of estimate. This applies to statistical methods as well as the semideterministic procedure described here. The difficulties in analysing very large floods, which are almost never measured properly in respect of either rainfall or runoff, have been referred to in previous sections of this chapter. These difficulties necessitate the adoption of conservative assumptions when designing against maximum floods.

In this section (6.8), the application of results is described and numerical examples are presented. Equations are repeated and some key figures and tables from this chapter and from Volume II are reproduced in part so that the examples may be followed within the section.

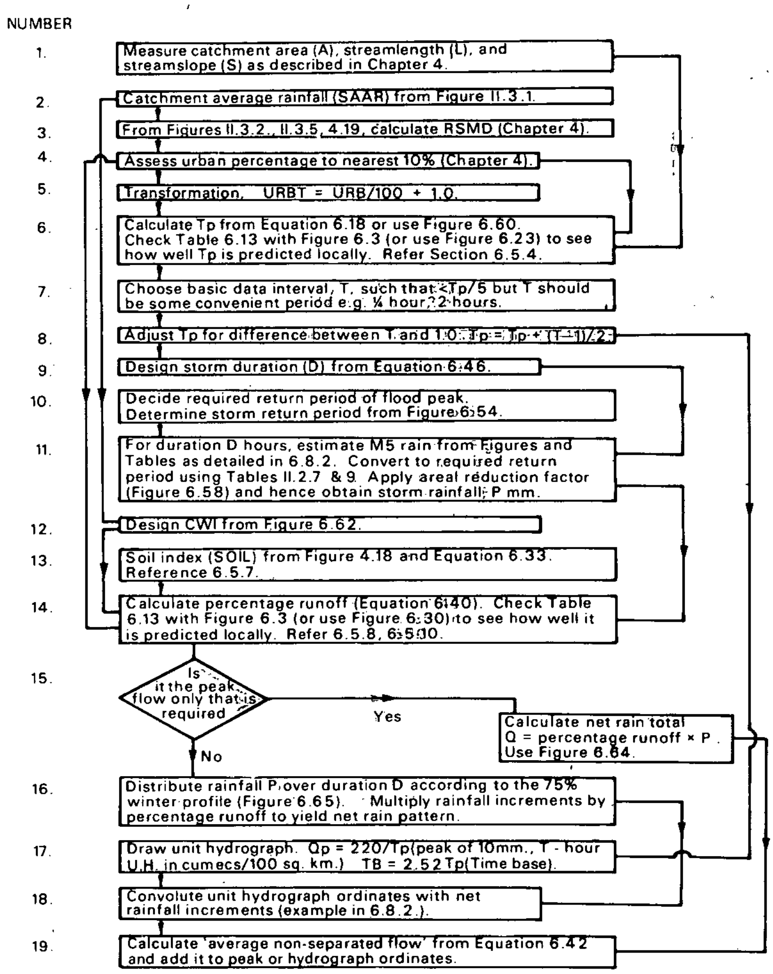


Fig 6.55 Flow chart of design procedure for flood with peak of specified return period.

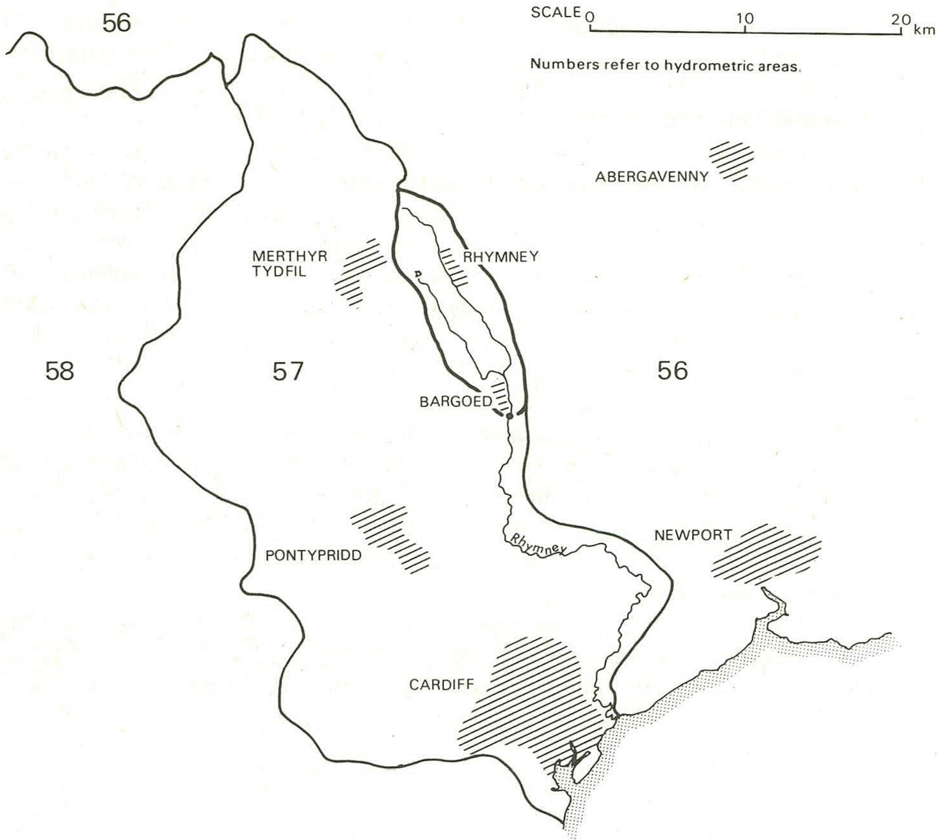


Fig 6.56 Location of example catchment; River Rhymney at Gilfach Bargoed.

The computations are presented mainly for the situation without rainfall or runoff data from the catchment or nearby catchments. If such data exist, the procedures may be modified to varying degrees and this is illustrated within the examples.

6.8.2 A design hydrograph with peak of specified return period

Figure 6.55 presents, in the form of a flow chart, the several steps in the specification of a hypothetical design hydrograph. A more detailed description now follows with an example of the calculations for the River Rhymney at Gilfach Bargoed. This catchment is on the eastern edge of the Glamorgan River Authority's area (Figure 6.56). The specific numerical values are given on the right of the page alongside the relevant description of the general procedure.

Step 1

Define topographic catchment on 1:25 000 map and measure area (A) by any convenient method. $A = 63.2$ km^2

Length (L) of main stream on 1:25 000 map should be measured with dividers set to 0.1 km step length as described in Section 4.2.2. Other methods of measurement will give different answers and should not be used. $L = 17.22$ km

Channel slope (4.2.2) is the average slope (S) between points 10 and 85% of the length of the main stream measured up from the outlet, i.e. $S = (H(85\%) - H(10\%))/0.75L$ where $H(\quad)$ are the altitudes (m) as interpolated between the contours.

$H(85\%) = 358.1$ m
$H(10\%) = 167.6$ m
$S = 14.7$ m/km

Step 2

The average annual rainfall (SAAR) is obtained from Figure II.3.1. The average for the catchment may be obtained by sampling at about 20 points equally spaced on a grid overlay and taking the arithmetic mean. Alternatively, the weighted areas technique can be used.

Figure 6.57a shows the relevant part of the map for the example catchment.

SAAR = 1530 mm

Step 3

Calculation of RSMD (Chapter 4).

The calculation of RSMD requires M5–2 day rainfall from Figure II.3.2, the ratio r (M5–60 min/M5–2 day) from Figure II.3.5, and the effective SMD (SMDBAR) from Figure 4.19. In all three cases, catchment average values

The parts of the three figures relevant to the example catchment are shown in Figure 6.57.

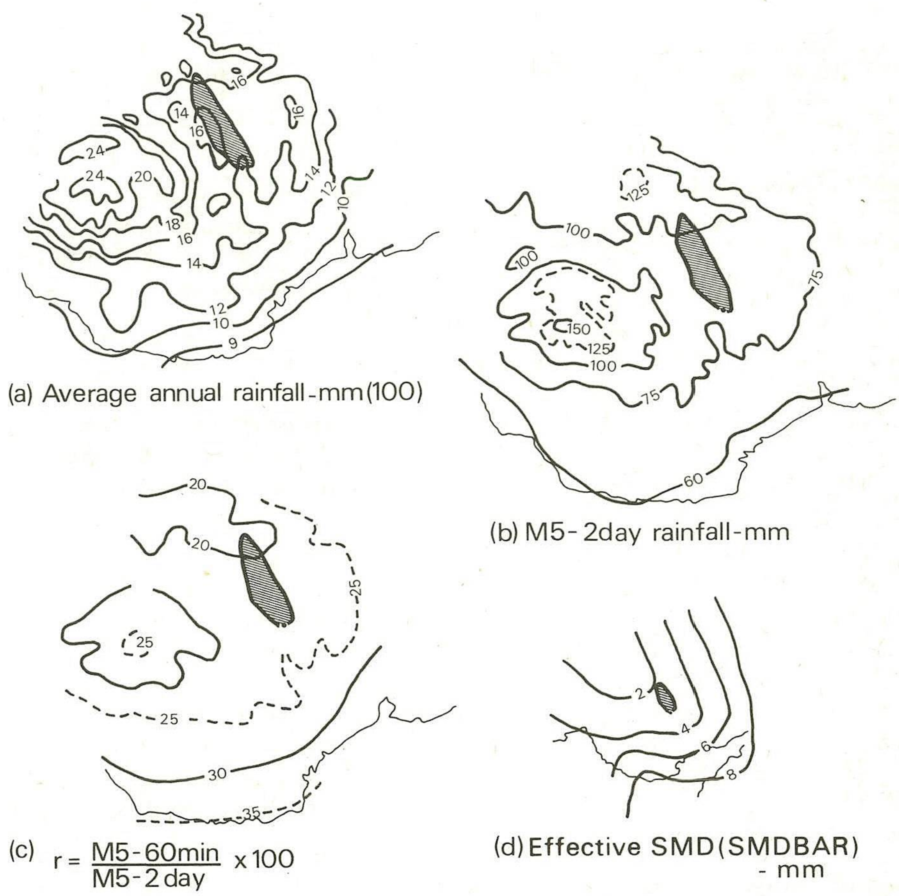


Fig 6.57 Climatic parameters extracted from folded maps—River Rhymney.

are required and these may be obtained by grid point sampling or weighted areas.

The relationship between RSMD and SAAR (standard annual average rainfall) shown in Figure 6.59 allows an initial estimate of RSMD to be made.

The ratio M5–24 hours/M5–2 day is determined, in terms of r, from Table II.3.10 (reproduced here as Table 6.21). M5–24 hours follows and this is converted to M5–1 day by dividing by 1.11 (Table II.3.1). The 1 day areal reduction factor (ARF) is obtained from Figure II.5.1 reproduced here as Figure 6.58.

RSMD = M5–1 day × ARF − SMDBAR

r = 21.5%
M5–24 h/M5–2 day = 80%
M5–2 day = 85.0 mm
M5–24 hours = 68.0 mm
M5–1 day = 61.3 mm
ARF (1 day) = 0.95
M5–1 day × ARF = 58.2 mm
SMDBAR = 3.0 mm
RSMD = 55.2 mm

n (hours) 1	2	4	6	12	24	48
12	18	26	33	49	72	106
15	21	30	37	53	75	106
18	25	34	41	56	77	106
21	28	38	45	60	80	106
24	31	41	48	63	81	106
27	35	44	51	65	83	106
30	38	48	55	68	85	106
33	41	51	57	71	87	106
36	44	54	60	73	88	106
39	47	57	63	75	89	106
42	50	60	66	77	91	106
45	53	63	68	79	92	106

Table 6.21 Ratio M5–n hour/M5–2 day in terms of ratio $r(n = 1 \text{ hour})$—based on Table II.3.10. Ratios are shown as percentages.

Step 4

The urban percentage of the catchment (URB) should be assessed from any convenient map. An estimate of the 'grey' areas on the 1:63 360 map is adequate.

URB = 5%

Step 5

The urban percentage variable is transformed for use in Step 6.

URBT = 1.0 + URB/100

URBT = 1.05

Step 6

The time to peak of the 1 hour unit hydrograph is calculated from Equation 6.18.

$$T_p = 46.6\, L^{0.14} S^{-0.38} \text{URBT}^{-1.99} \text{RSMD}^{-0.4}$$

T_p = 4.1 hours

Figure 6.60 is a nomogram for this equation.

If a direct estimate of LAG is available, a better estimate of T_p follows from Equation 6.14.

In the example catchment, a stage recorder has been installed at the outlet to act as a flood warning station. There

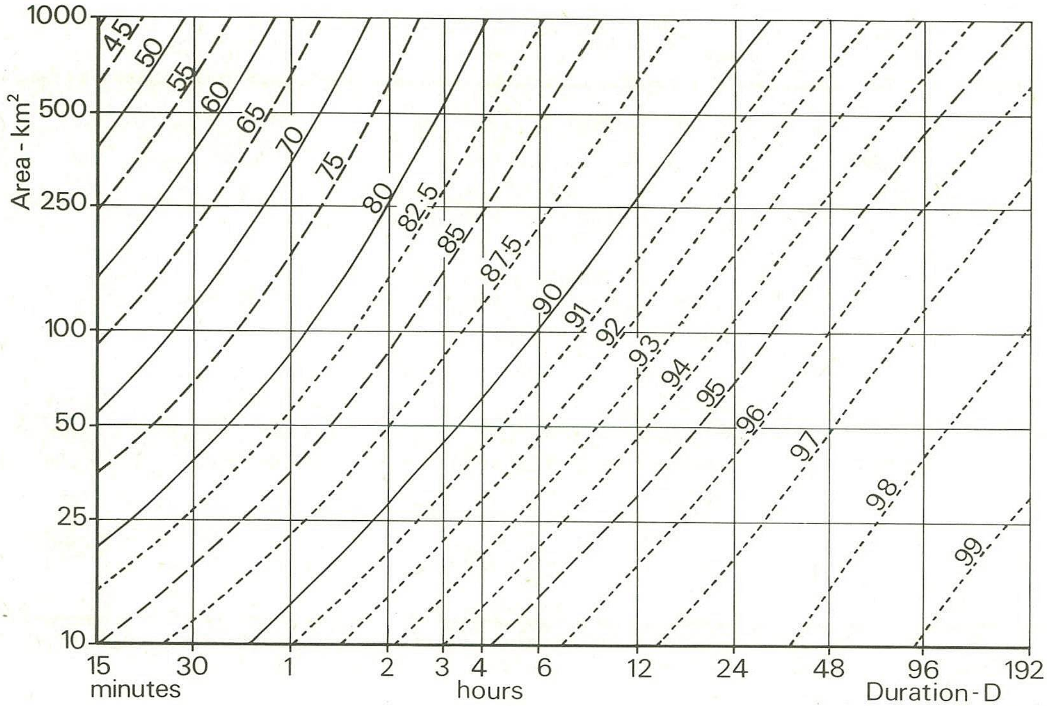
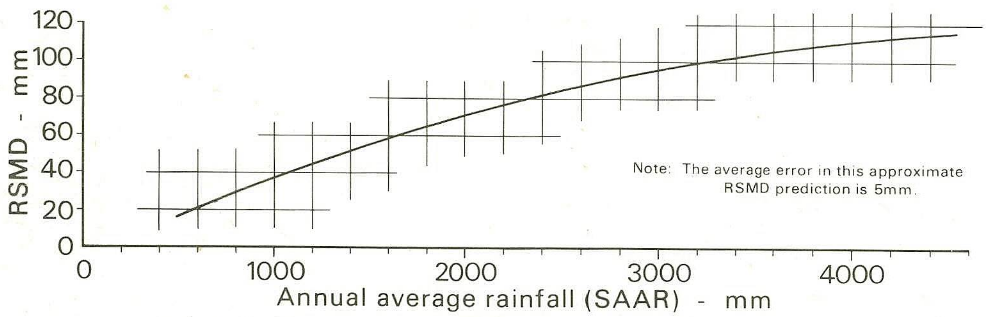


Fig 6.58 Areal reduction factor—part reproduced from Figure II.5.1.

Fig 6.59 Relationship between RSMD and SAAR.



T_p = 0.9 LAG

If an improved estimate is not available, it is advisable to refer to Figure 6.23 to see if T_p is badly predicted locally. If so, a correction to predicted T_p might be indicated.

Although catchment shape and overland slope factors did not enter the T_p regressions it is, of course, better to ensure that neighbouring catchments are of broadly similar configuration before applying such corrections.

is no rating curve relating stage and discharge but, with several recording raingauges in the vicinity, it is possible to estimate the lag of the catchment (Section 6.4.2) as 2.3 hours.

T_p = 2.1 hours

In the example it will be seen that, on three adjacent gauged catchments, T_p is underestimated by Equation 6.18. Figure 6.3 shows that the nearest catchment is No. 56/5, the Afon Llywd at Ponthir, and Table 6.13 shows that its predicted T_p is 4.0 hours compared with the observed 4.1 hours. The T_p prediction for the Rhymney might have been

Because analysis was based on catchments without lake storage, the presence of lakes or reservoirs may require a modification to T_p (see also 7.4.2).

modified if there had been a marked difference between observed and predicted values for the Afon Llywd.

Step 7

The basic data interval, T hours, should be chosen such that

$T \simeq T_p/5$

but T should also be some convenient number of hours or fraction of an hour.

$T = 0.5$ hour

Step 8

The value of T_p must be adjusted for application to the T hour unit hydrograph rather than the 1 hour unit hydrograph. This may be done in the same way that unit hydrographs were all standardised to 1 hour in the analyses (Section 6.4.8).

new T_p = old $T_p + (T-1)/2$

$T_p = 1.9$ hours

Step 9

The recommended (Section 6.7.6) design storm duration is obtained from Equation 6.46,

$D = (1.0 + \text{SAAR}/1000)T_p$

but it is convenient to take the nearest odd integer multiple of T.

$D = 4.5$ hours

In the case of a spillway design, where it is the reservoir outflow peak which is to be synthesised, it is suggested that the design storm duration should be increased by first adding the reservoir lag to T_P.

Step 10

The required return period of the flood must be decided at this stage.

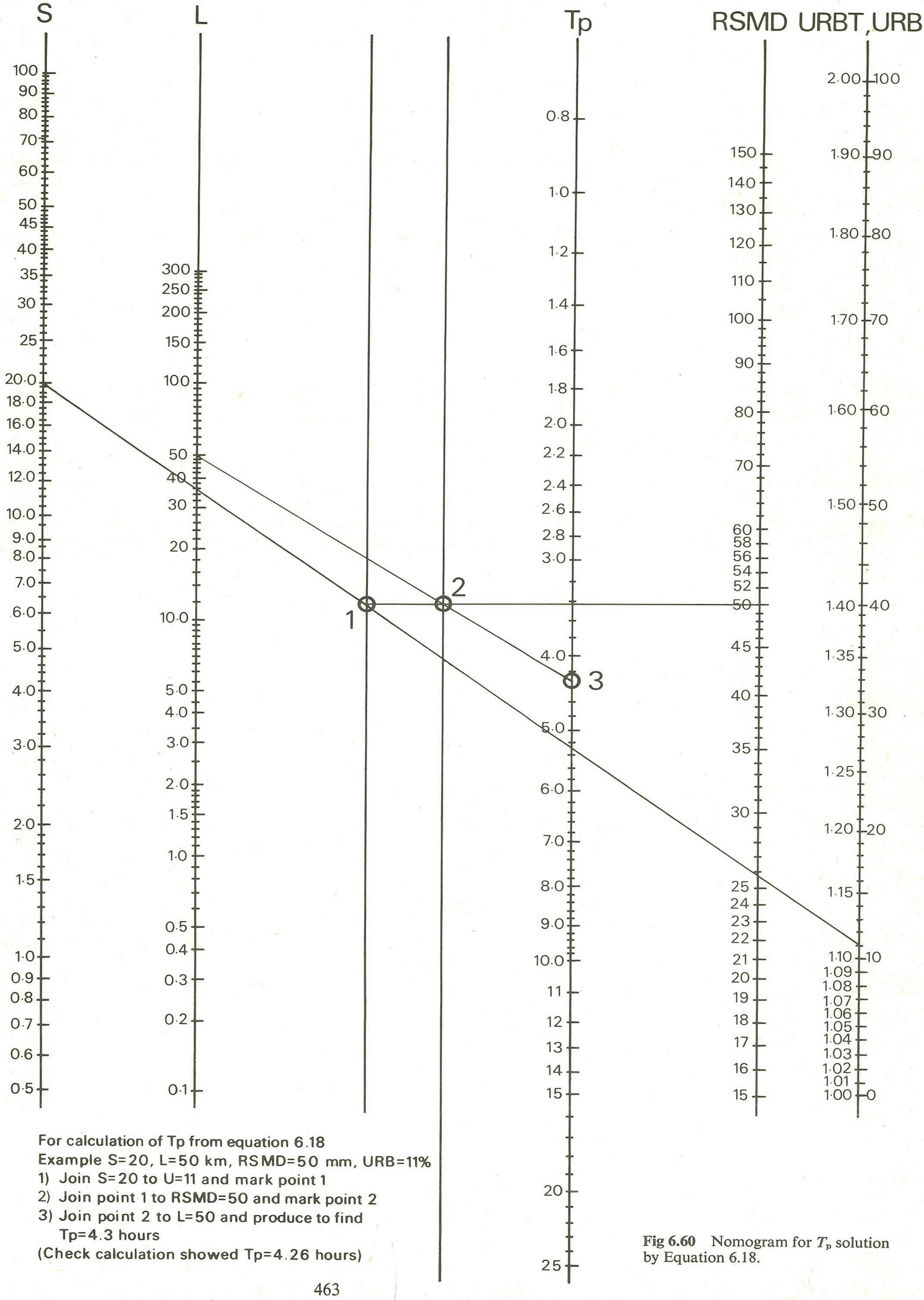
say, 1000 years

The recommended storm return period (Section 6.7.6) is obtained from Figure 6.61 (based on Figure 6.54). For return periods of 1000 years or more, it is suggested that the return period of storm (SRP) and flood can be equated.

SRP = 1000 years

Step 11

The particular combination of figures and tables required for calculating the design rainfall depends mainly on the duration but this will usually be less than 48 hours for which the following applies.



For calculation of Tp from equation 6.18
Example S=20, L=50 km, RSMD=50 mm, URB=11%
1) Join S=20 to U=11 and mark point 1
2) Join point 1 to RSMD=50 and mark point 2
3) Join point 2 to L=50 and produce to find Tp=4.3 hours
(Check calculation showed Tp=4.26 hours)

Fig 6.60 Nomogram for T_p solution by Equation 6.18.

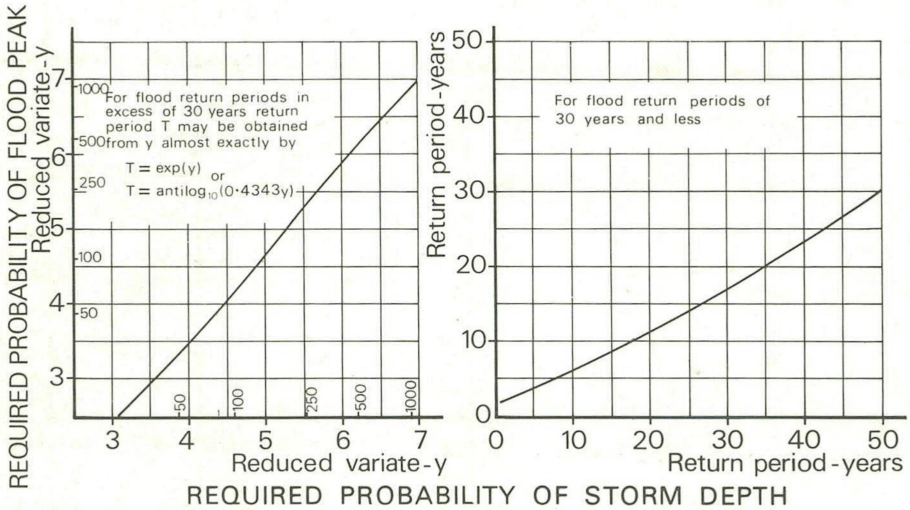


Fig 6.61 Recommended storm return period to yield flood peak of required return period by design method.

From Table II.3.10 (reproduced here as Table 6.21) the ratio M5(*D*)/M5–2 day may be extracted in terms of *r* (found in Step 3).

M5–4.5 hours/M5–2 day = 40.1%

This ratio is applied to M5–2 day, which has also been calculated in Step 3, to give M5(*D*).

M5(4.5 hours) = 34.1 mm

From Table II.2.7, partly reproduced here as Table 6.22, the growth factor M*T*/M5 is assessed for the storm return period and hence M*T* is estimated.

Growth factor = 3.17
M1000 = 108 mm

(Table II.2.7 applies to England and Wales, Table II.2.9 to Scotland and N. Ireland.)

This is a point rainfall estimate and is reduced to a catchment average estimate by applying an areal reduction factor obtained from Figure 6.58.

ARF = 0.91

The storm return period rain in *D* hours over the catchment = ARF × M*T* = *P*

P = 98.3 mm

M5 (mm)	Return period (years)			
	10^2	10^3	10^4	Est. max.
25	2.01	3.37	5.67	7.58
30	1.97	3.27	5.41	7.18
40	1.89	3.03	4.86	6.33
50	1.81	2.81	4.36	5.58
75	1.64	2.37	3.43	3.95
100	1.54	2.12	2.92	3.26
150	1.45	1.90	2.50	2.59
200	1.40	1.79	2.30	2.30
500	1.27	1.52	1.63	1.63

Table 6.22 Some rainfall growth factors for England and Wales (derived from parts of Tables II.2.7 and II.4.2).

Step 12

The recommended antecedent catchment condition (Section 6.7.6) is expressed by the design CWI which is read from Figure 6.62 (based on Figure 6.44 and Equation 6.43).

CWI = 125

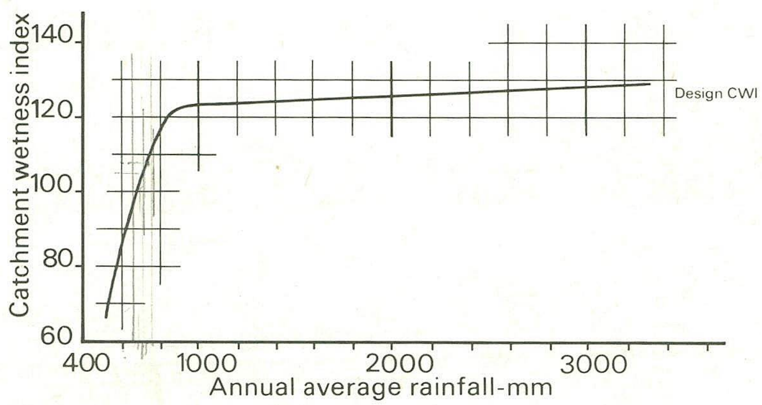


Fig 6.62 Recommended design values for catchment wetness index.

Step 13

The soil index (SOIL) is derived from Equation 6.33

SOIL = $(0.15\,S_1+0.30\,S_2+0.40\,S_3+0.45\,S_4+0.50\,S_5)/(1-S_u)$

as described in Section 6.5.7.

The fractions of the catchment occupied by the various soil classes S_1 to S_5 and the unclassified fraction S_u are determined from Figure 4.18.

Figure 6.63

$S_1 = 0.0$, $S_2 = 0.0$, $S_3 = 0.0$,
$S_4 = 0.2$, $S_5 = 0.8$, $S_u = 0.0$,
SOIL = 0.49

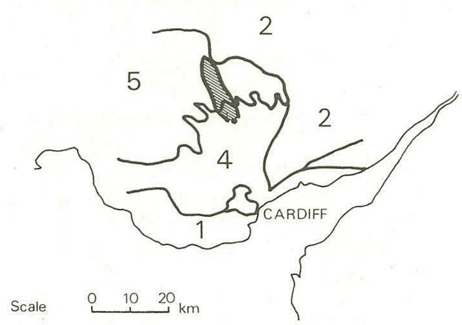


Fig 6.63 Soil classes—River Rhymney.

Step 14

The standard percentage runoff (SPR) is calculated from

SPR = 95.5 SOIL + 0.12 URB

SPR = 47.4%

The percentage runoff appropriate to the design event is calculated as

PR = SPR + 0.22 (CWI − 125) + 0.1 $(P-10)$.

(Equation 6.40 from Section 6.5.8).

PR = 56.2%

At this point runoff data from similar areas nearby may be used to modify the estimate.

For the example catchment, Figure 6.30 shows that percentage runoff on adjacent catchments is generally overestimated. Table 6.13 gives more detail. In addition, a recently established gauging station on the lower Rhymney has enabled some events to be studied and, for these, it was found that percentage runoff was, on average, overestimated by 9%, i.e. $PR_{est} - PR_{obs} = 9$.

It was decided to reduce the design value accordingly. PR = 47.2%

Step 15

If the peak flow is all that is required it may be determined directly from Figure 6.64. The design duration and T_p are divided by the basic data interval, T. With the recommended design storm profile, the peak flow (response runoff only) per unit area

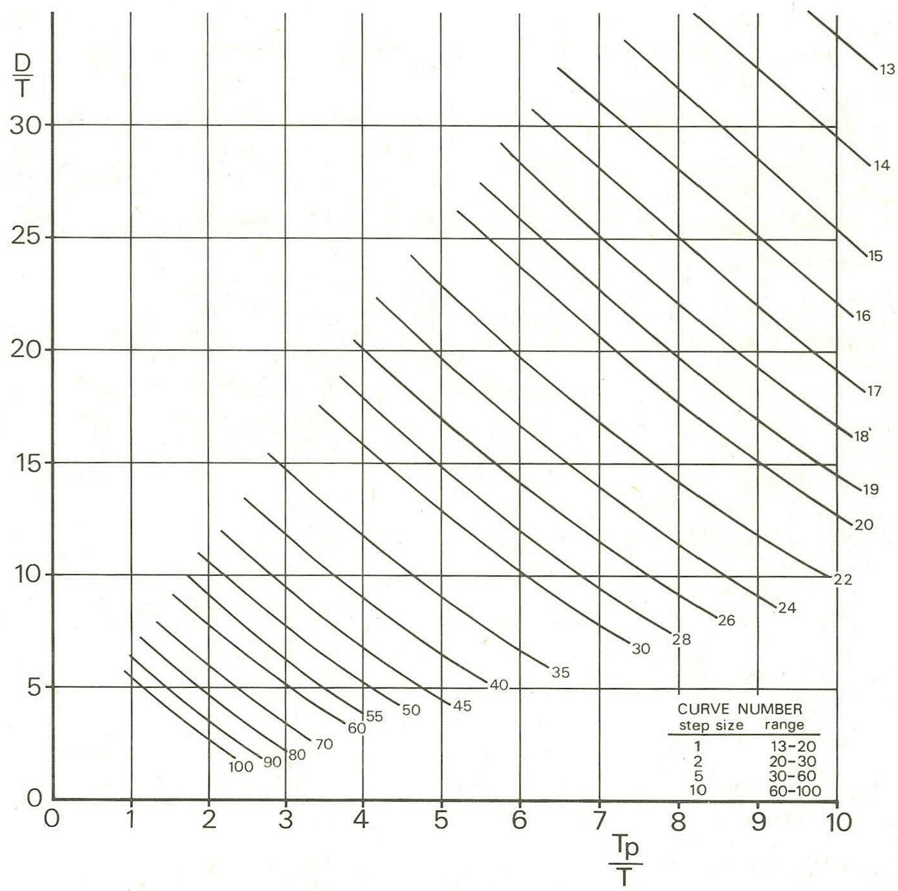


Fig 6.64 Diagram for direct determination of peak of synthesized response flow hydrograph.

per unit (net) rainfall is dependent on D and T_p only. The peak flow is derived from the curve number (CN) of Figure 6.64 thus:

$$\hat{q} = \frac{\text{CN} \times \text{AREA} \times P \times \text{PR}}{10^5 \times T}$$

$D/T = 9$
$T_p/T = 3.8$
$\text{CN} = 41$
$\hat{q} = 241$ cumecs

Steps 16 to 18 are needed to describe the complete hydrograph. Otherwise go to Step 19.

Step 16

The rainfall P is distributed over duration D according to the 75% winter profile of Table II.6.3. This is reproduced here, in graphical form, as Figure 6.65.

In the example case, T is 0.5 hour and D is 4.5 hours; the increments at 0.5 hour intervals should therefore be read from Figure 6.65 as differences between successive multiples of 11.1% of the duration. Table 6.23 gives the design storm profile and, after applying the constant percentage runoff PR, the net rain profile.

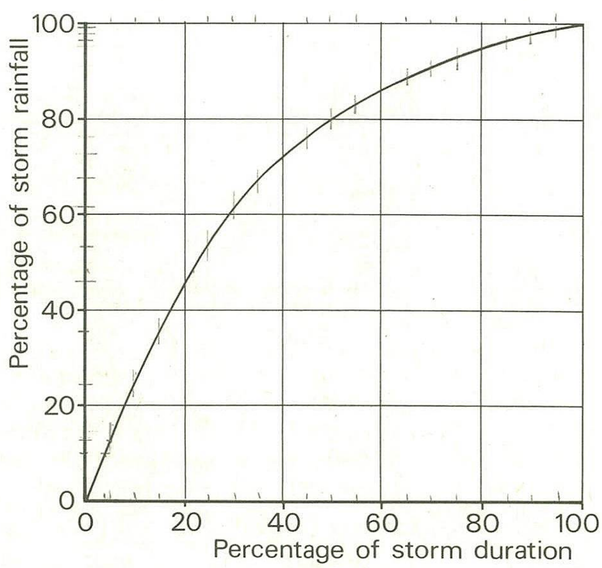


Fig 6.65 Recommended design storm profile—the 75% winter.

Table 6.23 Design storm and net rain distribution for 1000 year flood—River Rhymney at Gilfach Bargoed (interval length = 0.5 hour).

	Percentage of duration					Interval	Total rain (mm)	Net rain (mm)
	100	77.7	55.5	33.3	11.1			
						1	3.45	1.6
						2	4.90	2.3
						3	9.35	4.4
Rain %	100	93	83	64	26	4	18.65	8.8
Increment %	7	10	19	38	26	5	25.60	12.1
mm	6.9	9.8	18.7	37.3	25.6	6	18.65	8.8
						7	9.35	4.4
						8	4.90	2.3
						9	3.45	1.6

Step 17

The simple triangular unit hydrograph may be drawn

$Q_p = 220/T_p$ cumecs/100 km^2

Time base $= 2.52 \times T_p$

$Q_p = 116$ cumecs/100 km^2

TB = 4.8 hours

Figure 6.66 shows the synthetic triangular unit hydrograph for the Rhymney catchment.

Alternative unit hydrographs may be substituted at this point if there are some suitable data from the catchment.

No such data were available for this catchment.

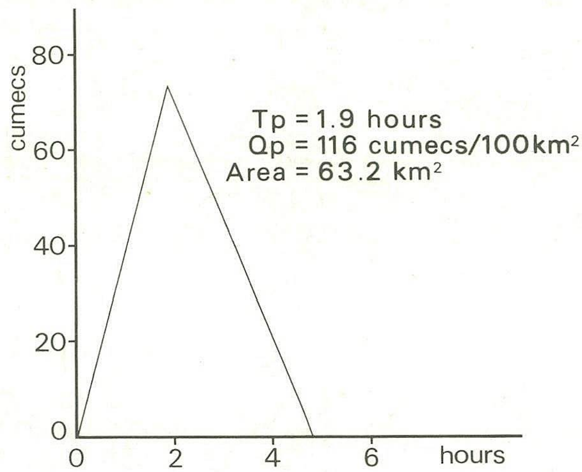


Fig 6.66 Synthetic triangular unit hydrograph—River Rhymney.

Step 18

The convolution of the unit hydrograph with the net rainfall pattern may be set out as a table.

The half-hourly ordinates of the half hour unit hydrograph are multiplied by AREA/100 to correct for the catchment area and are set out in the first row of the table. The left hand column shows the net rain increments divided by 10. (The unit hydrograph is for *10* mm of rain.) The unit hydrograph ordinates are all multiplied by the first net rain increment and the results entered directly below. They are then multiplied by the second increment and the results displaced one column to the right and so on. The column sums give the response runoff hydrograph. As the profile is symmetrical and the unit hydrograph a simple triangle, the peak of the response runoff hydrograph always occurs in the column

Table 6.24 illustrates the computation for the example catchment.

where the peak unit hydrograph ordinate is multiplied by the biggest net rain increment. Because of this, it was possible to provide Figure 6.64 for direct evaluation of the peak alone.

Table 6.24 Convolution of unit hydrograph and net rain profile—River Rhymney.

Net rain (cm)	Unit hydrograph (cumecs)										
	0	19	39	58	71	58	46	33	21	9	
0.16	0	3.0	6.2	9.3	11.4	9.3	7.4	5.3	3.4	1.4	—
0.23	—	0	4.4	9.0	13.3	16.3	13.3	10.6	7.6	4.8	2.1
0.44	—	—	0	8.4	17.2	25.5	31.3	25.5	20.3	14.5	9.2
0.88	—	—	—	0	16.7	34.3	51.0	62.5	51.0	40.5	29.0
1.21	—	—	—	—	0	23.0	47.2	70.2	85.9	70.2	55.7
0.88	—	—	—	—	—	0	16.7	34.3	51.0	62.5	51.0 etc.
0.44	—	—	—	—	—	—	0	8.4	17.2	25.5	31.3
0.23	—	—	—	—	—	—	—	0	4.4	9.0	13.3
0.16	—	—	—	—	—	—	—	—	0	3.0	6.2
Total 4.63	0	3.0	10.6	26.7	58.6	108.4	166.9	216.8	240.8	231.4	197.8

Step 19

The average nonseparated flow per km^2 is calculated from Equation 6.42.

$$\text{ANSF} = 0.00033\,(\text{CWI} - 125) + 0.00074\,\text{RSMD} + 0.003$$

This is added to the response runoff hydrograph.

ANSF = 0.045 cumecs/km^2

i.e. = 2.9 cumecs

Thus the 1000 year design flow for the example catchment is estimated by the unit hydrograph/loss method as 244 cumecs and the complete hydrograph is also obtained.

6.8.3 *The 'probable maximum' flood*

The concept of a probable maximum flood has attracted much controversy in the past. Opinion is still broadly divided between those who can visualise an upper limit to flow rates which can be estimated on physical grounds and those who prefer to retain statistical terminology and refer to an exceedance probability approaching zero. In practice, the apparently sharp division is blurred: in the rainfall studies (Volume II) the approach has been based largely on physical maximisation of recorded storms but these have been supported by limiting growth factors obtained from an examination of all recorded falls. In this way, the estimated maxima can be coupled with a return period which varies with storm duration and location. Although the association of a probable maximum event (theoretically near-zero exceedance probability) with a finite risk seems contradictory, the method provides realistic estimates of rainfall for use in extreme flood design.

For those who are familiar with American procedures and nomenclature it should be pointed out that no attempt is made here to distinguish between 'probable maximum' and 'standard project' floods (U.S. Army Corps of Engineers) or to specify lesser floods by varying assumptions regarding catchment condition (U.S. Bureau of Reclamation). It is assumed that, for nearly all design purposes in the United Kingdom, engineers will prefer to specify an exceedance probability but there will often be a need to examine the extreme flood and it is then appropriate to use the maximum rainfall estimates of Volume II, Chapter 4.

The purpose of this section is to discuss the antecedent conditions, storm profile and unit hydrograph which should be recommended for use with an extreme design rainfall. It is a subject of some difficulty; large expenditures and grave responsibilities are involved in an area of speculation to an extent which usually forces the adoption of ultra conservative assumptions. While recognising that the design engineer will exercise his own judgement in a particular case, the following recommendations are intended to comprise a realistic approach to extreme flood design based on the present understanding of the processes involved.

(*a*) *Spatial variation of rainfall.* Absolute maximisation requires that spatial variation of rainfall should be allowed and be matched with the distribution of soil classes on the catchment in order to maximise runoff production. However, the scale of this effect is controlled by the way in which the areal reduction factor (Figure 6.58) varies with area and duration and, in general, the effect is likely to be so small that the complications of separate subcatchments and unit hydrographs are not justified. In extreme cases, such as were discussed in Section 6.5.12, where there are urban areas at the bottom end of the catchment, it would be advisable to consider the responsive area alone as well as the catchment as a whole.

(*b*) *Storm movement and the unit hydrograph.* The direction of storm movement was ignored in analysis for reasons outlined in Section 6.3.3; it is one of the several sources of variation which could help explain the scatter of T_p values on a catchment. Inasmuch as storms moving downstream will produce unit hydrographs with smaller T_p values, the effect can be allowed for in an extreme design by using a unit hydrograph which is peakier than predicted by Equation 6.18. Over all catchments, the average ratio of minimum observed T_p to mean T_p is 0.67. Pending a more detailed study of the effect of storm movement on different catchments, it is recommended that this ratio be applied to predicted or recorded T_p values for use in extreme flood computation. Although this adjustment is associated here with a real source of variation (storm movement), it can also be viewed as an attempt partly to satisfy doubts arising from the tests on very large events (Section 6.6.3). Despite the error of prediction associated with Equation 6.18 itself, no further reduction in T_p is suggested unless there are data from or close to the design catchment.

(*c*) *Duration and profile of a maximum rainfall.* American practice is to distinguish between 'probable maximum precipitation' which is derived from a depth–duration set of design values based on all storm types and a 'probable maximum storm' being the maximisation and possible transposition of a single storm type. They specifically exclude the possibility of a storm being so structured that it could produce maximum values for all durations and all areas (Miller & Clark, 1965).

It is suggested that this exclusion cannot be applied in this country. Maximum rain intensities observed in the United States are several times greater than ours and there have been occasions, in the United Kingdom, when relatively severe localised thunderstorm activity has been associated with more general rain. Consequently, it is recommended that, in this country, a profile should be adopted which contains the estimated maximum fall in every duration centred at the peak of the storm profile. This profile lies between the 90 % winter profile (Figure II.6.2), and the 90 % summer profile (Figure II.6.1). The probability that an estimated maximum storm could be structured in this way varies with duration and location. It is more likely for storm durations of 12 hours or less in eastern Britain than for longer durations in the west. However, more investigation is needed before such generalisations can be translated into firm recommendations to relax the 'worst profile' assumption in some circumstances. The profile would be symmetrical and, on this basis, the 'critical' duration would be infinite but, for practical purposes, it is suggested that the same duration be used as recommended in Section 6.8.2 and that the effect of the preceding rains be taken into the calculation of a design value for CWI as described in paragraph (*e*) below.

(*d*) *Snowmelt.* Flood events which are predominantly the result of snowmelt are discussed in Chapter 7. The intention here is to discuss the possibility of snowmelt taking place during the estimated maximum rainstorm. Nowhere is the difficulty of defining the extreme flood more apparent. If the estimated maximum rainfall for a certain catchment is associated with a return period of 10 000 years, the chances of the 100 year snowmelt occurring in the same year are 1 in a million assuming independence and the chance of the two events occurring on the same *day* is even smaller. A physically derived maximum snowmelt rate has not been specified in Chapter II.7 but it is suggested there that 42 mm/day is a suitably rare value for design purposes. Although the chance of the maximum rainstorm and the maximum snowmelt occurring together can be regarded as near zero, in some design situations it cannot be ignored.

With large catchments having long times to peak and hence critical durations longer than 24 hours, it is necessary to see whether there is a possibility of sufficient snow lying to sustain the maximum melt rate for the full period. Figure II.7.2 shows the average annual maximum snow depth exceeded once in 2 years. Section II.7.4.1 suggests that the annual maximum depth of snow expected on average once in a 100 years is about 7.5 times the 2 year depths. Using an average density of 0.13 g/cm^3 (Section II.7.4.2), Figure II.5.2 can therefore be interpreted as an approximate guide to the 100 year depth of water equivalent. It is recommended that this combination of 42 mm/day snowmelt applied to the 100 year depth is a suitably rare occurrence for design purposes particularly when it is remembered that it is also being combined with a maximum rainstorm. Thus, in a design case, a melt of 42 mm/day could continue for one day anywhere in the United Kingdom but only in parts of Scotland and Northern England, where the 100 year water equivalent exceeds 210 mm, could it last for more than 5 days. According to the location therefore, Figure II.7.2 provides a design value of water equivalent which, when divided by 42, gives the number of days for which the 42 mm/day melt rate could continue. It can be applied as an additional rainfall occurring uniformly in time for this number of days and should be centred around the centre of the storm.

(*e*) *Antecedent conditions.* (*i*) No snowmelt. The choice of a design duration and profile was discussed in paragraph (*c*). It is suggested that estimated maximum rainfall should be assumed to fall in *all* durations centred on the peak of the storm profile. The same assumption can be used to find a suitable value for CWI at the start of the design duration (D) as calculated from Equation 6.46. Let the estimated maximum rain in D hours be P_D mm and, in $5D$ hours, P_{5D} mm. Half of the difference will occur in the first $2D$ hours of the $5D$ period and it may be assumed uniform. The CWI at the start of the $5D$ period can be taken at 125 (i.e.

SMD = API5 = zero) and its value at the end of the first $2D$ hours (the 'preceding rains') may be calculated. It would rarely exceed 180. Higher values of initial CWI have been observed but, if applied here, the implication would be that rainfall exceeding the estimated maximum was occurring over a longer duration.

(*ii*) Snowmelt assumed. When some snowmelt is assumed to occur with the estimated maximum rainfall it is logical to assume also that it could be taking place throughout the period of preceding rains; this will cause the design CWI to be slightly higher. The quantity of melt which would take place during the preceding period may be limited by the available snow depth as described in paragraph (*d*) above.

It is necessary to mention the possibility of 'frozen ground'. The March 1947 flood (snowmelt and rain) is commonly believed to have been aggravated by the long spell of cold weather which preceded it and which might have frozen the top layers of soil. If it is thought appropriate to make some allowance for this effect, a convenient way of doing so is to adjust the soil index to its maximum value of 0.5. Whether or not the allowance should be made must remain a matter for the judgement of the engineer. Extreme quantities of snowmelt and rainfall are already being combined and the rainfall is being distributed in time with the worst profile. However, pending a more detailed study of the variation between summer and winter estimated maxima for rainstorms of various durations, it is wise not to regard even the occurrence of an extreme thunderstorm over a frozen catchment with deep snow lying as physically impossible.

The procedure for estimating the 'probable maximum' flood on the West Lyn at Lynmouth will now be described. Steps 1–9 are worked in the same way as in Section 6.8.2. The specific numerical values for these steps are:

Area	= 23.5 km^2	SMDBAR	= 6.0 mm
Stream length	= 9.2 km	RSMD	= 55.0 mm
Stream slope	= 29.7 m/km	URB	= 0.0
SAAR	= 1500 mm	URBT	= 1.0
r	= 25%	T_p	= 3.1 hours
M5–24 hours/M5–2 day	= 82.1%	modified for	
M5–2 day	= 85.0 mm	'maximum' study	
M5–24 hours	= 69.8 mm	(×0.67)	= 2.1 hours
M5–1 day	= 62.9 mm	T	= 0.2 hour
ARF (1 day)	= 0.97	new T_p	= 1.7 hours
M5–1 day × ARF	= 61.0 mm	D	= 4.2 hours

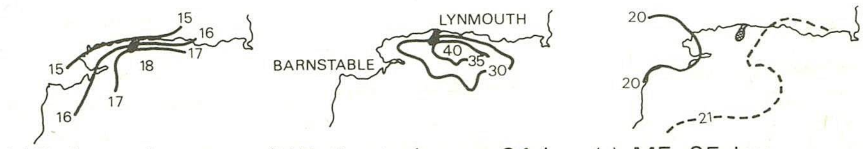
To avoid confusion with the normal procedure, subsequent step numbers will have the letter M after the number. The calculations for estimated maxima of various durations are based on Chapter 4 of Volume II.

Step 10M

Estimate the catchment average 2 hour and 24 hour maximum point rainfalls from Figures II.4.1 and and II.4.2 respectively.	Relevant parts reproduced in Figure 6.67. 2 hour EM 160 mm 24 hour EM 300 mm

Step 11M

Estimate ratio M5–25 days/SAAR from Figure II.3.4. Hence estimate M5–25 days.	Figure 6.67 M5–25 days/SAAR = 20.6% M5–25 days = 309 mm
From Table II.4.2 (Table 6.22)	Table 6.22



(a) Estimated max. 2-hr rainfall, 10's of mm (b) Estimated max. 24-hr rainfall, 10's of mm (c) $\frac{\text{M5-25day}}{\text{SAAR}} \times 100$

Fig 6.67 Climatic parameters extracted from folded maps—River West Lyn.

extract the relevant growth factor for the estimated maximum 25 day fall.

From Table II.4.3 find ratio of maximum 192 hours to that for 25 days and hence estimated 192 hour maximum. Relevant parts reproduced as Table 6.25a.

Also from Table 6.25a use the given ratios to calculate estimated maxima for durations between 24 and 192 hours.

From Table II.4.1 (Table 6.25b) calculate estimated maxima for durations less than 2 hours.

Growth factor = 1.92
EM 25 days = 593 mm
EM 192 hours/EM 25 days = 0.69
EM 192 hours = 409 mm

EM 48 hours = 336 mm
EM 72 hours = 351 mm
EM 96 hours = 366 mm

EM 10 min = 56.0 mm
EM 15 min = 73.6 mm
EM 30 min = 102.4 mm
EM 60 min = 129.6 mm

Table 6.25 Estimated rainfall maxima for various durations.

Average annual rainfall (hundreds of mm)	Ratio 192 hours/25 days	(a) Durations > 24 hours Ratio of stated duration (hours) to 24 hours			(b) Durations < 2 hours % of stated duration (min) to 2 hour						
		48	72	96	1	2	5	10	15	30	60
5– 6	0.84	1.10	1.13	1.17							
6– 8	0.80	1.10	1.13	1.17							
8–10	0.76	1.10	1.14	1.18	6	11	23	36	47	65	83
10–14	0.71	1.11	1.16	1.20							
14–20	0.68	1.12	1.18	1.24	6	11	22	34	45	62	79
20–28	0.65	1.14	1.23	1.32							
28–40	0.62	1.20	1.31	1.42	6	10	21	32	43	59	75
40–	0.60	1.23	1.35	1.48							

Finally plot the amount for various durations on double log paper. For durations between 2 and 24 hours a straight line is suggested. For the longer and shorter durations a smooth curve should be drawn.

For each duration, a different areal reduction factor will apply (Figure II.5.1 or Figure 6.58).

The areal rainfall (P) for the design duration is read from the figure.

(This full procedure is, of course, unnecessary on any particular catchment; duration of either 25 days or 10 min could be relevant but not both. It is illustrated here only for the sake of completeness.)

Figure 6.58 shows the estimated maximum point rainfall for various durations on the Lynmouth catchment.

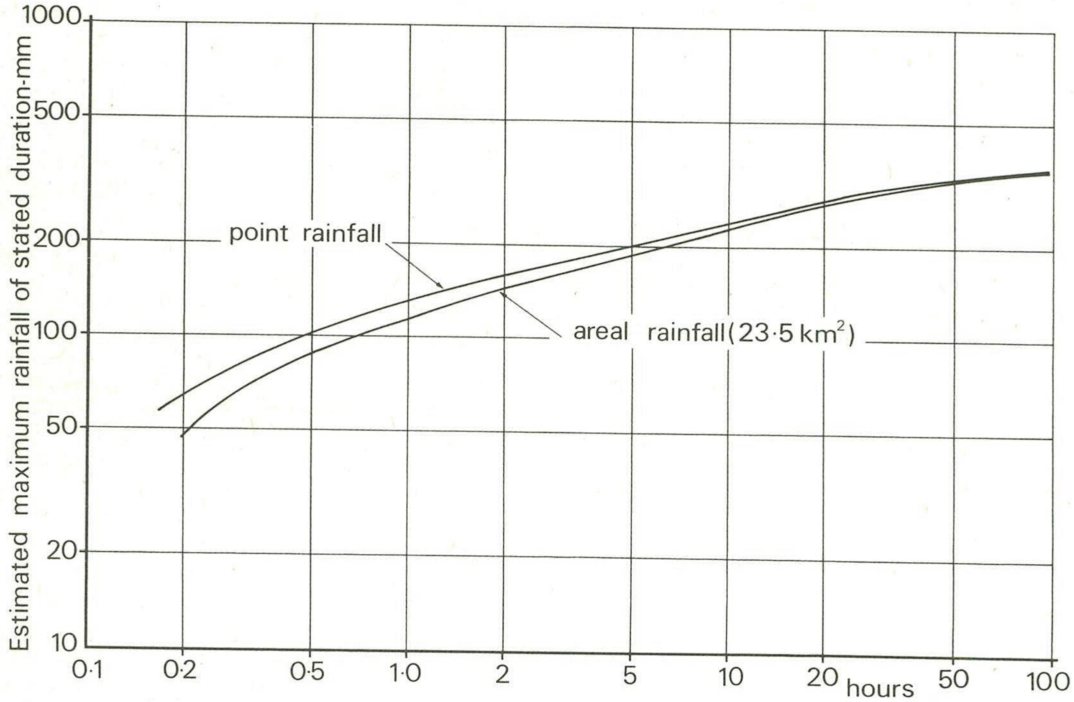
Catchment average estimates of the maxima are shown in the lower line of Figure 6.68. P for 4.2 hours = 180 mm

The snowmelt allowance may be added at this stage.

At 42 mm/day, the total snowmelt in the design duration amounts to 7 mm.

Total design precipitation = 187 mm

Fig 6.68 Estimated maximum falls for various durations—River West Lyn.



Step 12M

The design value for CWI is based on the assumption that the D hour storm occurs in the middle of the $5D$ hour storm.

It is therefore assumed that half the difference between the D hour and $5D$ hour maxima may fall in $2D$ hours preceding the design storm.

The CWI at the beginning of the $5D$ hour period is taken as 125. The rain plus snowmelt (Pa) falling in the next $2D$ hours is assumed to be uniform (rectangular profile) and the design CWI can be approximated as $125 + Pa \times 0.5^{D/24}$

The estimated maximum for 21 hours is read from Figure 6.68.

EM 21 hours = 280 mm

50 mm in 8.4 hours

The snowmelt allowance is added = 14 mm in 8.4 hours

Pa = 64 mm

CWI = 182

Steps 13 and 14 are the same as described in Section 6.8.2.

SOIL would be increased to 0.5 if 'frozen ground' was to be considered.

$S_1 = 0.0, S_2 = 0.6, S_3 = 0.0, S_4 = 0.0, S_5 = 0.4, S_u = 0.0,$ SOIL = 0.38, SPR = 36.3%, PR = 66.5%

In this example frozen ground has not been considered. If the soil index had been increased, the percentage runoff would be 78% rather than 66.5% and the peak flow would be about 330 cumecs.

Step 15 does not apply.

Step 16M

The estimated maximum rainfall is to be symmetrically distributed according to the extreme profile such that the estimated maximum occurs in every duration centred on the peak of storm profile.

Referring to Figure 6.68 (the numerical example) there are 48 mm in T (0.2) hours; 95 mm in $3T$ (0.6) hours; 114 mm in $5T$ (1.0) hours etc. The hyetograph is built up as shown in Figure 6.69; the net rain hyetograph is also shown

The snowmelt allowance is added as a uniform rate

Snowmelt allowance 0.33 mm/0.2 hour

Steps 17, 18, 19 are the same as described in Section 6.8.2.

Q_p = 129.4 cumecs/100 km^2

TB = 4.29 hours

ANSF = 0.060 cumecs/km^2 = 1.4 cumecs

Hydrograph shown in Figure 6.69

Peak = 280 cumecs (estimated maximum flow, River West Lyn at Lynmouth).

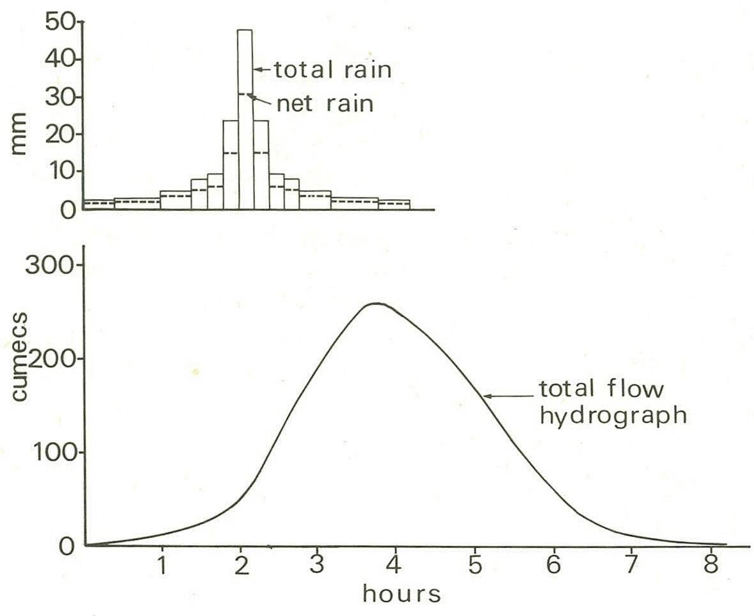


Fig 6.69 The estimated maximum event—River West Lyn.

6.8.4 Unit hydrograph derivation

If adequate rainfall and river flow records exist at the design site or somewhere near it on the same river, it is recommended that a number of unit hydrographs be determined. A minimum sample of five events is suggested but it is more important to use only those events where the rainfall distribution is either spatially uniform or at least typical of the topography.

It can sometimes be misleading to concentrate on clean looking isolated hydrographs; they may arise from small quantities of runoff perhaps originating uncharacteristically from only a part of the catchment.

To facilitate comparison with the results of this study it is suggested that the same method of analysis be employed as far as possible. The Institute of Hydrology is able to carry out specific studies if required, but for those wishing to perform their own analyses a brief review of the procedure is presented here. It is not proposed to repeat all the details given in Sections 6.3 and 6.4 but it may be helpful to outline the steps in the procedure and to clarify certain points.

Method of net rain separation. The method used in the analysis was based on the concept of a loss rate curve. Later developments provided grounds for the belief that a percentage based method of separation was more appropriate. This was tested in analysis and preferred for the reasons given in Section 6.5. It is suggested that, for equally successful application to a wide range of antecedent conditions, the percentage runoff should be assumed to increase with CWI through the storm. The procedure for this computation is as follows:

1 Separate response runoff as described in 6.4.3.

2 Tabulate the rainfall increments and calculate the CWI at each interval according to the definitions given in 6.4.4. API5 increases through the storm thus

$$\text{API5}_t = P_{t-1} \times 0.5^{T/48} + \text{API5}_{t-1} \times 0.5^{T/24}.$$

3 Multiply each rainfall increment by the CWI applying at the beginning of the interval. Sum these products through the event. Divide total response runoff by this sum to obtain factor F.

4 Multiply each CWI term by F to obtain percentage runoff and then by rain to give sequence of net rain increments.

This is illustrated in a numerical example in Table 6.26 where the SMD at the beginning of the event is 25 mm and no rain has fallen for 6 days so API5 is zero. Response runoff is 42 mm, data interval is 1 hour.

The API5 computation presented in Table 6.26 is consistent with the definition; with uniform rainfall the same answer for API5 would be achieved after 24 individual hourly calculations as after a single 24 hour calculation. However, when T is 1 hour or less, the decay factor ($0.5^{T/24}$) can be replaced by unity without affecting the net rain separation significantly; this greatly simplifies the calculations.

Time at start of interval	Total rain (mm)	SMD (mm)	API5 at start of interval	CWI	Rain × CWI	% runoff	net rain
0	5	25	0	100	500	19.3	1.0
1	18	20	0 + 4.9 = 4.9	110	1980	20.2	3.6
2	9	2	4.8+17.7 = 22.5	146	1314	27.2	2.5
3	23	0	21.8+ 8.9 = 30.7	156	3588	30.1	6.9
4	17	0	29.8+22.6 = 52.4	177	3009	34.2	5.8
5	34	0	50.7+16.7 = 67.4	192	6528	37.0	12.6
6	6	0	65.4+33.4 = 98.8	224	1344	43.2	2.6
7	0	0	96.0+ 5.9 = 101.9	227	0	43.8	0
8	0	0	98.8+ 0 = 98.8	224	0	43.2	0
9	0	0	96.0+ 0 = 96.0	221	0	42.6	0
10	5	0	93.1+ 0 = 93.1	218	1090	42.0	2.1
11	11	0	90.4+ 4.9 = 95.3	220	2420	42.5	4.7
	128				21773		41.8

$F = 42/21773$
$= 0.193 \times 10^{-2}$.

Table 6.26 Example of net rain computation for analysis purposes.

Method of unit hydrograph derivation. The basic data interval, T, will have been chosen to give not less than five ordinates on the rising limb of a typical hydrograph. The response runoff hydrograph is tabulated and, together with the net rain sequence, is presented as data to a unit hydrograph derivation routine (6.4.6). Matrix inversion followed by moving average smoothing was the particular technique used in this study; the different results which would arise from the use of other methods cannot yet be predicted. However, for those who wish to derive unit hydrographs without the benefit of a computer and its software, the Collins method (e.g., Wilson, 1969) is probably the simplest 'desk calculator' method and one which gives results similar to matrix inversion. Alteration of the data timing will be required in some cases (6.4.7) to produce a realistic unit hydrograph. The degree to which this is acceptable will depend on the reliability of the records and particularly the known accuracy of clock mechanisms. Unit hydrographs should be derived from, say, the five largest events with spatially near-uniform rainfall. Their peak values and times to peak should be averaged. The peaks should be aligned and average values calculated for the ordinates on either side. The several unit hydrographs will probably differ markedly but any search for a systematic variation with some measure of storm intensity should be done with caution (6.5.3).

6.8.5 *Application to short term forecasting*

The application of this chapter's results to short term hydrograph forecasting is very much the same as detailed in Section 6.8.2 except that net rain separation should be done as described in Section 6.8.4 instead of assuming a constant percentage runoff. The same applies to the attempted reproduction of recorded events (6.6.1).

6.8.6 *Description of computer program*

A FORTRAN program is presented which does some of the calculations described in Sections 6.8.2 and 6.8.3. Referring to Figure 6.55, it performs the last five steps, 15–19 inclusive. The program calculates and prints out the convolution of net rainfall and unit hydrograph ordinates for a variety of conditions.

Unit hydrograph ordinates can be specified, or calculated from T_p. The rainfall profile can be specified, or calculated from the 75% winter profile. Average nonseparated flow (ANSF) is calculated from standard base flow (SBF) and catchment wetness index (CWI). SBF is simply a name for the component of ANSF which is constant for the catchment, i.e. the second and third terms in Equation 6.42 (step 19 in 6.8.2). Percentage runoff can be made to increase throughout the storm as described in Section 6.8.4. The data format is set out below in the order that it is expected by the program.

Variable	Format	Remarks
TITLE	10A8	80 characters of title for the run
ICASE	I5	Case number; if zero, STOP
CASE TITLE	10A8	80 characters of title of this case.
AREA	7F10.0	Area of catchment (km^2)
T		Data interval (hours)

D		Design duration (hours)
P		Total rain (mm)
SPR		Standard percentage runoff
CWI		Catchment wetness index
SBF		Standard base flow (cumecs/km^2)
NUH	3I5, F5.1	No. of unit hydrograph values If >0, Read in specified UH ordinates = 0, UH ordinates as previous case <0, Use T_p to calculate UH ordinates
NHRF		No. of hourly rainfall values If >0, Read in specified rain profile = 0, Rain profile as previous case <0, Use 75% winter profile
IPRINC		If = 1, percentage runoff to be increased throughout storm If =0, percentage runoff to be constant
T_p		Time to peak of unit hydrograph (only there if NUH <0)
UNIT (J), J = 1, NUH	8F10.0	UNIT (J) is read in if NUH>0 (cumecs/100 km^2)
HRF (I), I = 1, NHRF	8F10.0	HRF (I) is read in if NHRF>0

Repeat from ICASE

Examples: A listing of a typical data set is shown. Five examples are presented:

1 Gilfach Test 1, 75WP, TP: The UH ordinates are calculated using T_p, from equations $Q_p = 220/T_p$, TB = 2.53 T_p, and NUH = INT(TB/T) + 1. Values of the 75% winter profile are incorporated in the program as a DATA statement. 20 values of 'PROF'.

2 Gilfach Test 2, UHB, SPEC. HRF: NUH = 0, thus the UH ordinates from the previous example are used. Seven values of hourly rainfall are read in.

3 Gilfach Test 3, SPEC. UH, 75WP: 21 values of UH ordinates are read in, and the 75% winter profile is used.

4 Lynmouth estimated maximum flood: T_p is used to calculate the UH ordinates, and 21 values of hourly rainfall are read in.

5 As 4 above but with percentage runoff increasing through storm (IPRINC = 1).

The results of applying the program to these data are shown and, finally, a listing of the program itself is included.

```
CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE
   1
GILFACH TEST1, 75WP, TP
      63.2       0.5       4.5       98.3      47.4      125.0     .0454
  -1   -1    0  1 9
   2
GILFACH TEST2, UHB, SPEC.HRF
      63.2       0.5       4.5       98.3      47.4      125.0     .0454
   1    7
        5.       10.       20.       30.       20.       10.        5
   5
GILFACH TEST3, SPEC.UH, 75WP
      63.2       0.5       4.5       98.3      47.4      125.0     .0454
  21   -1
       3.8       7.5   [illegible]   14.4      17.9       21.4      24.8      28.5
      28.3   [illegible]  24.[illegible]  21.9  19.2       17.3      15.0      12.7
      10.4   [illegible] [illegible]  3.5       1.1

LYNMOUTH ESTIMATED MAXIMUM FLOOD
   23.5         [illegible].2  4.2     137.0      36.3      175.0     0.0437
  -     21   0  1 7
       2.8       2.9       3.3       3.4       3.3        5.4       5.3       8.4
       9.8      23.9      48.3      23.9       9.8        8.4       5.3       5.4
       3.3       3.4       3.3       2.9       2.8

AS ABOVE BUT WITH PERCENTAGE RUNOFF INCREASING THROUGH STORM
   23.5         [illegible].2  4.2     137.0      36.3      175.0     0.0437
  -     21   1
       2.8       2.9       3.3       3.4       3.3        5.4       5.3       8.4
       9.8      23.9      48.3      23.9       9.8        8.4       5.3       5.4
       3.3       3.4       3.3       2.9       2.8
```

```
CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE


GILFACH TEST1, 75WP, TP

        AREA (SQ.KM.)                      63.2
        DATA INTERVAL (HR)                  0.5
        DESIGN DURATION (HR)                4.5
        TOTAL RAIN (MM)                    98.3
        PERCENTAGE RUNOFF                  56.2
        ANSF (CUMECS PER SQ.KM.)            0.05
```

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

TIME	TOTAL RAIN MM	NET RAIN MM	UNIT HYDROGRAPH ORDINATE	TOTAL HYDROGRAPH
0.00	3.39	1.90	0.00	2.87
0.50	5.08	2.86	30.47	6.54
1.00	9.07	5.10	60.94	15.70
1.50	18.68	10.50	91.41	34.68
2.00	25.89	14.56	111.81	72.68
2.50	18.68	10.50	91.89	132.03
3.00	9.07	5.10	71.98	201.09
3.50	5.08	2.86	52.06	260.29
4.00	3.39	1.90	32.14	288.97
4.50			12.23	277.54
5.00				237.04
5.50				184.59
6.00				128.33
6.50				76.27
7.00				39.40
7.50				18.87
8.00				8.94
8.50				4.34

CURVATURE AROUND PEAK = -160.412

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

GILFACH TEST2, UHR, SPEC.HRF

AREA (SQ.KM.)	63.2
DATA INTERVAL (HR)	0.5
DESIGN DURATION (HR)	4.5
TOTAL RAIN (MM)	98.3
PERCENTAGE RUNOFF	56.2
ANSF (CUMECS PER SQ.KM.)	0.05

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

TIME	TOTAL RAIN MM	NET RAIN MM	UNIT HYDROGRAPH ORDINATE	TOTAL HYDROGRAPH
0.00	5.00	2.81	0.00	2.87
0.50	10.00	5.62	30.47	8.28
1.00	20.00	11.25	60.94	24.53
1.50	30.00	16.87	91.41	62.43
2.00	20.00	11.25	111.81	131.02
2.50	10.00	5.62	91.89	210.53
3.00	5.00	2.81	71.98	279.38
3.50			52.06	314.25
4.00			32.14	298.98
4.50			12.23	251.49
5.00				189.24
5.50				124.74
6.00				70.04
6.50				32.23
7.00				12.93
7.50				5.04

CURVATURE AROUND PEAK = -200.548

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

GILFACH TEST3, SPEC.UH, 75WP

AREA (SQ.KM.)	63.2
DATA INTERVAL (HR)	0.5
DESIGN DURATION (HR)	4.5
TOTAL RAIN (MM)	98.3
PERCENTAGE RUNOFF	56.2
ANSF (CUMECS PER SQ.KM.)	0.05

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

TIME	TOTAL RAIN MM	NET RAIN MM	UNIT HYDROGRAPH ORDINATE	TOTAL HYDROGRAPH
0.00	3.39	1.90	3.80	3.33
0.50	5.08	2.86	7.30	4.43
1.00	9.07	5.10	10.90	6.72
1.50	18.68	10.50	14.40	11.44
2.00	25.89	14.56	17.90	19.47
2.50	18.68	10.50	21.40	29.79
3.00	9.07	5.10	24.80	41.14
3.50	5.08	2.86	28.50	53.07
4.00	3.39	1.90	28.80	64.98
4.50			26.50	75.95
5.00			24.20	85.45
5.50			21.90	92.06
6.00			19.20	93.93
6.50			17.30	91.26
7.00			15.00	85.77
7.50			12.70	78.79
8.00			10.40	70.97
8.50			8.00	63.03
9.00			5.80	55.12
9.50			3.50	47.13
10.00			1.10	39.07
10.50				31.19
11.00				23.64
11.50				16.65
12.00				10.77
12.50				6.74
13.00				4.55
13.50				3.49
14.00				3.00

CURVATURE AROUND PEAK = -18.183

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

LYNMOUTH ESTIMATED MAXIMUM FLOOD

AREA (SQ.KM.)	23.5
DATA INTERVAL (HR)	0.2
DESIGN DURATION (HR)	4.2
TOTAL RAIN (MM)	187.0
PERCENTAGE RUNOFF	65.0
ANSF (CUMECS PER SQ.KM.)	0.06

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

TIME	TOTAL RAIN MM	NET RAIN MM	UNIT HYDROGRAPH ORDINATE	TOTAL HYDROGRAPH
0.00	2.80	1.82	0.00	1.41
0.20	2.90	1.88	15.22	2.07
0.40	3.30	2.14	30.45	3.39
0.60	3.40	2.21	45.67	5.48
0.80	3.30	2.14	60.90	8.37
1.00	5.40	3.51	76.12	12.02
1.20	5.30	3.45	91.35	16.93
1.40	8.40	5.46	106.57	23.07
1.60	9.80	6.37	121.80	31.16
1.80	23.90	15.54	124.44	40.99
2.00	48.30	31.40	114.49	55.29
2.20	23.90	15.54	104.53	79.63
2.40	9.80	6.37	94.58	108.23
2.60	8.40	5.46	84.63	137.83
2.80	5.30	3.45	74.68	167.71
3.00	5.40	3.51	64.73	196.76
3.20	3.30	2.14	54.78	224.44
3.40	3.40	2.21	44.83	249.38
3.60	3.30	2.14	34.88	268.63
3.80	2.90	1.88	24.93	274.77
4.00	2.80	1.82	14.98	267.70
4.20			5.03	254.80
4.40				238.61
4.60				220.23
4.80				200.25
5.00				179.11
5.20				157.19
5.40				134.65
5.60				111.72
5.80				88.74
6.00				66.60
6.20				47.01
6.40				32.89
6.60				24.28
6.80				18.24
7.00				13.58
7.20				9.97
7.40				7.16
7.60				5.03
7.80				3.40
8.00				2.28
8.20				1.63

CURVATURE AROUND PEAK = -330.206

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

AS ABOVE BUT WITH PERCENTAGE RUNOFF INCREASING THROUGH STORM

PERCENTAGE RUNOFF INCREASING THROUGH STORM WITH CWI

AREA (SQ.KM.)	23.5
DATA INTERVAL (HR)	0.2
DESIGN DURATION (HR)	4.2
TOTAL RAIN (MM)	187.0
PERCENTAGE RUNOFF	65.0
ANSF (CUMECS PER SQ.KM.)	0.06

CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE

TIME	TOTAL RAIN MM	NET RAIN MM	UNIT HYDROGRAPH ORDINATE	TOTAL HYDROGRAPH
0.00	2.80	1.19	0.00	1.41
0.20	2.90	1.25	15.22	1.84
0.40	3.30	1.45	30.45	2.72
0.60	3.40	1.52	45.67	4.11
0.80	3.30	1.50	60.90	6.05
1.00	5.40	2.52	76.12	8.53
1.20	5.30	2.53	91.35	11.91
1.40	8.40	4.18	106.57	16.19
1.60	9.80	5.09	121.80	21.97
1.80	23.90	13.75	124.44	29.21
2.00	48.30	33.31	114.49	40.66
2.20	23.90	17.80	104.53	63.22
2.40	9.80	7.50	94.58	91.26
2.60	8.40	6.58	84.63	121.10
2.80	5.30	4.20	74.68	152.11
3.00	5.40	4.34	64.73	183.12
3.20	3.30	2.67	54.78	213.70
3.40	3.40	2.77	44.83	242.50
3.60	3.30	2.70	34.88	266.70
3.80	2.90	2.38	24.93	277.95
4.00	2.80	2.31	14.98	274.94
4.20			5.03	265.27
4.40				251.57
4.60				234.97
4.80				216.16
5.00				195.62
5.20				173.84
5.40				150.90
5.60				127.05
5.80				102.59
6.00				78.53
6.20				56.67
6.40				40.28
6.60				29.89
6.80				22.47
7.00				16.70
7.20				12.19
7.40				8.68
7.60				5.98
7.80				3.93
8.00				2.51
8.20				1.69

CURVATURE AROUND PEAK = -356 633

```
      DIMENSION PROF(20),UNIT(100),ERF(100),HRF(100),TITLE(10),
     *CXH(100),CASETITLE(10),RSFL(300)
      DATA PROF/12.5,24.0,34.5,45.0,53.0,60.0,66.5,72.0,76.0,79.5,82.5,
     *85.0,87.5,90.0,92.0,94.0,96.0,97.5,99.0,100.0/
      READ (1,100) TITLE
   99 READ (1,101) ICASE
      IF(ICASE.EQ.0) STOP
      READ (1,100) CASETITLE
      READ (1,102) AREA,T,D,P,SPR,CWI,SBF
      PR=SPR+0.22*(CWI-125.)+0.1*(P-10.)
      Q=P*PR/100.0
      ANSF=SBF+0.00033*(CWI-125.)
      READ (1,103) NUH,NHRF,IPRINC,TP
      IF(NUH.LE.0) GO TO 3
      READ (1,104)(UNIT(J),J=1,NUH)
      GO TO 2
    3 IF(NUH.EQ.0) GO TO 4
      QP=220.0/TP
      NUH=INT(2.53*TP/T)+1
      DO 5 J=1,NUH
      IF (FLOAT(J).GT.(TP/T+1.))GO TO 6
      UNIT(J)=(QP/TP)*FLOAT(J-1)*T
      GO TO 7
    6 UNIT(J)=QP*(1.-(FLOAT(J-1)*T-TP)/1.53/TP)
    7 IF(UNIT(J).LT.0.0)UNIT(J)=0.0
    5 CONTINUE
      GO TO 2
    4 NUH=NUHB
    2 NUHB=NUH
      IF(NHRF.GE.0)GO TO 19
   10 N=D/T
      PRIB=0.0
   24 DO 11 I=1,N,2
      S=(I*100.0)/N
      S1=S/5
      IS1=S1
      IF (I.EQ.N) GO TO 22
      PRI=PROF(IS1)+(S1-IS1)*(PROF(IS1+1)-PROF(IS1))
      GO TO 23
   22 PRI = PROF(IS1)
   23 PRI=PRI/100.
      IA=N/2+I/2+1
      IB=N/2-I/2+1
      HRF(IA)=(PRI*FLOAT(1+1/I)-PRIB)/2.*P
      HRF(IB)=HRF(IA)
   11 PRIB=PRI
      NHRF=N
      GO TO 12
   19 IF(NHRF.GT.0) GO TO 20
      NHRF=NHRFB
      GO TO 12
   20 READ(1,105)(HRF(I), I=1,NHRF)
   12 IF (IPRINC.NE.1) GO TO 30
      SCXH=0.0
      AK1=0.5**(T/24.)
      AK2=SQRT(AK1)
      IF (CWI.GT.125.) GO TO 26
      S=125.-CWI
      A=0.0
      GO TO 27
   26 S=0.0
      A=CWI-125.
   27 DO 25 I=1,NHRF
      S=S-HRF(I)
      IF (S.LT.0.0) S=0.0
      A=AK2*HRF(I)+AK1*A
      CWIR=125.-S+A
      IF (CWIR.LE.5.0) CWIR=5.0
      CXH(I) =CWIR*HRF(I)
   25 SCXH=SCXH+CXH(I)
      PF=Q/SCXH
   30 DO 13 I=1,NHRF
      IF (IPRINC.EQ.0) GO TO 29
      ERF(I)=PF*CXH(I)
      GO TO 13
   29 ERF(I)=HRF(I)*PR/100.
   13 CONTINUE
      NHRFB=NHRF
      NSFL=NUH+NHRF-1
      DO 8 IJ=1,NSFL
    8 RSFL(IJ)=ANSF*AREA
      DO 9 I=1,NHRF
      DO 9 J=1,NUH
      IJ=I+J-1
    9 RSFL(IJ)=RSFL(IJ)+ERF(I)*UNIT(J)*AREA/1000.0
      RSFLM=RSFL(1)
      IJM=1
      DO 15 IJ=2,NSFL
      IF(RSFL(IJ).LE.RSFLM) GO TO 15
      RSFLM=RSFL(IJ)
      IJM=IJ
   15 CONTINUE
      IF (IJM.LE.1) GO TO 31
      CURV=(RSFL(IJM+1)+RSFL(IJM-1)-2.*RSFLM)/T**2
```

```
   31 WRITE(2,200) TITLE
      WRITE(2,201) CASETITLE
      IF (IPRINC.EQ.1) WRITE (2,209)
      WRITE(2,202) AREA,T,D,P,PR,ANSF
      WRITE(2,203)
      WRITE (2,204)
      DO 21 I=1,NSFL
      TIME =FLOAT (I-1)*T
      WRITE (2,205) TIME,RSFL(I)
      IF (I.LE.NHRF) WRITE (2,206) HRF(I),ERF(I)
      IF (I.LE.NUH) WRITE (2,207) UNIT(I)
   21 CONTINUE
      WRITE (2,208) CURV
      GO TO 99
  100 FORMAT (10A8)
  101 FORMAT (I5)
  102 FORMAT (7F10.0)
  103 FORMAT (3I5,F5.1)
  104 FORMAT (8F10.0)
  105 FORMAT (8F10.1)
  200 FORMAT (1H1,1X,10A8,///)
  201 FORMAT (2X,10A8,//)
  202 FORMAT (10X,29HAREA (SQ.KM.)                  ,F5.1/,
     *10X,29HDATA INTERVAL (HR)            ,F5.1/,
     *10X,29HDESIGN DURATION (HR)          ,F5.1/,
     *10X,29HTOTAL RAIN (MM)               ,F5.1/,
     *10X,29HPERCENTAGE RUNOFF             ,F5.1/,
     *10X,29HANSF (CUMECS PER SQ.KM.)      ,F6.2//)
  203 FORMAT (52H CONVOLUTION OF UNIT HYDROGRAPH AND NET RAIN PROFILE/)
  204 FORMAT (1X,47HTIME      TOTAL      NET       UH ORD      TOTAL,
     */,11X,42HRAIN      RAIN       CUMECS PER  HYDROGRAPH/,
     *11X,30HMM         MM         100 SQ.KM./)
  205 FORMAT (F6.2,33X,F10.2)
  206 FORMAT (1H+,5X,2F10.2)
  207 FORMAT (1H+,25X,F10.2)
  208 FORMAT (/25H CURVATURE AROUND PEAK = ,F8.3)
  209 FORMAT(53H  PERCENTAGE RUNOFF INCREASING THROUGH STORM WITH CWI)
      END
```

6.9 References

BELL F.C. & OM KAR S. (1969) Characteristic response times in design flood estimation. *Journal of Hydrology*, **8**, 173–196.

BÉRAN M.A. (1973) *Estimation of design floods and the problem of equating the probability of rainfall and runoff.* International Association of Hydrological Sciences, Symposium on the design of water resources projects with inadequate data. June 1973, Madrid.

BERNARD M.M. (1935) An approach to determinate stream flow. *Transactions of the American Society of Civil Engineers*, **100**, 347–395.

BLEASDALE A. & DOUGLAS C.K. (1952) Storm over Exmoor on August 15, 1952. *Meteorological Magazine*, **81**, 353–367.

BRANSBY-WILLIAMS G. (1922) Flood discharge and the dimensions of spillways in India. *Engineer*, **34**, 321.

CHOW V.T. (1962) Hydrologic determination of waterway areas for the design of drainage structures in small drainage basins. *University of Illinois, Engineering Experiment Station Bulletin*, **462**.

CRAMÉR H. (1946) *Mathematical Methods of Statistics*. Princeton University Press.

CROSTHWAITE P.M. (1921) The Louth flood of 1920. *Transactions of the Institution of Water Engineers*, 204–217.

DOBBIE C.H. & WOLF P.O. (1953) The Lynmouth flood of August 1952. *Proceedings of the Institution of Civil Engineers*, Pt III, **2**, 522–588.

DUCKWORTH J.A. (1969) Bowland forest and Pendle floods. *Association of River Authorities Year Book*, 81–90.

ESPEY W.H., MORGAN C.W. & MASCH F.D. (1965) A study of some effects of urbanization on storm runoff from a small watershed. *Center for Research in Water Resources, University of Texas.*

GRAY D.M. (1961) Synthetic unit hydrographs for small watersheds. *Journal of Hydraulics Division, Proceedings of the American Society of Civil Engineers*, **87**, (HY4), 33–53.

GRAY D.M. (1964) *Physiographic characteristics and the runoff pattern.* Proceedings Hydrology Symposium, No. 4, 147–166. National Research Council, Canada.

GRINDLEY J. (1970) *Estimation and mapping of evaporation.* International Association of Scientific Hydrology, Symposium on the World Water Balance, Reading. *IASH Publication*, **92**, 200–213.

HALL M.J. (1973) *Synthetic unit hydrograph technique for the design of flood alleviation works in urban areas.* International Association of Hydrological Sciences. Symposium on the design of water resources projects with inadequate data, June 1973, Madrid.

HICKOK R.B., KEPPEL R.V. & RAFFERTY B.R. (1959) Hydrograph synthesis for small arid land watersheds. *Agricultural Engineering*, **40,** 608–611, 615.

HORTON R.E. (1940) An approach toward a physical interpretation of infiltration capacity. *Proceedings of the Soil Science Society of America*, **5**, 399–417.

HYDRAULICS RESEARCH STATION (1968) *Forest of Bowland and Pendle Floods of August 1967.* Report No. EX 382.

HYDRO-ELECTRIC COMMISSION (1970) *August 1970 Flood Data Report.* Hydrology Section, Hydro-Electric Commission, Tasmania.

JOHNSTONE D. & CROSS W.P. (1949) *Elements of Applied Hydrology.* The Ronald Press Co., New York.

KENNEDY R.J. & WATT W.E. (1967) *The relationship between lag time and the physical characteristics of drainage basins in Southern Ontario.* International Association of Scientific Hydrology, Symposium on floods and their computation, Leningrad. *IASH Publication*, **85,** 866–874.

KIRPICH Z.P. (1940) Time of concentration of small agricultural watersheds. *Civil Engineering (N.Y.)*, **10**, 362.

KOHLER M.A. & LINSLEY R.K. (1951) Predicting the runoff from storm rainfall. *U.S. Weather Bureau Research Paper*, 34.

LAW F. (1968) *The flood of 8 August 1967.* Fylde Water Board.

MANDEVILLE A.N. & O'DONNELL T. (1973) Introduction of time variance to linear conceptual catchment models. *Water Resources Research*, **9,** 298–310.

MCCARTHY G.T. (1938) *The unit hydrograph and flood routing.* Unpublished report of U.S. Engineer's Office, Providence, R.I.

MILLER D.L. & CLARK R.A. (1965) *Design of Small Dams* (Chap. II (Flood Studies), & Appendix A). U.S. Bureau of Reclamation.

MINSHALL N.E. (1960) Predicting storm runoff on small experimental watersheds. *Journal of Hydraulics Division, Proceedings of the American Society of Civil Engineers*, **86,** (HY8), 17–38.

MITCHELL W.D. (1972) *Model hydrographs.* United States Geological Survey Water Supply Paper 2005.

MOCKUS V. (1957) Use of storm and watershed characteristics in synthetic hydrograph analysis and application. *U.S. Soil Conservation Service.*

NASH J.E. (1960) A unit hydrograph study with particular reference to British catchments. *Proceedings of the Institution of Civil Engineers*, **17**, 249–282.

NASH J.E. (1966) *River Engineering and Water Conservation Works*, (ed. R.B. Thorn), Chapter 6, Applied flood hydrology. Butterworths, London.

Newnham E.V. (1921) Report on the thunderstorm which caused disastrous floods at Louth on May 29th 1920. Meteorological Office, *Professional Notes No. 17.*

O'Donnell T. (1966) Methods of computation in hydrograph analysis and synthesis. In: *Recent Trends in Hydrograph Synthesis*, Proceedings of Technical Meeting 21, Committee for Hydrological Research TNO, the Hague, 65–103.

Ordnance Survey Maps (1945) (i) Land classification (a) Scotland, (b) England and Wales. (ii) (a) Vegetation: reconnaissance survey of Scotland, (b) Vegetation: grasslands of England and Wales. Published by the Director General of the Ordnance Survey, Southampton.

Painter R.B. (1971) The hydrograph time-to-peak method of flood prediction. *Water and Water Engineering*, **75,** 235–237.

Pullen R.A. (1969) Synthetic unitgraphs for South Africa. *Hydrological Research Unit Report No. 3/69*, University of the Witwatersrand, Department of Civil Engineering.

Ramser C.E. (1927) Runoff from small agricultural areas. *Journal of Agricultural Research*, **34,** 797–823.

Sherman L.K. (1932) Streamflow from rainfall by unitgraph method. *Engineering News Record*, **108**, 501–505.

Snyder F.F. (1938) Synthetic unitgraphs. *Transactions of the American Geophysical Union*, **19**, Pt 1, 447–454.

Snyder W.M. (1955) Hydorgraph analysis by the method of least squares. *Proceedings of the American Society of Civil Engineers*, **81,** Separate No. 793.

Taylor A.B. & Schwarż H.E. (1952) Unit hydrograph lag and peak flow related to basin characteristics. *Transactions of the American Geophysical Union*, **33,** 235–246.

U.S. Army Corps of Engineers (1959) Flood hydrograph analyses and computations. *Engineering and design manuals E.M.* 1110-2-1405.

Watkins L.H. (1962) *The design of urban sewer systems.* Department of Scientific and Industrial Research, Road Research Technical Paper No. 55, HMSO.

Wilson, E.M. (1969) *Engineering Hydrology.* Macmillan, London, 2nd edn, 1974.

Wolf P.O. (1965) *Comparison of methods of flood estimation.* Institution of Civil Engineers, Symposium, River Flood Hydrology (publ. 1966), 1–23.

7 Supplementary studies: snowmelt runoff, conceptual catchment model and flood routing

7.1 Summary of investigations

7.1.1 Introduction

The results of several supporting studies are presented or discussed in this chapter; these are concerned with snowmelt runoff, conceptual modelling of isolated storm events, and flood routing. These studies were recommended by the Committee on Floods in the United Kingdom (Institution of Civil Engineers, 1967) either as part of the current investigation or as research; it proved convenient to treat them as separate but co-ordinated studies, using as far as possible data collected for the main investigation. Meteorological investigation of point snowmelt was undertaken by the Meteorological Office, but the analysis of runoff from snowmelt was carried out as an expansion of an existing research programme at the University of Newcastle-upon-Tyne. The development of a conceptual model for treating isolated storm events, as a variant of the more usual model using continuous data, was undertaken as part of the research programme of the Institute of Hydrology. The flood routing studies recommended were carried out at the Hydraulics Research Station, and are described in Volume III; the use of these methods is discussed here, with suggested ways of estimating certain parameters. Flood routing through a reservoir is an integral part of many design problems, and is dealt with briefly.

7.1.2 Snowmelt

The recommended snowmelt studies may be divided into the assessment of maximum snow depths and contents, and of potential snowmelt rates; and the examination of snow cover and flood records to assess the relative importance of snowmelt in different regions and to review methods of estimating snowmelt runoff in British conditions. The first part of these studies, by the Meteorological Office, is described in Chapter 7 of Volume II. The second part, carried out at the University of Newcastle-upon-Tyne, is described in the following section.

Snowmelt studies in the United Kingdom suffer from lack of records due to the sporadic snow cover over much of the country. Although the depth of snow lying has been recorded daily for some years, the water equivalent has only been measured recently at a limited number of sites; thus estimates of snowmelt based on successive daily snow depths are liable to error because of variations in snow density. On the other hand, estimates of snowmelt based on runoff measurement during periods when snow was known to be lying are catchment averages and are affected by variations of conditions over the catchment, by antecedent conditions, by infiltration losses and by storage attenuation. Until sufficient measurements of snow water equivalent have been made, the meteorological approach cannot provide reliable point snowmelt estimates for use in a catchment response model. Snowmelt runoff studies provide estimates comparable with rainfall floods after routing through a unit hydrograph/losses model.

The meteorological studies have provided a map of the median annual maximum snow depth and a summary of the depth frequency relationship. Estimates have been made of the density of lying snow and of the frequency distribution of annual maximum temperatures with snow lying.

Daily changes of snow depth were compared with the corresponding daily maximum temperatures to give a relationship which led to a first approximation to snowmelt rates for lowland catchments.

The snowmelt runoff studies showed that in each event the runoff was related to the temperature by a snowmelt coefficient. This coefficient varied between events with the total runoff. The corresponding coefficients vary between catchments with channel slope and, inversely, with catchment area. It is shown that frequency analyses of temperature and snowmelt coefficients can be combined to give a reasonable estimate of snowmelt runoff.

The combination of the meteorological and runoff studies and their significance to regional flood prediction are discussed in Section 7.2.5 following the account of the snowmelt runoff investigation.

7.1.3 Modelling of isolated events

The development of conceptual models of the rainfall–runoff relationship for use in flood prediction was recommended by the Institution of Civil Engineers' Committee on Floods in the United Kingdom (1967) as a research programme distinct from the three year investigation. During the course of the present study a part of this project was initiated to see whether the records of rainfall and runoff being collected for unit hydrograph studies could be used in 'short period' nonlinear models. This development was undertaken by the Analytical Section of the Institute of Hydrology, under the supervision of Professor T. O'Donnell of Imperial College, London, and Lancaster University, and followed previous work in this field (Mandeville *et al.*, 1970).

The objective was to examine a new type of nonlinear conceptual model dealing with isolated storm events only, as an alternative to the unit hydrograph method, as a means of estimating the flood resulting from a given rainfall storm on a catchment. Although the data collected for unit hydrograph analysis (Chapter 6) were used in this study, the choice of catchments was confined to those with a reasonable number of events. This project had possible advantages over the usual conceptual model approach because it was confined to flood events. The results of the study, presently based on 500 events on 21 basins, are described in Section 7.3.

A direct comparison of these results with the unit hydrograph approach is difficult because the model predicts total flow while the unit hydrograph deals with quick response runoff. Previous modelling work has shown that it is not easy to choose between models of different form with an equal number of parameters. The conceptual model and unit hydrograph approach might be expected to have similar success, because the number of parameters used is comparable; however, the unit hydrograph/losses model can be applied to an ungauged basin. The method of fitting is a major difference, being direct with the unit hydrograph and iterative with the model. The model provides the design engineer with an alternative tool which also has development potential.

7.1.4 Hydrograph routing

The recommended studies of flood routing methods for British rivers were carried out by the Hydraulics Research Station and are described in Volume III. Because the unit hydrograph study described in Chapter 6 of this volume was limited to catchments below 500 km^2, flood routing methods present a method of extending flood hydrographs to larger basins. When a design flood of a specific return period is required, it is necessary to use a uniform rainfall with an areal reduction factor based on the whole area; only when the maximum flood is required is it reasonable to allow for storm movement to maximise the combined outflow.

The routing of a design flood through a reservoir is a relatively simple operation, but as it may be necessary to repeat the calculation many times to optimise either spillway design or reservoir operating procedures it is suited to computer techniques. A program for this purpose is presented and discussed in Section 7.4.2.

7.2 Snowmelt runoff

7.2.1 Introduction

In the Institution of Civil Engineers' report *Flood Studies for the United Kingdom* (1967), the possible significance of snowmelt to flood discharge was recognised and recommendations were made for consideration of this significance in subsequent research. These recommendations included the examination of historical storms to assess the regional importance of the snowmelt contribution, the determination of the applicability of existing techniques to synthesising snowmelt flood discharges in the United Kingdom, and the development of methods for estimating maximum snowmelt rates for inclusion in probable maximum flood studies.

Some aspects of snowmelt flooding had been investigated in a small research programme at the University of Newcastle-upon-Tyne. In particular, the potential seriousness of a major snowmelt event in Britain had been noted, and the need for efficient snow survey methods and analytical techniques, to permit accurate forecasting of river floods, had become apparent. The research programme was expanded in 1971 with the financial support of the Natural Environment Research Council. Its objectives have been:

a to analyse historic records of snowmelt floods and by relating river flood discharges to causal factors to develop a suitable flood forecasting model;

b to establish representative catchments in different parts of the country where additional data, especially of snow water equivalents, could be collected during the programme period to aid improvement of model accuracy; and

c to combine the results of *a* and *b* to investigate variations of model parameters between catchments, the intention being to provide means of estimating parameters for catchments having no readily available data.

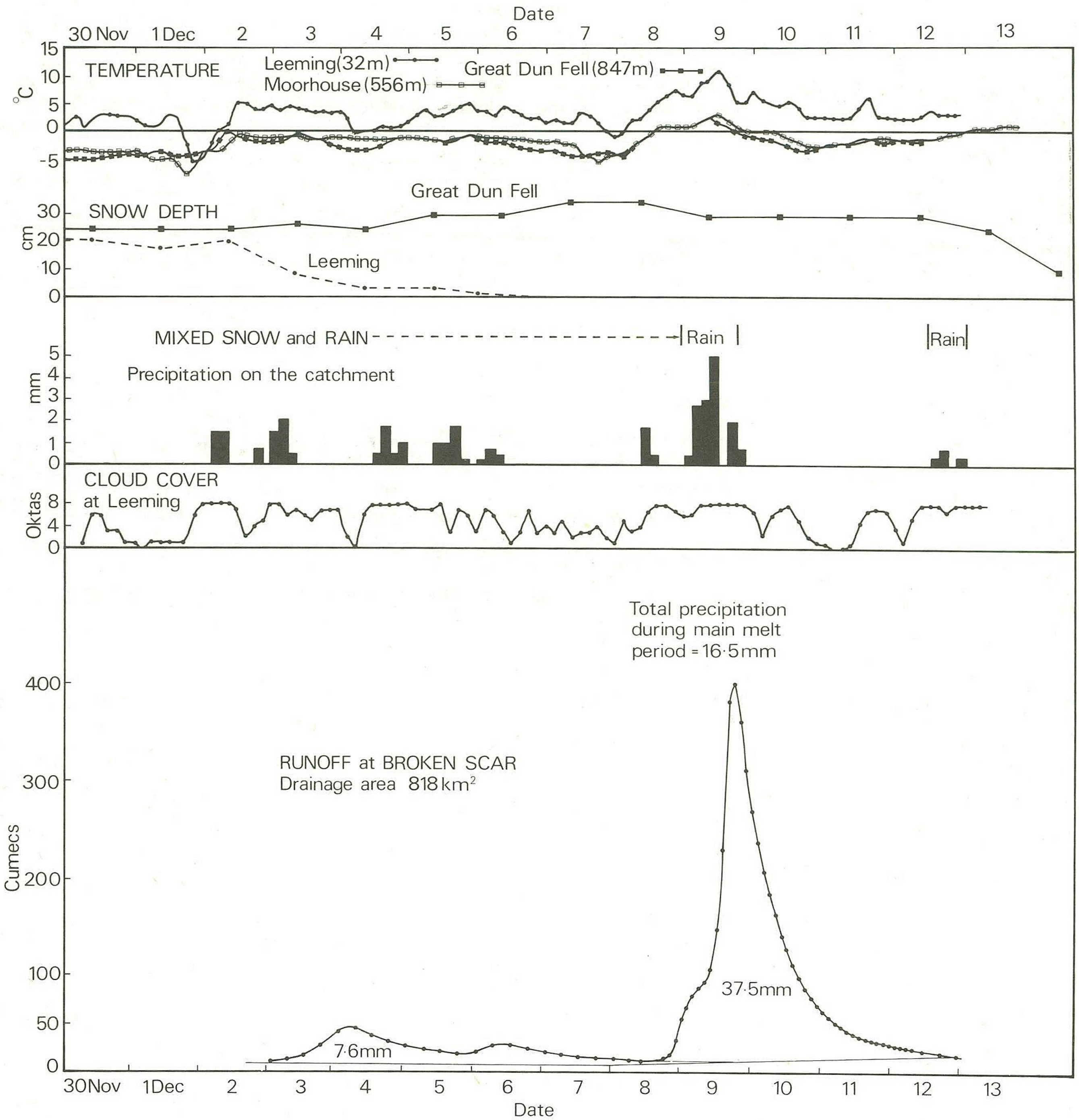
At the outset it was anticipated that although the research was specifically aligned to the problem of forecasting flood discharges, results would be useful in estimating for design. The research programme has provided information relevant to flood prediction. The account that follows outlines the nature of the snowmelt phenomenon and gives a simple method whereby prediction of floods may be attempted. Elaboration and refinement of simulation are also indicated. The significance of snowmelt to river flooding in Britain and the problem of estimation are discussed briefly.

7.2.2 Analyses and modelling of snowmelt floods

The nature of snowmelt runoff in Britain

A number of typical features of snowmelt runoff are recognisable. To explain these, an event which occurred at Broken Scar on the Tees in December 1965 is described, and illustrated in Figure 7.1. The catchment area down to Broken Scar is 818 km^2. Hydrometeorological conditions at the top and bottom of the catchment are represented by data from Moorhouse and Great Dun Fell, and from Leeming, respectively.

Fig 7.1 Snowmelt runoff event, November–December 1965, for the River Tees at Broken Scar.



Principal snowfalls occurred on 26 and 27 November and snow depths of 20–25 cm persisted over the entire catchment until 2 December. On that day there was a sharp rise in air temperature and snow depth at Leeming decreased steadily to zero by 7 December. It is clear from the river response that snow melted on only a small portion of the catchment and that most of the incident precipitation fell either as snow, or as rain which was absorbed by the snowpack. At middle and upper elevations, temperatures remained below zero until the afternoon of 8 December when rising temperatures were accompanied by overcast skies and substantial rain over the entire catchment. Although information on snow depth is inadequate to indicate snowmelt during the period a comparison of rainfall with runoff shows that snowmelt was the principal contributor to flood runoff. The combination of rain and melting snow resulted in high and, in some cases, record discharges in several rivers in northern England and the Midlands on 9 December; further south, rainfall was heavy and the snowmelt contribution relatively small.

The typical features of this event are summarised under the interrelated influences of topography, meteorology and snowpack.

Topography. Snowmelt is closely related to altitude. It occurs only between the snowline and the freezing level, and the greatest snowmelt runoff potential occurs when the entire catchment lies within these two levels. Water equivalent of the snow cover is often related to altitude. Aspect and slope have significant effects on local melting only during clear weather conditions when direct solar radiation contributes substantially to the total energy input.

Meteorology. Rapid snowmelt in Britain is most frequently brought about by a sudden influx of warm moist unstable air, and rain often accompanies the melt. The combined snowmelt and rainfall constitute large volumes of potential runoff and occasionally lead to severe flooding. Melt most commonly occurs under cloudy skies when the contribution of radiation to melt is small compared with the condensation and convective heat inputs. Since the last two are closely related to temperature, it is to be expected that for most snowmelt events a heat influx related to air temperature alone will adequately represent total energy input.

Snowpack conditions. A time lag exists from the time air temperature first rises above zero until the pack begins to release water at the ground surface. This is due initially to the heat energy required to bring the snowpack temperature up to 0°C before melt begins, and subsequently to the liquid water retention capacity of the snow and transmission time through the pack.

Snowmelt flood data

While illustrating the nature and complex processes involved in a snowmelt runoff event, the example also serves to indicate the type of data required if attempts are to be made to reconstruct historic snowmelt events or to forecast future ones. Often a very careful scrutiny of heat energy inputs, precipitation and streamflow is necessary even to determine whether snowmelt has contributed to flood flows. Lack of suitable data precludes the development of all but the simplest technique for simulating runoff. Daily values of snow depth may not generally provide sufficient basis for a forecasting method which could be developed for prediction purposes.

Flood forecasting will generally require at least the volume of snow lying prior to the melt (its water equivalent), precipitation on the pack during the melt period, and the heat energy input to the snowpack. For analyses streamflow data will also be required.

Snow. To provide data equivalent to those used for forecasting rainfall flood events the water equivalent of the snowpack is required at the onset of melt. A basis for determining the decline of the water equivalent with time is also needed.

Daily measurements of snow depth have been collected for the Snow Survey of Great Britain since the 1940s. These data are unfortunately of limited hydrological value, since in the absence of information on snow density, depths cannot be satisfactorily converted into water equivalents and potential melt runoff. Since 1967 certain meteorological stations have also measured water equivalents of snow on the ground, but the duration of the record and the spread of stations is such that the information is of practical value for only a limited number of events. The recent establishment by some river authorities of snow water equivalent surveys on a catchment basis will do much to alleviate the need but more surveys are still required and this should be considered an urgent need in some regions.

Specific measurements of the decline in water equivalent of the snow cover during the melt period are not made and the time distribution of the melt input must be estimated indirectly through the energy input.

Rain. It is necessary to distinguish between snow and rain. On occasions melt and rainfall may occur on the lower part of the catchment while snow is falling at higher elevations. Usually the proportion of each type of precipitation is estimated on the basis of an arbitrary temperature (0–1.5°C) and an associated lapse rate with altitude. Recording raingauges often do not operate during freezing conditions and may not be reinstalled until some time after the thaw.

Energy inputs. There are two basic methods for estimating the heat energy input to a catchment. By the energy budget method, separate estimates are made of the contribution to melt from the components of convection, condensation, radiation, heat from warm rain and ground heat. Data requirements may include radiation, vapour pressure, wind speed, cloud cover and sunshine; these are measured at only a limited number of stations and extrapolation over a catchment offers some difficulties. Alternatively for the temperature index or 'degree-day' method temperature data alone are used to estimate the total energy input. These (daily maxima and minima, 3 hourly screen temperatures, or thermograms) are available for a larger number of locations and areal extrapolation of these data can be done with reasonable accuracy.

Temperature index method

In view of the small amount of previous work on snowmelt runoff in Britain and the limitation imposed by data availability, snowmelt runoff has been investigated initially using a simple lumped system analysis in which the energy input is estimated by a simple temperature index and related directly to the streamflow output. Temperature index methods are well known and are based on the equation:

$$M = k(\theta-\theta_0) \qquad (7.1)$$

or

$$\Sigma M = k\Sigma(\theta-\theta_0) \qquad (7.1a)$$

where θ is measured air temperature; θ_0 is a base temperature at which snowmelt is assumed to start, usually taken as 0°C; k is a temperature index (degree-day or melt factor); M is variously taken as: (i) actual snowmelt at a point, (ii) water yield at the base of a snowpack at a point, (iii) runoff resulting from snowmelt.

In the absence of adequate data on point melt or yield rates, in this investigation M is taken as snowmelt runoff. The coefficient k therefore embraces in one action the yield of melt water and the following land and channel phases of drainage. Variations in percentage runoff or infiltration are included in the coefficient. The expression (7.1a) implies that a simple linear relationship exists between cumulative temperature and cumulative runoff which if graphed has a slope k. A more refined method of simulating or estimating snowmelt river discharges, in which snowmelt is still estimated by a temperature index method, is discussed later.

Experience in other parts of the world has shown that even where data are available to estimate the energy budget, the improvement in results over the use of a temperature index is comparatively small (Anderson, 1968). Temperature index methods still form the basis of most operational forecasting systems in the U.S.S.R. (Apollov, Kalinin & Komarov, 1970) and the United States. The SSARR model, for example, in use on the Columbia River system relies on the temperature index approach for forecasting although it uses the thermal budget approach for design flood determinations (Rockwood, 1972).

With this in mind and considering the present status of snow hydrology in Britain, there seems little reason to look beyond temperature index methods to determine effective snowmelt volumes.

Analyses

In the context of this lumped system, it is clear that k can vary during a flood event and also between events on the same catchment. To assess the magnitude of k and its variations between events and between catchments, several snowmelt floods on 16 catchments have been analysed.

Cumulative temperatures derived from 3 hourly screen measurements, supplemented by autographic records and adjusted to mean basin elevation using a lapse rate of 0.5°C per 100 m, were plotted against cumulative runoff. Figure 7.2 shows a typical plot for the River Tees at Broken Scar. In nearly all events there is an initial phase with low but increasing values of k, followed by a period of nearly constant slope extending up to the peak and in some cases beyond. The slope varies from one event to another. After the peak, there are marked variations between individual events and in some cases there are abrupt changes in slope.

As a first step in the improvement of the relationship, it was considered that a 'translation' lag (assumed constant) existed between inflow at the catchment surface and outflow. This lag was evaluated empirically for each catchment and the cumulative runoff at time (t) plotted against cumulative temperature at time $(t-\text{lag})$. This adjusted relationship (Figure 7.3) still displays variations in k between and during events. Reasons for these variations can be assessed in the light of the previous account of the snowmelt event on the River Tees

Conceptual variations in k. 1 Since temperature is not a perfect index of snowmelt potential the melt may vary with different air masses

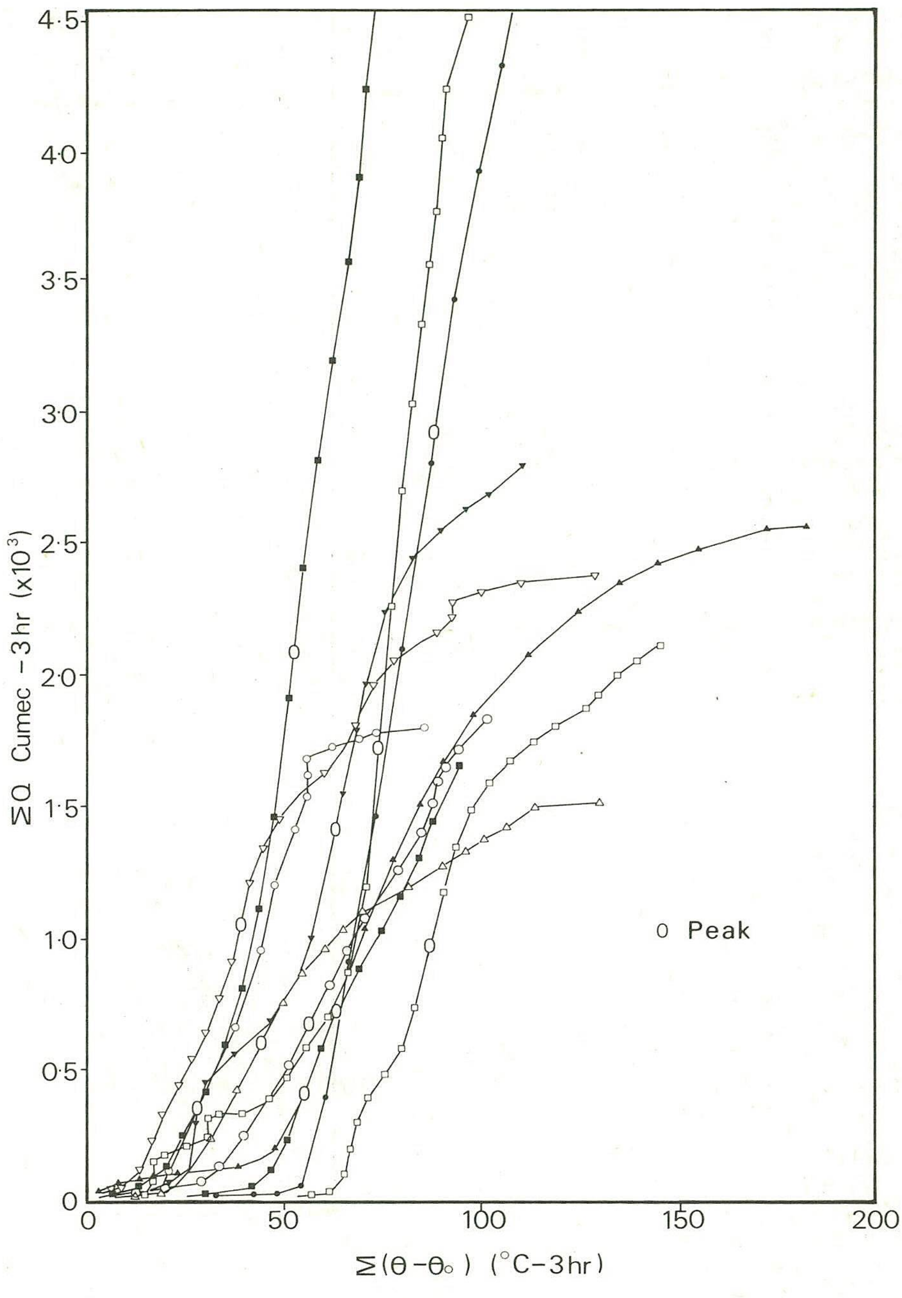


Fig 7.2 Graph of cumulative runoff against cumulative temperature, for the River Tees at Broken Scar.

over the catchment and hence with the variation in the relative contribution of individual melt components. For reasons noted above, the inaccuracy resulting from the use of a temperature index to predict point melts in Britain is probably small.

2 Further variations in k may result from the extrapolation of mean temperature from the observing stations to the mean catchment elevation. This is particularly true when a single low level snow free station is extrapolated to a predominantly snow covered catchment.

3 At the beginning of a period of positive temperatures some heat goes towards raising the snowpack to 0°C rather than melt.

4 The snowpack is able to store melt water until the liquid water retention capacity is reached. The pack then yields water at its base at a higher rate than melt. This factor along with 3 leads to an initially low

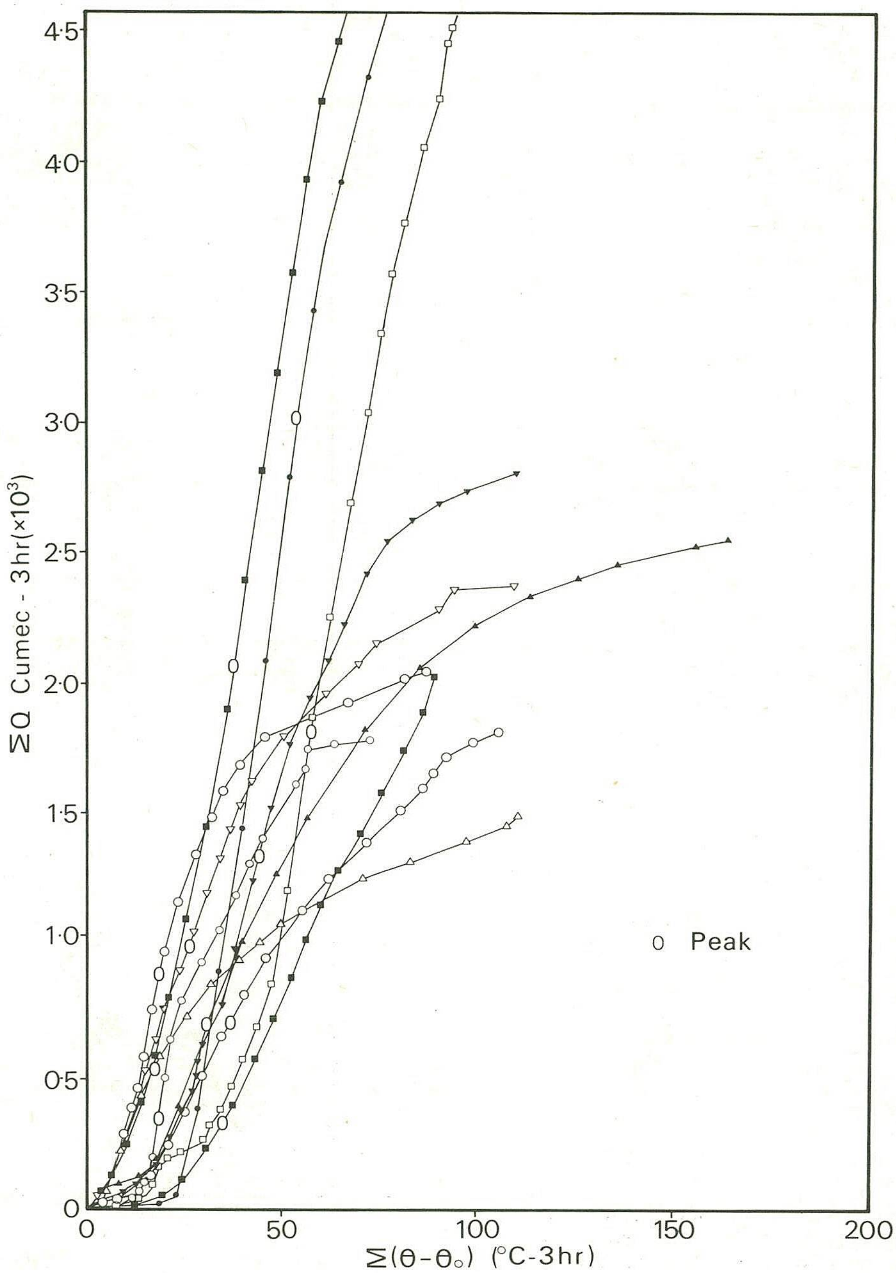


Fig 7.3 Graph of cumulative runoff against cumulative temperature (lagged 12 hours), for the River Tees at Broken Scar.

value of k. The duration of the resulting lag depends primarily on the water equivalent and density of the snowpack.

5 The proportion of the catchment contributing melt to the outflow point influences the value of the melt coefficient. In smaller events snow may be either absent from the lower part of the catchment or of such shallow depth that its contribution to flood runoff is virtually complete before that from the remoter parts of the catchment commences. In larger events the possibility of simultaneous contribution of melt to runoff from the entire catchment is higher. It is believed that variations in contributing area are the principal cause of the observed variation during the 'constant k' part of each event and that the effect is reflected in the flood volume.

6 When rain occurs during snowmelt the two contributions to the runoff hydrograph cannot be separated and the apparent melt factor will be increased. Rainfall during the early part of the melt period may rapidly satisfy the water retention capacity and also settle the snow cover and reduce that capacity. The initial lag in such events is often much reduced.

7 The input of water yield at the base of the snow pack is subject to similar modifications in its passage to the outflow point to that experienced by input from rainfall. Any variations in catchment antecedent conditions affect the runoff response and hence the magnitude of the melt factor.

Variation of k—between events. Despite the numerous influences on the melt factor, a phase of sensibly constant melt factor occurs surprisingly consistently (e.g. Figure 7.3). It is clear from results and theory that for predictive purposes the variation of k must be analysed. Records and theory suggest a number of factors to which k might be related. A linear regression analysis has been carried out between k of the constant phase and those factors with available data (Table 7.1).

Table 7.1 Results of linear regression analysis between melt factor and flood depth, rainfall during snowmelt, and season.

No.	Station name	Number of events	r Flood depth/ melt factor	r Rainfall/ melt factor	r Season/ melt factor	r Multiple (depth and rainfall)	se of k (after regression)
54/22	Severn at Plynlimon Weir	6	0.41	0.28	0.07	0.46	0.265
25/3	Trout Beck at Moorhouse	19	0.73	0.29	NT	0.73	0.305
46/5	East Dart at Bellever Bridge	2					
22/2	Coquet at Bygate	11	0.95	−0.27	0.30	0.97	0.064
25/6	Greta at Rutherford Bridge	15	0.60	0.37	NT	0.62	0.151
25/2	Tees at Dent Bank	20	0.41	0.43	NT	0.45	0.201
28/20	Churnet at Rocester	11	0.81	0.77	−0.06	0.91	0.012
46/3	Dart at Austins Bridge	7	0.92	0.86	0.06	0.93	0.053
45/2	Exe at Stoodleigh	5	0.20	0.77	−0.73	0.81	0.018
25/8	Tees at Barnard Castle	7	0.71	0.81	NT	0.91	0.029
54/14	Severn at Abermule	6	0.64	−0.13	0.36	0.66	0.026
28/11	Derwent at Matlock	8	0.91	−0.05	−0.49	0.91	0.001
27/28	Aire at Armley	7	0.91	0.85	−0.11	0.95	0.018
23/4	South Tyne at Haydon Bridge	9	0.55	0.64	−0.22	0.66	0.082
25/1	Tees at Broken Scar	20	0.86	0.63	NT	0.87	0.038
27/7	Ure at Westwick Lock	11	0.46	0.42	−0.09	0.50	0.053

NT = Not tested.

This reveals significant correlation between the melt factor and flood volume on most catchments and with liquid precipitation during the melt period on several catchments. No catchment shows a significant correlation between k and season as expressed by the week number from the winter solstice.

The high correlation between k and flood volume is probably a reflection of the contributing area in turn related to the volume and distribution of the snow pack. The analysis suggests that for most flood events the contribution of the lower part of the catchment is nearly complete before that from more distant parts commences (a feature which becomes more significant when refining the forecasting method to a spatially distributed model). If this is true, then in the absence of liquid precipitation during the melt period, k may attain a limiting upper value when the depth and distribution of the snowpack is such that the entire catchment is contributing to the outflow point at the optimum rate.

A typical relationship for the River Tees is shown graphically in Figure 7.4. The largest events with high values of k represent a melt input from the greater part of the catchment and may already be approaching an upper limit of k. In some instances anomalously high values of k occur in comparatively small events. These usually result from the persistence of temperatures near zero at mean catchment elevation with melt and rain at lower levels.

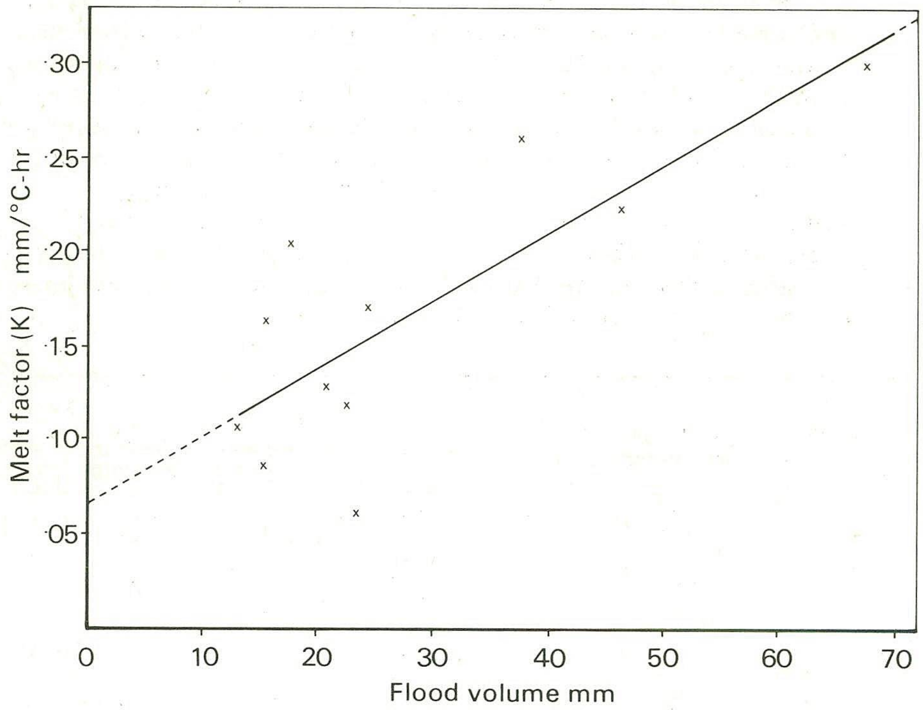


Fig 7.4 Relationship between melt factor (k) and flood volume (mm over catchment), for the River Tees at Broken Scar.

The correlation of k with rainfall during the melt is variable. Low correlation may occur on catchments where the events selected had a comparatively low rainfall input and the effects were obscured by the larger melt input. The use of liquid precipitation with flood volume in the regression is not entirely satisfactory since they are not independent but in the absence of actual data on snow quantities on the catchment there is little alternative to the use of stop-gap predictors.

The absence of correlation between k and season indicates that seasonal variations in solar radiation have little effect on the melt potential in British conditions.

Variation of k—between catchments. The results so far described permit the estimation of k for any one of the catchments studied when volume of runoff (snowmelt with rainfall) can be assessed. When estimates of snow lying and subsequent rainfall are available, this may allow a forecast of the resulting runoff to be made. They do not, however, allow the estimation of k and hence the prediction of snowmelt river discharges for other catchments. A further simple 'between catchment' analysis has therefore been attempted.

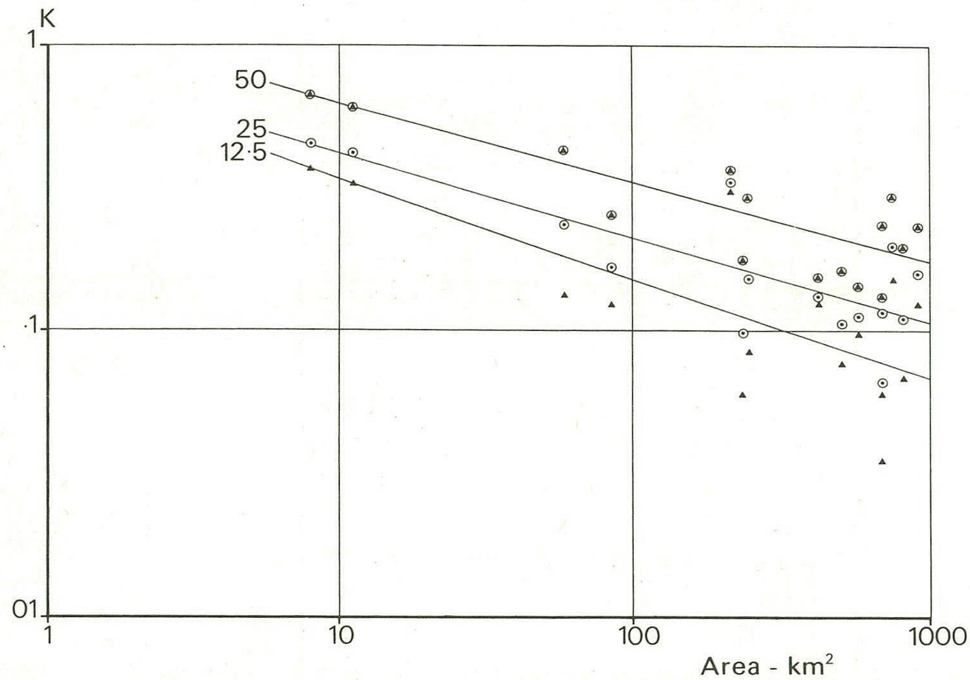
Calculated values of k combine the process of point melt with infiltration and catchment storage effects. Therefore, k values are generally much less than point melt coefficients. Since catchment storage generally relates to area, smaller k values are associated with larger catchments; values of k for small catchments will correspondingly approach point melt coefficient values.

Table 7.2 Estimation of melt factor for each catchment with given runoff depth.

No.	Station name	Area (km²)	Channel slope (S1085) (m/km)	Annual average rainfall (SAAR) (mm)	Stream frequency (STMFRQ)	Regression coefficient Depth (mm)	Regression coefficient Rainfall (mm)	Intercept	k at 12.5 mm depth Zero rainfall	k at 25 mm depth Zero rainfall	k at 50 mm depth Zero rainfall
54/22	Severn at Plynlimon Weir	8.05	63.47	2449	3.60	0.00736	−0.00916	0.26634	0.368	0.450	0.663
25/3	Trout Beck at Moorhouse	11.4	35.79	2182	2.89	0.00724	−0.00047	0.23870	0.329	0.420	0.603
46/5	East Dart at Bellever Bridge	21.5	22.60	2103	0.93	—	—	—	—	(0.276)	—
22/2	Coquet at Bygate	59.6	14.64	1053	2.43	0.00784	−0.01144	0.03541	0.133	0.231	0.428
25/6	Greta at Rutherford Bridge	86.2	1.68	1179	2.68	0.00331	0.00769	0.08352	0.124	0.166	0.251
25/2	Tees at Dent Bank	217	12.88	1717	3.08	0.00220	0.00584	0.27304	0.301	0.328	0.363
28/20	Churnet at Rocester	238	2.39	968	1.54	0.00309	−0.00157	0.02030	0.059	0.097	0.172
46/3	Dart at Austins Bridge	248	6.50	1821	0.78	0.00544	0.00118	0.01388	0.082	0.150	0.288
45/2	Exe at Stoodleigh	422	5.70	1402	0.85	0.00085	−0.01029	0.10880	0.124	0.130	0.151
25/8	Tees at Barnard Castle	509	9.83	1412	2.98	0.00218	0.00641	0.04882	0.076	0.104	0.158
54/14	Severn at Abermule	580	3.60	1257	1.83	0.00118	0.00095	0.08156	0.096	0.111	0.141
28/11	Derwent at Matlock	689	4.48	1105	1.46	0.00449	−0.00168	0.00279	0.059	0.115	0.227
27/28	Aire at Armley	692	1.90	1059	0.66	0.00249	0.00049	0.00319	0.034	0.065	0.127
23/4	South Tyne at Haydon Bridge	751	6.20	1172	2.08	0.00378	−0.01284	0.10180	0.149	0.196	0.290
25/1	Tees at Broken Scar	818	6.60	1207	2.34	0.00335	0.00138	0.02508	0.067	0.109	0.193
27/7	Ure at Westwick Lock	914	3.24	1163	1.16	0.00288	−0.00872	0.08540	0.121	0.157	0.227

To study the variation of k with catchment area it was necessary to estimate standardised values of k_r for constant runoff depths since, as already indicated, volume of runoff influences the melt factor. This was done for depths of 12.5, 25 and 50 mm (Table 7.2) using the previously derived regression equations from the 'between events' analysis. The expected inverse proportionality between area and k was then derived by plotting k_r against A on log/log co-ordinates (Figure 7.5a) from which a relationship of the form $k_r = aA^{-b}$ appears appropriate. Regression of k_r on other catchment characteristics has produced an equally good relationship with channel slope (Figure 7.5b and Section 7.2.5).

Fig 7.5b Relationship between melt factor (k) and channel slope (S1085) with given runoff depth (mm).



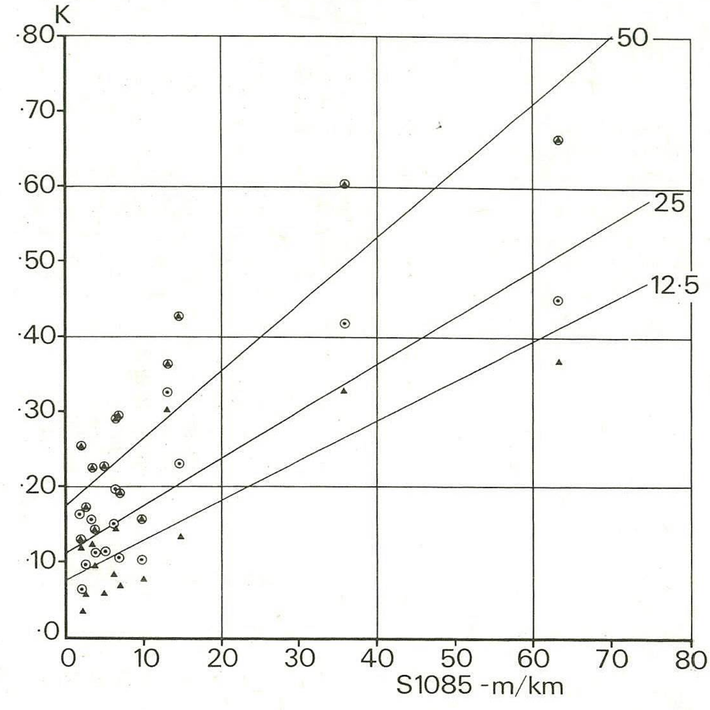


Fig 7.5b Relationship between melt factor (k) and channel slope (S1085) with given runoff depth (mm).

The three lines on Figure 7.5a representing runoff depths were interpolated visually. General scatter of data points about these lines is considered to be caused by:

a approximation adopted during the analysis;

b limited sample size in deriving the regression equations; and

c neglect of other catchment parameters which influence infiltration and storage.

The validity of interpolation should be tested by the addition of further data points.

Technique and illustrations of forecasting. Results of 'between events' and 'between catchments' analyses permit the forecasting of snowmelt flood discharges for any catchment in this country whether gauged or not. The technique may also be incorporated in flood estimation (Section 7.2.4). A three part procedure for the computation of river flood discharges is outlined below.

1 Values of k_r for the three runoff values 12.5, 25 and 50 mm respectively are read from Figure 7.5a corresponding to the catchment area A. Regression coefficients c and d relating k_r to depth R in the equation $k_r = cR+d$ are determined using these three values. (For a catchment with snowmelt runoff data already processed, values of c and d may be known and this part omitted.)

2 Snow survey data are necessary to estimate the catchment average water equivalent. After allowing for a coefficient of runoff (possibly near unity) the effective water equivalent R is substituted in the regression equation to determine an appropriate k_r for the event.

3 Observed or estimated temperature data at some fixed elevation point on the catchment are adjusted to mean catchment elevation (probably using a lapse rate of temperature of 0.59°C per 100 m). These temperatures are converted to a flood discharge histogram through the equation

$$Q = k_r \theta. \quad (7.2)$$

The computation should cease when all snow has effectively disappeared from the catchment. Unless a suitable snowpack melt coefficient is known this point will not be known and a subjective assessment will be necessary. The calculated histogram is displaced in time by a chosen interval to allow for lag effects.

An example using data recorded at Moorhouse and Broken Scar in February 1973 illustrates the technique. Calculation of k_r values is summarised in Table 7.3. Flood discharge computations were carried out, and observed and calculated discharges are shown on Figure 7.6.

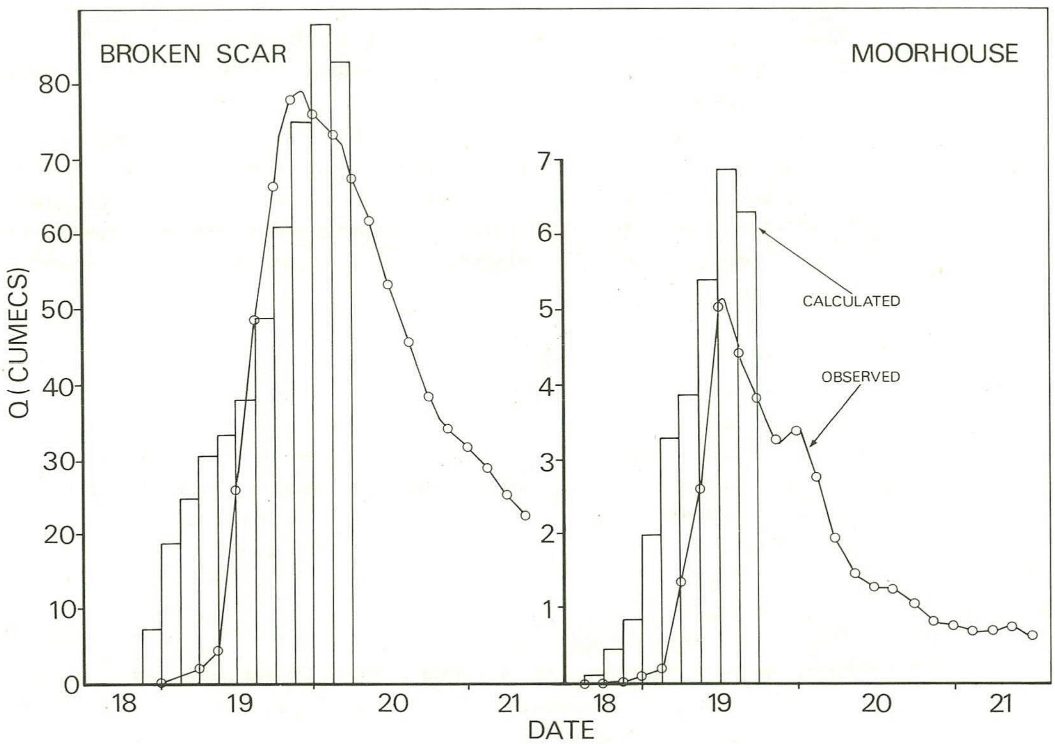
This example is not valid for testing accuracy in forecasting since data from these catchments were used in the original analysis. It nevertheless indicates a very simple approach to snowmelt flood forecasting, and indirectly to estimation, for catchments with little or no relevant data.

Table 7.3 Example of k_r estimation.

Station	Catchment area (km²)	k_r (mm per °C hour) Mean depth (mm) 12.5	25	50	Snow depth water equivalent R (mm)	Interpolated k for event (mm/°C hour)	Comparative k value from regression
Moorhouse	11.4	0.325	0.415	0.595	37.75	0.506	0.572
Broken Scar	818	0.062	0.102	0.178	11.75	0.060	0.064

The technique used in this illustration obviously oversimplifies the snowmelt runoff phenomenon; a number of implied approximations cannot be expected to lead to exact synthesis of a flood hydrograph. It ignores in particular the variation of k_r during the early part of the event and the influence that variation of spatially distributed input must have on hydrograph shape. For flood forecasting the technique may prove to be insufficiently precise and a more refined approach may be required. To a lesser degree the same may be said for flood estimation and design especially where the method is being used chiefly as an interface model to convert long term hydrometeorological data (generated or historical) into river discharge.

Fig 7.6 Calculated and observed runoff for the snowmelt event of February 1973 on the River Tees.



7.2.3 Conceptual model

Clearly the development of a more physically based model for snowmelt forecasting in Britain is constrained by data inadequacies. This is particularly true of data on snow water equivalents before and during melt. The use of energy input to indicate melt is also limited by the inability to determine accurately when the energy input ceased to cause melt. A network of snow water equivalent stations is therefore necessary for adequate calibration of a snowmelt runoff model.

Work carried out so far has shown that consideration of the distributed nature of energy and moisture inputs is fundamental to a model structure and also that account must be taken of the liquid water retention capacity of the snow and the resulting lag and transformation of the melt to yield at the base of the pack. The simulation model outlined in Figure 7.7 can now be based on a clearer understanding of these features and the problems of data inadequacy usually encountered. Many elements of the model are not original but have been adopted and modified to suit local conditions from procedures in use in the U.S.S.R. (e.g. Apollov *et al.*, 1970) and in the U.S.A. (Corps of Engineers, 1956).

For forecasting flows, required inputs to the model are temperature, rainfall and snow water equivalent, the first two from existing meteorological stations and the last from specially conducted snow surveys. The values are adjusted to represent the appropriate input for each of n elevation bands. For each elevation band, melt is determined by the temperature index equation. Initially the melt plus incident rainfall serves to bring the snow moisture level up to the retention capacity during which time the yield is assumed to be zero. After this point yield occurs at the rate given by Equation 7.2 and is always greater than the melt rate (m_{it}):

$$y_{it} = \frac{m_{it}}{1-\alpha} \tag{7.3}$$

where y_{it} is yield at base of snowpack for ith elevation band after time t, α is liquid water retention capacity $= h_w/h_{sw}$, h_w is depth of liquid water, h_{sw} is water equivalent of snow plus water.

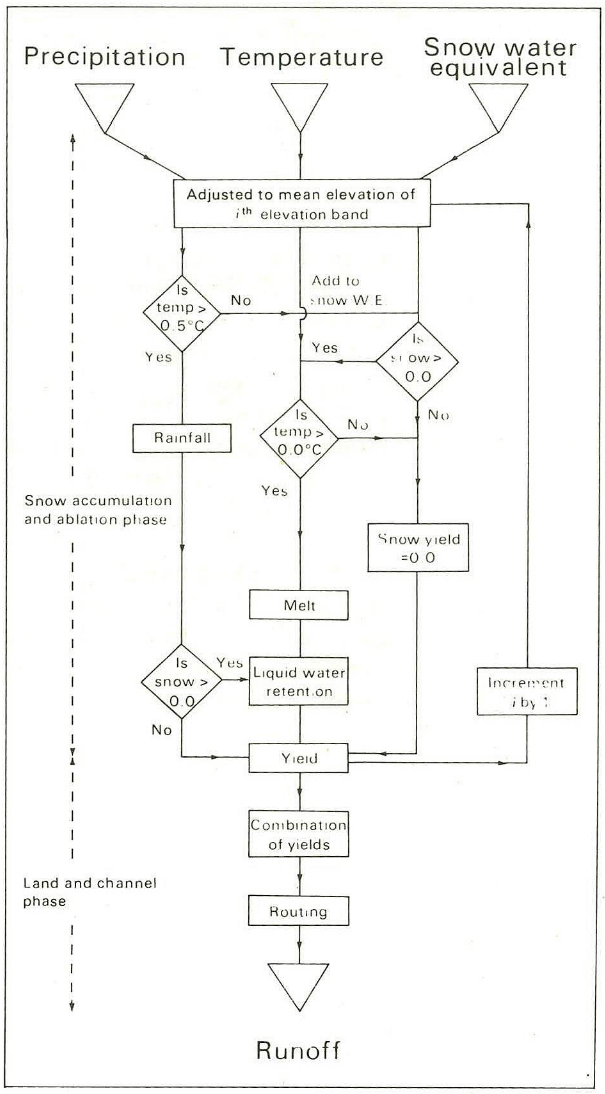


Fig 7.7 A conceptual simulation model of the snowmelt runoff process.

The value of α has been observed to range up to 0.5 with highest values observed at lower densities. Kovzel (1969) reports a relationship devised in the U.S.S.R. from observed data to predict α from density ρ

$$\alpha = \frac{0.11}{\rho} - 0.11 \quad (7.4)$$

where ρ is within the range 0.13–0.45 g/cm^3.

The yield becomes the rainfall when the snow depth decreases to zero. The yield represents the input to the catchment surface in the same sense as rainfall and it may be assumed that beyond this point, except on occasions when the ground is frozen, catchment response is similar to that for rainfall and therefore any existing procedure may be used in forecasting in combination with the snow phase of the model. Two basic approaches to modelling the land and channel phase are possible.

A The simpler method is to recombine the yields in each elevation band to give a lumped input and route through a lumped storage model. For example, an existing unit hydrograph derived from rainfall (Zoller & Lenz, 1958) may be used.

Combination of yields gives

$$Y_t = \frac{1}{A} \sum_{i=1}^{n} a_i y_{it}$$

where Y_t is total catchment yield after time t, A is catchment area, a_i is area of ith elevation band, y_{it} is yield for ith elevation band after time t.

Parameter identification methods developed in unit hydrograph studies may also be used in analysis for optimisation of the parameters of the snow phase.

B In view of the distributed inputs it would seem better to deal with the land and channel phase in terms of a distributed model, routing each element of input through the appropriate amount of lag and storage.

A computer simulation model incorporating the above features of the snow phase and linked to alternative systems of lumped or distributed storage routing has now been devised and is currently undergoing initial testing for parameter identification and comparison of results. The distributed model is similar to the catchment storage model of Laurenson (1964) but includes a procedure for dealing indirectly with input in terms of elevation bands.

The suggested model which uses established hydrological techniques is considered to account for the greater part of the variation in the melt response. It also leaves open the possibility of development within the framework as more data become available and as knowledge of the system improves.

7.2.4 Significance of snowmelt and design floods

The contribution of melting snow to flood runoff in Britain is not generally appreciated. This is due mainly to the relative infrequency of snowmelt. In addition, however, the contribution to floods is either underestimated or completely overlooked because significant proportions of many events consist of rainfall which thereby confuses analyses and interpretation. In essence two questions can be asked: first, how frequently does snowmelt contribute to flood discharge; and second, how important are the events which incorporate snowmelt?

The first of these questions has, in part, been answered by incidental investigations carried out during the research programme. In flood discharges, occurring on the South Tyne at Haydon Bridge from 1959 to 1969, the number of times snowmelt contributed to a flood event varied between 14 and 20% through the range of flood magnitude. Similarly, record peak discharges (up to 1966) in England and Wales listed in the *Surface Water Year Book of Great Britain* included an estimated 53 events out of a total of 329 (16%) affected by melting snow

Station No.	River	Gauging station	Area	Date
24/1	Wear	Sunderland Bridge	658	7. 3.1963
24/2	Gaunless	Bishop Auckland	93	7. 3.1963
24/4	Bedburn Beck	Bedburn	75	7. 3.1963
24/5	Browney	Burn Hall	178	7. 3.1963
25/1	Tees	Broken Scar	818	6. 3.1963
25/4	Skerne	South Park	250	6. 3.1963
25/7	Clow Beck	Croft	78	6. 3.1963
26/2	Hull	Hempholme Lock	378	15. 3.1964
27/1	Nidd	Hunsingore Weir	484	9.12.1965
27/2	Wharfe	Flint Mill Weir	759	16. 2.1950
27/5	Nidd	Gouthwaite Reservoir	114	10.12.1965
27/7	Ure	Westwick Lock	914	7. 3.1963
27/8	Swale	Leckby Grange	1350	7. 3.1963
27/24	Swale	Richmond	381	9.12.1965
27/26	Rother	Whittington	165	9.12.1965
27/28	Aire	Armley	692	9.12.1965
27/29	Calder	Elland	342	8. 2.1966
27/30	Dearne	Adwick	311	9. 2.1966
28/2	Blithe	Hamstall Ridware	162	16. 3.1947
29/2	Great Eau	Claythorpe Mill	77	29.11.1965
30/3	Bain	Fulsby Lock	197	29.11.1965
32/1	Nene	Orton	1630	18. 3.1947
32/2	Willow Brook	Fotheringhay	90	17. 3.1947
32/3	Harpers Brook	Old Mill Bridge	74	16. 3.1947
32/6	Kislingbury Branch	Upton	223	17. 3.1947
32/8	Whilton Branch	Dodford	107	16. 3.1947
33/1	Great Ouse	Brownshill Staunch	3030	16. 3.1947
33/2	Great Ouse	Bedford	1460	15. 3.1947
33/3	Cam	Bottisham	811	14. 3.1947
33/4	Lark	Isleham	466	18. 3.1952
33/8	Little Ouse	Thetford	702	16. 3.1947
33/20	Alconbury Brook	Brampton	202	16. 3.1964
33/21	Rhee	Burnt Mill Weir	303	11. 2.1966
33/22	Ivel	Blunham	541	10. 2.1967
33/23	Lea Brook	Beck Bridge	102	16. 3.1964
36/3	Box	Polstead Ford	54	15. 3.1964
36/6	Stour	Langham Mill	578	17. 3.1964
37/1	Roding	Redbridge	303	16. 3.1964
37/3	Ter	Crabbs Bridge	78	15. 3.1964
37/5	Colne	Lexden	238	16. 3.1964
37/6	Can	Beach's Mill	228	15. 3.1964
37/11	Chelmer	Churchend	73	15. 3.1964
37/12	Colne	Pool Street	65	15. 3.1964
37/13	Sandon Brook	Sandon Bridge	61	15. 3.1964
38/1	Lee	Feildes Weir	1040	16. 3.1947
38/5	Ash	Easneye	85	15. 3.1964
39/10	Colne	Denham	743	21. 3.1947
39/15	Whitewater	Lodge Farm	44	13. 3.1947
43/4	Bourne	Laverstock Mill	164	20. 1.1965
45/3	Culm	Woodmill	226	14. 2.1963
52/5	Tone	Bishops Hull	202	14. 2.1963
54/1	Severn	Bewdley	4330	21. 2.1947
55/1	Wye	Cadora	4040	20. 2.1947

This list gives 53 with a snowmelt contribution, which is 16.1% of total (329). This is a conservative estimate; there were several other occasions when snowmelt may well have contributed from the higher part of the catchment but for which snow records are not available.

Table 7.4 Stations in England and Wales with the highest gauged discharge up to 1966 contributed to by snowmelt (from *Surface Water Year Book of Great Britain, 1965–66*).

(Table 7.4). The events listed in Table 7.1 show that several significant snowmelt flood hydrographs can be identified on 16 well distributed catchments in periods between 5 and 15 years. Clearly snowmelt does contribute significantly to the hydrology of catchments in this country. Scottish rivers can be expected to reveal the same phenomenon.

In answer to the second question, the nature of snowmelt events in Britain suggests that for small catchments (<100 km^2), snowmelt alone will not generally be an important flood design criterion. This is to be appreciated by considering that 'high' intensities of snowmelt are of the order of 8 mm per hour whereas 'high' intensities of rainfall are generally of the order of 20 mm per hour. On rare occasions when the snowpack contains a high water equivalent a significant feature of snowmelt is its duration. The long periods of melt that result suggest that such an event will probably be more significant on larger catchments where times of concentration are of the same order of magnitude. More critical situations can arise when rainfall and snowmelt combine, producing an event which may be important to design even on small catchments. For larger catchments it may well be the major design criterion. Severity of the phenomenon becomes even more significant when high runoff coefficients due to frozen ground are possible. Historical records on a few larger catchments help to confirm these features (e.g. Ouse 1947, 1963; Severn 1947; Great Ouse 1947). It is important to note that such events refer to some of the most extreme floods ever to have been recorded in this country.

Historical events—extremes and maxima

Since it is argued that most serious snowmelt events will occur on large rather than small catchments, and since in this country flow records usually exist on large catchments, the use of historically recorded hydrographs for design purposes should be considered. A review of meteorological records will usually reveal the nature of the flood event and further data analysis may indicate the possibility of more critical temperatures and/or greater snowpack depths occurring. More extreme peak discharges can be estimated in a computation using a simple temperature index method, or a model, and estimated coefficients of runoff and k.

Regional surveys of flooding will indicate occasions when one or more rivers within a region have been subject to extreme volumes of runoff resulting from snowmelt (with rainfall). It will be apparent that such events would probably have occurred on other ungauged catchments within the region; in these instances regional interpolation seems appropriate. For a historic event interpolation could be achieved on a volumetric basis, assuming runoff coefficients to be similar to those of recorded hydrographs but allowing for the occurrence of more snow at higher elevations (see Figure 7.8). Information on this last factor is unfortunately scarce; runoff data must reflect this variation, so that if hydrographs of the same event are available at two or more sites the degree of variation can be deduced. Prediction of unrecorded flood hydrographs could thereafter follow the technique previously outlined.

Historical events—frequency

Similar reasoning can be used to determine a flood frequency relationship for snowmelt events. Clearly if sufficient runoff data exist for a river and

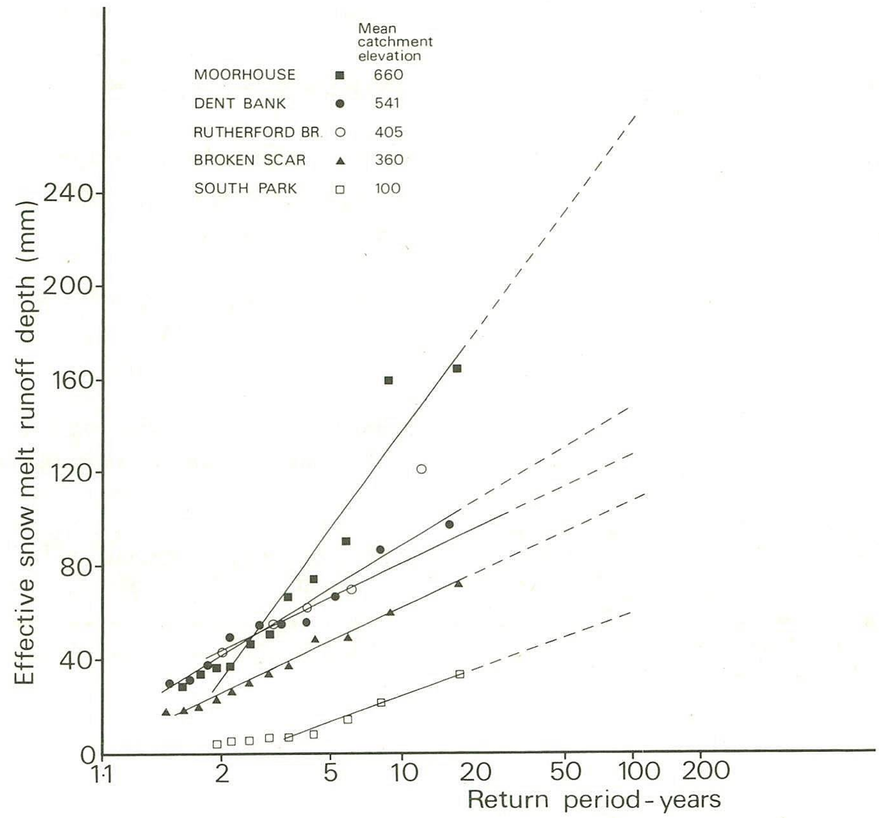


Fig 7.8 Frequency distribution of snowmelt runoff depths for Tees catchments.

events incorporating snowmelt are identifiable, then frequency analysis of flood peak discharges resulting from snowmelt (with rainfall) is possible. Comparison of such analyses with a general frequency analysis will provide information on the relative significance of snowmelt in a region.

Regional snowmelt flood frequency analyses also seem possible particularly in view of the generally widespread nature of snow cover and temperature rises during snowmelt events. Methods exist for grouping individual analyses into regional curves and are well known (see Chapter 2). Some of these methods are more reliable when applied to flood volumes than to peak discharges. This reasoning suggests that a bivariate analysis of temperature and flood volume would be particularly relevant to snowmelt events. A method is described and illustrated.

Previous analysis implies that the maximum flow Q_{max} is related to maximum temperature θ_{max} by

$$Q_{max} = k.\theta_{max}. \tag{7.5}$$

It may also be assumed that air temperature θ, and flood volume D (and hence k) can be represented as discrete variables. It is further assumed that the frequency distribution of these discrete variables can be determined using records of air temperature and flood volumes of snowmelt with rain (Figure 7.8). These are represented by $PR(\theta_i)$ and $PR(D_j)$ respectively, where $PR(\theta_i)$ means $PR(\theta_i - \Delta \leq \theta < \theta_i + \Delta)$ when 2Δ is the width of the class interval. Since k has been shown to be linearly related to D

$$PR(k_j) = PR(D_j). \tag{7.6}$$

By assuming temperature rise to be independent of flood volume,

$$PR(Q_{max} = k.\theta_{max}) = \Sigma\, PR(\theta_i).PR(k_j) \tag{7.7}$$

where the summation only includes terms containing values of θ and k satisfying the condition $Q_{max} = \theta_i.k_j$ and such that the cumulative runoff computed from the temperature series up to the time Q_{max} is reached should not exceed the flood volume D corresponding to the k value.

The cumulative probability function is then estimated from

$$PR(Q_{max} \leq k.\theta_{max}) \simeq \sum_{Q_{max} < \theta_i.k_j} \left(\sum_{i,j} PR(\theta_i).PR(k_j)\right). \tag{7.8}$$

By way of illustration flood volume data and air temperature data (provided by the Meteorological Office), relevant to the Moorhouse catchment, have been analysed. The basic matrix, abbreviated, is shown in Table 7.5 in which corresponding values of Q_{max} and $PR(Q_{max})$ are indicated on the left and right of pairs of numbers corresponding to each element. Cumulative probabilities formed from these results are shown in Table 7.6, where small corrections have been applied to the values to correct for using discrete data. These data are shown plotted in Figure 7.9 and are compared with a frequency plot of observed snowmelt flood peak discharges; the agreement is considered to be satisfactory. There is a small discrepancy of estimate between the two sets of data which is believed to be mainly due to sampling error.

Table 7.5 Matrix of calculated peak discharges (mm per hour) in bold and their respective probabilities for Trout Beck at Moorhouse, e.g. **0.891** $= \theta_1 k_1 = 3.6 \times 0.2475$ and $0.25 = PR(\theta_1)\, PR(k_1) = 0.5 \times 0.5$.

	k_j	0.2475		0.5505		0.675		. .	1.803		1.891		2.129	
	$PR(k_j)$	0.5		0.167		0.083		. .	0.0083		0.005		0.02	
θ_i	$PR(\theta_i)$													
3.6	0.5	**0.891**	0.25	**1.982**	0.083	**2.430**	0.041	. .	.	.	.	.	.	.
4.35	0.167	**1.077**	0.083	**2.395**	0.028	**2.936**	0.0138	. .	.	.	.	.	.	.
4.75	0.083	**1.176**	0.041	**2.615**	0.0138	**3.206**	0.0069	. .	.	.	.	.	.	.
.	.	.	.	.	.	.	.	. .	.	.	.	.	.	.
.	.	.	.	.	.	.	.	. .	.	.	.	.	.	.
7.475	0.0083	.	.	.	.	.	.	. .	**13.477**	0.0001	**14.135**	0.00004	**15.914**	0.0002
7.775	0.005	.	.	.	.	.	.	. .	**14.018**	0.00004	**14.703**	0.00002	**16.553**	0.0001
8.0	0.02	.	.	.	.	.	.	. .	**14.424**	0.0002	**15.128**	0.0001	**17.032**	0.0004

Table 7.6 Calculated cumulative frequency distribution of snowmelt peak discharges (cumecs). Trout Beck at Moorhouse.

Q	3.12	4.16	6.27	7.99	9.21	11.05	11.95
$PR(Q)$	0.248	0.397	0.497	0.649	0.695	0.747	0.797
Q	13.56	15.41	16.84	18.15	18.72	20.09	20.61
$PR(Q)$	0.824	0.863	0.883	0.904	0.911	0.923	0.934
Q	22.00	23.72	24.35	25.78	29.29	32.00	35.23
$PR(Q)$	0.945	0.954	0.968	0.974	0.982	0.988	0.993

Hydrometeorological methods

If future hydrometric work in this country were to produce sufficient information on snow depths and water equivalents, in addition to the already existing air temperature and rainfall data, then it is conceivable that flood prediction for snowmelt events could be achieved without having any regional river discharge data. Because river flow records exist, the advantage of such methods for flood prediction must remain open to question. They will certainly give rise to additional problems, not least of which will be the estimation of runoff coefficient, and the possible combinations and spatial distribution of rainfall, snow water equivalent, and temperature. Therefore, although the potential use of hydrometeorological methods (as defined above) in estimating snowmelt discharges is scientifically interesting, the major reason for developing snow survey procedures must rest with the flood forecasting demand.

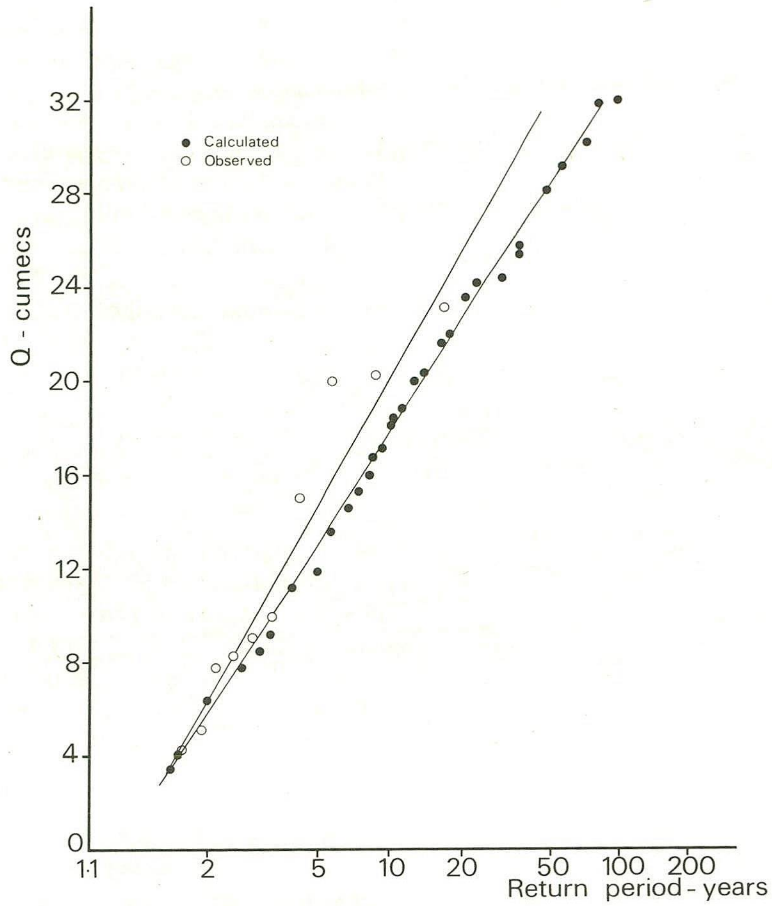


Fig 7.9 Calculated and observed frequency distributions of snowmelt flood peaks. Trout Beck at Moorhouse.

7.2.5 Use of snowmelt studies in flood prediction

The following preliminary attempt at regional snowmelt flood prediction is based on a combination of the meteorological studies of point snowmelt and the snowmelt runoff studies on specific catchments. A summary of the relevant findings of each study follows.

Meteorological studies

In the meteorological studies, the median annual maximum snow depth is mapped (Figure II.7.2) from 1946–64 records for about 100 stations, and it is suggested that a snow depth frequency relationship based on these studies (Table II.7.1) indicates the distribution at any station. A frequency table of annual maximum temperatures with snow lying (Table II.7.3) is based on about 15 years of record from each of 11 stations. A comparison of daily maximum temperatures (θ) with daily changes in snow depth leads to an expression for the point snowmelt rate $R = 12.1\ \theta\rho$ mm water per day, where ρ is the density of lying snow. A typical value for this density is 0.13 g/cm^3. This expression is combined with the temperature frequency analysis to estimate point snowmelt rates. It has been assumed that snow density increases with temperature, and that the snowmelt rate increases proportionally with both density and temperature. Evidence of severe winters with deep snow accumulation, and evidence of snow density in Germany during 1963, lead to the estimate that point snowmelt rates of at least 42 mm per day might occur in most lowland parts of Britain.

Point snowmelt rates, like design rainfall intensities, require routing through a model such as the unit hydrograph/losses model described in Chapter 6. However, higher runoff percentages may be appropriate for snowmelt conditions, and snowmelt is likely to continue longer than rainfall at a high rate and to extend over wider areas. Thus, although a snowmelt rate of 42 mm per day seems small by comparison with an extreme 24 hour rainfall of over 250 mm over most of south east England (Figure II.4.2), the two figures are not directly comparable, especially for larger catchments where it is known that snowmelt floods can be important.

Runoff studies

Studies based on measured runoff showed that in any one event the cumulative runoff was proportional to the cumulative temperature during the main snowmelt period so that $\Sigma Q = k\Sigma\theta$. In any one catchment the snowmelt coefficient k varied with the total runoff, and also with rainfall during the snowmelt period. It is suggested that the increase of k with total runoff may reflect an increasing fraction of the basin contributing to the runoff and that a limiting value of k may be reached. The relationship between k and the total expected runoff can be used to forecast the distribution of runoff from a temperature sequence when the effective snowmelt volume is known.

The k values predicted for various depths were found to vary between catchments inversely with area. Given the runoff depth and catchment area, k can be predicted to transform a temperature sequence to a runoff hydrograph. The values of k for small basins are expected to approach point melt coefficients. This apparent increase of k with snowmelt volume implies that snowmelt cannot be related simply to temperature with snow lying, without taking into account the snow depth. An estimated snowmelt runoff frequency distribution at a sample site was based on the joint probability of temperature and of snowmelt coefficient k, which was related to volumes of snowmelt with rainfall; this compared reasonably with the distribution of recorded snowmelt floods. While such analysis cannot take the place of recorded flows, this detailed example suggested that reasonable estimates of snowmelt runoff frequency might be obtained from regional estimates of temperature and snowmelt volume by assuming that they were independent.

Combination of meteorological and runoff studies

Because the meteorological estimates are primarily applicable to lowland catchments and do not allow for variation in snow depth over the country, while the runoff studies provide a means of forecasting or predicting snowmelt runoff, given the volume of snow in storage, an attempt at regional flood prediction requires a combination of the results of both studies. A tentative method of using these results, based on the limited measurements presently available, provides estimates of snowmelt floods for different parts of the country which appear to be of the right order of magnitude.

For a given catchment where $\Sigma Q = k\Sigma\theta$ for each event, $Q_{max} = k\theta_{max}$ where k varies with total runoff (Table 7.2). If the total runoff in the pure snowmelt event is equated with the product of snow depth (S) and density (ρ), then $k = f(S\rho)$ and $Q_{max} = \theta_{max} f(S\rho)$. $S\rho$ and θ_{max} could be measured but records are sparse at present. If S_T is the annual maximum snow depth of T year return period, S_2 has been mapped (Figure II.7.2) and $S_T:S_2$ can be estimated from Table II.7.1. The corresponding k_2 and k_T can be derived from $k = f(S\rho)$, given ρ. The maximum temperature $\theta_{max,T}$ of a given return period can be estimated from Table II.7.3 on the assumption that similar temperatures over snow might occur throughout the country.

Thus, $Q_{max} = k\theta_{max}$, where the frequency distribution of each variable might be estimated for a given site. Moreover, because the predicted k for a given depth has been found to be related to catchment characteristics, k_2 or k_T could be estimated for an ungauged basin. The k values corresponding to various depths are given in Table 7.2, together with certain catchment characteristics for the basins studied. Table 7.7 gives the correlation matrix for k values and catchment characteristics, showing that k is related to both area and slope in the small set of 15 stations. These relationships are shown in Table 7.8 and illustrated in Figure 7.5(a) and (b).

Table 7.7 Correlation between snowmelt coefficients (k) and catchment characteristics.

	Natural				Logarithmic			
	AREA	S1085	STMFRQ	SAAR	AREA	S1085	STMFRQ	SAAR
AREA	1.000	−0.558	−0.499	−0.556	1.000	−0.719	−0.531	−0.673
S1085	−0.558	1.000	0.643	0.853	−0.719	1.000	0.582	0.798
STMFRQ	−0.499	0.643	1.000	0.500	−0.531	0.582	1.000	0.349
SAAR	−0.556	0.853	0.500	1.000	−0.673	0.798	0.349	1.000
k 12.5	−0.588	0.838	0.691	0.831	−0.711	0.759	0.629	0.735
k 25	−0.639	0.874	0.691	0.829	−0.791	0.814	0.626	0.735
k 50	−0.663	0.894	0.641	0.786	−0.828	0.806	0.573	0.668

The data for this analysis are drawn from Table 7.2 (15 catchments).

Table 7.8 Regression of k on area and slope.

	Coeff.	seb	R	see	Const.
Natural regression of k on slope (S1085)					
k 12.5	0.00528	0.00096	0.838	0.0594	0.07846
k 25	0.00626	0.00096	0.874	0.0600	0.11395
k 50	0.00886	0.00123	0.894	0.0767	0.17985
Logarithmic regression of k on AREA					
k 12.5	−0.3185	0.0873	0.711	0.2154	−0.1967
k 25	−0.2913	0.0626	0.791	0.1543	−0.1067
k 50	−0.2793	0.0526	0.828	0.1296	0.0560

Prediction of snowmelt floods

As an index of snowmelt runoff, the median annual snowmelt flood was estimated for the isopleth intervals of Figure II.7.2, and the results are presented in Table 7.9. The median snow depth and typical density were combined to give the water equivalent; the corresponding k values for areas of 10, 100, and 1000 km^2 were deduced from Figure 7.5(a), while k values were also derived from Figure 7.5(b) using typical values of channel slope selected by comparing Figure II.7.2 with Figure 4.4. The products of these k values with temperatures of 2 year return period (Table II.7.3) give estimates of the median annual maximum snowmelt runoff in mm per hour and in cumecs per 1000 km². These figures might be used to calibrate the intervals of Figure II.7.2, but it will be noted that the runoff from small catchments, estimated from area, greatly exceeds estimates based on general channel slope. Because the small areas of Figure 7.5 represent steep upland basins, the extrapolation to small lowland catchments is not based on direct data and is also sensitive to whether slope or area is chosen as the independent variable.

The snowmelt floods of other return periods could in theory be deduced by deriving the distribution of $Q = k\theta$ from the bivariate distribution of k and θ, but the data do not justify such a complex calculation. However, a simple approach to a Q:T curve would be to select a median value for the snow depth and density, and to vary the return period of the temperature; this would imply a ratio Q_T:Q_2 equal to the ratio θ_T:θ_2 from Table II.7.3. The alternative combination of a median temperature (θ_2) and a snow depth and k of a varying return period gives results which depend on whether k is predicted from area or slope. When catchment area is used to predict k values corresponding to snow depths of different return period and median density, the results were similar to those from k_2 and θ_T; when slope was used to predict k, a steeper curve

Table 7.9 Provisional snowmelt estimates based on k values related to slope and area.

(a) Estimates based on typical slope.

Median snow depth (mm)	Median water equivalent (mm)	Typical slope (S1085) (m/km)	Typical median k (mm per hour °C)	Typical snowmelt runoff corresponding to various return periods (mm per hour) 2	10	100	(cumecs per 1000 km²) 2	10	100
50	6.5	1	0.066	0.28	0.41	0.57	78	114	158
100	13	2	0.090	0.38	0.56	0.77	106	156	214
200	26	5	0.15	0.63	0.93	1.29	175	258	358
300	39	10	0.22	0.92	1.36	1.89	256	378	525

The median snow depths correspond to the intervals of Figure II.7.2; the median water equivalents are calculated from a density of 0.13; the typical slopes were estimated by comparing Figure II.7.2 with Figure 4.4. The k values are estimated from Figure 7.5(b). The snowmelt estimates are based on temperatures of 4.6, 6.2 and 8.6°C (Table II.7.3).

(b) Estimates based on area.

Median snow depth (mm)	Median water equivalent (mm)	Estimated k (mm per hour °C)	Estimated snowmelt runoff corresponding to various areas and return periods (mm per hour) 2	10	100	(cumecs per 1000 km²) 2	10	100
Area 10 km²								
50	6.5	0.29	1.22	1.80	2.49	339	500	692
100	13	0.35	1.47	2.17	3.01	408	603	836
200	26	0.42	1.76	2.60	3.61	489	722	1000
300	39	0.52	2.18	3.22	4.47	606	895	1240
Area 100 km²								
50	6.5	0.12	0.50	0.74	1.03	139	206	286
100	13	0.15	0.63	0.93	1.29	175	258	358
200	26	0.21	0.88	1.30	1.81	244	361	503
300	39	0.26	1.09	1.61	2.24	303	447	622
Area 1000 km²								
50	6.5	0.051	0.21	0.32	0.44	58	89	122
100	13	0.066	0.28	0.41	0.57	78	114	158
200	26	0.11	0.46	0.68	0.95	128	189	264
300	39	0.14	0.59	0.87	1.20	164	242	333

The k values are estimated from Figure 7.5(a).

for Q_T was obtained. Either approach assumes that the annual maximum temperature over snow occurs in the same snowmelt event as the annual maximum snow depth, but that between years temperature is independent

Table 7.10 Snowmelt floods estimated for selected stations from annual maximum temperature series, with *k* estimated from median snow depth and area or slope.

Station	Median snow depth (cm)	Median water equiv. (mm)	Area (km²)	Slope (m/km)	*k* est. from area	*k* est. from slope	Snowmelt flood for various return periods est. from area (cumecs)			Snowmelt flood for various return periods est. from slope (cumecs)			Overall flood for various return periods est. from records (cumecs)		
							2	10	100	2	10	100	2	10	100
8/3	31	40.3	534	3.07	0.170	0.178	106	156	217	111	163	227	102	158	228
10/1	30	39.0	448	3.48	0.175	0.177	92	135	188	92	137	189	49	76	110
20/1	10	13.0	307	6.03	0.100	0.112	36	53	73	40	59	82	57	88	127
24/3	27	35.1	172	14.33	0.210	0.245	42	62	86	49	72	100	119	185	267
27/1	6	7.8	484	4.01	0.070	0.086	40	58	81	49	72	100	128	200	288
27/14	9	11.7	679	3.59	0.074	0.096	59	87	120	76	113	156	91	142	205
27/810	9	11.7	899	1.98	0.066	0.088	69	102	142	93	137	190	207	323	466
31/2	8	10.4	342	2.12	0.090	0.085	36	53	74	34	50	70	16	24	35
37/1	4	5.2	303	1.22	0.071	0.067	25	37	52	24	35	48	21	33	47
39/6	3	3.9	365	1.90	0.060	0.066	26	38	52	28	42	58	12	20	28
41/3	5	6.5	130	2.70	0.110	0.077	17	25	34	12	17	24	37	58	84
46/2	5	6.5	381	9.83	0.072	0.108	32	47	63	48	71	98	192	299	431
54/7	9	11.7	319	3.30	0.094	0.095	35	52	71	35	52	72	42	65	94
68/1	7	9.1	609	1.30	0.066	0.078	47	69	96	55	82	113	57	89	128
84/4	20	26.0	742	1.74	0.120	0.127	104	153	212	110	163	226	252	393	567

The *k* value corresponding to median water equivalent is estimated from area (Figure 7.5(a)) or from slope (Figure 7.5(b)). The snowmelt flood is the product of *k*, area and temperatures of 4.2, 6.2 and 8.6°C corresponding to 2, 10 and 100 years return period (Table II.7.3). The overall flood estimates are based on annual maximum floods at the site.

of snow depth and density. If temperature and snow depth are not independent, and if the snowmelt coefficient k does not reach a limiting value, then the ratio $Q_T:Q_2$ would increase even faster.

To test the method of prediction for an ungauged catchment in a given part of the country, snowmelt estimates were made for 15 selected gauging stations in catchments between 100 and 1000 km^2 (see Table 7.10). The median annual maximum snowmelt flood was predicted using channel slope and area as alternatives, with similar results. On average the median snowmelt flood was less than the overall flood from records, which of course it must be; however in five cases the snowmelt flood exceeded the overall flood and in two cases was twice as large (these cases were both in south east England). The conclusion must be that the method provides a prediction which is of the right order of magnitude, but which is no substitute for detailed study of records at a site and cannot yet be applied to lowland basins.

Conclusions

Can any conclusions be drawn that are relevant to regional flood prediction? The snowmelt flood appears to be greater relative to the overall flood in the drier east of the country than the west, and this is supported by the generalised estimates of Table 7.9. The $Q:T$ curves based on temperature alone are similar to the overall flood $Q:T$ curves, so the estimates would give the same impression at other return periods.

The extrapolation from the upland basins where the snowmelt events were examined to lowland basins in the south east may be vitiated by differences in permeability and runoff fraction as well as slope, but the importance of snowmelt floods in lowland Britain has been illustrated in 1947 (Table 7.4). The physical argument that the duration of snowmelt compared with rainfall makes snowmelt floods likely to be critical in large basins seems reasonable, but Table 7.4 shows that record floods on all types of basin may be affected by snowmelt.

It is concluded, however, that separate analysis of snowmelt and rainfall floods has statistical and practical disadvantages and is largely unnecessary for design purposes. It is in practice extremely difficult to differentiate between the two types of flood, as many recorded winter floods contain some proportion of snowmelt runoff and detailed study of weather records is required even to determine whether snowmelt contributed to a given event.

If it were possible to classify all events into those which were predominantly caused by rainfall or by snowmelt, there are disadvantages in treating the floods at a single station as drawn from two populations, as the records must be used to estimate the parameters of two distributions (1.2.6 and 2.3.2). It seems no less appropriate in this country to treat floods arising from snowmelt and rainfall as one series than to combine those arising from frontal or convectional storms.

It is suggested that the statistical analysis of the record from a single station (Chapter 2) should include both types of floods. The derivation of a regional curve already does so because the longer records used should include a representative sample of snowmelt floods; it is worth noting that the proportion of the extreme floods containing snowmelt was not exceptional. If the shape of the hydrograph is particularly relevant to a design problem, historic snowmelt floods may provide useful evidence.

The catchment response approach, in which the unit hydrograph/losses model is used to convert rainfall of a selected return period to a design hydrograph (Chapter 6), is more appropriate to smaller catchments. Because the design technique is based on recorded floods, the recommended choice of duration and return period of design storm and of antecedent conditions makes implicit allowance for snowmelt events in the flood records. If it is important to predict snowmelt floods specifically, a method of estimating snowmelt coefficients from specific snowmelt events has been presented for use with snow and temperature estimates. This method would make it possible to make some estimate of the maximum snowmelt, but at present not enough is known about maximum temperatures over snow, maximum snow depths and densities, and whether or not there is a limit to snowmelt coefficients as these increase; there is therefore insufficient evidence at present for generalised estimates of maximum snowmelt. However, the maximum flood at a site will either be caused predominantly by rainfall or predominantly by snowmelt. The inclusion of some snowmelt in estimating the maximum rainfall flood is discussed in Section 6.8.3; the maximum rainfall and flood may be compared with possible snowmelt runoff and snowmelt rates to determine whether the possibility that snowmelt runoff could provide the maximum is sufficient to justify a detailed investigation. This might be the case in permeable areas of lowland Britain where freezing conditions may lead to temporary impermeability either because the ground is frozen below the snow cover or through the partial melting and refreezing of early, well compacted snow. A small chalk catchment in southern England produced in 1947 a flow 10 times its mean annual flood, and the estimation of the maximum flood from a normally permeable catchment should take account of this coinciding with a snowmelt flood by adjustment of the soil index.

7.3 Conceptual catchment modelling of isolated storm events

7.3.1 Introduction

A simple four parameter conceptual model of catchment response has been developed with the object of modelling only isolated storm events and thus providing a useful tool for flood estimation. Most previous conceptual modelling has been concerned with fitting continuous long term records in which flood events are small in number. The isolated event model (IEM) is concerned only with such flood events. The IEM project was carried out in parallel with the unit hydrograph studies which have formed a major part of the Flood Studies project. In fact the IEM project was deliberately planned so as to take advantage of the data being collected for the unit hydrograph work.

At its present stage of development, use of the isolated event model is confined to gauged catchments. In essence, the model is fitted to such catchments by finding those optimum values of the model parameters which enable the model to reproduce existing isolated events to an acceptable degree of accuracy. Design rainstorms can then be routed through the fitted model to yield flood hydrographs.

The optimum parameter values are found by an automatic computer optimisation technique. It is envisaged that this phase of the use of the IEM technique could best be done at the Institute of Hydrology using the sophisticated program on the IH computer system. Once the required optimum parameter values have been found, routing design rainstorms through the fitted model could be done by the design engineer himself using simple programs. The optimisation phase could also be handled by the design engineer after experience of the method if he has access to an appropriate computer system.

A future development of the IEM project could aim to provide a design method for the ungauged catchment, for instance by regional correlation and field measurements giving information on model parameter values to be used.

The following sections fall into two broad groups, a 'research and development' group summarising the work done in developing and verifying the technique, and an 'application' group setting out the steps to be taken by the user in implementing the technique. The intention of the latter group is to provide the design engineer with sufficient material for use of the method with the minimum of need to refer to the 'R & D' group of sections.

Section 7.3.2 describes briefly the catchments and data used to develop the model and the IEM technique. Data from each catchment were divided into a number of separate storm events, each storm event containing rainfall, river flow, antecedent rainfall and initial soil moisture data.

The isolated event model used is simple and deterministic, has a small number of parameters, and the current version, IEM4, is described in Section 7.3.3. The initial conditions of a catchment at the start of a storm are a major factor in determining the volume of subsequent runoff, and this model makes use of the daily soil moisture deficits calculated by the Meteorological Office.

The methods used in fitting the model to the data are described in Section 7.3.4. A fitting function is defined to estimate objectively the difference between an observed hydrograph and the hydrograph reconstructed by the model. Optimisation techniques adjust the parameter values of the model so that the fitting function is minimised.

After considerable development work on a sequence of model structures and strategies of optimisation, the present model (IEM4) has been found acceptable on the proving data used, consisting of 500 events from 21 catchments, and a two stage optimisation strategy has been adopted. The first of these stages, described in Section 7.3.5, applies an abridged version of model IEM4 to a given catchment, using individual storm events one at a time. The major concern here is whether the model is able to fit the shape of each observed hydrograph. The data for poorly fitted hydrographs are checked for quality, and events of inadequate standard are excluded from the next stage of fitting. Snowmelt events discovered during this stage are also excluded.

In the second stage of fitting, described in Section 7.3.6, the complete model IEM4 is applied to all the events from a single catchment as a group. The major concern here is to establish the relation between the volume of runoff and the initial conditions on the catchment. Measures of the differences between the observed and reconstructed hydrograph (namely the differences in peak discharge, volume of runoff, and timing of the peak) are produced. Results for those catchments and events used in the R & D phase are summarised in Section 7.3.6.

The instructions for the use of the isolated event model technique are contained in Section 7.3.7. Details are given of the data required, with a short discussion of the question of initial catchment conditions. All steps in the calculation of the output hydrograph are listed, and a computer program incorporating these is appended.

To conclude this introduction, the main points to emerge so far from this IEM project will be summarised and future studies outlined. A simple four parameter conceptual model has been developed to relate, for isolated events, the storm rainfall and initial catchment conditions to the resulting outflow hydrograph. The isolated event model and technique were developed using 500 isolated storm events from 21 catchments. The model is not designed to be applied to events involving snowfall or frozen ground. The optimum values of the parameters of the model, when it is applied to the data for a given catchment, are determined by automatic optimisation methods. The standard design procedure incorporates loss estimation and unit hydrograph derivation; the IEM technique provides an alternative method but is suitable only for gauged catchments.

The IEM technique appears to be more successful for steep upland catchments than for slow lowland rivers. It is possible that considerable improvement in the latter cases may be obtained by the addition of a time–area element to the basic model. Other changes that are being incorporated into further versions of the model are: variation of the runoff ratio during a storm; adjustment of the exponent in the nonlinear storage/discharge relation to give more concave recessions; allowance for the variation of the storage/discharge coefficient between different events; and more detailed study of the relation between runoff and initial soil moisture deficit. Finally, it is proposed to investigate further the relation between frequencies of peak flows and of rainfall intensities using long series of stochastically generated rainfall sequences.

7.3.2 Data used for development

The same isolated storm event data used in the unit hydrograph project (Chapter 6) provided the data bank for the IEM project. The data for each storm event consisted of four types: two types describe what occurs during the storm (namely the streamflow hydrograph and rainfall hyetograph, normally at hourly intervals); the two other data sets describe the conditions on the catchment before the storm (namely daily values of the rainfall and of the soil moisture deficit for the 28 days preceding the storm date). Full details of the storm event data are given in Volume IV.

Of the total number of isolated storm events available, approximately one third (500) were used in the IEM project. Twenty catchments were selected to give a good coverage over England and Wales, with one additional catchment in Scotland (Figure 7.10 and Table 7.11). Only those catchments with a large number of events, preferably more than 20, were used. The size of the catchments ranged from 1.52 km^2 to 443 km^2.

7.3.3 Structure of the isolated event model

Several versions of the basic conceptual model first postulated were studied, each version incorporating minor modifications leading to improvements in the model performance. The prime requirement was to keep the number of model parameters small. This task was made easier

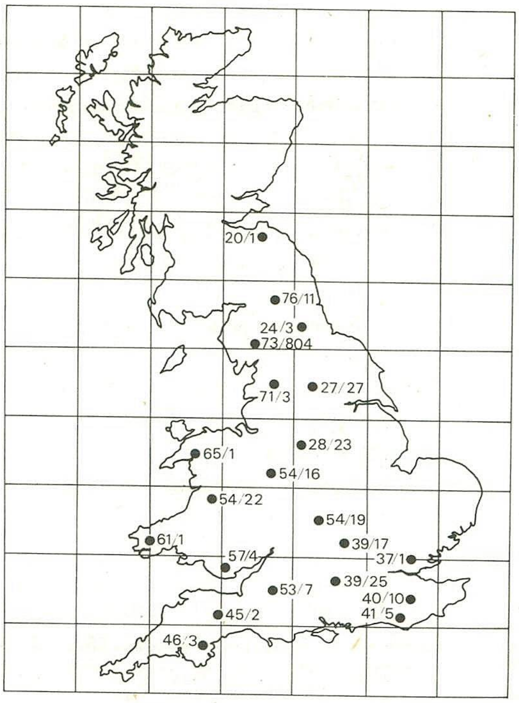


Fig 7.10 Geographical position of catchments used in IEM project.

because a flood event model does not need the many infiltration and groundwater parameters required to deal with long recessions and dry weather conditions as in conventional continuous record models. The version finally adopted, the IEM4 model, has four parameters and is illustrated in Figure 7.11. Two of the parameters (PERC and PERI) determine the volume of rainfall converted to runoff, and the remaining two (DEL and AC) control the routing of that runoff volume. The volume conversion procedure is based on the contributing area concept, where the area of a

Table 7.11 List of catchments used in IEM project.

Catchment number	Catchment name	Area (km²)	Number of events	Objective function values (see Section 7.3.5)	
				$\bar{R}_E$	$\bar{P}_E$
20/1	Tyne at East Linton	307.0	10	0.967	0.147
24/3	Wear at Stanhope	172.0	21	0.900	0.327
27/27	Wharfe at Ilkley	443.0	24	0.934	0.184
28/23	Wye at Ashford	154.0	11	0.907	0.117
37/1	Roding at Redbridge	303.0	14	0.779	0.256
39/17	Ray at Grendon Underwood	18.6	67	0.899	0.276
39/25	Enborne at Brimpton	152.8	17	0.857	0.216
40/10	Eden at Penshurst	224.0	29	0.866	0.219
41/5	Ouse at Goldbridge	182.0	24	0.883	0.174
45/2	Exe at Stoodleigh	422.0	22	0.932	0.170
46/3	Dart at Austins Bridge	248.0	23	0.928	0.227
53/7	Frome at Tellisford	262.0	21	0.911	0.170
54/16	Roden at Rodington	259.0	20	0.830	0.168
54/19	Avon at Stareton	347.0	19	0.823	0.228
54/22	Severn at Plynlimon Weir	8.05	26	0.929	0.226
57/4	Cynon at Abercynon	109.0	19	0.946	0.137
61/1	Western Cleddau at Prendergast Mill	197.6	25	0.926	0.172
65/1	Glaslyn at Beddgelert	68.6	21	0.969	0.155
71/3	Croasdale Beck at Croasdale Flume	10.4	24	0.938	0.279
73/804	Brathay at Brathay Hall	57.5	19	0.953	0.183
76/11	Coal Burn at Coalburn	1.52	44	0.908	0.229
Number of catchments 21				Total number of events 500	

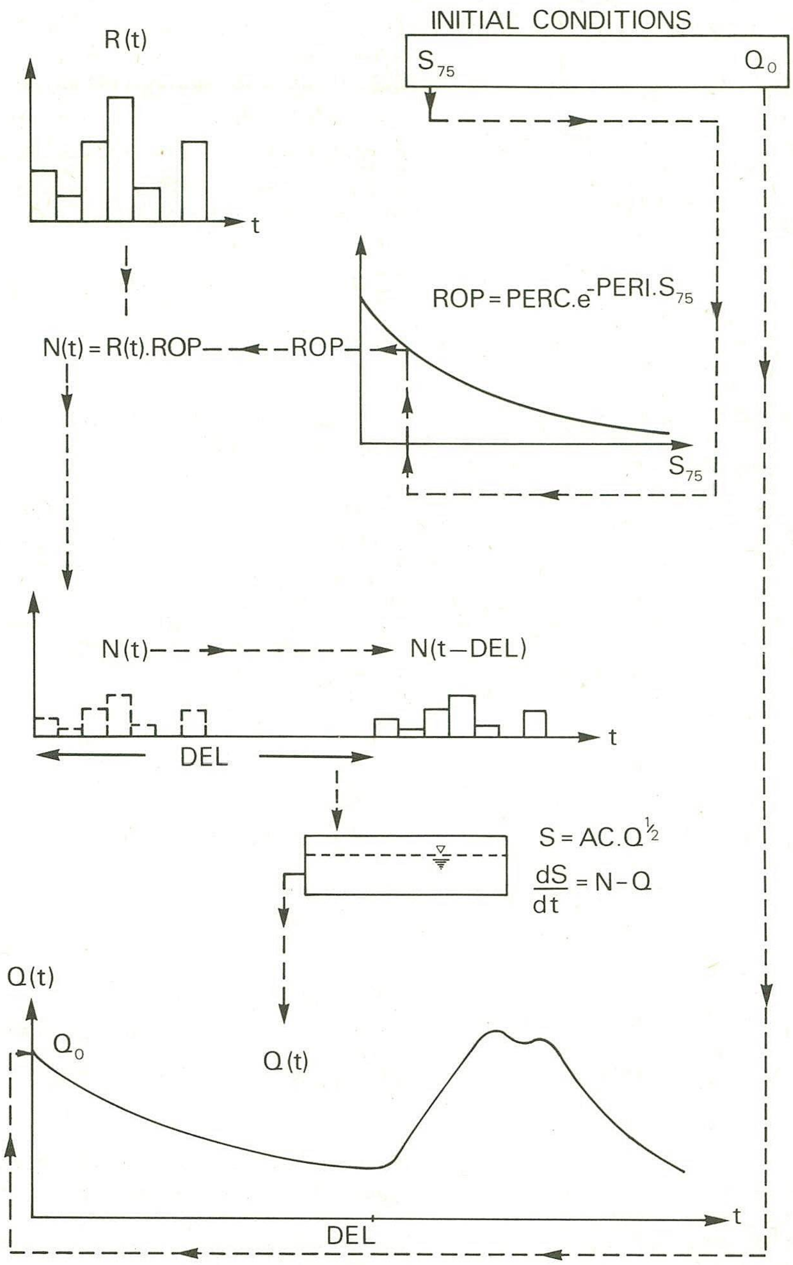


Fig 7.11 The four parameter model IEM4.

catchment contributing to runoff is a function of initial catchment conditions. The net rainfall, $N(t)$, is taken as a proportion of the gross rainfall, $R(t)$, the proportion ROP, the runoff ratio, being constant for any one event.

Intuitively, ROP was expected to depend on initial conditions at the start of the storm. The best relation found for the 21 catchments and 500 events was between ROP and S_{75}, the initial soil moisture deficit over the area with 75 mm root constant, and has the form

$$\text{ROP} = \text{PERC}.e^{-\text{PERI}.\,S_{75}} \tag{7.9}$$

where PERC and PERI are two parameters to be determined for a given catchment. The value of S_{75} on the storm date was adjusted by any rainfall occurring before the start of the main storm.

The main routing element of the IEM4 model is a reservoir with the nonlinear storage/discharge relation $S = \text{AC}.Q^{\frac{1}{2}}$, where S is storage and

Q is discharge. The net rainfall, $N(t)$, is first delayed by a time shift, DEL, before being routed through the nonlinear reservoir. Taking Q_0 as the initial value of the discharge at the start of the event, the nonlinear reservoir is allowed to recede with zero input for a period DEL. The net rainfall, $N(t)$, is then used as input to the nonlinear reservoir, and the complete runoff hydrograph, $Q(t)$, obtained, as shown in Figure 7.11. DEL and AC are the two remaining parameters to be determined.

7.3.4 *Model fitting criteria and optimisation*

Fitting criteria

Given a set of n observed discharge values Q_i ($i = 1,2,\ldots n$), and the corresponding observed input rainfall and initial conditions, specific numerical values may be given to the parameters of a conceptual catchment model and the input data routed through the model to reconstruct another set of discharge values Q_i' ($i = 1,2,\ldots n$). Some objective function that measures the disagreement between the observed and reconstructed hydrographs is required, and for this project a first choice was simply the objective function:

$$F^2 = \sum_{i=1}^{n}(Q_i - Q_i')^2. \tag{7.10}$$

To compare the fit of the model for one catchment over records of different lengths or over records from different catchments, it is preferable to express the function F^2 in dimensionless form, and two versions are generally employed. The first is

$$P = \frac{\left(\dfrac{F^2}{n}\right)^{\frac{1}{2}}}{\bar{Q}} \tag{7.11}$$

which relates the root mean square difference to $\bar{Q}$, the mean of the observed discharges. The second version is

$$R = \frac{{F_0}^2 - F^2}{{F_0}^2} \tag{7.12}$$

where ${F_0}^2$, referred to as the initial sum of squares, is a measure of the deviations about the mean in the observed record,

$${F_0}^2 = \sum_{i=1}^{n}(Q_i - \bar{Q})^2. \tag{7.13}$$

P and R are analogous to the coefficients of variation and determination respectively in linear statistical theory. For a perfect fit between observed and reconstructed hydrographs, F^2 and P take the value zero, and R takes the value unity.

In the IEM project F^2, P and R normally refer to a group of events on a catchment, but for a comparison over the hydrograph of a single event, the notation F_E^2, P_E and R_E is used.

Optimisation of parameters

For each set of values assigned to the parameters of a model a value of the objective function F^2 can be calculated for a given data set. Different sets of parameter values give different values of F^2; when F^2 takes a minimum value the model will then give the closest agreement between the observed and reconstructed hydrographs, and the relevant parameter values are optimum.

Optimisation techniques are iterative procedures for automatically finding that set of parameter values which will give the minimum value of a function of those parameters. The Rosenbrock 'rotating co-ordinate' optimisation method was used in the IEM project, since it has been found to be efficient and stable when applied to hydrological conceptual models. In the version used for the IEM project, each parameter is constrained to lie within a certain range. The limits of the range are subjectively chosen to straddle the expected optimum value of the parameter, and also to constrain the parameter to physically realistic limits; for example, PERC would be normally expected to lie within the range zero to unity. The optimum value for F^2 is considered reached when the maximum change in any one parameter does not exceed one thousandth of its range.

Although these optimisation techniques would be impractical except with the aid of a computer, one reason for restricting the total number of parameters in the isolated event model is that optimisation techniques work less efficiently and consume a larger amount of computer time as the number of parameters is increased. Small gains in the goodness of fit obtained using models with more parameters were usually vitiated by optimisation problems and heavily increased computer time.

The order in which the parameters of a model are optimised also has a bearing on the efficiency of the procedure, and normally those parameters which have most effect on the magnitude of the fitting criteria should be optimised first. In the IEM4 model the parameters controlling the volume of runoff dominated those controlling the shape of the hydrograph, and the order of optimisation was in sequence PERC, PERI, DEL and AC.

Fitting strategy for IEM4 model

A two stage fitting strategy emerged from the development work as being the most effective procedure to follow. Fitting first to individual events on a catchment allows a screening of the quality of those events, leading to a rejection of nonacceptable events. An abridged version of the model is used for this first stage fitting. The second stage fitting is done on all the acceptable events on a catchment as a group, and uses the full four parameter IEM4 model.

7.3.5 Fitting the IEM4 model to individual events

Method

In the first stage of the fitting process, an abridged version of the IEM4 model is applied to each event in turn. The runoff ratio, ROP, is itself treated as a parameter and an optimum value of ROP determined for each event which gives the best agreement between the runoff volume of the observed hydrograph and that reconstructed by the model. At the same time, optimum values of the routing parameters, AC and DEL, are found. For each event the fitting criteria P_E and R_E are calculated, and each event is then inspected in detail, particularly those with poor values of the fitting criteria or anomalous parameter values.

Results

Examples of typical events and fitting results are given in Tables 7.12 and 7.13 for catchment No. 39/25, the Enborne at Brimpton. Table 7.12 contains details of each storm event and certain properties of the observed streamflow. Table 7.13 gives the result of applying the abridged version of the IEM4 model to each event. Optimised values of ROP and of the two parameters AC and DEL are given in columns b, c and d respectively. The total net rainfall, ΣN, is given in column e. The errors in the three most important statistics of a hydrograph, namely the maximum discharge, the runoff volume under the hydrograph, and the time of the peak are given in columns g, i and j. The objective measure of fit, F_E^2, between the observed hydrograph and reconstructed hydrograph is given in column k, together with the dimensionless forms, R_E and P_E, in columns l and m. Comments after scrutiny of the results and checking of the data quality are given in column n. The events in column n were rejected for the second stage of fitting.

As a guide to the general performance of the abridged version of the IEM4 model, the mean values of the dimensionless fitting criteria, $\bar{R}_E$ and $\bar{P}_E$, were calculated for the group of events on each of the 21 catchments, and are given in Table 7.11.

Discussion

In fitting the abridged model to individual events one is principally concerned with the shape of the hydrograph. The first question to be posed is whether the model is sufficiently flexible to fit all the hydrographs in the data selected. From Table 7.11 it is apparent that the mean fitting criterion, $\bar{R}_E$, ranges from 0.779 to 0.969 where $\bar{R}_E = 1.00$ indicates a perfect fit. Those catchments with poor values of $\bar{R}_E$ are seen to be the large slow flowing rivers of the Midlands and south east, such as the Avon at Stareton or the Eden at Penshurst. For those catchments the reconstructed hydrographs tend to be much too spiky in comparison with the observed hydrographs, and the model is clearly inadequate. Those catchments with high values of $\bar{R}_E$ tend to be the small steep upland catchments such as the Glaslyn or Croasdale Beck, and for such catchments the model is normally successful. The fit for the remainder of the catchments falls within these two extremes.

The second question to be posed is whether the optimum values of the parameters are consistent between different events from the same catchment. Inspection of Table 7.13 indicates that the parameter DEL appears consistent, excluding those events with poor quality data, as noted in column n, but that the parameter AC tends to vary between events. This conclusion is true of the great majority of the remaining catchments.

7.3.6 Fitting the IEM4 model to groups of events

Method

Certain events need to be rejected at the end of the individual event fitting stage before the second stage of the model fitting. It is important that the reasons for the rejection are valid ones and are as objective as

Table 7.12 Details of isolated storm events for catchment No. 39/25 the Enborne at Brimpton (area 152.8 km², time interval 1.0 hour).

a	b	c	d	e	f	g	h	i	j	k	l	m
Storm event number	Date of event	Number of hourly rainfall volumes	Total gross rainfall ΣR (mm)	Rainfall preceding start of storm on date of event (mm)	Soil moisture deficit over 75 mm root constant area S_{75} (mm)	First observed flow Q_0 (cumecs)	Number of hourly stream flow ordinates n	Total observed runoff volume ΣQ (10^3 m³)	Maximum observed discharge (cumecs)	Maximum discharge ranked in order of magnitude	Mean discharge $\bar{Q}$ (cumecs)	Initial sum of squares $\Sigma(Q-\bar{Q})^2$ (cumecs)²
1	15.10.1967	32	38.8	0.0	−25.4	1.690	72	1338	17.30	5	5.16	2050
2	30.10.1967	21	14.8	0.0	−25.4	2.030	42	746	11.10	9	4.94	457
3	18.12.1967	17	14.7	0.0	0.0	0.944	62	615	6.01	14	2.76	154
4	13.1.1968	4	4.5	0.0	0.0	1.580	76	1586	14.10	6	5.80	1515
5	5.2.1968	29	17.2	0.0	0.0	3.530	65	1048	9.04	12	4.48	253
6	13.2.1968	14	11.3	0.0	0.8	1.660	72	847	7.51	13	3.27	196
7	24.5.1968	44	23.3	0.0	31.2	0.634	54	395	5.71	17	2.03	111
8	26.6.1968	45	27.4	0.1	−25.4	0.645	96	563	5.93	15	1.63	175
9	14.9.1968	73	85.6	0.0	5.3	0.373	106	4085	26.20	1	10.70	6790
10	27.10.1968	58	22.1	0.0	3.3	0.881	72	796	9.27	11	3.07	527
11	28.11.1968	56	18.2	0.1	0.0	1.590	84	1026	5.91	16	3.39	218
12	17.12.1968	24	11.5	0.0	0.0	3.950	48	1346	12.50	7	7.79	493
13	21.12.1968	5	17.3	0.0	0.0	5.730	42	1573	18.70	4	10.40	1090
14	24.12.1968	21	20.0	0.0	0.0	3.500	60	1339	10.80	10	6.20	439
15	17.1.1969	1	13.4	0.0	0.0	4.390	48	1100	11.70	8	6.39	361
16	11.3.1969	87	39.8	0.0	6.6	2.730	107	3027	21.50	3	7.86	3420
17	22.1.1971	31	19.7	0.0	0.0	7.450	48	2177	23.30	2	12.60	1950

Table 7.13 Results of fitting abridged IEM4 model to individual events on catchment No. 39/25 the Enborne at Brimpton (area 152.8 km², time interval 1.0 hour).

a	b	c	d	e	f	g	h	i	j	k	l	m	n
Storm event number	Runoff ratio ROP	Storage/discharge coefficient AC (mm. hour)$^{\frac{1}{2}}$	Routing delay DEL (hour)	Total net rainfall ΣN (mm)	Maximum reconstructed discharge (cumecs)	Error in maximum discharge (%)	Total reconstructed runoff volume $\Sigma Q'$ (10^3m^3)	Error in total runoff volume (%)	Error in time of peak (hour)	Objective function F_E^2 (cumecs)^2	Objective function R_E	Objective function P_E	Comments on data anomalies
1	0.257	12.8	8.69	9.98	18.80	8.8	1359	1.6	−5	366.0	0.836	0.419	Soil moisture data absent
2	0.371	11.0	3.41	5.49	11.90	7.0	721	−3.3	−4	61.1	0.866	0.244	Soil moisture data absent
3	0.304	13.3	4.36	4.48	5.91	−1.7	607	−1.3	0	1.7	0.989	0.060	
4	3.300	32.7	9.53	14.90	15.20	7.5	1540	−2.9	−8	248.0	0.836	0.312	Snowfall
5	0.331	18.2	4.32	5.70	9.68	7.1	1046	−0.1	−5	29.0	0.885	0.149	
6	0.500	19.3	2.88	5.65	7.23	−3.7	835	−1.5	−1	6.3	0.968	0.091	
7	0.133	8.8	4.56	3.10	4.97	−12.9	387	−2.2	1	4.5	0.960	0.142	
8	0.136	8.3	5.47	3.73	5.88	−0.9	572	1.6	−3	14.5	0.917	0.238	Soil moisture data absent
9	0.279	18.1	21.30	23.90	29.30	11.7	2951	−27.7	−6	4800.0	0.293	0.629	Rainfall data suspect
10	0.291	11.4	5.66	6.44	9.82	5.9	839	5.4	1	30.7	0.942	0.213	
11	0.386	17.3	3.25	7.02	6.33	7.1	1007	−1.9	0	8.2	0.962	0.092	
12	1.010	34.6	−5.67	11.70	12.30	−1.5	1370	1.8	−8	71.6	0.855	0.157	Rainfall data suspect
13	0.725	33.0	4.47	12.50	18.60	−0.9	1574	0.1	−8	311.0	0.715	0.262	
14	0.431	19.2	5.31	8.63	10.90	1.4	1330	−0.7	−5	3.8	0.991	0.041	
15	0.602	29.1	6.46	8.07	12.00	2.5	1081	−1.8	−6	52.3	0.855	0.164	
16	0.512	18.5	10.50	20.40	22.30	4.0	3059	1.1	−5	487.0	0.857	0.272	
17	0.764	15.9	8.21	15.10	26.40	13.5	2112	−3.0	−1	293.0	0.850	0.196	

possible. Only those events with data found to be incorrect or with data missing, or those involving snowmelt, should be rejected. Events which appear merely 'odd' or inexplicably poorly fitted should not be excluded until some adequate evidence for rejection can be furnished.

As stated in Section 7.3.3, the empirical function finally chosen to relate the runoff ratio, ROP, for an event to initial catchment conditions at the start of that event, was

$$\text{ROP} = \text{PERC}.e^{-\text{PERI}.\,S_{75}} \qquad (7.9)$$

where S_{75} is the initial soil moisture deficit over the catchment area with 75 mm root constant, and PERC and PERI are two parameters whose values are catchment dependent and have to be found in the optimising process.

Equation 7.9 is a compromise between a desire to fit the proving data well and the need to keep the relationship between the runoff ratio and initial conditions both as simple as possible and with the minimum number of parameters. The degree of compromise may be seen in Figure 7.12, typical of the results for the 21 catchments used in the proving stage. The scatter of points is appreciable, particularly for those events with low initial soil moisture deficit typical of wet winter conditions.

In the second stage of fitting, the IEM4 model is fitted to all the acceptable events on a catchment *as a group*, and optimum values of the set of four parameters (PERC, PERI, DEL and AC) are found for that catchment in one optimisation run. The objective function to be minimised is now the sum of the function values for the individual events.

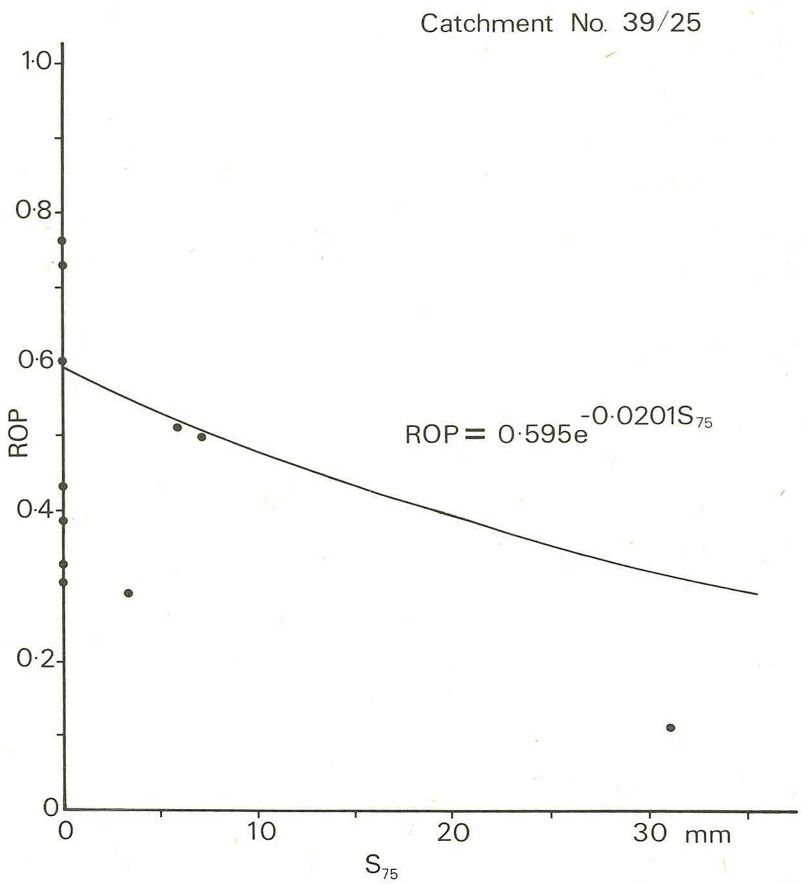


Fig 7.12 Relation between runoff ratio and initial soil moisture deficit for catchment No. 39/25.

Table 7.14 gives a synopsis of the results obtained with the proving data for the nonrejected events from the first stage fitting described in the previous section. Item 4 in Table 7.14 gives an indication of the deviations between the three most important characteristics of the individual recorded hydrographs and those reconstructed by the group fitted model, namely the maximum discharge, the total runoff volume, and the timing of the peak. The mean error, in which positive and negative values tend to cancel out, indicates the bias in the estimates. The standard deviation indicates the spread about the mean. Negative errors in the timing of the peak indicate that the reconstructed peak is earlier than the observed peak.

Table 7.14 Summary of results of applying modelIEM4 to the group of events on catchment No. 39/25. The Enborne at Brimpton.

1. Basic standards		
(a) Area	152.8	km^2
(b) Time interval	1.0	hour
(c) Total number of storm events	11	—
(d) Mean discharge	5.42	cumecs
(e) Initial sum of squares F_0^2	15 145	$(cumecs)^2$
2. Fitting criteria for group fitting		
(a) Objective function F^2	3 412	$(cumecs)^2$
(b) Objective function R	0.775	—
(c) Objective function P	0.403	—
3. Optimum parameter values for group fitting		
(a) PERC	0.595	—
(b) PERI	0.0201	mm^{-1}
(c) AC	27.6	(mm. hour)^{1}
(d) DEL	5.27	hour
4. Hydrograph statistics (group fitted model used to reconstruct individual events)		
(a) Mean % error in maximum discharge	0.798	—
(b) Standard deviation of % error in maximum discharge	22.5	—
(c) Mean % error in total runoff volume	13.7	—
(d) Standard deviation of % error in total runoff volume	22.1	—
(e) Mean error in time of peak	−1.91	hour
(f) Standard deviation of error in time of peak	4.53	hour

In Table 7.15 greater detail may be found of the comparison between the recorded hydrographs and those reconstructed by the fitted model. The events are listed in descending order of magnitude of the maximum discharge. The definition of total runoff volume is taken as the total runoff between the start and end of the observed hydrograph, and as such depends on a subjective choice of end points. A better comparison might be between the maximum runoff volumes in a fixed time interval, such as 24 hours or some multiple thereof.

Table 7.15 Comparison between individual recorded and reconstructed hydrograph characteristics for catchment No. 39/25 (group fitted model used for reconstruction). The Enborne at Brimpton (area 152.8 km², time interval 1.0 hour).

Storm event number	Maximum observed discharge (cumecs)	Maximum reconstructed discharge (cumecs)	Error in maximum discharge (%)	Total observed runoff volume (10^3 m^3)	Total reconstructed runoff volume (10^3 m^3)	Error in total runoff volume (%)	Error in time of peak (hour)
17	23.30	15.00	−35.5	2177	1657	−23.9	− 3
16	21.50	16.80	−21.5	3027	2968	− 2.0	−10
13	18.70	17.20	− 8.2	1573	1421	− 9.7	− 7
15	11.70	12.40	6.2	1100	1104	0.3	− 7
14	10.80	12.50	16.1	1339	1584	18.3	1
10	9.27	9.75	5.1	796	1107	39.0	0
5	9.04	12.40	36.6	1048	1501	43.3	− 4
6	7.51	6.17	−17.9	847	841	−0.8	2
3	6.01	6.95	15.6	615	829	34.8	2
11	5.91	7.59	28.4	1026	1268	23.5	2
7	5.71	4.79	−16.1	395	504	27.6	3

It should be stressed that the reconstructed hydrographs of the individual events were obtained from the model fitted over *all* the events. No one event is necessarily fitted optimally, but the group of events as a whole is so fitted. Although fitting by individual events would give much smaller differences than those shown in Table 7.15, it is group fitted parameter values that are needed for design purposes.

Table 7.16, similar to Table 7.14, gives summaries of the results of fitting the IEM4 model to the proving data from all the 21 catchments. A disadvantage of expressing the errors as percentages is illustrated by the results for catchment No. 39/17, where the overestimate of some minor peaks dominates the results.

Discussion

In establishing the worth of any catchment modelling technique, possibly the severest examination is the split record test. How well does a model fitted with part of the records on a catchment reconstruct the remaining records not used in the fitting process? In this project, there were in general insufficient events to divide the group on a particular catchment into two parts for adequate split record tests. However, this was done for 57 out of the total 67 events on catchment No. 39/17, the Ray at Grendon Underwood. The IEM4 model was fitted to the first group of 29 events, and the optimum parameter values found (Table 7.17). These parameter values were then used in the model to reconstruct the 28 events of the second group; the groups were then reversed and the procedure repeated.

Table 7.17 illustrates a close consistency between the two sets of optimum parameter values. Consequently, when the model takes the optimum set of parameter values found on one group, and is used to reconstruct the hydrographs on the second group, the efficiency obtained (R and P) is only slightly worse than that obtained when the model is fitted to the second group. The stability of these split record tests creates confidence in the IEM4 model.

7.3.7 *Implementing the isolated event model technique*

As pointed out in Section 7.3.1, it is envisaged that until users of the isolated event model technique have gained sufficient experience of the optimisation part of the method (and have access to adequately large

Table 7.17 Split record tests applied to data from catchment No. 39/17.

	Model fitted on Group 2 Events reconstructed on Group 1	Model fitted on Group 1 Events reconstructed on Group 2
Optimum parameter values		
PERC	0.746	0.757
PERI	0.0150	0.0168
DEL	5.37	5.43
AC	11.6	12.9
Group 1 events	Error criteria of reconstructed peaks $R = 0.785$, $P = 0.611$	Error criteria of fitted peaks $R = 0.800$, $P = 0.588$
	% error of highest six peaks −23.5, +14.9, −1.4, −8.6, −22.6, −41.4	% error of highest six peaks −29.2, +5.5, −11.3, −11.6, −29.0, −48.9
Group 2 events	Error criteria of fitted peaks $R = 0.838$, $P = 0.684$	Error criteria of reconstructed peaks $R = 0.825$, $P = 0.711$
	% error of highest six peaks −3.1, −18.2, −24.3, −16.7, +41.3, +18.6	% error of highest six peaks −9.7, −24.2, −50.5, −22.7, +25.0, +15.4

computer systems), the task of fitting the IEM4 model to any given catchment could best be carried out at the Institute of Hydrology. Again, it should be stressed that at its present stage of development, the isolated event modelling technique can cope only with gauged catchments with an adequate number (not less than ten) of sizeable isolated events not involving snowfall or frozen ground conditions. No correlations are yet available for specifying values of the four model parameters from catchment characteristics. Within these constraints, the application of the fitted IEM4 model either to reconstruct a recorded event or to yield a design hydrograph consists of assembling the required input data and processing those data through a computer program of the model.

Data

The following input data, shown with their program names, are required:
1 Catchment reference number, NCATCH;
2 Catchment area in km^2, AREA;
3 Optimised parameter values, PERC, PERI, AC, DEL;
4 The number of hourly rainfall blocks in the hyetograph, NR;
5 Rainfall hyetograph in blocks of rainfall measured in mm over the catchment, $R(I)$, $(I = 1,2,\ldots,NR)$. The rainfall should be the total falling between clock hours, e.g. 0900–1000, 1000–1100, 1100–1200, etc;
6 Initial discharge of the river in cumecs, Q_0. This should coincide with the clock hour at the start of the hourly rainfall amounts;
7 Soil moisture deficit over 75 mm root constant area at the start of the event, S_{75}. Values of S_{75} at 0900 GMT are available from the Meteorological Office, and are also published regularly in map form;
8 The number of hourly ordinates required in the output discharge hydrograph, NQ. This should normally be at least twice the value of $(\text{DEL} + NR)$.

The design storm

For the case of a design storm information about the initial conditions (Q_0 and S_{75}) on the catchment will, of course, be lacking. Probably the two cases of most interest are an 'average' condition, and a 'worst' condition. For S_{75} estimates, an average condition may well be taken as the median of observed values for the relevant time of year taken over a period of record, either daily or monthly figures being considered. The worst condition is simple $S_{75} = 0$. For Q_0 estimates, the average condition may simply be the average monthly mean flow at the relevant time of year over a period of record. The worst initial flow is more difficult to define. Perhaps the relevant highest mean monthly flow might be suitable. Several alternative values of the 'worst' Q_0 could be used to show that the peak flow due to a design rainfall hyetograph may not be particularly sensitive to the initial flow value.

The design rainfall hyetograph is, of course, a problem in itself, although strictly outside the scope of this chapter. With the aid of Volume II, the designer should perhaps choose a set of design storms having different characteristics of amount, duration, and shape, and route them through the fitted model. Allied to this is the very difficult problem of the frequency of any design flood (see Chapter 6). The likelihood of 'average' or 'worst' initial conditions of Q_0 and S_{75} coinciding either

Section	Variable	Units	20/1	24/3	27/27	28/23	37/1	39/17	39/25	40/10
Basic standards	Area	km^2	307	172	443	154	303	18.57	152.8	224
	Time interval	hour	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	Total number of storm events		10	15	22	11	12	67	11	23
	Mean discharge	cumecs	22.4	20.9	58.8	9.81	12.0	0.686	5.42	11.3
	Initial sum of squares F_0^2	(cumecs)^2	305 820	571 960	3 431 500	26 124	59 621	6148.0	15 145	129 840
Fitting criteria	Objective function F^2	(cumecs)^2	28 190	154 070	1 046 700	3215	17 513	1550.3	3412	22 180
	Objective function R		0.908	0.731	0.695	0.877	0.706	0.748	0.775	0.829
	Objective function P		0.307	0.655	0.485	0.237	0.347	0.851	0.403	0.307
Optimum parameter values	PERC		0.538	0.534	0.754	0.721	1.14	0.798	0.595	0.778
	PERI	mm^{-1}	0.00490	0.000	0.0411	0.0396	0.0138	0.00995	0.0201	0.0140
	AC	(mm. hour)$^{\frac{1}{2}}$	19.5	10.4	21.0	81.9	70.0	15.0	27.6	32.3
	DEL	hour	4.51	0.386	3.42	2.37	3.52	4.68	5.27	7.31
Hydrograph statistics	Mean % error in maximum discharge		−9.0	−31.1	−20.8	−10.4	−14.9	90.4	0.8	4.4
	Standard deviation of % error in maximum discharge		20.3	24.3	33.0	19.0	22.9	211.8	22.5	21.4
	Mean % error in total runoff volume		1.9	−13.1	−9.0	7.1	−2.9	120.6	13.7	−4.4
	Standard deviation of % error in total runoff volume		21.6	22.4	18.9	15.7	17.1	257.3	22.1	14.7
	Mean error in time of peak	hour	−0.4	1.3	0.2	5.4	−11.3	−1.9	−1.9	−5.6
	Standard deviation of error in time of peak	hour	2.0	1.3	2.0	8.5	12.9	6.5	4.5	5.9

Section	Variable	Units	41/5	45/2	46/3	53/7	54/16	54/19	54/22	57/4	61/1	65/1	71/3	73/804	76/11
Basic standards	Area	km^2	182	422	248	262	259	347	8.05	109	197.6	68.6	10.4	57.5	1.52
	Time interval	hour	1.0	1.0	1.0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0
	Total number of storm events		18	17	16	16	11	16	25	12	21	12	16	15	32
	Mean discharge	cumecs	13.2	59.9	37.8	18.9	8.09	12.8	1.66	15.9	20.7	19.4	2.55	18.7	0.248
	Initial sum of squares F_0^2	(cumecs)^2	102 120	1 908 600	697 400	236 520	18 887	196 570	5275.1	156 090	146 970	212 730	8470.7	339 420	158.94
Fitting criteria	Objective function F^2	(cumecs)^2	19 940	186 400	114 500	48 290	4127	38 290	921.7	15 320	41 330	14 127	1374.6	34 260	16.92
	Objective function R		0.805	0.902	0.836	0.796	0.781	0.805	0.825	0.902	0.719	0.934	0.838	0.899	0.894
	Objective function P		0.329	0.252	0.385	0.427	0.258	0.363	0.444	0.280	0.353	0.248	0.511	0.322	0.451
Optimum parameter values	PERC		0.784	0.938	0.495	0.463	0.727	0.631	0.689	0.687	0.550	0.568	0.678	0.892	0.879
	PERI	mm^{-1}	0.0105	0.0133	0.00722	0.000005	0.0115	0.00833	0.00171	0.0476	0.00793	0.00524	0.00626	0.00328	0.0113
	AC	(mm. hour)$^{\frac{1}{2}}$	29.3	48.4	11.0	17.4	57.8	34.7	12.8	29.2	22.6	21.2	8.19	25.3	10.3
	DEL	hour	8.31	2.33	2.10	5.41	10.4	21.8	0.0905	1.79	2.27	0.462	1.11	4.47	0.573
Hydrograph statistics	Mean % error in maximum discharge		−1.6	−21.9	−12.1	9.3	2.9	8.5	−9.8	−9.0	−6.6	3.7	−5.6	0.7	−8.9
	Standard deviation of % error in maximum discharge		20.9	13.4	13.8	44.1	14.3	46.9	40.5	18.3	29.3	11.6	29.2	19.8	23.1
	Mean % error in total runoff volume		−3.9	14.5	4.9	10.2	3.7	1.1	23.4	3.3	8.6	4.4	−2.8	10.7	3.2
	Standard deviation of % error in total runoff volume		13.8	22.7	17.6	45.6	13.1	26.5	63.0	13.4	38.5	11.3	23.8	13.8	17.4
	Mean error in time of peak	hour	−3.8	−0.4	−0.1	−0.2	−12.4	−8.4	0.2	3.1	−0.9	−0.3	−0.4	0.3	−0.3
	Standard deviation of error in time of peak	hour	3.5	1.9	1.3	3.2	7.2	9.4	1.5	5.2	2.6	1.2	1.9	2.2	1.1

Table 7.16 Summary of results of applying IEM4 model. Fitting the IEM4 model to groups of events from catchments listed.

with each other or with a given design rainfall of designated frequency is difficult to quantify in terms of frequency or return period. Current research efforts in stochastic rainfall data generation may ultimately lead to a solution of this difficult problem.

Computing the flood hydrograph

The following steps are necessary to obtain the output hydrograph $Q(I)$, $(I = 1, 2, \ldots, NQ)$ measured in cumecs and timed on the clock hours beginning from the start of the rainfall hyetograph.

1 Obtain the value of the runoff ratio, ROP, from the relation:
$\text{ROP} = \text{PERC}.e^{-\text{PERI}.\,S_{75}}$

2 Multiply all the amounts in the gross rainfall hyetograph, $R(I)$, by ROP to obtain the net rainfall hyetograph, $N(I)$:
$N(I) = \text{ROP}.R(I)$ for $I = 1, 2, \ldots, NR$.

3 Convert the initial discharge, Q_0, to mm per hour by multiplying by 3.6/AREA.

4 The net rainfall, $N(I)$, starts entering the nonlinear reservoir a period DEL after the start of the event. There is thus zero input to the reservoir for a period DEL followed by hourly amounts, $N(I)$, thereafter. Generally, DEL will not be an integer, so the times for calculating the outflow hydrograph from the reservoir will not be clock hours. The procedure used is first to separate the value of the parameter DEL into two parts: (a) the nearest whole integer NDEL less than DEL; (b) the fractional part remaining, FDEL, i.e. $\text{FDEL} = \text{DEL} - \text{NDEL}$ $(0 < \text{FDEL} < 1)$.

5 Calculate the discharge, Q_1, occurring at FDEL hours after the start of the event using the following relationship developed from the continuity equation and nonlinear storage/discharge relation $S = \text{AC}.Q^{\frac{1}{2}}$ for initial flow, Q_0, and zero input over FDEL hours:

$$Q_1 = \frac{Q_0}{\left(1 + \frac{\text{FDEL}.(Q_0)^{\frac{1}{2}}}{\text{AC}}\right)^2}$$

6 Starting from this discharge, Q_1, apply input to the nonlinear reservoir in three consecutive separate phases as follows; firstly, zero input for NDEL hours; next, the net rainfall hyetograph, $N(I)$, $(I = 1, 2, \ldots, NR)$ for NR hours; finally, zero input for the remaining (NQ-NDEL-NR) hours, to give a complete hydrograph in excess of NQ hours. Again, using the continuity and reservoir equations, the discharge at the end of one time interval, Q_{i+1}, is given in terms of the discharge at the start of the interval, Q_i, by one of two equations depending on whether or not there is zero input or rainfall into the nonlinear reservoir over the interval:

(a) if there is zero input, $N(I) = 0$,

$$Q_{i+1} = \frac{Q_i}{\left(1 + \frac{(Q_i)^{\frac{1}{2}}}{\text{AC}}\right)^2}$$

(b) if $N(I) > 0$,

$$Q_{i+1} = N(I)\left(\frac{(Q_i)^{\frac{1}{2}} + (N(I))^{\frac{1}{2}}.H}{(N(I))^{\frac{1}{2}} + (Q_i)^{\frac{1}{2}}.H}\right)^2$$

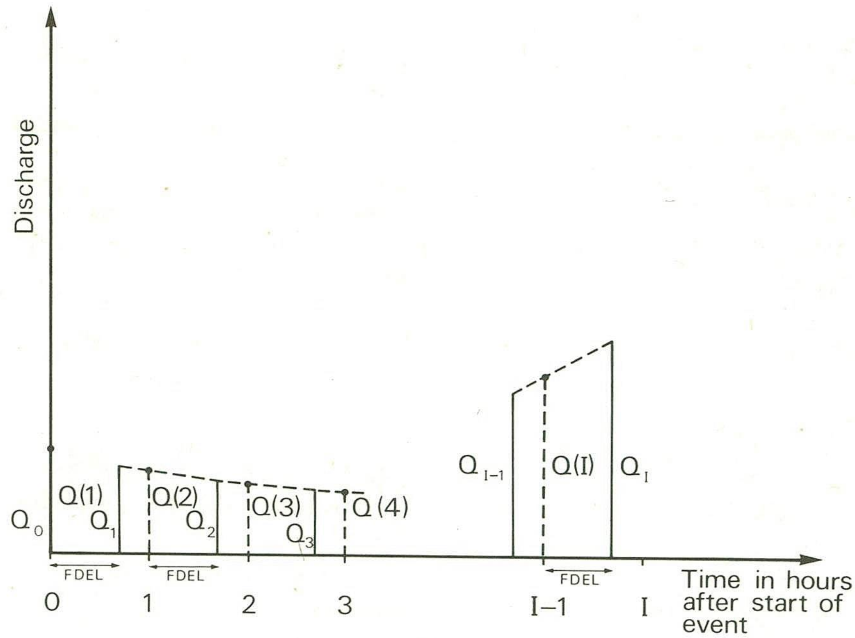


Fig 7.13 Linear interpolation procedure used to estimate discharge on the clock hours.

where

$$H = \tanh\left(\frac{(N(I))^{\frac{1}{2}}}{\mathrm{AC}}\right).$$

7 These Q_i values start with Q_1 at time FDEL and continue with Q_2, Q_3, ..., at times (FDEL + 1), (FDEL + 2), ..., after the start of the storm. To determine the hydrograph ordinates, $Q(I)$, (I = 1, 2, ... NQ), on the

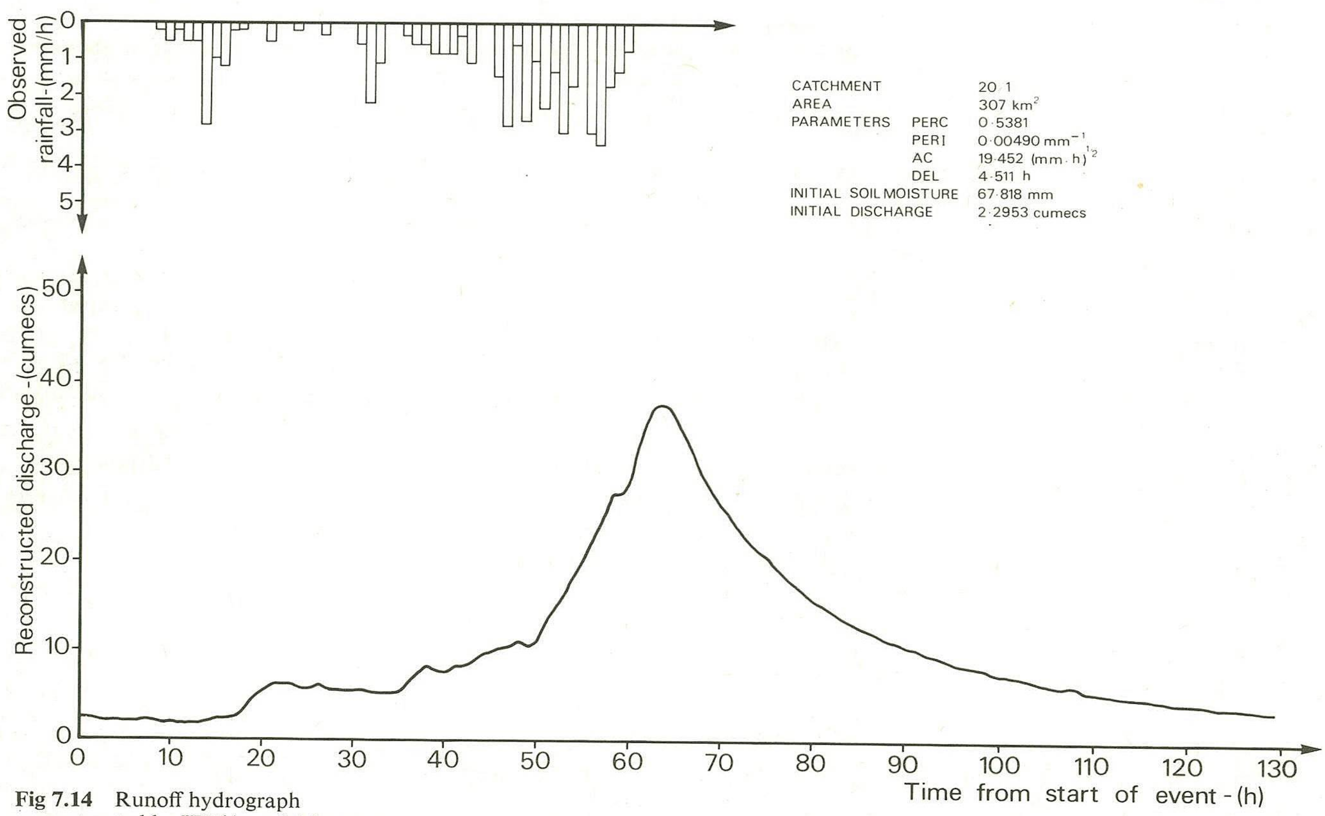


Fig 7.14 Runoff hydrograph reconstructed by IEM4 model for event from catchment No. 20/1.

clock hours, the first ordinate $Q(1)$ is taken equal to Q_0, and then linear interpolation is used:

$$Q(I) = Q_{I-1}.\text{FDEL} + Q_I.(1-\text{FDEL})$$

where Q_{I-1} and Q_I are the ordinates calculated from 6(a) and (b) before and after the clock hour ordinate $Q(I)$. This interpolation is illustrated in Figure 7.13.

8 Convert the clock hour hydrograph ordinates to cumecs by multiplying by AREA/3.6.

Computer program

The calculations set out above are rather tedious for any lengthy storm and are best carried out by computer. A short program has been written in standard FORTRAN to carry out the calculations and this is given in Table 7.18. In Table 7.19 an example of the input data has been given for one event on catchment 20/1 and the corresponding hydrograph output shown in Table 7.20. This storm event is illustrated in Figure 7.14.

```
C
C        ROUTINE TO APPLY ISOLATED STORM EVENT MODEL IEM4 TO SINGLE RAIN
C        FALL STORM . GIVEN CATCHMENT AREA AND PARAMETER VALUES,INITIAL
C        CONDITIONS ON THE CATCHMENT, RAINFALL HYETOGRAPH, THE MODEL
C        GIVES OUTPUT HYDROGRAPH OF DESIRED LENGTH .
C
      DIMENSION R(100),Q(250)
C
C        READ CATCHMENT INFORMATION.    NCATCH = NUMBER OF CATCHMENT
C         AREA = CATCHMENT AREA IN SQ.KM.     PERC,PERI,AC,DEL  =  VALUES
C         OF MODEL PARAMETERS
C
      READ (5,100) NCATCH,AREA,PERC,PERI,AC,DEL
 100  FORMAT( I5,5X,5F10.0 )
C
C        REAL INITIAL CONDITIONS ON CATCHMENT .    S3 = SOILMOISTURE
C         DEFICIT IN MM.    QO = RIVER DISCHARGE IN CUMECS
C
      READ (5,101) S3,QO
 101  FORMAT( 2F10.0 )
C
C        REAL RAINFALL HYETOGRAPH AND LENGTH OF HYDROGRAPH REQUIRED IN
C         OUTPUT .     NR =  NUMBER OF ORDINATES IN RAINFALL HYETOGRAPH
C         NQ = NUMBER OF ORDINATES REQUIRED IN HYDROGRAPH
C         R(I) = HOURLY RAINFALL HYETOGRAPH MEASURED IN MM.
C
      READ (5,102) NR,NQ,( R(I),I=1,NR )
 102  FORMAT( 2I5/(12F6.0) )
C
C         WRITE OUT HEADING, CATCHMENT INFORMATION, INITIAL CONDITIONS
C          AND RAINFALL HYETOGRAPH
C
      WRITE (6,200) NCATCH,AREA,PERC,PERI,AC,DEL,S3,QO,(R(I),I=1,NR)
 200  FORMAT(1H1,    26X,67HAPPLICATION OF ISOLATED EVENT MODEL IEM4 TO A
     1 SINGLE RAINFALL STORM/26X,67(1H*)////10X,16HCATCHMENT NUMBER,I10,
     220X,4HAREA, F10.3,7H  SQ KM    ////10X,16HPARAMETER VALUES,4X,
     36H PERC   ,F8.4 , 4X, 6H PERI , F8.5 , 6H   1/MM , 4X,
     44H AC  ,   F8.3 ,14H   (MM*HR)**0.5,4X ,5H DEL   , F8.3 ,4H  HR
     5////10X,18HINITIAL CONDITIONS,10X,21HSOILMOISTURE DEFICIT , F8.3,
     64H  MM ,10X,16HRIVER DISCHARGE  , F8.4 ,8H  CUMECS , ////46X,
     727H RAINFALL HYETOGRAPH  (MM)    //  ( 12F10.2/) )
C
C        CALCULATE THE VALUE OF RUNOFF RATIO ROP
C
      ROP = PERC*EXP(-1.0*PERI*S3)
C
C        OBTAIN THE NET RAINFALL HYETOGRAPH
C
      DO 1 I=1,NR
 1    R(I)=ROP*R(I)
C
C        CONVERT INITIAL DISCHARGE TO MM./HOUR
C
      DIS=QO*3.6/AREA
C
C        DIVIDE PARAMETER DEL INTO ITS COMPONENTS
C
      NDEL= INT(DEL)
      FDEL= DEL - FLOAT(NDEL)
C
C        CALCULATE THE DISCHARGE AT FDEL HOURS AFTER START TIME
```

Table 7.18 Listing of computer program to calculate single runoff event using IEM4 model.

```
C
      SQDIS=SQRT(DIS)
      DN=1.0 + FDEL*SQDIS/AC
      DIS=DIS/DN**2
      SQDIS=SQRT(DIS)
      PDIS=DIS
      Q(1)=QD
C
C       CALCULATE THE OUTPUT HYDROGRAPH Q(I)
C
      DO 10 I=1,ND
      IF (I-NDEL) 2,2,6
 6    IF (I-(NDEL+NR)) 3,3,2
C
C       CALCULATE THE OUTPUT DISCHARGE FOR ZERO INPUT
C
 2    DN = 1.0 + SQDIS/AC
      DIS= DIS/DN**2
      SQDIS = SQRT(DIS)
      QCAL=0.0
      SQQCAL=0.0
      GO TO 5
C
C           DETERMINE WHETHER RAINFALL INPUT IS ZERO OR NON-ZERO
C
 3    IP=I-NDEL
      QCAL=R(IP)
      IF (QCAL) 7,7,4
 7    QCAL=0.0
      SQQCAL=0.0
      GO TO 2
C
C           CALCULATE THE OUTPUT DISCHARGE FOR POSITIVE INPUT
C
 4    SQQCAL=SQRT(QCAL)
      TIN = TANH(SQQCAL/AC)
      D=SQDIS + SQQCAL*TIN
      E=SQQCAL + SQDIS*TIN
      DIS=QCAL*(D/E)**2
      SQDIS = SQRT(DIS)
C
C      DETERMINE DISCHARGE ON THE CLOCK HOUR USING LINEAR INTERPOLATION
C
 5    IQ=I+1
      Q(IQ)=(PDIS*FDEL + (1.0 -FDEL)*DIS)
      PDIS = DIS
C
C       CONVERT DISCHARGES TO CUMECS
C
 10   Q(IQ)=Q(IQ)*AREA/3.6
C
C       WRITE OUT HYDROGRAPH OF DISCHARGES Q(I)
C
      WRITE (6,201) (Q(I),I=1,NQ)
 201  FORMAT(   ///1H ,43X,32H DISCHARGE HYDROGRAPH  (CUMECS)  //(12F10
     1.4/) )
      STOP
      END
```

Table 7.19 Input data required to calculate runoff hydrograph for event from catchment No. 20/1.

```
20001           307.0  0.538055  0.004899   19.4523    4.5114
   67.8180    2.2953
   60  130
   0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.2    0.5    0.2    0.5
   0.5    2.8    0.9    1.2    0.2    0.2    0.0    0.0    0.5    0.0    0.0    0.2
   0.0    0.0    0.3    0.0    0.0    0.0    0.6    2.2    1.1    0.0    0.0    0.3
   0.6    0.6    0.8    0.8    0.8    0.3    1.1    0.0    0.0    1.4    2.8    0.6
   2.7    1.0    2.3    1.3    3.0    1.7    0.0    3.0    3.3    1.7    1.3    0.7
```

```
              APPLICATION OF ISOLATED EVENT MODEL IEM4 TO A SINGLE RAINFALL STORM
              ******************************************************************

CATCHMENT NUMBER     20001                      AREA    307.000   SQ KM

PARAMETER VALUES     PERC    .5381    PERI   .00490  1/MM     AC   19.452  (MM*HR)**0.5     DEL    4.511  HR

INITIAL CONDITIONS           SOILMOISTURE DEFICIT    67.818  MM       RIVER DISCHARGE    2.2953  CUMECS

                                  RAINFALL HYETOGRAPH  (MM).

 .00     .00     .00     .00     .00     .00     .00     .00     .20     .50     .20     .50
 .50    2.80     .90    1.20     .20     .20     .00     .00     .50     .00     .00     .20
 .00     .00     .30     .00     .00     .00     .60    2.20    1.10     .00     .00     .30
 .60     .60     .80     .80     .80     .30    1.10     .00     .00    1.40    2.80     .60
2.70    1.00    2.30    1.30    3.00    1.70     .00    3.00    3.30    1.70    1.30     .70
```

DISCHARGE HYDROGRAPH (CUMECS)

2.2953	2.2572	2.2199	2.1835	2.1480	2.1134	2.0796	2.0466	2.0144	1.9829	1.9522	1.9222
1.8929	1.9132	2.0624	2.2155	2.3705	2.6158	3.6594	4.9163	5.6663	6.1339	6.1431	6.0680
5.9050	5.9627	6.0281	5.8666	5.7961	5.7322	5.5824	5.5627	5.5495	5.4067	5.2694	5.3819
6.4839	7.8712	8.1962	7.9411	7.8446	8.0575	8.4251	8.8951	9.4733	10.0494	10.3352	10.7839
11.0562	10.6579	11.0938	13.3215	15.0578	16.7667	18.7800	20.4876	22.4343	24.9876	27.8916	27.9331
29.1367	33.6392	36.7467	37.5824	37.2911	35.6409	33.3841	31.3351	29.4692	27.7651	26.2047	24.7723
23.4542	22.2386	21.1152	20.0747	19.1094	18.2120	17.3764	16.5970	15.8689	15.1877	14.5495	13.9506
13.3880	12.8597	12.3603	11.8902	11.4465	11.0272	10.6304	10.2548	9.8987	9.5608	9.2399	8.9349
8.6448	8.3686	8.1054	7.8544	7.6150	7.3863	7.1677	6.9587	6.7587	6.5673	6.3838	6.2079
6.0392	5.8773	5.7218	5.5724	5.4288	5.2906	5.1577	5.0297	4.9064	4.7876	4.6731	4.5626
4.4560	4.3531	4.2537	4.1577	4.0649	3.9751	3.8883	3.8043	3.7230	3.6447		

Table 7.20 Output data showing runoff hydrograph for event from catchment No. 20/1.

7.4 Hydrograph routing

7.4.1 Routing a hydrograph down a river channel

Volume III presents a survey of flood routing methods for British rivers produced by the Hydraulics Research Station. A comprehensive review of alternative methods is presented and the theoretical side of the subject is thoroughly discussed. The Muskingum–Cunge method is recommended for most situations.

The methods of Chapter 6 of this volume enable a synthetic flood hydrograph to be produced for any site. The catchments used in analysis were limited to areas below 500 km^2 and application to larger catchments must be done with caution.

Because it incorporates aspects of the channel and flood plain geometry which are ignored in the unit hydrograph analyses, a river flood routing technique provides a more satisfactory way of synthesising a hydrograph for the larger catchment. It is particularly suitable where there is a long river reach but little tributary inflow along it. Lateral inflow complicates the routing but, if significant, also makes it necessary to synthesise additional hydrographs and forces difficult assumptions about the temporal and spatial patterns of rainfall over areas much larger than 500 km^2.

The main point to note is that, if uniform rainfall is to be assumed over the larger area, an areal reduction factor appropriate to this larger area should be used when calculating the rain expected over the subcatchments.

In estimation of the maximum flood it may be advisable to allow for a realistic storm movement which gives the worst combination of synthetic hydrographs from the separate subcatchments. For lesser floods this would be unreasonable.

It is shown, in Volume III, that to route a hydrograph down a river channel, two parameters of the hydrograph are required for evaluation of the routing coefficient; these are the peak and the curvature around the peak. It is shown (III.5.1, Case 1,C) that the curvature can be expressed in terms of the peak and flow values at equal times (t_0) either side of the peak. If the design hydrograph has been synthesised as described in Section 6.8 then t_0 can be taken as T, the basic data interval, whether or not T is less than 1 hour. The computer program described in Section 6.8.6 gives the curvature of the peak on this basis.

7.4.2 *Routing a design flood through a reservoir*

The hydrograph computed by the methods of Chapter 6 may be intended as the inflow design hydrograph for a reservoir. If the reservoir is to be a large one the hydrograph should be synthesised for those points on the stream network which are intersected by the normal operating level of the proposed reservoir. The reservoir causes stream lengths to be decreased and slope increased so that T_p estimates could be significantly lower than estimates made at the dam site. Although several small subcatchments would be formed in practice, they may be replaced by a composite catchment where the streams join together just before entering the reservoir. Thus, stream length and slope are taken from the largest subcatchment and the other topographic and climatic characteristics are taken from the combined area.

For spillway design, the inflow hydrograph (plus rain falling on the water surface) is required to be routed through the reservoir. The Institution of Civil Engineers' report, *Floods in Relation to Reservoir Practice* gives two methods of routing (Appendix 1, pp 56–61 in the 1960 reprint). A simple graphical method is illustrated and also an arithmetic iterative technique. Wilson (1969, 2nd edn 1974) describes the well known graphical technique known as the Puls method.

The basic relationship between inflow, outflow and storage is expressed in difference form as

$$(I_t+I_{t+T}).T/2 - (O_t+O_{t+T}).T/2 = S_{t+T}-S_t \tag{7.14}$$

where I is the inflow rate, O the outflow rate and S is the volume in storage. T is the length of the time interval and subscripts t and t+T refer to the beginning and end of the interval respectively. The graphical method used in the 1933 report relies on the simpler relationship

$$I_{t+T}.T - O_{t+T}.T = S_{t+T}-S_t \tag{7.15}$$

and, while it is recognised that the use of average inflows and outflows during each time interval would be more accurate, it is maintained in the report that such a refinement is unnecessary providing the time interval, T, is small enough.

The basic equation may be rewritten

$$T.O_{t+T}+2\,S_{t+T} = T.(I_t+I_{t+T}-O_t)+2\,S_t. \tag{7.16}$$

With an assumption of initial level and hence storage and outflow at $t = 0$ everything on the right is known and the sum of the two terms on the left is therefore known. Graphical methods for finding the level at $t+T$ depend on prior determination of the relationship between the composite term $T.O+2\,S$ and one or other of its parts. Arithmetic iteration methods involve a search for the value of level (H) which, from known relationships between H and O and between H and S, produces the required value for $T.O+2\,S$. In either case, having found the correct values of O and S, the calculation moves on to the next time interval. As both techniques are presented, with variations, in many sources it is unnecessary to illustrate the detailed calculations by graphs or tables. However, reservoir routing is a computation which, because it may be done many times for different combinations of spillway type or length and for various release, abstraction or initial level assumptions, is well suited to the computer. It is hoped that the basic FORTRAN routing described below will prove useful for this purpose.

FORTRAN program

The program calculates and outputs the reservoir level, storage and outflow at the end of each time interval used in the inflow hydrograph.

The method of calculation is as follows: At the beginning of an interval the storage and outflow are known. The inflows at the beginning and end of the interval are both known. It is possible to calculate

$$\text{FHC} = S_t + (I_t + I_{t+T} - O_t).T/2$$

T in seconds, I,O in cumecs, S in m^3 (FHC is the name of the variable used in the program to represent this summation of known quantities). As shown above, this equals

$$O_{t+T}.T/2 + S_{t+T}.$$

Both O and S are functions of the level, H.

Thus

$$f(H_{t+T}) - \text{FHC} = 0.$$

$f(H_{t+T})$ and $f'(H_{t+T})$ are evaluated with an initial assumption for H_{t+T} (for the first interval HSTART+0.1; thereafter $2\ H_t - H_{t-T}$). An improved estimate of H_{t+T} follows:

new $H_{t+T} = H_{t+T} - f(H_{t+T})/f'(H_{t+T})$ (Newton formula)

and this iteration is continued until successive estimates of H_{t+T} change by 1 mm or less.

Data

The data format is set out below in the order that it is expected by the program; it is followed by an explanation and examples.

	Variable	Format	Remarks
	TITLE(I), I = 1, 10	10A8	80 character title of the run
	ICASE	I2	Case number; if zero, STOP
	CASE(I), I = 1, 10	10A8	80 character title of this case
Inflow hydrograph:			
	NQI	I3	No. of equally spaced ordinates of inflow hydrograph; if zero, use same hydrograph as previous case
	TINT	F3.0	Time interval (hours) between ordinates of inflow hydrograph
	QI(I), I = 1, NQI	10F8.0	The inflow hydrograph (cumecs); 10 values per card
Outflow data:			
	NOP	I2	No. of separate outflow facilities; if zero, use same combination of such facilities as previous case

	Variable	Format	Remarks
I = 1, NOP	B(I)	3F8.0, I1, F7.0, F8.0	Primary coefficient
	HZ(I)		Level (m) of zero flow
	X(I)		Exponent
	NC		No. of points defining variation of secondary coefficient
	C(I, 1)		Secondary coefficient
	HR(I, 1)		Level at which C(I, 1) applies; no outflow below this level
J = 2, NC	C(I, J)	2F8.0	Further pairs of secondary coefficient values and levels
	HR(I, J)		No outflow above HR (I, NC)
Storage curve:	NSH	I2	No. of points defining storage–height curve; if zero, use same curve as previous case
I = 1, NSH	S(I)	2F8.0	Storage (million m^3)
	H(I)		Level (m)
	HSTART	F8.0	Assumed initial level (m)
	Repeat from ICASE		

The inflow hydrograph is given by a number (up to 200) of equally spaced ordinate values (cumecs) between which it is assumed to vary linearly. In the first example below there are 23 flows at 2 hour intervals.

The outflow is calculated as the sum of up to 10 separate facilities each being specified by the formula

$$O = B.C.(H-HZ)^X$$

where O is the outflow in cumecs, B is a primary coefficient such as weir crest length, C is a secondary coefficient allowed to vary linearly from C_J to C_{J+1} as H changes from HR_J to HR_{J+1}, H is the level for which O is calculated, HZ is the level for which O is zero, X is an exponent.

In the first example given below a combination of four separate outflow facilities is illustrated. These are:

a automatic gated spillway with crest length of 20 m at elevation 150 m; discharge coefficient of 2.1. Gate starts to open at 151 m and is fully open at 153 m; during opening the outflow is proportional to head above 151 m. This can be considered as two separate facilities:

1 Gate opening between 151 and 153 m. B equals outflow at 153 m divided by 2 m (the opening range). $B = 2.1 \times 20 \times 3^{1.5}/2 = 109$; $C = X = 1$; $HZ = 151$ m.

2 Gate fully open. $B = 20$ m; $C = 2.1$ at $HR = 153$ m; $HZ = 150.0$ m; $X = 1.5$.

b ungated spillway with crest length 50 m at elevation 154 m; discharge coefficient 1.71 at 154 m, 1.86 at 155 m, 1.98 at 156 m, 2.08 at 157 m, 2.14 at 158 m, 2.20 at 170 m. This coefficient is assumed to vary linearly at intermediate levels. As outflow is presumed to be zero for levels outside the range, it is important to ensure that a coefficient is specified for a level above the expected peak.

c submerged outlet with valve area of 1 m^2, valve coefficient 0.75, valve axis elevation 130 m, valve closes when reservoir level reaches 151 m. Orifice formula applied, $Q = 0.75\ A\ \sqrt{2g(H-HZ)}$ ($g = 9.81$ m sec^{-2}), $B = A\ \sqrt{2g} = 4.43$, $C = 0.75$, $X = 0.5$, $HZ = 130.0$.

d constant abstraction of 2 cumecs whenever level exceeds 140 m. $B = 2$, $C = 1$, $X = 0$, $HZ = 140.0$.

In case *d* the example listing shows that, when $NC = 1$, $HR(I,1)$ can be omitted if it is the same as $HZ(I)$.

The storage curve is specified as a series of pairs (up to 20) of values of storage (million m^3) and level (m). A linear relationship is assumed between each pair of points and extrapolation at either end is based on the slope of the first or last segment. In the first example there are five pairs of points defining the storage–outflow curve.

The final piece of data required is the initial water level and, in the first example, it is taken as 149 m.

The second example is the one used by Wilson (2nd edn 1974) to illustrate the graphical Puls method.

The third example is the same as the second but with a longer spillway (100 m). It illustrates the use of zeros in the appropriate place to retain data from the previous case.

The Newton convergence formula may fail to find the correct level at an abrupt discontinuity in the outflow–level relationship. Program trials suggest that realistic combinations of outflow facilities will cause no difficulties and that two or three iterations normally suffice.

```
TEST ROUTING AS ILLUSTRATION TO 7.4.2 IN VOL.1
 1
EXAMPLE 1 - 4 OUTLET FACILITIES FIRST OF WHICH IS TREATED AS 2
 23
2.0
   0.0     10.0   25.     45      215.   435.    810.    865.    555.    420.
 325.     245.    175.    115.    65.     40.     25.     15.     10.     5.
   5.      2.      1.
 5
  100.0    151.0    1.0 2   1.0   151.0
 1.0      155.0
   20.0    150.0    1.5 1   2.1   153.0
   50.0    154.0    1.5 6   1.71  154.0
 1.86     155.0
 1.93     156.0
 2.03     157.0.
 2.14     158.0
 2.20     170.0
   4.43    150.0    0.5 2   0.75  130.0
 0.75     151.0
   2.0     140.0    0.0 1   1.0
 5
   0.1     90.0
   5.0    100.
  80.     130.
 140.     150.
 170.     160.
   149.0
 2
EXAMPLE 2 - FROM WILSON(1969 SECOND EDITION,1974)
 22
6.0
   50.      75.     180.     350.    430.    520.    505.    445.    360.    290.
  250.     210.     175.     140.    110.     85.     65.     55.     50.     45.
   40.      38.
 2
   50.8     54.0     0.5  1   0.8
   72.5     66.0     1.5  1   2.2
 4
   20.      57.6
   40.      62.1
   60.      66.4
   80.      70.1
  63.5
 3
EXAMPLE 3 - AS EX.2 BUT WITH LONGER SPILLWAY
0
 2
   50.8     54.      0.5  1   0.3
  100.0     66.0     1.5  1   2.2
0
  63.5
0
****
```

TEST ROUTING AS ILLUSTRATION TO 7.4.2 IN VOL.1
CASE NO. 1
EXAMPLE 1 - 4 OUTLET FACILITIES FIRST OF WHICH IS TREATED AS 2

TIME (HRS.)	INFLOW (CUMECS)	OUTFLOW (CUMECS)	LEVEL (METRES)	STORAGE (MILLION CU.METRES)
0.00	0.00	16.48	149.00	137.00
2.00	10.00	16.47	148.97	136.92
4.00	25.00	16.47	148.97	136.92
6.00	45.00	16.49	149.02	137.06
8.00	215.00	16.59	149.29	137.87
10.00	435.00	16.87	150.03	140.09
12.00	810.00	53.47	151.44	144.33
14.00	865.00	232.92	153.12	149.35
16.00	555.00	355.6[illegible]	154.11	152.34
18.00	420.00	415.47	154.37	153.11
20.00	325.00	392.22	154.31	152.92
22.00	245.00	352.79	154.10	152.29
24.00	175.00	313.26	153.80	151.40
26.00	115.00	276.93	153.45	150.34
28.00	65.00	227.49	153.07	149.20
30.00	40.00	186.75	152.69	148.08
32.00	25.00	151.07	152.37	147.10
34.00	15.00	120.75	152.09	146.27
36.00	10.00	95.71	151.86	145.58
38.00	5.00	75.31	151.67	145.02
40.00	3.00	58.31	151.52	144.56
42.00	2.00	45.73	151.40	144.20
44.00	1.00	35.54	151.31	143.92
46.00	0.00	27.45	151.23	143.70
48.00	0.00	[illegible]	151.18	143.53
50.00	0.00	15.21	151.13	143.39
52.00	0.00	12.45	151.10	143.29
54.00	0.00	9.58	151.07	143.21
56.00	0.00	7.36	151.05	143.15
58.00	0.00	5.65	151.03	143.10
60.00	0.00	4.35	151.02	143.06
62.00	0.00	3.34	151.01	143.04

TEST ROUTING AS ILLUSTRATION TO 7.4.2 IN VOL 1
CASE NO. 2
EXAMPLE 2 - FROM WILSON(1969 SECOND EDITION,1974)

TIME (HRS.)	INFLOW (CUMECS)	OUTFLOW (CUMECS)	LEVEL (METRES)	STORAGE (MILLION CU.METRES)
0.00	50.00	125.26	63.50	46.51
6.00	75.00	123.35	63.21	45.18
12.00	180.00	123.46	63.23	45.26
18.00	350.00	127.74	63.88	48.27
24.00	450.00	135.55	65.13	54.07
30.00	520.00	210.39	66.55	60.82
36.00	505.00	415.35	67.37	65.26
42.00	445.00	455.89	67.55	66.21
48.00	360.00	416.42	67.41	65.47
54.00	290.00	351.35	67.18	64.20
60.00	250.00	296.74	66.96	63.04
66.00	210.00	254.30	66.78	62.05
72.00	175.00	218.72	66.60	61.09
78.00	140.00	186.78	66.42	60.11
84.00	110.00	157.0[illegible]	66.21	59.10
90.00	85.00	140.60	65.97	57.99
96.00	65.00	138.82	65.67	56.60
102.00	55.00	136.66	65.31	54.92
108.00	50.00	134.31	64.92	53.12
114.00	45.00	131.84	64.52	51.28
120.00	40.00	129.25	64.12	49.37
126.00	38.00	126.59	63.70	47.45
132.00	0.00	123.34	63.21	45.16
138.00	0.00	119.50	62.65	42.54
144.00	0.00	115.67	62.10	40.00
150.00	0.00	111.65	61.55	37.55
156.00	0.00	107.64	61.02	35.18
162.00	0.00	103.63	60.50	32.90
168.00	0.00	99.61	60.01	30.70
174.00	0.00	95.60	59.53	28.59
180.00	0.00	91.59	59.08	26.57

```
TEST ROUTING AS ILLUSTRATION TO 7.4.2 IN VOL.1
CASE NO.   3
EXAMPLE 3 - AS EX.2 BUT WITH LONGER SPILLWAY

 TIME      INFLOW     OUTFLOW     LEVEL      STORAGE
 (HRS.)   (CUMECS)   (CUMECS)   (METRES)   (MILLION CU.METRES)
   0.00     50.00     125.26      63.50      46.51
   6.00     75.00     123.35      63.21      45.18
  12.00    180.00     123.48      63.23      45.26
  18.00    350.00     127.74      63.88      48.27
  24.00    450.00     135.55      65.13      54.07
  30.00    520.00     226.99      66.52      60.64
  36.00    505.00     445.88      67.22      64.46
  42.00    445.00     471.87      67.29      64.82
  48.00    360.00     452.85      67.13      63.97
  54.00    290.00     341.83      66.93      62.84
  60.00    250.00     287.22      66.75      61.88
  66.00    210.00     246.54      66.60      61.08
  72.00    175.00     210.99      66.46      60.30
  78.00    140.00     177.59      66.29      59.51
  84.00    110.00     149.92      66.11      58.67
  90.00     85.00     144.15      65.89      57.64
  96.00     65.00     138.39      65.59      56.25
 102.00     55.00     136.22      65.24      54.59
 108.00     50.00     133.88      64.85      52.80
 114.00     45.00     131.42      64.46      50.96
 120.00     40.00     128.84      64.15      49.07
 126.00     38.00     126.17      63.64      47.16
 132.00      0.00     122.92      63.15      44.88
 138.00      0.00     119.39      62.59      42.26
 144.00      0.00     115.24      62.04      39.73
 150.00      0.00     111.22      61.49      37.29
 156.00      0.00     107.21      60.96      34.93
 162.00      0.00     103.29      60.45      32.66
 168.00      0.00      99.13      59.96      30.47
 174.00      0.00      95.17      59.48      28.37
 180.00      0.00      91.15      59.03      26.36
```

```
      DIMENSION QI(200),CASE(10),TITLE(10)
      COMMON/A/NCA(10),NOP,B(10),HZ(10),X(10),C(10,10),HR(10,10)
     */B/S(20),H(20),NSH
      READ (1,100) TITLE
    1 READ (1,101) ICASE
      IF (ICASE.EQ.0) STOP
      READ (1,100) CASE
      READ (1,102) NQI
      IF (NQI.EQ.0) GO TO 6
      NQIB=NQI
      READ (1,106) TINT,(QI(I),I=1,NQI)
    6 READ (1,101) NOP
      IF (NOP.EQ.0) GO TO 2
      NOPB=NOP
      DO 3 I=1,NOP
      READ (1,103) B(I),HZ(I),X(I),NC,(C(I,1),HR(I,1))
      IF (NC.GT.1) READ (1,104) (C(I,J),HR(I,J),J=2,NC)
      IF (I.EQ.1) HZMIN=HZ(1)
      IF (HZ(I).LT.HZMIN) HZMIN=HZ(I)
    3 NCA(I)=NC
    2 READ (1,101) NSH
      IF (NSH.EQ.0) GO TO 4
      NSHB=NSH
      READ (1,104) (S(I),H(I),I=1,NSH)
    4 READ (1,103) HSTART
      NQI=NQIB
      NOP=NOPB
      NSH=NSHB
      WRITE (2,200) TITLE,ICASE,CASE
      NQI1=NQI+1
      NQI10=NQI+10
      DO 5 I=NQI1,NQI10
    5 QI(I)=0.0
      CALL OUTFLOW (HSTART,Q01,Q0D)
      CALL STORE (HSTART,S1,SSD)
      HDIFF=0.1
      H1=HSTART
      NQI9=NQI+9
      DO 10 I=1,NQI9
      FHC=S1+((QI(I)+QI(I+1))/2. - Q01/2.)*TINT*3600.
      H2=H1+HDIFF
      S1M=S1/1000000.
   11 CALL OUTFLOW (H2,Q02,Q0D)
      IF (H2.LT.HZMIN) H2=HZMIN
      CALL STORE (H2,S2,SSD)
```

```
8   FH=S2+Q02*TINT*1800.-FHC
    FHD=SSD+QOD*TINT*1800.
    HAD=FH/FHD
    IF (ABS(HAD).LE.0.001.OR.H2.EQ.HZMIN) GO TO 12
    H2=H2-HAD
    LOOPS=LOOPS+1
    IF (LOOPS.LE.50) GO TO 11
    WRITE (2,202) H2
    STOP
12  HDIFF=H2-H1
    TIME=TINT*FLOAT(I-1)
    WRITE (2,201)TIME,QI(I),Q01,H1,S1M
    H1=H2
    S1=S2
10  Q01=Q02
    GO TO 1



100 FORMAT (10A8)
101 FORMAT (I2)
102 FORMAT (I3)
103 FORMAT (3F8.0,I1,F7.0,F8.0)
104 FORMAT (2F8.0)
106 FORMAT (F3.0/(10F8.0))
200 FORMAT (1H1,///,1X,10A8,/,9H CASE NO.,I3,/,1X,10A8,//
   * 47H  TIME     INFLOW    OUTFLOW    LEVEL      STORAGE,/
   * 59H  (HRS.)   (CUMECS)  (CUMECS)   (METRES)   (MILLION CU.METRES))
201 FORMAT (1X,F7.2,F9.2,F10.2,F9.2,F10.2)
202 FORMAT (25H  51 LOOPS NEEDED AT H2= ,F7.2,20H ; THERE IS PROBABLY,
   *54H AN ABRUPT DISCONTINUITY IN THE LEVEL-OUTFLOW RELATION   )
    END


    SUBROUTINE STORE (HH,SS,SSD)
    COMMON /B/S(20),H(20),NSH
    DO 2 I=2,NSH
    NI=I
    IF (HH.LE.H(I)) GO TO 3
2   CONTINUE
    NI=NSH
5   I=NI
    SI=S(I-1)*1000000.
    HI=H(I-1)
    SSD=(S(I)*1.00000.-SI)/(H(I)-HI)
    SS=SI+SSD*(HH-HI)
    RETURN
    END


    SUBROUTINE OUTFLOW (H,Q,QD)
    COMMON /A/NCA(10),NOP,B(10),HZ(10),X(10),C(10,10),HR(10,10)
    Q=0.0
    QD=0.0
    DO 4 I=1,NOP
    NC=NCA(I)
    IF (H.LT.HZ(I).OR.H.LT.HR(I,1)) GO TO 4
    IF (NC.GT.1) GO TO 5
    COEFF=C(I,1)
    GO TO 6
5   DO 2 J=1,NC
    NJ=J
    IF (H.LT.HR(I,J)) GO TO 3
2   CONTINUE
    GO TO 4
3   J=NJ
    CIJ=C(I,J-1)
    HRIJ=HR(I,J-1)
    COEFF=CIJ+(C(I,J)-CIJ)*(H-HRIJ)/(HR(I,J)-HRIJ)
6   QD=QD+B(I)*COEFF*X(I)*(H-HZ(I))**(X(I)-1.)
    Q=Q+B(I)*COEFF*(H-HZ(I))**X(I)
4   CONTINUE
    RETURN
    END

    FINISH
```

7.5 References

ANDERSON E.A. (1968) Developing and testing of snowpack energy balance equations. *Water Resources Research*, **4**, 19–37.

APOLLOV B.A., KALININ G.P. & KOMAROV V.D. (1970) *Hydrological Forecasting*. Israel Program for Scientific Translations. Jerusalem (translated from Russian).

CORPS OF ENGINEERS, U.S. ARMY (1956) *Snow Hydrology*. North Pacific Division, Portland, Oregon.

INSTITUTION OF CIVIL ENGINEERS (1967) *Flood Studies for the United Kingdom*. Report of the Committee on Floods in the United Kingdom.

JOHNSON P. (1966) Flooding from snowmelt. *Civil Engineering and Public Works Review*, **61**, 747–750.

JOHNSON P. & ARCHER D.R. (1972) *The significance of snow in Britain*. International Association of Hydrological Sciences, Symposium on the Role of Snow and Ice in Hydrology, Banff, IAHS Publications, **107**, publ. 1973, 1098–1105.

KOVZEL A.G. (1969) *A method for the computation of water yield from snow during snowmelt period*. International Association of Scientific Hydrology, Symposium on Floods and their computation, Leningrad, IASH Publications, **85**, 598–607.

LAURENSON E.M. (1964) A catchment storage model for runoff routing. *Journal of Hydrology*, **2**, 141–163.

MANDEVILLE A.N., O'CONNELL P.E., SUTCLIFFE J.V. & NASH J.E. (1970) River flow forecasting through conceptual models: Part III—The Ray catchment at Grendon Underwood. *Journal of Hydrology*, **11**, 109–128.

ROCKWOOD D.M. (1972) *New techniques in forecasting runoff from snow*. International Association of Hydrological Sciences, Symposium on the Role of Snow and Ice in Hydrology, Banff, IAHS Publications **107**, publ. 1973, 1058–1061.

WATER RESOURCES BOARD AND SCOTTISH DEVELOPMENT DEPARTMENT (1971) *The Surface Water Year Book of Great Britain, 1965–66*. HMSO, London.

WILSON E.M. (1969, 2nd edn 1974) *Engineering Hydrology*. Macmillan, London.

ZOLLER J.H. & LENZ A.T. (1958) Snowmelt runoff. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, **84, (HY6)**, Paper 1834, 1–31.

8 Future research and investigation needs

8.1 Introduction

The recommendations for the present study recognised that the need to complete the investigations in a reasonable time would restrict the study to the use and development of existing techniques. It was foreseen, however, that the need for further work would become apparent as the investigations progressed and it was suggested that these should be reported on to guide future research work and data acquisition programmes. In this chapter recommendations are collected from the contributors to all four volumes of the report with no particular attempt to allocate priorities; by no means all are related to research and the requirements for further data are emphasised. It is hoped that users will report on their experience as it is expected that local information will lead to refinements in the techniques.

In compiling these guidelines the requirements of current design practice and possible future trends have both been considered. Following the example of the design of water resource systems it seems possible that future emphasis in the design of flood protection works will shift away from the 'single site' approach to consider the whole of the affected river system. This new design approach will introduce many problems both in statistical and physical modelling of river behaviour.

Hydrological investigations (including snowmelt) are covered in Section 8.2, meteorological studies in 8.3 and hydraulic research in 8.4. Each section includes recommendations for acquiring the necessary field data and, where appropriate, suggestions for storing the collected information for ready access and use.

8.2 Hydrological investigations

A distinction is drawn between flood prediction and forecasting which, like prediction, involves estimating extreme river discharges under specified conditions. However, forecasting requires a current solution to a problem which can be specified in detail, whereas flood prediction usually relates to a hypothetical and often simplified situation. The research requirements are to some extent similar although the emphasis is on physically based hydrological models for forecasting and towards statistical modelling for flood prediction.

8.2.1 Data requirements

An accurate stage discharge relationship covering the full range is essential. The records of many stations could not be used in this study because the rating extended only to the modular limit of a structure or else to bankfull level. Various rating methods are appropriate to differing circumstances:

a direct measurement on the flood plain;
b correlation with a local rated point;
c hydraulic flood routing from a more distant point (Section 8.4);
d slope area based on crest gauge records or from air photographs.

All methods depend on a flood plain survey to a large scale, and on well laid contingency planning by the gauging authority. It should be remembered that the information may be used to upgrade all past flood records and not solely to improve future records.

Volume IV, Chapter 4, contains the product of the team's search for historical data. There must remain numerous untapped sources of semi-quantitative information in newspaper and county record offices, and reservoir records of water undertakings, which would permit flood ranking over long periods. Measurements of reservoir level or intakes would produce useful flood records for many sites which now only provide runoff volumes. Continuous monitoring of reservoir level and outflow would permit reconstruction of the inflow record, although instantaneous discharge accuracy would not be high especially where the reservoir area is large in relation to the catchment area. However, these catchments are often in important areas for flood design and the information provided on runoff percentage and lag time would be valuable (IV.2.1.8). Reservoir records could provide useful records for the study of floods of various durations (I.5).

The importance of a central flood archive must be stressed. Only when data are collected uniformly can large scale analyses be undertaken. The programme of microfilming river flow charts and maintaining a file of background information on each gauged catchment, which was begun for the present study, is currently continuing and should be extended. The collation of flood records from the whole country provided a mass of material for various types of analysis; collaboration in this study with the Irish authorities was not only of mutual scientific benefit but also increased the pool of records, and should continue where appropriate.

One possible way of extending the flood data collection network would be for the regional water authorities to nominate a number (say five per authority area) of catchments representative of local topography and conditions, to provide high quality data for continued studies into catchment response and, in the course of time, into statistical methods. These catchments should be equipped with full range flow measuring facilities and a minimum of one recording raingauge per 50 km^2. Snow depth stations with sites for measuring snow water equivalent should be provided. It is recommended that the flow gauging station should be equipped with a chart recorder as well as a punched tape recorder. The recorder should be set above any foreseen flood level and full range accuracy should be maintained by use of a pen reversal device. Associated crest gauge records up and down stream of the recorder would assist with rating the section to maximum flood level. Such an installation would provide valuable data not only for future investigations but also for the region's flood warning system.

The data storage needs for the study of water resources and of floods are not identical; research may reconcile the differences. An investigation of suitable data intervals to retain flood information to any desired precision could be undertaken; similar studies have been undertaken in the past for runoff volumes. The punched tape records should be reprocessed to extract a 'peaks over a threshold' and flood event record. Considerable quality control may be necessary to remove clock and punching errors before processing.

Following the introduction of punched tape recorders there has been a tendency to neglect the annotation of the analogue charts, which it is recommended should continue (IV.2.1.4). Apart from the obvious benefits of knowing time and level adjustments, the charts are a convenient place for noting any other information on the current state of catchment or river. Such chart annotations have proved invaluable in the current study for augmenting the pen trace information.

There is also need for information on the economic and social effects of flood damage, which, although not hydrological, have an important impact on flood design.

8.2.2 Statistical analyses of floods

The recommendations of I.2 are based on empirical and theoretical information; further investigation could modify the results or permit firmer recommendations.

Choice between different models

The relative theoretical merits of the three approaches—time series, peaks over a threshold and annual maxima—need to be further explored, in particular to decide whether the use of low flows as well as flood peaks leads to better identification of model or distribution form, or to more precise estimates of the T year flood. The peaks over a threshold models have not been extensively tested. The statistical algebra involved in allowing rates of peak occurrence and distributions of peak magnitudes to be different for each season has been outlined in Section 2.7; it remains to be seen whether these models can be usefully employed in practice. The same models are applicable to the study of flood durations or volumes as well as peak magnitudes.

Choice between different distributions

With the sample sizes available in hydrology the usual goodness of fit indices are not sufficiently informative (I.2.4). More than one measure of agreement between sample and distribution needs to be indicated. Thus, a suitable index might consist of two or more components measuring for example:
a the average scatter of the sample data about the population curve;
b the degree of curvature in the sample plot relative to the population curve;
c the agreement at the upper end with the population curve.
It would be of great practical value to know what error is involved when $Q(T)$ is estimated from a sample of the three parameter general extreme value distribution under the assumption that the two parameter EV1 distribution obtains.

Pooling data into regions

Figure 2.14 shows that there are large differences between the growth curves of different regions and it would be interesting to know whether the differences between these and the Great Britain curve of Figure 2.16 could be accounted for by sampling errors. This investigation might require solution by Monte Carlo rather than analytical methods.

Alternative ways of pooling the observations into consistent sets could be sought. In the current study the mean annual flood was used and the passage of time will enable this to be more accurately estimated.

Moreover, the growth rate of floods was found to relate to the growth of rainfall maxima and this might suggest a means of forming coherent regional sets, as considerable success was achieved by pooling meteorological data (II.2.3).

Sorting data according to origin

It has been suggested that flood data might be sorted according to magnitude, season or climatic origin and each series studied separately. Theoretical and practical difficulties have been mentioned in 1.2.3.2. Little attention has been paid to floods in particular seasons of the year. Such information may be required for risk evaluation during a short construction period or in circumstances where there is an artificial seasonal variation in the catchment conditions due for instance to reservoir operating rules. Statistical analysis of the interdependence of rainfall magnitudes and catchment conditions and losses should continue.

Flood volumes

Many of these comments and suggestions apply equally to flood volumes. Little success was achieved in relating reduction curve parameters to catchment characteristics in the present study (I.5). A further study in which more data were used and particular attention paid to matching the data source for both long and short duration floods should produce useful results. A practical research project would be to compare the flood hydrograph obtained by superimposing flood volumes of differing durations but of the same return period, with a design flood from the rainfall/unit hydrograph approach.

Time plays an important role in determining many design factors including flood volumes. It would be useful to relate the various time factors involved in surface water hydrology, such as catchment lag and unit hydrograph response (I.6.5.3), time series structure (I.2.9), flood volume reduction curve (I.5) and data storage interval (I.8.2.1).

8.2.3 Data augmentation and extension

A plea has been made in this report to make use of all available relevant information in deriving design floods and to this end I.3 and I.2.8 include a variety of ways of incorporating regional or historical information into the flood synthesis. These methods are relatively untried in practice and a period of consolidation is recommended to discover their merits. However, there is ample scope for research into each topic. Examples are:
a substitution of a more realistic distribution of exceedances into the method of extending the peaks over a threshold series (I.3.3);
b the use of regression to improve the variability estimate (I.3.2.1);
c the use of different regression equations for each month in data extension by regression (I.3.2);
d a simulation exercise to investigate the correlation criteria for successful use of *monthly* regression to predict *annual* maxima;
e the combination of several sources of information in Bayesian estimation;

f the use of the time series approach to generate long synthetic flow sequences on which to test different operational flood control strategies.

An important requirement under *e* and for combined estimation generally is a critical examination of standard error formulae for different estimates of $Q(T)$. Of major importance are the relative magnitudes of standard errors of different methods so that the weights to be applied to each estimate in a combined estimate can be assigned correctly.

8.2.4 *Catchment response*

Despite past research into the physical description of hydrological processes large gaps in understanding remain. Improvements in forecasting accuracy will result from continued research into linear (I.6) and nonlinear models (I.7.3). Their use in flood prediction is linked with the frequency of catchment state variables and rainfall depth so further study into the dependence of the response on these factors will result in more accurate predictions of the T year flood hydrograph.

Storm losses

The current studies have shown that the ability to predict losses is crucial to the prediction of most floods, and soil class was the factor of most importance in predicting losses. Certain catchments of apparently similar soil type were found in practice to give markedly different runoff percentages. It is suggested that a more detailed study should be made of the variations in the soil/cover/slope complex.

The variation of runoff percentage around its average catchment value is influenced primarily by antecedent condition in which soil moisture deficit is an important component. One way in which accuracy might be improved would be to modify the Meteorological Office's simple accounting model for obtaining SMD to allow for the effects of runoff and differing catchment constants on the balance. Comparison could be made between such re-estimated SMD values and observed soil moisture for use in operational flood forecasting. There is scope for further analysis on the lines of I.6.5.5 into loss separation alternatives. A useful additional source of data for future analysis would be events with intense rainfall but low runoff. The inclusion of such extra events would possibly improve the accuracy of prediction and might also yield evidence of nonlinearity of response.

Catchment lags and rainfall variation in time and space

The current study provided no consistent explanation for the lag time variations between different events on the same catchment. Useful results may possibly have to await analysis of data from upgraded catchments of the type referred to in 8.2.1. A more direct investigation into the effect of urbanisation on lag times than was possible in the present study should be carried out.

The results of I.6.5.8 support the contributing area concept as a working hypothesis under United Kingdom conditions. This could imply that the contributions to a flood from different portions of a catchment depend on their drainage network. Thus, for flood forecasting purposes spatial variability of rainfall input and catchment wetness should be taken into account. The dependence of response on the drainage network suggests a possible means of distributing the response for use with the flood routing methods of 8.4.3. Such a development is required if forecasting is to keep pace with the improvements in rainfall and catchment monitoring made possible by telemetering and radar.

Design needs

Ways of extending the applicability of catchment response models to flood design in very large catchments would be valuable. Statistical problems of summing rare combinations from different portions of the catchment will arise as well as timing problems from spatially varying storms. Where a reservoir is to be designed its presence in the catchment alters the response not only through the balancing effect of the storage but also through its effect on travel times down tributary streams. A recommended design practice is needed for this situation.

The current study investigated the effects of varying rainfall statistics and catchment state on flood prediction using a catchment response model (I.6.7). Further work remains to be done to develop this simulation technique. Consideration should be given to the effects of two conflicting storm definitions as used for depth statistics and duration statistics in the simulation. No allowance was made for seasonal effects or for spatial rainfall variation. The technique could be applied to deduce the distribution of flood indices other than the peak discharge, for example flood volume or reservoir spillway length.

In operating a flood warning scheme two separate streams of information will be made continuously available; telemetered river flow information, and a forecast of the flow based on previous conditions. A method of continuous correction of the forecast based upon the time series properties of the errors would be valuable. An associated decision rule based on the error could show when an updating of the forecasting model parameters becomes necessary.

8.2.5 Regional analysis

An important part of the current study has been the regression of observed flood values on catchment characteristics to make predictions at ungauged points. It should be possible to make improvements to the form of the regressions in the light of the spatial distribution of the residuals between observation and prediction. Research should take account of the physical justification for a particular form, coherence between the residual patterns, and further stations and years of record to reduce the effect of the sampling error in assessing the residual. The detailed study of soils proposed above (8.2.4) could also be valuable in regional analysis.

This is a field where the systematic application of the design techniques will itself lead to refinements as experience in a region is accumulated. After applying the various techniques to a number of flood designs and comparing the results with the available data, local variations may become apparent and it might be possible to pin-point the factors mainly responsible for the variation. However, for this purpose the interpretations and assumptions made leading to the design flood should be recorded and reported back to the authors of this study. It is hoped that a series of seminars will provide the necessary forum for these exchanges.

8.2.6 Snowmelt studies

Experience has shown that for flood estimation on larger catchments snowmelt and rainfall events can be lumped together without error. However, greater understanding of the snow phase of the hydrological cycle is necessary for estimation on smaller catchments and particularly for flood forecasting. Research so far has shown that data availability is the principal limiting factor in snowmelt modelling.

The measurement of snowfall is extremely difficult. Care has to be taken to ensure that the data collected are hydrologically useful so that the different processes of accumulation, redistribution, metamorphism, partial melt and final melt can all be modelled together. Thus, for example, a general network of shielded snow gauges, although of benefit in selected high snowfall areas, is not sufficient to fulfill current needs.

What is required is a programme of collection of snow/water equivalent data. Because of the organisational problems involved in either issuing snow core samplers to rainfall observers or deploying members of hydrometric staff at short notice to undertake snow surveys, as for current metering during floods, the programme would best be carried out by the regional water authorities and by analogous bodies in Scotland and Ireland. Research will be necessary into efficient sampling procedures so that a minimum number of point measurements may be confidently translated into areal values taking into account the areal variation in the properties of the snowpack. This variation is related to wind speed and direction, temperature, and catchment elevation, aspect and cover. Once the basic networks are established and data become available renewed development of snowmelt runoff modelling can continue on a firmer basis.

8.3 Meteorological studies

8.3.1 Introduction

Volume II shows how, for any location, the T year return period rainfall of any duration may be estimated. Subsidiary topics are maximum rainfall, snowfall and storm profile. The following suggestions are confined to flood design applications although there is a vast field of application for meteorological information in other topics, e.g. 'time lost' for building, recreation and communications. As with river flows there is need for further and better basic climatic data (8.3.2), improvements to statistical estimates (8.3.3) as well as further study of rainfall variation in time and space (8.3.4).

8.3.2 Data storage and storm classification

Despite the large number of daily and recording raingauges in the United Kingdom, their distribution is uneven with the poorest coverage in upland regions. In mountainous areas the average annual rainfall, for example, has been determined through correlations with topography; however, the aim must be for more rainfall measurements in such regions. The relatively small number of rainfall stations in some of the wettest regions of the British Isles also has important repercussions on studies of extreme rainfall events. A long term solution is a radar network for nationwide rainfall measurement and for the forecasting of rainfall a few hours ahead.

The application of radar measurements of rainfall to flood design problems requires further consideration. Research is required to develop new, or extend existing, techniques for the statistical and climatological analysis of the vast quantities of radar data.

The detailed analysis of very large quantities of continuous recording raingauge data has never been undertaken except for hourly totals and monthly, seasonal and annual maxima for a variety of durations. The safe archiving of available raingauge charts on microfilm is an important first step. A new technique of precision encoding and pattern recognition (PEPR), developed at the Nuclear Physics Laboratory, Oxford, will enable many thousands of years of rainfall charts to be translated into minute by minute rainfall data in a form suitable for reading directly into a digital computer. The recent introduction of magnetic tape devices suitable for logging rainfall events, via a tipping bucket type raingauge, also offers possibilities for a better supply of good quality, short duration, rainfall data. It is planned to develop objective techniques of quality control for these short duration rainfall data.

There is a requirement for further synoptic studies so that the main weather types could be arranged according to severity or dynamic activity. In this way it should be possible to relate local heavy rainfall to operational numerical weather forecasts. This might lead to improvements in the forecasting of heavy rainfall over restricted areas for flood warning purposes. It might also provide an objective way of sorting the records for statistical analysis and by later recombination improve rainfall depth–area–duration statistics. This is analogous to the technique outlined in 8.2.2 for dividing up a sample according to its physical origin.

8.3.3 Statistical analysis of point rainfall depths

The work described in II.2 and II.3 on the standardisation and amalgamation of monthly, seasonal and annual maxima should be continued and extended to other meteorological elements such as snow and the elements of probable maximum precipitation (II.4). There will be an increasing need for methods of numerical simulation of the climatic inputs to rainfall/runoff models (8.2). Account must be taken of the different processes apparently responsible for short and long wet spells. In the first instance simulation of a sequence of daily rainfall totals should be attempted, although where the design requires a shorter time scale a means of distributing the total will be required (8.3.4). A generalised technique for simulating continuous rainfall sequences over an area of widely varying rainfall would be very valuable.

8.3.4 Time and space variation of rainfall

Until recently little account has been taken in flood prediction of the time variation of point rainfall intensity, and virtually none of spatial variation beyond the application of an areal reduction factor. In future, results of radar rainfall mapping will allow numerical models to incorporate areal cohesion and storm movement. However, until then averaged results, such as the areal reduction factor information (II.5) and storm profiles (II.6), will remain in use and should be given further attention. Although the modelling of storm profiles is difficult, indices of profile more flexible than those described in II.6 should be studied and allowance should be made for possible variation with duration, season and storm type. Likewise the areal reduction factor information should be supplemented by a study of seasonal and annual maximum areal falls for 1, 2, 4 and 8 day durations for representative catchment areas.

Such methods of defining the variation should be tailored to the needs of flood prediction. For numerical simulation and for flood forecasting more complex indices are needed which allow departures from the average from event to event. The simulation problem is the most difficult as ideally the synthesised sequence should preserve the known statistical properties of the maxima.

8.3.5 Rainfall cycles

It was noted during the present study that there were very prominent cyclic variations in the incidence of heavy rainfall. For short durations of a few hours, for example, there is a pronounced 10–12 year cycle, with a factor of 2 between the peaks and troughs of rainfall amounts. These effects were visible for falls of longer duration, and even in annual falls with an amplitude of about 20% of the mean. An investigation is required to establish the degree of space and time coherence of these cyclical effects and their effects on flood design should be studied.

8.4 Research needs in flood hydraulics

8.4.1 Introduction

The main impact of hydraulic principles to the study has been in flood wave routing from an upstream to a downstream site. The recommended technique is of the 'storage' type in that the dynamic terms of the flood routing equations are neglected or implicit in other parameters. Much remains to be learned of the techniques in particular applications (8.4.2) although with the advent of computers the possibility now exists for the solution of the full equations in particular circumstances (8.4.3).

The data requirements for flood routing studies are essentially those for hydrological research as outlined in 8.2.1 with particular emphasis on flood plain surveys, not only near the gauging station but over an entire river reach.

8.4.2 Development of existing techniques through application

It is to be expected that one of the main applications of the present study will be to flood warning schemes. Rivers with a large number of tributaries and which have a large catchment area present difficult flood warning problems. The effects of moving storms over such a catchment can produce radically different water levels at a downstream site. This synchronisation problem could be solved from statistical or physical considerations.

Other possible uses for flood routing techniques are the improvement of rating curve accuracy (8.2.1) and the profile of the flood wave following a dam break.

The development of a method to calculate flood wave speed accurately would lead to improved flood routing (III.2). It should be possible to make use of the Kleitz–Seddon equation $\bar{c} = \mathrm{d}Q/\mathrm{d}A$ to improve estimates of $\bar{c}$ at sections where Q is known as a function of A. There are two problems which follow. The first is that it is very difficult to estimate how Q varies with A when the flow is in the region of bankfull and above bankfull. Consequently, there are large errors in the calculation of $\bar{c}$ for these flows; but even when $\bar{c}$ has been found as a function of Q, there is then the problem of estimating the speed of the peak of the flood wave. This speed depends on the shape of the flood hydrograph in a rather complicated way which is imperfectly understood. Obviously, where there are no records of flood wave speed, the use of current methods for calculating the speed introduces large errors into the predictions of the flood routing methods, and it would be valuable to have these errors reduced.

8.4.3 Development of new flood routing techniques

The position has been reached where a numerical solution of the comprehensive Saint–Venant equations is practicable in some circumstances. If the river reach is divided into subreaches by sections, and the cross section geometry, boundary roughness and lateral inflow are known for each section, then given the inflow hydrograph including any tributary inflow, and the location and representation of intermediate and downstream hydraulic controls, the equations may be solved at discrete time intervals by finite difference techniques. Such a method would enable predictions to be made of water levels as well as discharges along the whole of a reach. The model would include head losses at bridges, bends, weirs, locks and natural control points, and allow for backwater effects due to tides and large tributaries. In addition, the model could be used to study how the flow in the channel interacts with the flow over the flood plain; it is this problem which is one of the main difficulties in developing a viable model. The collection of relevant field data would be valuable here.

The current treatment of lateral inflow by such a model is rather crude. There exists the possibility that given the *relative* values of the parameters for the unit hydrographs, the mathematical flood routing model can then be used to optimise these values from flow data for the main channel. Such a procedure may well be more accurate than determining initially the absolute values of the unit hydrograph parameters.

8.5 Conclusions

Continuing research is required into many aspects of river flooding. The general trend in hydrological theory and practice has been to develop empirical, statistical, physical and systems methods which some researchers have tried to reconcile with each other. The advent of computers has made it easier to combine the various classes of 'models' so that in future it may be possible to use a synthesis of empirical data, of fluid and solid mechanics and thermodynamics, of statistical and of systems techniques to provide comprehensive methods of hydrological analysis.

The past decade has seen a 'data explosion' and this will continue especially as telemetering and remote sensing techniques come into general use. The experience of the study has shown that quite as important as the instrumentation and collection of field data is the processing and archiving of the information. This aspect requires very careful planning so that high grade information is available to users not envisaged at the time of setting up the system.

The need for continuity and uniformity is needed in equal measure in the use of the design techniques. If users were encouraged to feed back results of practical applications it is felt that regional detail could be filled in. It is believed that this could be brought about by using standard computer programs and by organising a series of flood design seminars. By this means, advances in hydrological techniques could be incorporated in flood design at shorter intervals than between 1933 and 1975.