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in five volumes

Volume III

Flood Routing Studies

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Preface

This volume, which forms Volume III of the Flood Studies Report, examines various routing methods to determine which method or methods would be most suitable for routing floods in British rivers. The work was carried out as part of the research programme of the Hydraulics Research Station and was undertaken by Dr R. K. Price under the general direction of Mr A. J. M. Harrison.

The members of the Flood Studies Steering Committee were the late Mr M. Nixon (Chairman) who was succeeded as Chairman by Mr E. J. K. Chapman (ICE) in 1974, Mr G. Cole (MAFF), Mr V. K. Collinge (WRB), Mr D. Fiddes (TRRL), Mr J. Harding (MO) succeeded by Mr R. Murray and Mr J. F. Keers, Mr A. F. Jenkinson (MO), Mr M. A. Lynn (OPW, Dublin), Mr M. Mansell-Moullin, Dr J. S. G. McCulloch (Director, IH), Mr J. C. Munro (SDD), Dr J. V. Sutcliffe (IH), Mr J. I. Taylor (ARA), Mr S. F. White (DE) and Professor P. O. Wolf.

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Notation

A	wetted cross-sectional area
a	part of the diffusion coefficient in the convection–diffusion equation
b	subscript denoting a bankfull value
C_1, C_2, C_3, C_4	parameters used in the finite difference Muskingum equation
c	convection speed
c	subscript denoting a variable for the channel
$\bar{c}_{ave}$	average value of $\bar{c}$ over a range of values for Q
$F(\tau, x)$	$\equiv Q(t,x)$
f	subscript denoting a variable for the flood plain
f_1, f_2	functions defining the theoretical curves for $\bar{c}$ and Q
G	function used to simplify analysis
g	acceleration due to gravity
J	space label of downstream boundary
J′	space label of foot of characteristic curve used in the downstream boundary condition
j	subscript denoting a finite difference variable evaluated at the jth space node
K	storage constant in the Muskingum method
L	length of reach
M	number of subreaches in the calculation of α
m	subscript denoting a variable for the mth subreach
max	subscript denoting a maximum value for the variable
min	subscript denoting a minimum value for the variable
N	half the number of time steps in each finite difference method
n	Manning roughness coefficient
n	superscript denoting a finite difference variable evaluated at the nth time node
p	subscript denoting a peak value for the variable
p_r	parameter used in the elementary flood wave solution for the linear convection–diffusion equation
Q	discharge
Q_a	finite difference average discharge
Q_{amp}	amplitude of variable part of synthetic hydrograph
Q_{base}	base flow for synthetic hydrograph
Q_I	inflow discharge to reach
Q_O	outflow discharge from reach
Q^*	attenuation of peak discharge
Q_x	cut-off discharge for drainage off the flood plain
q	lateral inflow per unit length
q^*	lateral inflow per unit length from the flood plain to the channel
q_{amp}	amplitude of lateral inflow function
q_r	parameter used in the elementary flood wave solution for the linear convection–diffusion equation
R	hydraulic radius
S	storage in a reach of river
s	bottom slope of channel
s_{fr}	friction slope
T	time scale
T_p	recorded time-of-travel of peak
T_{tot}	total time to simulate flood

t	time
t_p	time-to-peak of hydrograph
v_q	downstream velocity component of lateral inflow
X	length scale
x	distance from upstream section of reach
y	depth
1,2	subscripts denoting first and second order solutions of the convection–diffusion equation.

The superposition of a bar denotes either a scale for the variable or the mean value along the reach.

Greek symbols

α	attenuation parameter
β	parameter used in the synthetic hydrograph
γ_r	parameter used in the elementary flood wave solution for the linear convection–diffusion equation
Δt	finite difference time increment
Δx	finite difference space increment
ε	parameter in the Muskingum method
θ	a contribution to the convection speed from the flow along the flood plain
κ	parameter used in the calculation of the speed–discharge curve
λ	ratio of inundated width of flood plain and channel to width of channel
μ	diffusion coefficient
σ	sinuosity of channel with respect to flood plain
τ	characteristic time variable
ω	convection speed.

1 Choice of a flood routing method

1.1 Introduction

The prediction of a design flood hydrograph at a particular site on a river may be based on the derivation of a discharge or stage hydrograph at an upstream section, together with a method to route this hydrograph along the rest of the river. In order to limit this investigation to cases where the assumptions like uniform rainfall may be reasonably valid, the derivation of unit hydrographs has been limited to catchments with an area less than 500 km^2. Consequently, flood routing methods provide a useful tool for the analysis of flooding in all but the smaller catchments, particularly where the shape of the hydrograph as well as the peak value is required. The volume concentrates on an examination of various flood routing methods to determine which method or methods is most suitable for use in British rivers. It is therefore assumed that a discharge hydrograph at an upstream section of a particular river is available from records at a gauging station, or has been derived using the hydrological methods described in Volume I, and that information is required about how the flood defined by this discharge hydrograph affects discharges at one or several sections downstream. As will become apparent, this problem is basically one of open channel hydraulics.

1.2 Flood routing methods

The importance of being able to route floods accurately is also reflected in the large number of flood routing methods which have been developed since the year 1900. These methods can readily be sorted into three groups:
a hydrological or storage methods;
b those methods based on a convection-diffusion equation; and
c methods using a numerical solution of the full Saint–Venant equations for gradually varying flow in open channels.

The methods in group *a* are the most numerous, and, in general, the most simple of all flood routing methods. They are termed 'hydrological methods' because they concentrate on the concept of storage for the flood water and do not directly include the effects of resistance to the flow. So the routing of a flood by a hydrological method in a given reach of river is based on the continuity equation which equates the rate of change of the storage, $\mathrm{d}S/\mathrm{d}t$, in the reach to the difference between the inflow, Q_{I}, at the upstream section and the outflow, Q_{O}, at the downstream section:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = Q_{\mathrm{I}} - Q_{\mathrm{O}}. \tag{1.1}$$

The method then recommends a second, algebraic relationship between the storage and both the inflow and the outflow. This enables a solution to be found for the outflow when the inflow is given. One of the most popular and satisfactory methods of this type is known as the Muskingum method, which was originated by McCarthy (1938). The method uses the linear algebraic relationship:

$$S = K[\varepsilon Q_{\mathrm{I}} + (1-\varepsilon)Q_{\mathrm{O}}] \tag{1.2}$$

where K is termed the storage parameter, and ε relates the relative importance of the inflow and the outflow. The actual values for the two parameters have to be determined from the channel characteristics under study. A variety of graphical and step-by-step techniques has been suggested for the Muskingum and similar storage routing methods; see Chow (1959, p. 609).

One of the disadvantages with the hydrological methods is that they assume a unique relationship between the stage and the discharge along the reach. This is contrary to observations of natural floods which show that the discharge for a particular stage when the flood level is increasing is larger than the discharge for the same stage when the flood level is decreasing. This phenomenon can be displayed graphically in the well-known loop rating curve (Figure 1.1). Directly related to this non-uniqueness in the stage–discharge relationship for the reach is the attenuation of

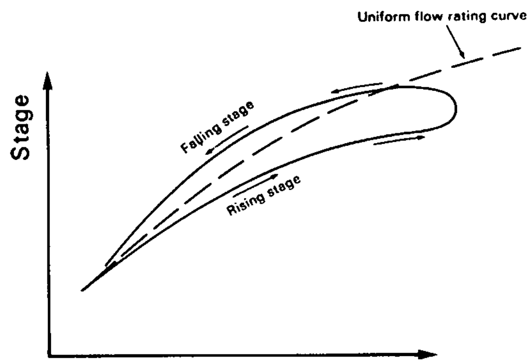


Fig 1.1 Loop rating curve.

the peak discharge along the reach. Because the Muskingum method predicts such an attenuation it is not immediately apparent how this can be reconciled with the assumption of a unique stage–discharge relationship. So, engineers turned to the equation expressing the principle of conservation of momentum. This equation, often called the dynamic equation, includes the effect of resistance to the flow and replaces the algebraic relationships such as Equation 1.2. The continuity and dynamic equations can be written in the form†

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x} = q \tag{1.3}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = gA\left(s-\frac{\partial y}{\partial x}-s_{\mathrm{fr}}\right)+qv_{\mathrm{q}}. \tag{1.4}$$

Here, A is the wetted cross-sectional area at a distance x from the upstream section of the reach, Q is the discharge, q is the lateral inflow per unit length, g is the acceleration due to gravity, s is the bottom slope of the channel, $\partial y/\partial x$ is the slope of the water surface defined relative to the bottom of the channel, s_{fr} is the friction slope, and v_{q} is the downstream component of velocity along the channel for the lateral inflow. Equations 1.3 and 1.4 are usually referred to as the Saint–Venant equations for gradually varying flow in open channels.

As Equation 1.4 stands, it is too complicated to solve analytically for an arbitrary flood in a natural river. Fortunately however, some of the terms in the equation are usually sufficiently small that they can be neglected. Because of this Lighthill & Whitham (1955) were able to show that flood propagation can be described in terms of kinematic rather than dynamic waves. Here a kinematic wave is a wave which has a constant amplitude and which possesses only one velocity at each point of the wave,

†Henderson (1966, p. 285) gives a succinct derivation of Equations 1.3 and 1.4, but note that Henderson writes the equations in terms of the stage and velocity rather than the cross-sectional area and the discharge as here.

in contrast to a dynamic wave which has at least two velocities. By treating a flood as a kinematic wave to the first approximation, and by including modifications to this wave due to the diffusion induced by the water surface slope, $\partial y/\partial x$, Lighthill & Whitham outlined a new flood routing method which they termed the kinematic wave method. In effect, their method is based on a convection-diffusion equation, such as

$$\frac{\partial y}{\partial t}+\omega\frac{\partial y}{\partial x}=\mu\frac{\partial^2 y}{\partial x^2} \tag{1.5}$$

which is written in the characteristic form

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\mu\frac{\partial^2 y}{\partial x^2} \tag{1.6}$$

with the characteristic curve given by

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\omega. \tag{1.7}$$

For simplicity, ω and μ are usually regarded as constant parameters. It is the diffusion term which introduces an attenuation of the peak stage along the reach.

Because of its basic equation, the kinematic wave method belongs to group *b* above. However, it was Hayami (1951) who first produced a flood routing method based on a linear convection-diffusion equation. He argued that floods in natural river channels are affected by the irregularities in the channel geometry. To include the effect of these irregularities Hayami proposed the linear convection-diffusion equation with an arbitrary value for μ. Because he knew of no way to calculate this value of μ directly from the channel geometry, Hayami suggested that μ should be determined by a trial-and-error comparison of results using his method with records of previous floods in the river under study. Once the value of the diffusion coefficient is known, and the parameter ω is defined as the speed of a flood peak, Hayami's diffusion method gives good agreement with natural floods which have similar peak discharges. The uncertainty in the value of μ remains however as a major disadvantage with the diffusion method.

As explained by Hayami, the irregularities in the width of the river define, in effect, a series of reservoirs which increase the storage capacity of the river. This phenomenon is accentuated when the flood water flows out over an associated flood plain. Because the diffusion method effectively routes floods in a river which has a regular cross-section, bottom slope, and roughness, and which is equivalent to the natural river, the increase in the storage capacity due to the irregularities in the natural river can be viewed as a change in the geometry and roughness of the equivalent river model. It is shown in Chapter 2 that these changes can in fact be quantified, so that the parameters for the diffusion method, and μ in particular, can be determined without requiring a trial-and-error application of the method.

Another disadvantage with the kinematic wave and diffusion methods is their use of fixed values of the parameters ω and μ. Suppose that ω and μ have been found so that the speed of travel and the magnitude of the peak discharge along the reach are correctly simulated. Then these values for ω and μ can be considerably different from the corresponding values determined for an overbank flood. So, although values of the parameters can be obtained to route a range of previous floods, the extrapolation of these values to deal with a possible larger range of floods can lead to significant errors. Thomas & Wormleaton (1970) have made a numerical study of floods in an upper reach of the River Dee (Wales) using the diffusion method with the stage as the dependent variable. Their results indicate how difficult it is to isolate fixed values for the convection velocity and diffusion coefficient when applying the method to a wide range of floods in a particular river. A way of overcoming this difficulty is to define ω and μ as functions of the stage or discharge in the river. A new flood routing method based on the generalised convection-diffusion equation with the discharge as the dependent variable and ω and μ as functions of discharge is presented in Chapter 2.

Despite the fact that the difficulties referred to above can be removed, there remains one more significant problem in using the diffusion methods, namely the inclusion of discharges from major tributaries. The difficulty which arises in this case is how to prevent what is in effect a discrete lateral inflow upsetting calculations using the governing equation. The simplest solution to the problem is to route a flood from tributary to tributary, summing the discharge hydrographs from the main channel and the tributary at the confluence. But, whereas this procedure is satisfactory for a well gauged river, there will inevitably be questions of accuracy in applying the diffusion methods to rivers which are not well monitored.

It has already been mentioned above that it is the water surface slope, $\partial y/\partial x$, which induces a diffusion of the kinematic wave solution, and that one of the consequences of this diffusion is an attenuation of the peak stage or discharge along the reach. A formula for the attenuation of the peak stage for a flood in a regular channel was discovered by Forchheimer (1930), many years before Hayami proposed the diffusion method. Forchheimer showed that the attenuation is directly related to the curvature of the peak of the upstream stage hydrograph. More recently, Henderson (1963) has shown how the acceleration and convection of momentum terms on the left hand side of Equation 1.4 also contribute to the attenuation of the peak stage. The importance of these latter terms increases as the Froude number for the flow increases. So, for steep rivers, Henderson points out that all the terms in the dynamic equation may become important. However, the attenuation of the peak stage is approximately inversely proportional to the square of the bottom slope, so that the magnitude of the attenuation in steep rivers is not as important as that in rivers with small bottom slopes. A similar attenuation formula and analysis can be derived for the attenuation of the peak discharge.

The attenuation formula can also be regarded as a consequence of the second order solution for the convection–diffusion equation. For this reason it is convenient to classify the use of the formula as a flood routing method in group *b*. In addition, because it is now possible to quantify and include the effect of irregularities in the geometry of the natural river in the convection–diffusion equation, a similar improvement can be made to the attenuation formula. This is explained in more detail in Chapter 2.

Although the Muskingum method, already referred to above, ignores dynamic effects on the flood wave, Cunge (1969) has shown that it is possible to improve the method so that it gives a good approximation to the solution of the linear convection–diffusion equation. This improvement is made on the basis of the finite difference equation for the Muskingum method. The finite difference scheme introduces an arbitrary diffusion and Cunge has identified the magnitude of this diffusion with that of the corresponding term in the diffusion method. In this way he is able to define the parameters of the Muskingum method in terms of the parameters for the diffusion method. As shown in Chapter 3, the Muskingum–Cunge method can be used to find the attenuation of the peak discharge with a high degree of precision. In addition, it can be commented here that the Muskingum method does not have the difficulties with tributaries that the diffusion methods have. Consequently, there will be advantages in using the Muskingum–Cunge method for rivers which have major tributaries and which are not well gauged.

Because of the limitations in the analytical flood routing methods which have been proposed and the availability of powerful digital computers, increasing attention has been paid to numerical solutions of the full Saint–Venant equations. These numerical flood routing methods which come under group *c* above, differ from each other primarily in the technique used to solve the differential equations. With sufficient storage in the computer, the methods can be extended to include as much detail of the geometrical characteristics of the channel and flood plain as required. Problems do arise in isolating friction parameters along the channel and the flood plain, and in specifying the head losses for the flow to and from the flood plain and over walls and hedges. And a concern for detail in such a model can obscure an overall description of the flow. So although one can anticipate that a flood routing method of group *c* is likely to be more accurate than any of the other methods given sufficient data, the simpler flood routing techniques are usually adequate for many purposes. However, the numerical methods do become an important tool if both levels and discharges are required continuously along a reach of river. In this case the simpler flood routing techniques are cumbersome, and their poorer accuracy compared with the numerical methods may be significant.

1.3 Choosing a flood routing method

Faced with such a range of flood routing methods, the choice of a suitable method for routing a flood in a particular British river is at first sight formidable. Inevitably there are two major factors affecting the choice, namely the information required from the method, and the data available about the geometry of the natural river and previous floods.

The results from a flood routing study will of course be dictated by the nature of the overall project. For example, if a building is being constructed on the flood plain such that the building will not significantly affect the flooding characteristics of the river, but will itself be sensitive to flooding, the engineer will be concerned with, say, the peak level of a flood hydrograph at the construction site. He may also be interested in how long the flood will be above a certain level, in which case he will need to know the shape of the stage hydrograph. Similar information with respect to flood discharge hydrographs will be required when designing a spillway for an onstream reservoir. Here the engineer may be concerned principally with the rising part of the discharge hydrograph and with its shape near the peak. If, however, alterations are being proposed, such as a flood alleviation scheme, which will alter the flooding characteristics of the river, then a knowledge of peak levels and discharges from certain design floods, and possibly the associated hydrographs, will be necessary not only at the sites where the improvements are to be made but also at sections far downstream. When information is required at discrete sections which are more than, say, 20 times the width of the flood plain apart, then the routing of a flood between each section can be regarded as a 'black box' problem: the interest is only in the input and output to each reach. In this case one of the simpler flood routing methods from groups *a* or *b*, which routes a flood along a simplified equivalent river model instead of a complex model of the irregular natural river, is usually sufficient. However, when more detailed information on flooding is required along the river, including water levels and mean velocities, then one has to resort to one of the much more complicated numerical methods of group *c*. There does exist the possibility of using a method from group *b* with a method from say, group *c*, to supply the detailed, local information which the former method is unable to produce, but it is not usual to combine such methods at present.

Another important aspect of the information obtained from a flood routing method is the accuracy of the results. This accuracy will of course be a function of the accuracy of the data and the method itself. If it is supposed that the accuracy of the input data for the method can be considered separately, then errors in the results will depend principally on the suitability of the basic equations to describe the phenomenon of flood propagation. A brief survey of the various methods has been made in Section 1.2 above, and the advantages and disadvantages are discussed more thoroughly in the remaining chapters of this volume. It is sufficient to note here that in general it is the errors arising from the unsuitability of terms in the basic equations which are the most difficult to eliminate: numerical analysis is sufficiently well advanced that errors generated by the solution techniques—including finite difference schemes for the equations, boundary conditions and data handling techniques—need not be too great a problem.

The amount and quality of data from the natural river, both for the geometry of the channel and flood plain, and for previous flood discharges and levels is another significant factor in the choice of a flood routing method. Fortunately, it is usually possible to obtain general geometrical information about a British river from survey maps. This information will include, say, the length of a reach, the slope of the channel, and the plan area of the flood plain. It is still necessary however to know something about the speed of flood peaks along the reach, and the corresponding peak discharges, particularly if water inundates the flood plain. If this information is of poor quality then nothing is gained by using a flood routing method more complicated than a simple storage method. Fortunately, most British rivers have at least one gauging station which can be used to produce a typical discharge hydrograph for the river and peak discharges for previous floods. However, it is common for the error in the calculated discharges to be more than about 0.2 of the actual discharge, particularly for high flows when the error can be much greater. This large error is due to the difficulties in extrapolating the rating curve for flow over an adjacent flood plain. If there is no gauging station available, either a gauging station has to be built or discharge hydrographs for previous floods have to be derived at the upstream section of the reach from some hydrological catchment model as described in Volume I of this report. Of course, if a detailed numerical model is to be developed, the amount of proving data increases with the amount of detail required. Such data can be expensive to obtain and depend, in the case of a flood plain study, on the occurrence of appropriate floods during the course of the investigation. It should also be remembered that if the parameters for a given method have been determined for a particular range of floods, then the use of the method with the same parameters for larger floods can introduce errors.

This brief discussion of the factors affecting the choice of a flood routing method has so far neglected how the characteristics of British rivers and flooding in these rivers affect such a choice of method. It is appropriate then to conclude this chapter with a description of these characteristics.

1.4 Characteristics of British rivers

Table 1.1 contains a summary of data from a number of British rivers and several foreign rivers. Perhaps the most outstanding characteristics of the British rivers are the relatively short lengths and large average slopes. Defining the upstream section of a river as that section which has a catchment area of 500 km^2 and the furthest downstream section at the tidal limit, then there is no river in Britain which has a length greater than 210 km. In fact, besides the rivers Severn, Thames, Wye and Trent, all other British rivers have lengths less than 110 km. Closely related to the length of a river is its average slope, defined as the difference between the levels at the upstream and downstream sections of the river, divided by the length of the river. As the rivers are relatively short in length, they have fairly steep average slopes of the order of 10^{-3}. The Great Ouse has the smallest average slope of the larger rivers, namely 2.1×10^{-4}.

The maximum recorded discharges in British rivers vary considerably from river to river, and in several rivers the maximum discharge may occur far upstream and not at the tidal limit. In general, the greatest discharges have occurred in rivers with steep slopes, namely the rivers Tay, Tyne, Tweed and Dee (Scotland). The Tay is exceptional in that it has a mean annual discharge of 154 $m^3 s^{-1}$, which is far larger than the mean annual discharge for any other British river, and has had flood flows of up to 1419 $m^3 s^{-1}$. Longer rivers like the Wye and Trent, do not appear to have exceeded a discharge of about 1000 $m^3 s^{-1}$ at their downstream sections, and the maximum recorded discharge in the Severn at Bewdley is only 671 $m^3 s^{-1}$. Notice that, in general, the longer rivers have higher peak discharges at upstream sections. Evidently the peak discharge of a flood in these rivers attenuates as the flood moves downstream.

Most river systems in Britain are complex, having several tributaries with significant mean annual discharges. Typical of these river systems are the Great Ouse and the Yorkshire Ouse. A few river systems like the Wye are more straightforward, the Wye having only two major tributaries along most of its length. However, besides the discrete discharges from the major tributaries to the main river, there is a significant increase in the discharge of certain rivers due to the lateral runoff from the catchments along the rivers, including minor tributaries and the local aquifer. For example, there is no major tributary between Erwood and Belmont on the Wye, but the difference in the mean annual discharges at these two stations is 9.76 $m^3 s^{-1}$, or 27% of the mean annual flow at the upstream station at Erwood. This gives an average lateral inflow of about 0.4 $m^3 s^{-1} km^{-1}$. Again, the magnitude of the lateral inflow, being a function of the local rainfall, varies from river to river. Some rivers in the east of England, such as the Nene and Trent, usually have a negligible lateral inflow, though under snowmelt conditions the lateral inflow for such rivers can be large.

The magnitude and variation of the width of the flood plain along a river is another important factor to be considered in a flood routing study. A convenient definition of the flood plain width is an area per unit length of river, as measured from the area inundated by the largest recorded

River	Length (km)	Mean annual discharge ($m^3 s^{-1}$)	Catch-ment area (km^2)	Peak dis-charge ($m^3 s^{-1}$)	Bankfull discharge ($m^3 s^{-1}$)	Bank-full width (m)	Bank-full depth (m)	Average slope ($\times 10^{-3}$)	Slope at upstream section ($\times 10^{-3}$)	Slope at tidal limit ($\times 10^{-3}$)	No. of major tribs	Est. average lateral inflow ($m^3 s^{-1} km^{-1}$)
Spey	103†	60.51 Boat O'Brig	2850	1598				2.132	3.788	1.965	3	3×10^{-1}
		9.55 Ruthven Bridge	534	224								
Dee	77	36.17 Woodend	1370	1134				3.521§	4.740§	0.685§	4	
Tay	60‡	154.46 Ballathie	4590	1419				1.905§	3.165§	0.893§	3	
		51.34 Pitnacree	1150	559								
Tweed	74	81.22 Norham	4390	1193				1.786	1.453	0.568	5	2×10^{-1}
		16.31 Peebles	694	1079								
Tyne	42	44.23 Bywell	2180	1497	410 Barrasford	64	3	1.832	1.960	0.848	3	
		18.08 Haydon Bridge	751	545								
Tees	77	16.98 Broken Scar	818	667				2.688	4.310	0.741	4	
Wharfe	77	15.80 Flint Mill	759	417				1.786	3.788		5	
Trent	124	23.75 Colwick	7490	810	333 Nottingham	55	4	0.581	0.709		7	Negligible
		11.32 Yoxall	1230	1265‖								
Nene	84	8.09 Orton	1630	382				0.617	1.692	0.410§	3	4×10^{-2}
Great Ouse	106	9.06 Bedford	1460	140	74	26	2.5	0.213	0.472§	0.094§	10	
		8.54 Harrold Mill	1320	183	65	30	2.8					
Thames	177	66.85 Teddington	9870	1065	315 Kingston	78	3.3	0.375	0.394	0.073	12	
		12.11 Eynsham	1620	83	136 Day's Weir	46	2.8					
Exe	34	15.59 Thorverton	601	457				1.859	2.959	1.637	2	
Severn	130	62‖ Bewdley	4330	671	284	52	3.9	0.379	1.482	0.152	12	2×10^{-1}
		11.69 Abermule	580	617	200 Montford	36	4.2					
Wye	93	69.39 Cadora	4040	926	510	64	5.0	1.264	3.953	0.269	4	4×10^{-1}
		34.84 Erwood	1280	1206	440	61.5	3.6					2×10^{-1}
Usk	29	28.17 Chain Bridge	912	655				2.283	0.807	0.054	—	4×10^{-1}
Dee	45	31.00 Erbistock	1040	627	170	51	2.0	1.786	0.301	0.301§	5	3×10^{-1}
Clyde	26	38.16 Blairston	1710	577				1.802§	4.740§	0.455§	4	
		17.16 Sills	742	443								
Mississippi	6414	2623‖ Alton Ill.						0.370				
		15906‖ Vicksburg Miss.										
Rhine	1335	991										
Danube	2832	6515						0.110				

flood for that river. Notice that this width is not necessarily the same as the geographical width of the flood plain. For some of the longer British rivers the flood plain width can reach a local maximum in excess of 2 km, though the average width along the whole length of these rivers is less than 0.5 km. For many rivers the flood plain is artificially controlled by flood banks and other flood control features. The embankments for some rivers, such as the Great Ouse in the fenland, produce a normal water level which can be above the surrounding flood plain. In this latter case the overtopping of the embankments leads to flooding over a very wide area, with water which is only returned to the main channel by pumping over a period of months.

One of the major effects of an inundated flood plain is to change considerably the shape of the flood hydrograph at sections along the river. This is explained in Section 2.6 below. But it can be mentioned here that the most important change induced by a large flood plain on the shape of a flood hydrograph is the attenuation of the peak discharge. This attenuation may not of course be observed if there is a large lateral inflow to the river, and in general the peak discharges of floods in the smaller British rivers amplify along the rivers because of the lateral inflow due to direct runoff from the surrounding catchment and from tributaries.

1.5 Flood routing in a British river

From the discussion above it follows that an appropriate flood routing method for British rivers should be able to route accurately floods in a reach which typically has a slope of 10^{-3} and which is up to 100 km long. In addition, the method should be sufficiently versatile to treat the case of flooding in a river with extensive flood plains. Not surprisingly the last restriction is severe, and a new flood routing method has been derived for this case in the following chapter. However, as will become apparent from Chapter 3, the simple flood routing methods, and in particular the Muskingum–Cunge method, have much to recommend them for general use in British rivers.

In the following chapter a more comprehensive study is made of the theoretical basis of flood routing. This study leads to the proposal of a new formula for the attenuation of the peak discharge for a flood along a reach with extensive flood plains, together with the development of a new flood routing method as mentioned above. Chapter 3 then concentrates on a comparison of three of the most appropriate flood routing methods. Finally, a strategy for flood routing in British rivers is outlined in Chapter 4. If desired, Chapter 2 may be omitted on a first reading.

†has a large loch in the catchment
‡includes Loch Tay
§from 1 inch to 1 mile map
‖estimated.

Table 1.1 Data for some British and foreign rivers.

2 Theory of flood routing

2.1 Introduction

It is usual in developing a theory of flood routing to begin with the simplest models, such as hydrological models, and to proceed to more complicated models once the deficiencies of a particular model have been understood. This is the approach adopted in the brief survey of flood routing methods in Chapter 1. However, to emphasise that all flood routing methods are based on, or can be shown to depend on, hydraulic principles, the theory of flood routing in this chapter is developed from the Saint–Venant equations for gradually varying flow in open channels. Necessarily, some algebra is required in Sections 2.4, 2.7 and 2.12 to ensure a proper development of the theory. These sections are so arranged that they may be omitted on a first reading of the chapter. Also, an effort has been made throughout the chapter to specify the underlying assumptions in each step of the argument. Many of the assumptions may appear crude or severe to those who have no experience of flood routing. The accurate results which can however be obtained by even a very simple flood routing method indicate that the assumptions are realistic and that simplicity has much to recommend it.

2.2 Mathematical modelling of flood flows

Each flood routing method is based on some model for the river and its associated flow. The hydrological methods regard the river as a 'black box' with the storage in the box depending on the inflow and outflow. Necessarily the black box has one or more parameters, the values of which are peculiar to the river being studied. One of the best ways to find the parameters is to simulate the model on an analogue computer and to vary the parameters until the best fit is obtained between the predicted and recorded hydrographs for a calibration flood in the natural river. The approach adopted in this type of model is equivalent to saying that the flow in the river is much too complex to be modelled in detail and that it is sufficient to assume some arbitrary functional relationship between the outflow and inflow to the reach, with the one restraint that the total amount of water stored in the reach is conserved. Although this approach appears crude, it should be recognised that at some stage a similar approach is inevitable in the derivation of any mathematical model of flow in a river. As already indicated, the major difficulty is the complexity of the flow, in view of both the irregular nature of the boundaries in the river, and the turbulent motion of the water. So simplifying assumptions have to be made *ab initio* to make the problem tractable.

The Saint–Venant equations describe the one-dimensional bulk flow of water in a river (Brutsaert, 1971). In the derivation of the equations it is assumed that the variation of the mean velocity across a section in the river is not important, and the water surface slope varies gradually along the river so that the pressure is approximately hydrostatic. The effect of friction on the flow is generally simulated by an empirical term for the friction slope. It is usual to adopt the Manning or Chézy form for this term.

The roughness coefficient in the term for the friction slope can be regarded as a proving parameter. Just as the boundary roughness in a physical model of a hydraulic problem has to be adjusted so that water levels in the model agree with the corresponding levels in the natural river, so the roughness coefficient in a mathematical model can be varied to the same end. An estimate of the roughness coefficient can of course be made simply from a knowledge of the natural river channel by noting such points as the boundary texture, the character of the banks, whether there is weed growth in the channel, the variability in the shape of the cross-section and the sinuosity of the channel. Inevitably, because of all the factors affecting the magnitude of a roughness coefficient there is uncertainty in assigning a value to the coefficient for a particular river channel. For this reason, it is preferable where possible to use other methods of estimating the roughness coefficient, or better still to use another proving parameter. It is shown below that in the case of the simpler flood routing methods the speed of flood peaks is generally more convenient than the roughness coefficient. This is because the speeds of previous flood peaks in a river are often known and because the simpler methods can all be derived in terms of the speed. Another advantage in using the speed as a proving parameter is that observations of the speed of flood peaks include the direct effect of irregularities in the width and bottom slope of the channel. These irregularities, which have a length scale along the river of say, many times the width of the channel, can possibly be included in a detailed numerical model if the roughness coefficient can be found for each small subreach. But because the simpler flood routing methods make simplifications to the geometry of the river channel and in effect use an 'equivalent' channel with average values along the reach for the parameters describing the geometrical characteristics of the natural channel, these methods can include implicitly the effect of irregularities. As explained in Chapter 1, it was this sort of consideration which led Hayami to propose his diffusion method.

The extension of the models to include storage and flow over an associated flood plain increases the complexity of the problem. For example, if the river channel in the model is regarded as straight, the plan geometry of the flood plain will be distorted. This makes the exchange of water and momentum between the channel and the flood plain difficult to simulate. One way round this difficulty is to regard the flood plain simply as an extension of the channel. This, however, is inappropriate because of the large difference between typical velocities in the channel and over the flood plain. But once it is accepted that the flow over the flood plain is to be regarded as separate from the flow in the channel, then the exchange of water and momentum between the channel and flood plain has to be considered.

Obviously the effect of channel meanders on the exchange can be extremely complicated, and there would be little hope, even with present day computers, of correctly simulating in detail the processes involved over a long reach of river. A considerable amount of research has been concentrated in recent years on these processes, particularly in the USSR. The two main processes are a direct convection of momentum via the flow to and from the channel, and a diffusion effect. This latter effect arises from the difference in the velocity of the flow on the flood plain and the flow in the channel. Obviously the velocity in the channel will generally be greater than the velocity of the water over the flood plain immediately adjacent to the channel. This velocity difference produces vortices which diffuse and are convected from the channel to the flood plain. In turn this leads to a small reduction of the velocity in the channel and a larger increase in the velocity over the flood plain. Zheleznyakov (1971) has made a thorough study of this phenomenon, which has been termed, somewhat inappropriately, the 'kinematic effect'.

Another important feature of flow over a flood plain is the flow normal to the general direction of the river channel. In a numerical model this difficulty can be partly overcome by using a separate equation to describe flow to and from the channel. This equation might include a term with a structure similar to the formula for the discharge over a broad crested weir. In a simple flood routing model, however, it does not seem possible to include transverse flow over the flood plain as a distinct feature and still preserve a necessary simplicity in the model. The analytical development of the flood routing model below assumes that the flows in the channel and over the flood plain are distinct but related by the condition that the water level in the channel is the same as that over the flood plain and is uniform across a section. This is equivalent to saying that the lateral flow over the flood plain is instantaneous.

2.3 Basic equations

The Saint-Venant equations for gradually varying flow in open channels are

continuity:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x} = q \tag{2.1}$$

momentum:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = Ag\left(s-\frac{\partial y}{\partial x}-s_{fr}\right)+qv_q \tag{2.2}$$

where A is the wetted cross-sectional area, Q is the discharge, q is the lateral inflow/unit length, g is the acceleration due to gravity, s is the bottom slope of the channel, y is the depth, s_{fr} is the friction slope, and v_q is the velocity component of q along the channel in the downstream direction (Figure 2.1).

A good approximation for s_{fr} is given by the Strickler–Manning formula:

$$s_{fr} = \frac{Q^2n^2}{A^2R^{4/3}} \tag{2.3}$$

where R is the hydraulic radius and n is the Manning roughness coefficient.

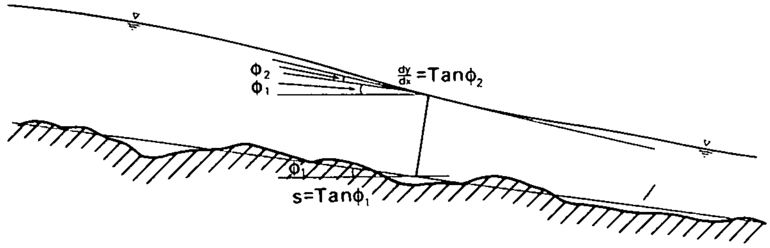


Fig 2.1 Definition sketch.

Equations 2.1 and 2.2 have no exact analytical solution relevant to flood wave propagation other than the monoclinal wave in a uniform channel which is infinitely wide (Henderson, 1966, p. 372). This wave travels with constant speed and tends to a fixed depth downstream and a larger fixed depth upstream. In the absence of a more general exact solution, it is necessary to resort to approximate analytical solutions of the basic equations. This is possible because some of the terms in Equation 2.2 are less important than others. Following Henderson (1966, p. 364), the relative importance of the terms in the basic equations can be deduced from an order of magnitude analysis.

Because the flow is one-dimensional in space, and because the lateral inflow is often negligible, the terms on the left hand side of Equation 2.1 must have a similar magnitude. Formally, if X and T are the length and time scales for a flood wave respectively,

$$\frac{\bar{A}}{T} = \frac{\bar{Q}}{X} \tag{2.4}$$

where the superposition of a bar above a variable denotes the scale for that variable. It now follows from Equation 2.2 that

$$\left|\frac{\partial Q}{\partial t}\right| \bigg/ \left|\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)\right| = \frac{\bar{Q}}{T} \bigg/ \frac{\bar{Q}^2}{X\bar{A}} = 1 \tag{2.5}$$

using Equation 2.4. So the local acceleration and convection of momentum terms in Equation 2.2 are of the same order. To make a further comparison of the terms in Equation 2.2 it is necessary to have estimates of the values for the various scales which are typical for British rivers. Set

$$\begin{aligned}
\bar{W} &= 60 \text{ m} & \bar{g} &= 10 \text{ m s}^{-1} \\
\bar{s} &= 10^{-3} & \bar{y} &= 5 \text{ m} \\
\bar{n} &= 4 \times 10^{-2} & \bar{q} &= 10^{-4} \text{ m}^2 \text{ s}^{-1} \\
\bar{A} &= 300 \text{ m}^2 & T &= 10^5 \text{ s} \\
\bar{Q} &= 500 \text{ m}^3 \text{ s}^{-1} & \bar{v}_q &= 0.5 \text{ m s}^{-1}.
\end{aligned} \tag{2.6}$$

With these data

$$\begin{aligned}
X &= \bar{Q}T/\bar{A} \simeq 1.7 \times 10^5 \text{ m} \simeq 200 \text{ km} \\
|gAs_{fr}|/|gAs| &= \bar{s}_{fr}/\bar{s} \sim 0.9 \\
\left|gA\frac{\partial y}{\partial x}\right| \bigg/ |gAs| &= \frac{\bar{y}}{X} \bigg/ \bar{s} \sim 2.0 \times 10^{-2} \\
\left|\frac{\partial Q}{\partial t}\right| \bigg/ |gAs| &= \frac{\bar{Q}}{T} \bigg/ g\bar{A}\bar{s} \sim 1.7 \times 10^{-3} \\
|qv_q|/|gAs| &= \bar{q}\bar{v}_q/g\bar{A}\bar{s} \sim 1.7 \times 10^{-5} \\
|q| \bigg/ \left|\frac{\partial A}{\partial t}\right| &= \bar{q} \bigg/ \frac{\bar{A}}{T} \sim 0.3 \times 10^{-1}.
\end{aligned} \tag{2.7}$$

These relationships show that:

i the momentum of the flow in the river is governed primarily by the bottom and friction slopes, and is modified by the water surface slope, $\partial y/\partial x$, which is defined relative to the bottom slope of the channel;

ii the acceleration and convection of momentum terms can be ignored;

iii the contribution to the momentum in the main channel from tributaries and lateral inflow can also be ignored;

iv the lateral inflow from small tributaries and direct runoff can be significant under snowmelt conditions, but in general its effect is small; and

v the length scale of a flood wave is considerably greater than the lengths of most British rivers.

Conclusion *ii* would not of course be true for flow in steep rivers, and the conclusion may also be violated locally for flow through bridges, weirs and other obstructions in the river. However, in the latter case it can often be assumed that head losses at such obstructions are included in the appropriate values for the parameters of a particular flood routing method.

Using the above conclusions, the following equations can be recommended for the routing of inbank floods in British rivers:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x} = q \tag{2.8}$$

$$O = s-\frac{\partial y}{\partial x}-\frac{Q^2 n^2}{A^2 R^{4/3}}\ . \tag{2.9}$$

Equation 2.9 can now be solved to give Q explicitly, and Q can then be substituted directly into Equation 2.8 to give an equation involving A (or y) alone:

$$\frac{\partial A}{\partial t}+\frac{\partial}{\partial x}\left[\frac{1}{n}AR^{2/3}\left(s-\frac{\partial y}{\partial x}\right)^{\frac{1}{2}}\right] = q \tag{2.10}$$

where y is a prescribed function of x and A. However, Equation 2.10 is inappropriate for an analytical study of flood routing because the peak value of A is strongly dependent on the local channel geometry. This difficulty, which will be explained in more detail below, is avoided if Q is the dependent variable rather than A.

2.4 Equations for flow in channel-flood plain systems†

Previous steady flow studies of flooding over flood plains have assumed that the storage and flow over the flood plain can be introduced by separating the total discharge along the river into a discharge Q_c in the channel and a discharge Q_f over the flood plain (Zheleznyakov, 1971). As commented in Section 2.2, such a distinction is crude from a local viewpoint due to such geometrical features as the bifurcation of the flood plain and the meanders in the channel. However, over a long reach this division of the total discharge is a good approximation to reality. Next, it is assumed that the water level across the flood plain normal to the general direction of the main channel is uniform and the same as that in the channel. Then, if Equations 2.8 and 2.9 can also be taken to describe the flow over the flood plain, the equations describing the flow in the whole system are, for the channel:

$$\frac{\partial A_c}{\partial t}+\frac{\partial Q_c}{\partial x} = q^* \tag{2.11}$$

$$O = s-\frac{\partial y_c}{\partial x}-\frac{Q_c^2 n_c^2}{A_c^2 R_c^{4/3}} \tag{2.12}$$

and for the flood plain:

$$\sigma\frac{\partial A_f}{\partial t}+\frac{\partial Q_f}{\partial x} = -q^*+q \tag{2.13}$$

†This section may be omitted on a first reading of this chapter.

$$O = \sigma^3\left(s-\frac{\partial y_c}{\partial x}\right)-\frac{Q_f^2 n_f^2}{A_f^2 R_f^{4/3}} \tag{2.14}$$

where q^* is the lateral inflow per unit length from the flood plain to the channel. As subscripts, c and f refer to variables for the channel and the flood plain respectively. The sinuosity, σ, is defined as the ratio of the length of the channel to the length of the prototype flood plain in the mean direction of the channel. Note that A_f refers to the wetted cross-sectional area of the flood plain in the model. Because the channel in the model is regarded as straight and the plan area of the flood plain in the model is the same as that in the prototype, the width of the flood plain in the model will be $1/\sigma$ times the width of the prototype flood plain (Figure 2.2).

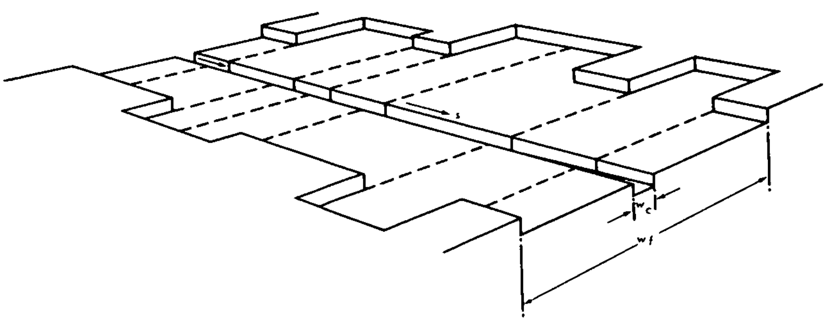


Fig 2.2 Schematic channel-flood plain model.

Equations 2.11 and 2.13 can be combined to give

$$\frac{\partial}{\partial t}(A_c+\sigma A_f)+\frac{\partial Q}{\partial x} = q. \tag{2.15}$$

The problem now is to replace $A_c+\sigma A_f$ by a function form involving Q. Because the level of the water surface across a section of the river and flood plain is taken as uniform, it is possible to express A_f as a function of A_c. So, Equation 2.15 can be rewritten as

$$\left(1+\sigma\frac{\partial A_f}{\partial A_c}\right)\frac{\partial A_c}{\partial t}+\frac{\partial Q}{\partial x} = q \tag{2.16}$$

where

$$\frac{\partial A_f}{\partial A_c} = \frac{W_f}{W_c}. \tag{2.17}$$

Here W_c and W_f are the inundated widths of the channel and the flood plain respectively. Next, from Equations 2.12 and 2.14,

$$Q \equiv Q_c+Q_f = \left[\frac{A_c R_c^{2/3}}{n_c}+\sigma^{3/2}\frac{A_f R_f^{2/3}}{n_f}\right]\left(s-\frac{\partial y_c}{\partial x}\right)^{\frac{1}{2}} \tag{2.18}$$

or, differentiating this equation with respect to t,

$$\frac{\partial Q}{\partial t} = \left[\frac{R_c^{2/3}}{n_c}\left(1+\frac{2}{3}\frac{A_c}{R_c}\frac{\partial R_c}{\partial A_c}\right)+\frac{\sigma^{3/2}}{n_f}\frac{\partial}{\partial A_c}(A_f R_f^{2/3})\right]\left(s-\frac{\partial y_c}{\partial x}\right)^{\frac{1}{2}}\frac{\partial A_c}{\partial t}$$

$$-\tfrac{1}{2}Q\left(s-\frac{\partial y_c}{\partial x}\right)^{-1}\frac{\partial}{\partial x}\left(\frac{1}{W_c}\frac{\partial A_c}{\partial t}\right). \tag{2.19}$$

By eliminating $\partial A_c/\partial t$ between Equations 2.16 and 2.19, and assuming that the channel has a large width to depth ratio

$$\frac{\partial Q}{\partial t}+c\left(1-\frac{1}{s}\frac{\partial y_c}{\partial x}\right)^{3/10}\left(\frac{\partial Q}{\partial x}-q\right) =$$
$$-\tfrac{1}{2}Q\left(s-\frac{\partial y_c}{\partial x}\right)^{-1}\frac{\partial}{\partial x}\left[\frac{1}{\lambda W_c}\left(\frac{\partial Q}{\partial x}-q\right)\right] \tag{2.20}$$

where

$$c = \frac{Q}{\lambda(Q_c n_c)^{3/5}\,W_c^{2/5}\,s^{3/10}}\left\{1+\frac{2}{3}\frac{A_c}{R_c}\frac{\partial R_c}{\partial A_c}+\theta\right\} \tag{2.21}$$

$$\lambda = 1+\sigma W_f/W_c \tag{2.22}$$

and

$$\theta = \sigma^{3/2}\frac{A_f R_f^{2/3}}{n_f}\left[\frac{A_c}{A_f}\frac{W_f}{W_c}\left(1+\frac{2}{3}\frac{A_f}{R_f}\frac{\partial R_f}{\partial A_f}\right)-\left(1+\frac{2}{3}\frac{A_c}{R_c}\frac{\partial R_c}{\partial A_c}\right)\right]$$
$$\cdot\left[\frac{A_c R_c^{2/3}}{n_c}+\sigma^{3/2}\frac{A_f R_f^{2/3}}{n_f}\right]^{-1}. \tag{2.23}$$

If the flood plain is not inundated or, for the purposes of this study, if the discharge is less than the bankfull discharge Q_b, $\lambda = 1$ and $\theta = 0$. For simplicity it is assumed that the water surface width of the channel when the flood plain is inundated is the bankfull width.

Finally, when q is effectively uniform along the river, and $|\partial y_c/\partial x|$ is small compared with s, then Equation 2.20 becomes

$$\frac{\partial Q}{\partial t}+c\frac{\partial Q}{\partial x} = Q\frac{\partial}{\partial x}\left(\alpha\frac{\partial Q}{\partial x}\right)+\frac{\alpha Q}{s}\frac{\mathrm{d}s}{\mathrm{d}x}\frac{\partial Q}{\partial x}+\frac{3}{5}\alpha_c\left(\frac{\partial Q}{\partial x}\right)^2+cq \tag{2.24}$$

where

$$\alpha = (2\lambda W_c s)^{-1}. \tag{2.25}$$

2.5 Convection–diffusion equation

The flood routing equation derived in the previous section (Equation 2.24) is basically a convection-diffusion equation. The second term on the left hand side of the equation describes a convection with speed c of the quantity Q. This change in time of the local value of Q is modified by the terms on the right hand side of Equation 2.24, which can be regarded as describing a diffusion of Q. Similarly, the convection of $\int_0^x q\,\mathrm{d}x'$ with speed c also affects the local value of Q.

Equation 2.24, or a similar equation with the stage as the dependent variable, can be shown to be the basis for most of the simpler flood routing methods.† These models generally assume that c, α and αQ are constant in space and time, and that the third term on the right hand side of Equation 2.24 is negligible; for example, see Cunge's improvement of the Muskingum method (Cunge, 1969), the kinematic wave method (Lighthill & Whitham, 1955) and the diffusion method (Hayami, 1951; Thomas & Wormleaton, 1970; Price, 1973a). These assumptions make the convection-diffusion equation linear, and it is then comparatively easy to obtain an exact analytical solution of the equation (Hayami, 1951). However, the expressions for c and α in the previous section indicate that these parameters are properly functions of several variables, including Q and x. In particular, the magnitude of the parameters will vary considerably when

†This is true even for the hydrological or storage routing methods, if they are to produce accurate results. See Section 2.15 and Cunge (1969).

there is flooding of an associated flood plain. It follows that in such a case of flooding it is better where possible to know the functional forms for c and a. Although c is defined by an equation (Equation 2.21) in the previous section, the structure of the terms in that equation makes the task of finding c as an analytical function of, say, Q and x rather difficult. An alternative and more convenient approach is to calculate c from records of the time of peak for several floods at each end of the reach. This procedure is of course more accurate the longer the reach. If there is also a gauging station with a good rating curve at some section of the river then the values for the speed of each flood peak can be correlated with the corresponding peak discharges. In this way c can be found as an averaged function, $\bar{c}$, of Q alone.

It remains now to calculate the appropriate functional form for a in terms of Q alone. Values for a for particular floods, with a defined in terms of average values for the bottom slope and the maximum width of the river (Equation 2.25) could be used, but such values for a would ignore the variations in λW_c and s along the reach. And it is precisely these variations which Hayami (1951) argued are important and should be included in a. Inevitably, to obtain this averaged value of a some sort of analytical solution of Equation 2.24 has to be derived for an arbitrary flood wave in a given river. Fortunately, it is possible to adapt Hayashi's (1965) method of solving the Saint–Venant equations to the solution of Equation 2.24, and by making some appropriate assumptions, to obtain an explicit expression for a. The solution uses a technique known as perturbation analysis.

2.6 First order solution

Because the terms on the right hand side of Equation 2.24 can be regarded as being significantly smaller than the terms on the left hand side, it follows that to the first order Equation 2.24 can be written as

$$\frac{\partial Q_1}{\partial t}+c_1\frac{\partial Q_1}{\partial x} = 0 \tag{2.26}$$

where Q_1 is the first order term in the expansion for Q, and

$$c_1 \equiv c(Q_1, x). \tag{2.27}$$

Equation 2.26 implies that Q_1 is a function of a single characteristic variable τ, defined by

$$\tau = t-\int_0^x \frac{dx'}{c_1}. \tag{2.28}$$

For clarity, set

$$Q_1\,(x, t) \equiv F_1(\tau). \tag{2.29}$$

$F_1(t)$ is the function for the discharge hydrograph at the upstream section of the reach.

Equation 2.26 describes a kinematic wave moving with a velocity c_1 (Lighthill & Whitham, 1955). The peak discharge for this wave is unaffected by variations in the channel geometry. However, as the speed of the wave is a function both of distance along the reach and the discharge, the variations in c_1 directly affect the shape of the discharge hydrograph for the kinematic wave at sections along the river. To illustrate what happens, consider a given discharge hydrograph at the upstream section of a reach of a river with extensive flood plains and assume that Equation 2.26 is an adequate description of the motion of the flood wave. The discharge hydrograph at a downstream section will have three important features (Figure 2.3).

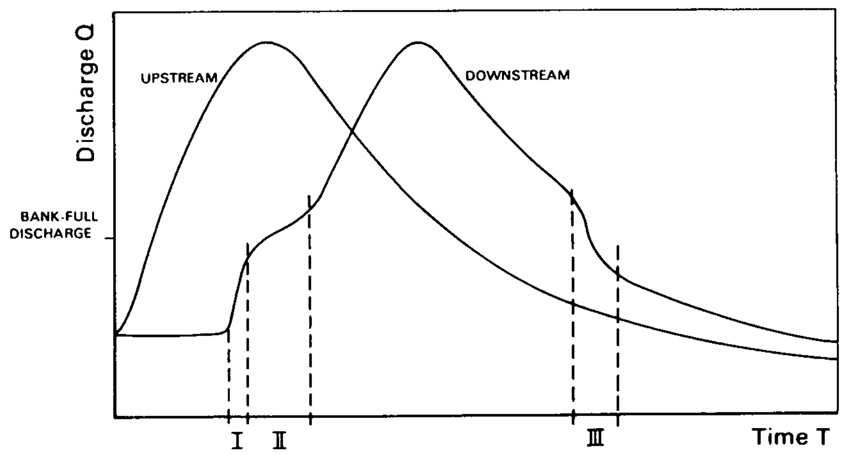


Fig 2.3 Deformation of the discharge hydrograph for a kinematic wave.

Suppose in the first instance that the kinematic wave is entirely contained within the banks of a rectangular channel. It can be shown from Equation 2.21 that the speed of the crest of the wave is then greater than the speed of the foot of the wave. So, the rising part of the discharge hydrograph at the downstream section will be steeper than the corresponding part of the discharge hydrograph at the upstream section. This is also true in general for the inbank part of a kinematic wave which inundates the flood plain (region I in Figure 2.3). Observations of floods in prototype channels show a similar feature.

Next, consider what happens to the kinematic wave when the flood plain is inundated. From Equation 2.21, if the flood plain is infinitely rough, $\theta = 0$ and the speed of the kinematic wave will be considerably reduced for that part of the wave above bankfull, depending on the magnitude of λ. Even when $\theta \neq 0$ it is found that c_1 is smaller than it would be if there was no flood plain. This reduction in the speed for the overbank part of the kinematic wave leads to a distinct flattening of the rising part of the discharge hydrograph at the downstream section in the region of the bankfull discharge; see region II. This feature is very pronounced if the flood plain is flat and can be assumed to be bounded by vertical walls. Correspondingly, in region III where the water is receding from the flood plain, the discharge hydrograph is markedly steeper than the same part of the hydrograph at the upstream section. In practice however, region III is not as pronounced as in Figure 2.3. This is because the drainage off the flood plain tends to smooth the curve in this region. As may be expected, the effect of the drainage off a flat flood plain is more important than the drainage off a flood plain which slopes towards the river.

2.7 Second order solution†

The terms on the right hand side of Equation 2.24 modify the kinematic wave solution. To find the additional small term Q_1 it is necessary to

†This section may be omitted on a first reading of this chapter.

extract the second order equation from Equation 2.24;

$$\frac{\partial Q_2}{\partial t}+c_1\frac{\partial Q_2}{\partial x}+c_2\frac{\partial Q_1}{\partial x} = Q_1\left[\frac{\partial}{\partial x}\left(\alpha_1\frac{\partial Q_1}{\partial x}\right)+\frac{\alpha_1}{s}\frac{ds}{dx}\frac{\partial Q_1}{\partial x}\right]$$
$$+\frac{3}{5}\alpha_{1c}\left(\frac{\partial Q_1}{\partial x}\right)^2+c_1 q \tag{2.30}$$

where

$$\alpha_1 \equiv \alpha(Q_1, x). \tag{2.31}$$

Equation 2.30 can be solved as follows.

As Q_2 is a function both of τ and x, expressions can be obtained for the partial derivatives of Q_2:

$$\frac{\partial Q_2}{\partial t} = \phi\frac{\partial F_2}{\partial \tau} \tag{2.32}$$

$$\frac{\partial Q_2}{\partial x} = -\frac{\phi}{c_1}\frac{\partial F_2}{\partial \tau}+\frac{\partial F_2}{\partial x}. \tag{2.33}$$

Here, $Q_2(x, t) \equiv F_2(\tau, x)$ and the function ϕ is defined by

$$\phi(\tau, x) = \left\{1-\int_0^x\frac{1}{c_1^2}\frac{\partial c_1}{\partial F_1}\,dx'\,\frac{dF_1}{d\tau}\right\}^{-1} \tag{2.34}$$

It can similarly be shown that

$$\frac{1}{\alpha_1}\frac{\partial}{\partial x}\left(\alpha_1\frac{\partial Q_1}{\partial x}\right)+\frac{1}{s}\frac{ds}{dx}\frac{\partial Q_1}{\partial x}+\frac{3}{5}\alpha_c\left(\frac{\partial Q_1}{\partial x}\right)^2$$
$$= \frac{\phi^2}{c_1^2}\left\{\phi\frac{d^2F_1}{d\tau^2}-\left(\frac{dF_1}{d\tau}\right)^2\left\langle\frac{2}{c_1}\frac{\partial c_1}{\partial F_1}+\frac{1}{\alpha_1}\frac{\partial \alpha_1}{\partial F_1}-\frac{3}{5}\frac{\alpha_{1c}}{F_1}\right.\right.$$
$$\left.-\int_0^x\left[\frac{1}{c_1^2}\frac{\partial^2 c_1}{\partial F_1^2}-\frac{2}{c_1^3}\left(\frac{\partial c_1}{\partial F_1}\right)^2\right]dx'\,.\,\phi\frac{dF_1}{d\tau}\right\rangle$$
$$\left.+\frac{1}{\phi}\frac{dF_1}{d\tau}\left[\frac{\partial c_1}{\partial x}-\frac{c_1}{\alpha_1}\frac{\partial \alpha_1}{\partial x}-\frac{c_1}{s}\frac{ds}{dx}\right]\right\} = \frac{\phi}{c_1^2}G(\tau, x). \tag{2.35}$$

By substituting the expressions above in Equation 2.30 it follows that

$$\frac{\partial}{\partial x}\left(\frac{F_2}{\phi}\right) = \frac{1}{\phi}q(\tau, x)+\tfrac{1}{2}\frac{F_1 G}{\lambda W_c s c_1^3}. \tag{2.36}$$

This equation can be integrated to give

$$Q_2 \equiv F_2 = \phi\int_0^x\frac{1}{\phi}q\,dx'+\tfrac{1}{2}\phi F_1\int_0^x\frac{G\,dx'}{\lambda W_c s c_1^3}. \tag{2.37}$$

The expression for Q_2 in Equation 2.37 is an exact solution of Equation 2.30. However, the complexity of the function $G(\tau, x)$ makes it difficult to develop the solution for practical purposes. So three additional assumptions are made:

i the reach is sufficiently short so that $\phi \sim 1$;

ii the contribution from the terms in Equation 2.35 involving $\partial c_1/\partial x$, $\partial \alpha_1/\partial x$ and ds/dx to the second integral in Equation 2.37 can be neglected; and

iii c_1 is a separable function of x and Q_1.

These assumptions may appear at first to be severe and so they require further comment.

The first assumption can obviously be made applicable by reducing the length of the reach being considered. The actual length of the reach will of course depend on the geometrical characteristics of the channel through the quantity $\partial c_1/\partial F_1$, and on the maximum slope of the discharge hydrograph (Equation 2.34). If $\partial c_1/\partial F_1$ is small, as is the case for rivers with wide and reasonably flat flood plains, the maximum length of the reach which satisfies the first assumption can be considerable.

The second assumption is more difficult to justify, and it may well be that the term involving $\partial c_1/\partial x$, $\partial \alpha_1/\partial x$ and $\mathrm{d}s/\mathrm{d}x$ are not always negligible. However, if the flood plain is not too irregular, the changes of sign in these functions along the reach will tend to make the contribution from the term involving these quantities fairly small.

Finally, the third assumption can readily be justified when there is little or no flow along the flood plain. By introducing an average speed $\bar{c}(Q)$ and regarding $s^{-3/10} \sim s^{-1/3}$

$$c(Q, x) = \bar{c}(Q) \frac{s^{1/3}(x)}{\lambda(Q, x)} \frac{1}{L} \int_0^L \frac{\lambda(Q, x')}{s^{1/3}(x')} \mathrm{d}x' \tag{2.38}$$

where L is the length of the reach. It is now assumed that the definition of $\bar{c}$ as a function of Q in Equation 2.38 gives an adequate definition of c for any reach, even when there is flow along an associated flood plain.

The substitution for c from Equation 2.38 in Equation 2.37, together with the first two assumptions above, gives

$$Q_2 = \int_0^x q\mathrm{d}x' + \frac{F_1}{\bar{c}_1} \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\alpha'}{\bar{c}_1^2} \frac{\mathrm{d}F_1}{\mathrm{d}\tau}\right) + \frac{3}{5} \frac{\alpha_c'}{\bar{c}_1^3} \left(\frac{\mathrm{d}F_1}{\mathrm{d}\tau}\right)^2 \tag{2.39}$$

where

$$\alpha'(Q_1, x) = \frac{1}{2\overline{W}_c} \left\{\frac{1}{L} \int_0^L \frac{\lambda}{s^{1/3}} \mathrm{d}x'\right\}^{-3} \int_0^x \left(\frac{\lambda}{s}\right)^2 \mathrm{d}x'. \tag{2.40}$$

Here the channel width W_c is assumed to be approximately uniform along the channel.

2.8 Attenuation of peak discharge

At the peak of the discharge hydrograph for any section $\partial Q/\partial t = 0$. If the reach is sufficiently short so that $\partial Q/\partial \tau \simeq 0$ at the peak, or alternatively $\mathrm{d}F_1/\mathrm{d}\tau \simeq 0$, then the attenuation, Q^*, of the peak discharge is given approximately by

$$Q^* = \frac{\alpha'(F_1, L)}{\bar{c}_1^3} F_1 \left|\frac{\mathrm{d}^2 F_1}{\mathrm{d}\tau^2}\right| \tag{2.41}$$

where all the functions on the right hand side of Equation 2.41 are evaluated for the peak discharge at the upstream section and it is assumed that $q = 0$. Because of the close connection of α' with the attenuation of the peak discharge it is convenient to call $\alpha \equiv \alpha'(F_1, L)$ the attenuation parameter for the reach.

The attenuation of a flood wave was first discussed analytically by Forchheimer (1930), who considered the attenuation of the peak stage along a prismatic channel. Because Forchheimer's channel was prismatic, the integral in the expression for α above did not appear in his derivation. Van der Made (1968) extended Forchheimer's analysis to include the effects of overbank flooding, but he too avoided an integral formulation for the attenuation by using the concept of a 'stream carrying width'. The importance of the integrals in Equation 2.40 is that they include a contribution to the attenuation from irregularities in the width of the flood plain and the channel slope along the natural river. For example, if the second integral were replaced by the square of the average value of λ/s along the reach times the length of the reach, a significant contribution to the attenuation of a flood wave would be lost. Analytically, as the integrand contains the square of λ/s, the integral is a minimum when λ/s is uniform along the reach. In physical terms this result implies that for a given reach a short wide flood plain will induce a larger attenuation of an overbank flood than a flood plain with the same area uniformly distributed along the reach, provided the values for $\bar{c}_1$ are similar in both cases. This conclusion is reinforced by the fact that where the flood plain is wide, s is usually smaller than the average value, $\bar{s}$, for the whole reach.

It is important to remember that the flood routing solution above has only been developed to the second order: that is, the application of the solution should ideally be restricted to reaches along which the predicted attenuation is less than, say, 10% of the original peak discharge. The need for such a restriction on the use of the solution is evident from the theoretical work of Hayami (1951) and the computations of Di Silvio (1969), which indicate that in prismatic channels the rate of attenuation appears to decrease exponentially with distance downstream. As may be expected, when a flood wave attenuates, the curvature at the peak of the hydrograph is reduced at sections along the river, so reducing the rate at which the wave attenuates further. Hayami's theoretical solutions show that this reduction in the rate of attenuation is partly a consequence of the term in Equation 2.39 involving $d^2F_1/d\tau^2$. Following Hayami (1951) and the theoretical work of Hayashi (1965), the formula for the attenuation of the peak discharge in Equation 2.40 can be regarded as the first order term from the alternative formula

$$Q^* = F_1\left\{1-\exp\left[\frac{\alpha_1}{\bar{c}_1^3}\frac{d^2F_1}{d\tau^2}\right]\right\}. \qquad (2.42)$$

However, although this formula may be more accurate than Equation 2.41 for the attenuation of a flood wave in a long reach, it should be observed that the higher order solutions of Equation 2.24 also play an important part in determining how the flood wave attenuates. It can be anticipated that the third order solution for the attenuation will include third and fourth order derivatives of F_1 at the peak of the upstream hydrograph. So, for a long reach Equation 2.42 will be accurate to the second order only. In addition, Equation 2.39 shows that over a long reach the effect of changes in $\bar{c}_1$ and α_1 with F_1 can be important. In particular, if $d\bar{c}_1/dF_1 > 0$ for values of F_1 in the neighbourhood of the crest of a flood wave, then the decrease in the rate of attenuation will be less than if $d\bar{c}_1/dF_1 < 0$. So, for these reasons, when the predicted attenuation along the reach is more than, say, 10% of the original peak discharge, it is preferable to return to the flood routing equation, Equation 2.24, and to solve it numerically for the propagation of the entire hydrograph along the reach. In this way a more accurate result will be obtained for the attenuation than by using the formula in Equation 2.41.

2.9 Variable parameter diffusion method

If c in Equation 2.24 is defined in terms of the average speed, $\bar{c}(Q)$, along the reach, then α has to be defined in terms of a similar averaged value for the attenuation parameter. Consequently, Equation 2.24 becomes

$$\frac{\partial Q}{\partial t}+\bar{c}\frac{\partial Q}{\partial x} = Q\frac{\partial}{\partial x}\left(\frac{\alpha}{L}\frac{\partial Q}{\partial x}\right)+\frac{3\alpha_c}{5L}\left(\frac{\partial Q}{\partial x}\right)^2+\bar{c}q. \tag{2.43}$$

It is now assumed that $\bar{c}$, and α defined by Equation 2.40 with $x = L$, provide an adequate definition of what can be termed the equivalent river model, even though the attenuation predicted by Equation 2.41 may be greater than 10% of the original peak discharge.

The next objective is to solve Equation 2.43 for a particular flood hydrograph, given prescribed functional forms for $\bar{c}$ and α. However, it is apparent from Equation 2.43 that any solution for the routing of a hydrograph using this equation is liable to be sensitive to the functional form for $\mathrm{d}\alpha/\mathrm{d}Q$. In addition, because it happens that the curve for α is generally much more difficult to calculate for a particular river than the corresponding curve for $\bar{c}$, it was decided to confine attention to the equation

$$\frac{\partial Q}{\partial t}+\bar{c}\frac{\partial Q}{\partial x} = \frac{\alpha}{L}Q\frac{\partial^2 Q}{\partial x^2}+\bar{c}q. \tag{2.44}$$

The flood routing method based on Equation 2.44 with $\bar{c}$ and α as prescribed functions of Q can be termed the variable parameter diffusion method.

2.10 Calculation of the attenuation parameter

The most convenient way of evaluating α for a given reach of river is to divide the reach into a number of sub-reaches, so that the geographical width of the prototype flood plain in each sub-reach is approximately uniform. α can then be written as

$$\alpha(Q) = \tfrac{1}{2}\left\{\frac{1}{L}\sum_{\mathrm{m}=1}^{\mathrm{M}}\frac{P_{\mathrm{m}}}{s_{\mathrm{m}}^{1/3}}\right\}^{-3}\sum_{\mathrm{m}=1}^{\mathrm{M}}\left(\frac{P_{\mathrm{m}}^2}{L_{\mathrm{m}}s_{\mathrm{m}}^2}\right) \tag{2.45}$$

where P_{m} is the plan area of the inundated flood plain and the channel in the mth sub-reach, and L_{m} and s_{m} are the corresponding length and bottom slope of the channel. It has again been assumed that the width of the channel, W_{c}, is approximately uniform along the reach.

$\alpha(Q)$ can readily be found for the largest recorded flood if limits of flooding on the flood plain are known. In addition, α can be calculated for a small inbank flood from

$$\alpha = \frac{1}{2W_{\mathrm{c}}}\left\{\frac{1}{L}\sum_{\mathrm{m}=1}^{\mathrm{M}}\frac{L_{\mathrm{m}}}{s_{\mathrm{m}}^{1/3}}\right\}^{-3}\sum_{\mathrm{m}=1}^{\mathrm{M}}\left(\frac{L_{\mathrm{m}}}{s_{\mathrm{m}}^2}\right). \tag{2.46}$$

However, intermediate values for $\alpha(Q)$ are much more difficult to obtain unless there are data available on the extent of flooding by different overbank floods. If the data are not available, then the curve for $\alpha(Q)$ between the inbank and extreme flood values has to be estimated. The present investigation has indicated that irregularities in the channel width may tend to make α approximately constant for $Q < \bar{Q}_{\mathrm{b}}$, where $\bar{Q}_{\mathrm{b}}$ is the average value for the bankfull discharge along the reach. Obviously the shape of the curve for $Q > \bar{Q}_b$ depends to a large extent on the flatness or otherwise of the flood plan.

$\alpha(Q)$ has been obtained for the reach of the River Wye (Herefordshire) between Erwood and Belmont. As flooding in this reach forms the major case study in Chapter 3, it is sufficient at this stage to draw attention to the relevant curve for $\alpha(Q)$ in Figure 2.4.

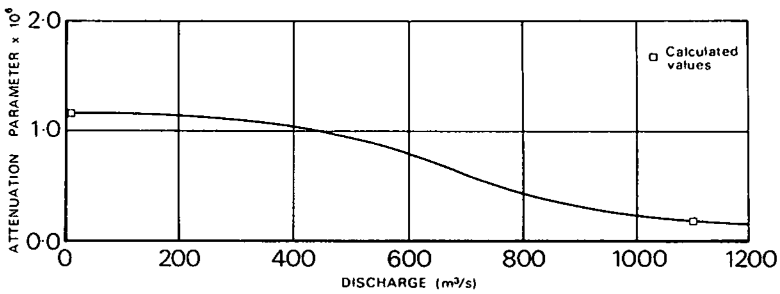


Fig 2.4 Attenuation parameter for the Erwood to Belmont reach of the River Wye.

2.11 Calculation of the convection speed

The procedure for calculating the speed $\bar{c}$ from records of previous floods is facilitated by having good stage recording stations along the river, and at least one gauging station with a reasonably accurate rating curve. If there is no reliable rating equation for any station along the river it may still be possible to find $\bar{c}(Q)$. In this case, unit hydrograph or some similar theory has to be used, both to obtain the peak discharges of previous floods to correlate with the observed speeds of those floods, and to generate discharge hydrographs at the upstream section as the input for the flood routing method. For the River Wye, data were extracted from the gauging stations at Erwood and Belmont. Both of these stations have rating equations which are reasonably accurate even for high flows, and so the flow data from the reach between the stations are of good quality.

It should be emphasised that $\bar{c}(Q)$ is properly defined as the average speed along a reach of the flood wave with peak discharge Q under the condition that there is no attenuation. This condition is equivalent to the requirement that $\bar{c}(Q)$ is the speed derived from the equations for steady flow with discharge Q. When there is attenuation of the peak discharge the observed speed of the flood peak is a function not only of $\bar{c}$ but also of the shape of the discharge hydrograph. For example, Hayami (1951) has shown from a theoretical treatment of the convection-diffusion equation for the stage, in which ω and μ, the convection and diffusion parameters, are assumed constant, that flood waves of short periods propagate with speeds greater than ω. If the observed speed of the flood wave is L/T_p where T_p is the travel time of the peak along the reach, then a correction to L/T_p can be derived from Hayami's analysis to give

$$\omega = \frac{L}{T_p} - \frac{2\alpha}{L^2} Q^* \tag{2.47}$$

(see the Appendix at the end of this chapter). In addition, because $\bar{c}$ is a function of Q, and because in longer reaches it is not necessarily true that $dF_1/d\tau \simeq 0$ at the peak of the hydrograph for a downstream section, then L/T_p is a function of $d\bar{c}/dQ$ and possibly $d\bar{\alpha}/dQ$.

Experiments (Price, 1973b) with numerical models of flooding in synthetic rivers indicate that there is a strong dependence of $\bar{c}$ on $d(L/T_p)/dQ$ and on the attenuation Q^*. For example, it is found that the curves for L/T_p and $\bar{c}(Q)$ intersect where $d(L/T_p)/dQ \sim 0$. Further, the deviation between the two curves is greatest when the floods are peaky; in other words, when the attenuation is large. Consequently, it is suggested that $\bar{c}$ should be defined by

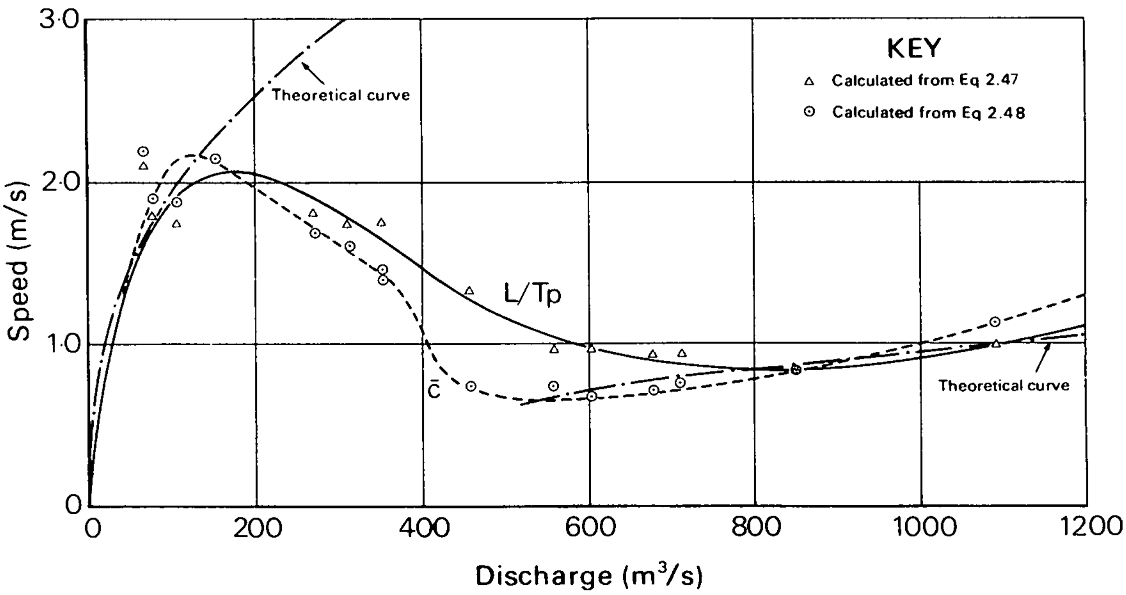
$$\bar{c} = \omega + Q^* \frac{d}{dQ}\left(\frac{L}{T_p}\right) \tag{2.48}$$

where ω is given by Equation 2.47.

Naturally the definition of $\bar{c}$ by Equation 2.48 is not entirely satisfactory. One of the main objections to Equation 2.48 is that $\bar{c}$ is now a function of Q^*, which is certainly not unique for a given $\bar{Q}_p$. However, like the assumptions of unit hydrograph theory, it can be assumed that in the mean Q^* is proportional to Q_p. The success of the flood routing model below in predicting floods in the River Wye appears to establish that Equation 2.48 is a reasonable definition of $\bar{c}$ in Equation 2.44.

Figure 2.5 shows the points for L/T_p and $\bar{c}(\bar{Q}_p)$ for the reach of the River Wye between Erwood and Belmont together with the corresponding estimates of the curves through these points.

Fig 2.5 Speed–discharge curves for the Erwood to Belmont reach of the River Wye.



The shape of, say, the $\bar{c}$ curve in Figure 2.5 is typical of almost any reach of a natural river. There are a number of points of interest. For example, $\bar{c}$ has a maximum value for a discharge which is usually less than the average bankfull discharge along the reach. This shows that small inbank floods will travel considerably faster than a flood which is just bankfull. The main reason for this effect is that the river channel generally has a more irregular surface width as the depth of water increases, and the irregularities increase the effective storage of the channel. This storage is magnified when water begins to pond up on the flood plain. So for some discharge greater than the bankfull discharge $\bar{c}$ will be a minimum. Here the river is most efficient at storing water and attenuating flood peaks. As the discharge increases there is an effective flow of water along the flood plain and the speed also increases. For extreme discharges the whole of the flood plain begins to act like the main channel.

The estimation of L/T_p for floods strongly affected by lateral runoff along the reach, or by a tributary with a significant discharge, can be difficult. If the runoff or the discharge from the tributary is reasonably steady then it is advisable to plot L/T_p against the value of Q_p equal to the estimated average peak discharge with no lateral inflow.

Where local records for the times of flood peaks at the upstream and downstream ends of the reach are not available or only one or two times are known, the curve defining $\bar{c}(Q)$ has to be determined using Equations 2.18 and 2.21. However, it has already been commented that the use of the theoretical Equations 2.18 and 2.21 to define the speed is extremely difficult due to the need to specify the roughness coefficients. Whereas the Manning's n can be estimated with some degree of precision for a natural river channel, the values of n for the flood plain can have large variations depending on the texture of the surface and the presence of obstructions such as trees and hedges. In addition there is some doubt about the value of Manning's n for the boundary of the channel flow when there is overbank flooding. Zheleznyakov (1971) has indicated that the roughness due to the shear between the flow in the channel and over the flood plain can play a significant role in reducing the total discharge in the river. Because of these difficulties it is emphasised that the following method for producing a synthetic speed–discharge curve should be used with caution.

In the case of a reasonably flat flood plain and a wide channel, $\bar{c}$ and Q are approximately given by

$$\bar{c} = \frac{\bar{c}_b}{\bar{\lambda}}[z^{2/3} + \kappa(z-1)^{2/3}] = \frac{\bar{c}_b}{\bar{\lambda}} f_1(z, \kappa) \tag{2.49}$$

and

$$Q = \bar{Q}_b\,[z^{5/3} + \kappa(z-1)^{5/3}] = \bar{Q}_b f_2(z, \kappa) \tag{2.50}$$

where

$$z = \frac{\bar{y}}{\bar{y}_b} \qquad \kappa = \bar{\sigma}^{3/2}\frac{\overline{W}_f \bar{n}_c}{\overline{W}_c \bar{n}_f}$$
$$\bar{c}_b = \frac{5}{3}\frac{\bar{s}^{\frac{1}{2}}\bar{y}_b^{2/3}}{\bar{n}_c} \qquad \bar{Q}_b = \frac{s^{\frac{1}{2}}\,\overline{W}_c \bar{y}_b^{5/3}}{\bar{n}_c}. \tag{2.51}$$

Here, $\bar{c}_b/\bar{\lambda}$ and $\bar{Q}_b$ are the bankfull values for $\bar{c}$ and Q, and $\bar{y}$ and $\bar{y}_b$ are the average depth and bankfull depth along the reach. It is suggested that $\bar{y}_b$ should be defined by $\bar{A}_b/\overline{W}_c$, where $\bar{A}_b$ is an average bankfull area, preferably measured off cross-sectional data, and $\overline{W}_c$ is an estimated width for the channel averaged over depth. This means that $\bar{\lambda}$ is defined by $\overline{W}_b/\overline{W}_c$ when $\bar{y} = \bar{y}_b$ ($z = 1$). Note that $\overline{W}_b/\overline{W}_c$ is only equal to unity when the channel has a perfectly rectangular cross-section. The parameters κ, $\bar{c}_b$ and $\bar{Q}_b$ can readily be calculated from prototype data with estimates for $\bar{n}_c$ and $\bar{n}_f$. If values for $\bar{c}_b$ and $\bar{Q}_b$ are known from the prototype, then $\bar{n}_c$ should be calculated directly from the expressions in Equation 2.51. Similarly, if an isolated record of $\bar{c}$ and Q exists for a large flood, it is preferable to use this record to find κ and hence to calculate $\bar{n}_f$. If such a record is not available, it is suggested that κ be chosen between 0.1 and 0.2, depending on whether the flood plain is regular or irregular along the river and whether the flow is relatively free of obstructions. The curves for an inbank flood with $\bar{\lambda} = 1$ and an overbank flood with $\bar{\lambda}$ given some predetermined value greater than unity, can then be drawn using the curves for f_1 and f_2 in Figure 2.6.

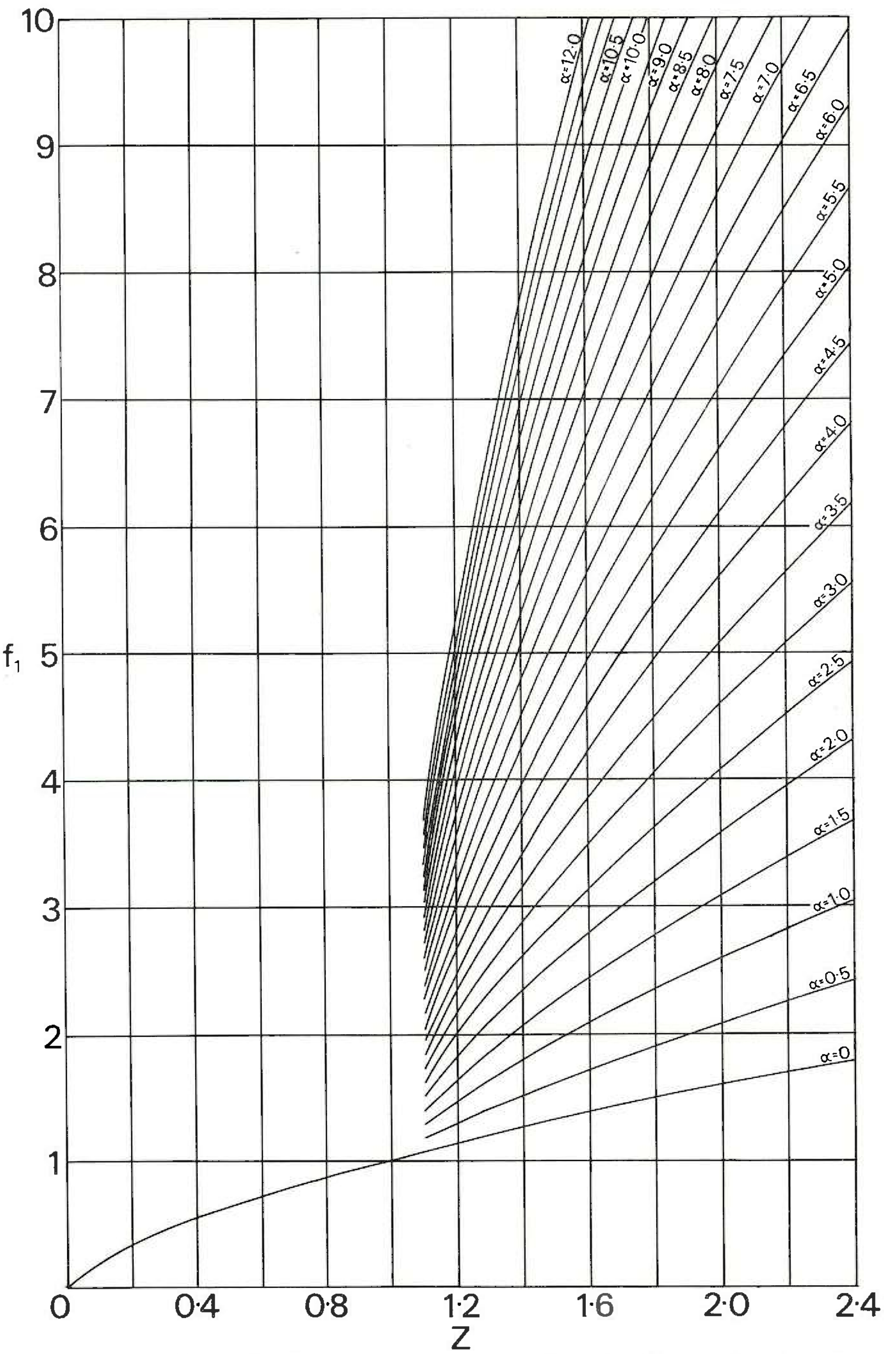


Fig **2.6a** Design curves to calculate theoretical speed–discharge curve.

Figure 2.5 shows the theoretical curves for $\bar{c}$, derived from data for the Erwood to Belmont reach. The following data were used:

Assumed	Derived	
$\overline{W}_c = 50$ m	$\bar{n}_c = 0.0374$	
$\bar{s} = 0.00088$	$\bar{c}_b = 3.33$ m s^{-1}	
$\bar{y}_b = 4.0$ m	$\bar{\lambda} = 8.18$	(2.52)
$Q_b = 400$ m^3 s^{-1}	$\kappa = 1.680$	
$\bar{\sigma} = 1.1$	$\bar{n}_f = 0.184$	
$\overline{W}_f = 359$ m		

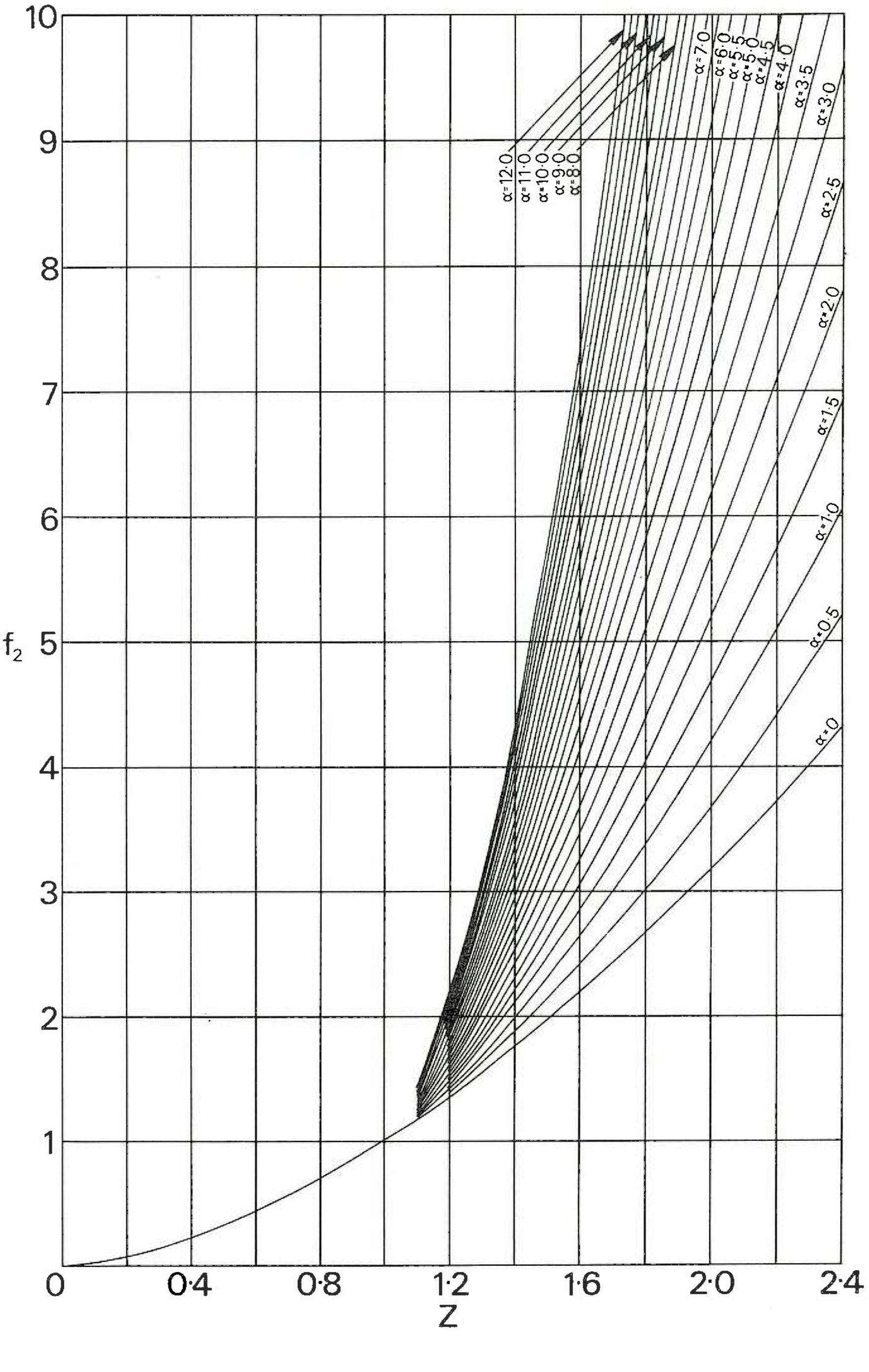


Fig 2.6b Design curves to calculate theoretical speed–discharge curve.

For an overbank flood

$\bar{c} \; = 1.01 \text{ m s}^{-1}$

$Q \; = 1090 \text{ m}^3 \text{ s}^{-1}$

Again there does not appear to be any precise way of finding the shape of the speed–discharge curve for the intermediate floods and this part of the curve has to be estimated.

2.12 Numerical technique

The equation to be solved numerically is

$$\frac{\partial Q}{\partial t}+\bar{c}\frac{\partial Q}{\partial x} = \frac{\alpha}{L}Q\frac{\partial^2 Q}{\partial x^2}+\bar{c}q. \tag{2.53}$$

At the upstream boundary, Q is prescribed as a function of t, and at the downstream boundary a 'free' condition is used. This free boundary condition is based on the characteristic form of Equation 2.53

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\alpha}{L}Q\frac{\partial^2 Q}{\partial x^2}+\bar{c}q \tag{2.54}$$

with the characteristic curve given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \bar{c}. \tag{2.55}$$

A numerical solution of Equation 2.53 for the equivalent river model can be obtained by writing the equation in finite difference form using the implicit Crank–Nicholson scheme (Richtmyer & Morton, 1967):

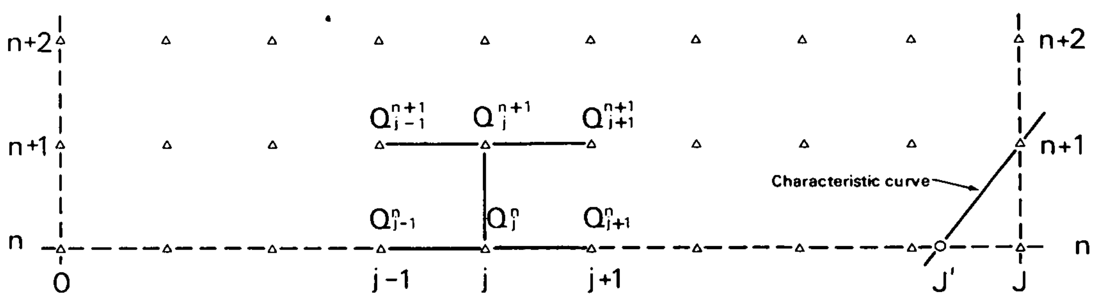
$$\begin{aligned}\Omega_j \equiv{}& Q_j^{n+1}-Q_j^n+\frac{\Delta t}{4\Delta x}\,\bar{c}(Q_a)[Q_{j+1}^{n+1}-Q_{j-1}^{n+1}+Q_{j+1}^n-Q_{j-1}^n]\\ &-\bar{c}(Q_a)q_j^{n+1}-\frac{\Delta t}{2L\Delta x^2}\,\alpha(Q_a)Q_a[Q_{j+1}^{n+1}-2Q_j^{n+1}+Q_{j-1}^{n+1}\\ &+Q_{j+1}^n-2Q_j^n+Q_{j-1}^n] = 0\end{aligned} \tag{2.56}$$

for all $1 \leq j \leq J-1$, where

$$Q_a = \tfrac{1}{2}[Q_j^{n+1}+Q_j^n]. \tag{2.57}$$

Ω_j denotes the finite difference expression in Equation 2.56 and J is the label for the downstream boundary. The subscript j refers to a variable evaluated at the point distance $j\Delta x$ downstream of the upstream boundary, where Δx is the space step. Similarly, the superscript n refers to the variable evaluated at the time $n\Delta t$ after the beginning of the calculations, where Δt is the time step; see Figure 2.7.

Fig 2.7 Finite difference net.



The boundary conditions for Equation 2.53 are
initial condition:

$$Q_j^0 = Q_{\text{init}} = \text{constant for } 0 \leq j \leq J \tag{2.58}$$

upstream condition:

$$Q_0^n = F_1(n\Delta t) \text{ for } 0 \leq n \tag{2.59}$$

downstream condition:

$$Q_J^{n+1} = Q_{J'}^{n} + \left[\bar{c}_{J'}^{n} \, q_{J'}^{n} + \frac{\alpha_{J'}^{n}}{L} \, Q_{J'}^{n} \left(\frac{\partial^2 Q}{\partial x^2} \right)_{J'}^{n} \right] \Delta t \tag{2.60}$$

from Equation 2.54. $F_1(t)$ above is the recorded discharge hydrograph which is used as the input for the model at the upstream boundary. J′ in Equation 2.60 refers to the point at time $n\Delta t$ on the characteristic curve through the point $\{J\Delta x, (n+1)\Delta t\}$. The distance, $\Delta x'$, of the point labelled J′ from the downstream boundary is given approximately by

$$\Delta x' = \bar{c}_{J'}^{n} \, \Delta t \tag{2.61}$$

from Equation 2.55. $Q_{J'}^{n}$ is then calculated from a quadratic spline through Q_J^n, Q_{J-1}^n and Q_{J-2}^n using $\Delta x'$. But as $\Delta x'$ is itself a function of $Q_{J'}^{n}$ it is necessary to iterate to find an accurate value for $Q_{J'}^{n}$. Once $Q_{J'}^{n}$ is known, values for $\bar{c}_{J'}^{n}$, $\alpha_{J'}^{n}$ and $(\partial^2 Q/\partial x^2)_{J'}^{n}$ can be evaluated and substituted in Equation 2.60 to find Q_J^{n+1}.

Given the values for Q_O^{n+1}, Q_J^{n+1} and the $\{Q_j^n\}$ it remains to solve the set of non-linear simultaneous equations in the $\{Q_j^{n+1}\}$. The most convenient way of solving these equations is to use the generalised Newton iteration procedure (Amein and Fang, 1970). This procedure involves the evaluation of Ω_j for estimated values of the $\{Q_j^{n+1}\}$. The $\{Q_j^{n+1}\}$ are then replaced by the set $\{Q_j^{n+1} + \mathrm{d}Q_j^{n+1}\}$, where the $\{\mathrm{d}Q_j^{n+1}\}$ are the solution of the simultaneous linear equations

$$a_{j,j-1} \, \mathrm{d}Q_{j-1}^{n+1} + a_{j,j} \, \mathrm{d}Q_j^{n+1} + a_{j,j+1} \, \mathrm{d}Q_{j+1}^{n+1} = \Omega_j. \tag{2.62}$$

The matrix $\{a_{j,k}\}$ is defined by

$$a_{j,k} = 0 \qquad k = 1, 2, \ldots, j-2$$

$$a_{j,j-1} = \left[\frac{\Delta t}{4\Delta x} \bar{c} + \frac{\Delta t}{2L\Delta x^2} \alpha Q_a \right]$$

$$\begin{aligned} a_{j,j} = {} & 1 + \frac{\Delta t}{8\Delta x} \frac{\mathrm{d}\bar{c}}{\mathrm{d}Q_a} \left[Q_{j+1}^{n+1} - Q_{j-1}^{n+1} + Q_{j+1}^{n} - Q_{j-1}^{n} \right] \\ & - \frac{\Delta t}{2} \frac{\mathrm{d}\bar{c}}{\mathrm{d}Q_a} q_j^{n+1} - \frac{\Delta t}{4L\Delta x^2} \left[Q_a \frac{\mathrm{d}\alpha}{\mathrm{d}Q_a} + \alpha \right] \left[Q_{j+1}^{n+1} - 2Q_j^{n+1} + Q_{j-1}^{n+1} \right. \\ & \left. + Q_{j+1}^{n} - 2Q_j^{n} + Q_{j-1}^{n} \right] + \frac{\Delta t}{L\Delta x^2} \alpha Q_a. \end{aligned} \tag{2.63}$$

$$a_{j,j+1} = \frac{\Delta t}{4\Delta x} \bar{c} - \frac{\Delta t}{2L\Delta x^2} \alpha Q_a$$

$$a_{j,k} = 0 \qquad k = j+2, \ldots, J.$$

Because of the banded nature of the matrix $\{a_{j,k}\}$ it is a simple matter to set up a Gaussian elimination procedure to solve the linear equations. First define

$$a'_{2,2} = a_{2,2} \qquad a'_{2,3} = a_{2,3}$$

$$a'_{j,j-1} = 0$$

$$a'_{j,j} = a_{j,j} - \frac{a_{j,j-1} \, a'_{j-1,j+1}}{a'_{j-1,j}} \tag{2.64}$$

$$a'_{j,j+1} = a_{j,j+1}$$

$$\Omega'_j = \Omega_j - \frac{\Omega'_{j-1} \, a'_{j-1,j+1}}{a'_{j-1,j}}$$

for j = 3, 4, . . . , J − 1. Then

$$dQ^{n+1}_{J-1} = \frac{\Omega'_{J-1}}{a'_{J-1,J-1}} \tag{2.65}$$

and

$$dQ^{n+1}_{j} = (\Omega'_j - a_{j,j+1}\,dQ^{n+1}_{j+1})/a'_{j,j} \tag{2.66}$$

for j = J − 2, J − 3, . . . , 2. So it is a matter of sweeping in each direction along the band of the matrix $\{a_{j,k}\}$. When the values of the $\{dQ^{n+1}_j\}$ have been calculated, the new values for $\{Q^{n+1}_j\}$ are evaluated and another set of values for the $\{\Omega_j\}$ are found. The iteration continues until the maximum value of $|dQ^{n+1}_j|$ for a particular iteration is less than a certain error value. As a rough guide this error value can be taken as 10^{-4} times the peak discharge of the largest floods in the particular river. For example, in the River Wye between Erwood and Belmont, large floods have peak discharges of the order of 10^3 m^3 s^{-1}. This gives an error value for the iteration of 10^{-1} m^3 s^{-1}.

It will be observed that the forms of the derivatives for $\bar{c}$ and α with respect to Q_a in Equation 2.63 have not been specified in detail. One way of deriving expressions for $d\bar{c}/dQ_a$ and $d\alpha/dQ_a$ would be to define quadratic splines through four of the data points adjacent to Q_a for each of $\bar{c}$ and α to differentiate the resulting quadratic equation and so evaluate $d\bar{c}/dQ$ and $d\alpha/dQ$ at $Q = Q_a$. However, small errors in the data for $\bar{c}$ and α can produce incorrect large values for the gradients of $\bar{c}$ and α. These large values inevitably upset the iteration process described above. So to avoid such errors it is preferable to use smoothed quadratic curves. In other words, a quadratic curve is fitted through, say, the four adjacent points to Q_a for both $\bar{c}$ and α by the method of least squares. $\bar{c}$, α, dc/dQ_a and $d\alpha/dQ_a$ are all calculated from the equations for these new quadratic curves. The details of the procedure for calculating the coefficients of the equations can be found in Section 5.3 in the computer program, FLOODS2, under the subroutines DATIN and FIT.

For maximum accuracy of the implicit finite difference scheme above, Δx and Δt should be chosen so that

$$\frac{\Delta x}{\Delta t} \geq \bar{c}_{\text{ave}} \tag{2.67}$$

where $\bar{c}_{\text{ave}}$ is an average value for $\bar{c}$ defined over the anticipated range of values for Q. However, there are two additional constraints on Δt which have to be satisfied. The first constraint is to ensure that enough detail of the hydrograph at the upstream boundary is fed into the model. Because the peak of the hydrograph is usually the most important feature, it is suggested here that Δt is chosen so that

$$\Delta t < 2\left(\frac{1}{Q}\frac{d^2Q}{dt^2}\right)^{-\frac{1}{2}} \tag{2.68}$$

where the right hand side of this inequality is evaluated at the peak of the upstream hydrograph. The second constraint on Δt comes from the downstream boundary condition. Basically, it is necessary that the length of the characteristic used at the downstream boundary between the old and new time levels is sufficiently small so that the characteristic curve is approximately a straight line. Such a condition is given analytically by

$$\Delta t < \frac{\bar{c}_{min}}{\left|\frac{dQ}{dt}\right|_{max} \left|\frac{d\bar{c}}{dQ}\right|_{max}} \tag{2.69}$$

where $|dQ/dt|_{max}$ is the maximum gradient of the upstream hydrograph. $\bar{c}_{min}$ and $|d\bar{c}/dQ|_{max}$ are the minimum and maximum values of $\bar{c}$ and $|d\bar{c}/dQ|$ in the range of discharge anticipated at the downstream boundary. In practice, Δt should be determined first from Equations 2.68 and 2.69, and Δx should then be calculated from Equation 2.67. Finally, Δx is adjusted so that $L/\Delta x$ is an integer.

2.13 Application of the flood routing method

The method described in the previous sections has been applied to floods in a number of British rivers. These cases are discussed in the following chapter. It is sufficient here to draw attention to results for two floods in the Erwood to Belmont reach of the River Wye. Figures 2.8 and 2.9 show the recorded and predicted hydrographs at Belmont for an inbank flood and a large overbank flood respectively. In both cases the agreement between the observed and theoretical hydrographs is good on the rising part of the hydrograph. In addition, the predicted peak discharge and time of arrival of the peak are in good agreement with the records of both floods.

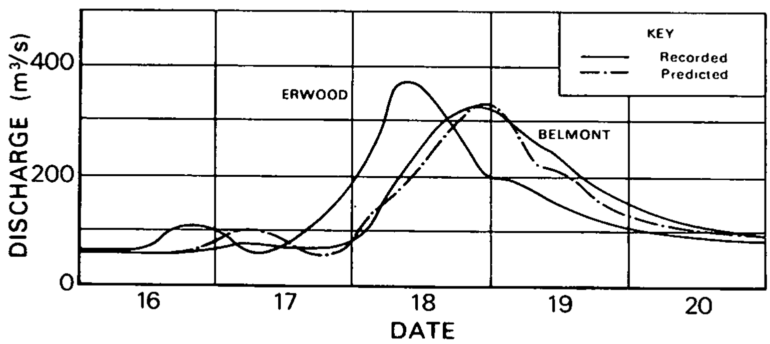


Fig 2.8 Inbank flood for the River Wye, January 1969.

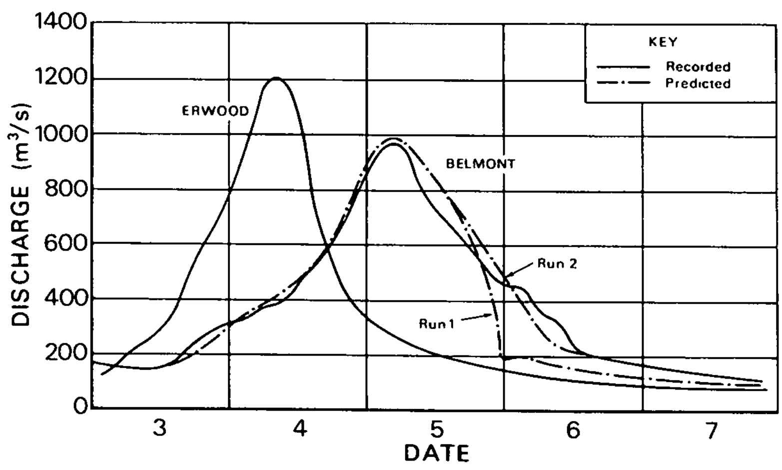


Fig 2.9 Overbank flood for the River Wye, December 1960.

There is, however, some departure in the predicted hydrograph from the recorded hydrograph for the large flood as the discharge is decreasing.

Two main reasons can be given for this disagreement. The first reason is a tendency for the implicit finite difference scheme to become inaccurate. This inaccuracy occurs when $\partial Q/\partial x$ is large. For example, it was noted in the discussion of the kinematic wave as the first order solution of the convection-diffusion equation that, when the water is receding from the flood plain, the change in the speed of the kinematic wave produces a steepening of the curve for the downstream hydrograph. Correspondingly, if the change in the speed with discharge is large and the reach is sufficiently long, then $\partial Q/\partial x$ can be infinite. Analytically, it can be shown that this situation is equivalent to the condition that ϕ, as defined in Equation 2.34, is infinite. Obviously, the implicit finite difference scheme will not be able to deal with this case.

The second reason for the disagreement between the recorded and predicted hydrographs is that the inertia terms in the dynamic equation (Equation 2.2), and the neglected terms in Equation 2.43 may become important when there is extensive inundation of a flood plain. As yet, there is no evidence that the inertia terms are important in this particular case. In addition, a flood routing method based on Equation 2.43 instead of Equation 2.44, gives only a marginal improvement in accuracy (Price, 1973a). It is possible that a third reason should be adduced for the disagreement between the hydrographs, namely that the drainage off the flood plain in nature leads to a violation of the condition, assumed above, that the water surface across the flood plain is uniform and has the same level as the water surface in the channel. The condition, of course, is not unreasonable when the water level is rising in the channel, as the flow of water on to the flood plain is governed primarily by the rate of rise of the water level. However, when the water level is falling, the drainage off the flood plain into the channel is controlled more by the flood plain characteristics, such as the lateral slope and roughness, than by the rate of fall of the water level in the channel. It can therefore be argued that the retention of water on the flood plain when the water level is falling has two effects. The first effect is that the total discharge along the river (when the water level is above bankfull) is less than if the water level was uniform across the river at each section. Similarly, when the water level in the channel is at or below bankfull, the drainage off the flood plain tends to increase the discharge along the river. Consequently, the assumption of a uniform water level across the river is questionable for rivers with large flood plains.

It would appear therefore that a condition limiting the change in the speed of the flood wave with discharge when the flood level is falling may help both to avoid the numerical inaccuracy in the finite difference scheme and to be more realistic physically. Run 2 in Figure 2.9 illustrates the result of fixing $\bar{c}$ when the discharge at any point along the equivalent river model falls below a certain value, Q_x, which in this case is 400 m^3 s^{-1}. The condition is only applied however when the discharge has previously exceeded some value greater than the bankfull discharge in the natural river. It will be seen that there is a significant improvement in the predicted hydrograph for the inbank part of the flood.

It is evident from the results in this section and in the following chapter that the variable parameter diffusion method holds considerable promise for application to floods in rivers with inundation of extensive flood plains.

2.14 Linear diffusion method

The linear diffusion method as used by Hayami (1951), Thomas & Wormleaton (1970, 1971) and a number of others with the stage as the dependent variable, is based on the equation

$$\frac{\partial Q}{\partial t}+\omega\frac{\partial Q}{\partial x} = \mu\frac{\partial^2 Q}{\partial x^2}+\omega q \tag{2.70}$$

where both ω and μ are regarded as constant parameters. In this case μ is defined by

$$\mu = \frac{\alpha(\bar{Q}_p)}{L}\bar{Q}_p. \tag{2.71}$$

ω is again defined by Equation 2.47. Because Equation 2.71 is strictly linear, the corresponding finite difference equation can be solved directly, without using the generalised Newton iteration procedure. Again, a set of equations similar to Equation 2.62 can be derived with the $\{Q_j^{n+1}\}$ in place of the $\{dQ_j^{n+1}\}$. The same Gaussian elimination procedure gives the most efficient way of solving the equations.

2.15 Muskingum–Cunge method

The improvement of the Muskingum method is presented in detail by Cunge (1969). However, a brief description is given here for the sake of completeness.

Suppose that the inflow at the upstream section of a reach is given by Q_j and the outflow by Q_{j+1}. Then from the Muskingum equations (Equations 1.1 and 1.2) it follows that

$$K\frac{d}{dt}\{\varepsilon Q_j+(1-\varepsilon)Q_{j+1}\} = Q_j-Q_{j+1}. \tag{2.72}$$

This equation can be rewritten in the finite difference form:

$$\begin{aligned}&\frac{K}{\Delta t}\{\varepsilon Q_j^{n+1}+(1-\varepsilon)Q_{j+1}^{n+1}-\varepsilon Q_j^n-(1-\varepsilon)Q_{j+1}^n\}\\ &\quad = \tfrac{1}{2}\{Q_j^{n+1}-Q_{j+1}^{n+1}+Q_j^n-Q_{j+1}^n\}.\end{aligned} \tag{2.73}$$

Now if K is defined by

$$K = \frac{\Delta x}{\omega} \tag{2.74}$$

where ω is the average speed of the flood peak and Δx is the length of the reach, then it can be seen that Equation 2.73 is a finite difference representation of the kinematic wave equation

$$\frac{\partial Q}{\partial t}+\omega\frac{\partial Q}{\partial x} = 0. \tag{2.75}$$

So with K defined as in Equation 2.74 it remains to calculate ε. At first sight there is no obvious form for ε, but Cunge observed that by expressing the $\{Q_j^n\}$ in terms of their Taylor expansions Equation 2.73 is also a finite difference representation of the equation

$$\frac{\partial Q}{\partial t}+\omega\frac{\partial Q}{\partial x} = \mu\frac{\partial^2 Q}{\partial x^2} \tag{2.76}$$

when

$$\mu = (\tfrac{1}{2}-\varepsilon)\omega\Delta x. \tag{2.77}$$

As $\mu = \alpha\bar{Q}_p/L$ from Equation 2.71, it follows that

$$\varepsilon = \tfrac{1}{2}-\frac{\alpha\bar{Q}_p}{L\omega\Delta x} \tag{2.78}$$

where L is now the length of the whole reach which is divided into a number of sub-reaches, each of length Δx. Again α is the value of the attenuation parameter corresponding to the discharge $\bar{Q}_p$, and ω is the speed as defined by Equation 2.47. Cunge originally derived Equation 2.78 in terms of the average slope and width of the channel. α/L in Equation 2.78 replaces Cunge's factor $(2s\bar{W})^{-1}$.

Once K and ε have been determined, the discharge hydrograph at the downstream end of the reach is determined from the recurrence relationship

$$Q_{j+1}^{n+1} = C_1\, Q_j^n + C_2\, Q_j^{n+1} + C_3\, Q_{j+1}^n + C_4 \tag{2.79}$$

where

$$C_1 = \frac{K\varepsilon+\frac{1}{2}\Delta t}{K(1-\varepsilon)+\frac{1}{2}\Delta t} \qquad C_2 = \frac{\frac{1}{2}\Delta t-K\varepsilon}{K(1-\varepsilon)+\frac{1}{2}\Delta t}$$

$$C_3 = \frac{K(1-\varepsilon)-\frac{1}{2}\Delta t}{K(1-\varepsilon)+\frac{1}{2}\Delta t} \qquad C_4 = \frac{q\Delta t\Delta x}{K(1-\varepsilon)+\frac{1}{2}\Delta t}. \tag{2.80}$$

The accuracy of the finite difference scheme in Equation 2.79 depends largely on the magnitude of ε. For floods in British rivers Δx can be set equal to $L/10$ and ε is then calculated from Equation 2.78. Then the relevant value of $\Delta x/(\omega\Delta t)$ is read off the curve in Figure 2.10 for the particular ε, and the corresponding value for Δt is determined. In practice, of course, the reach length L will be divided into a number of sub-reaches, each of length Δx, and Δt can be chosen as an integral number of hours so that $\Delta x/(\omega\Delta t)$ lies below the curve in Figure 2.10. This is sufficient to ensure accuracy of the method.

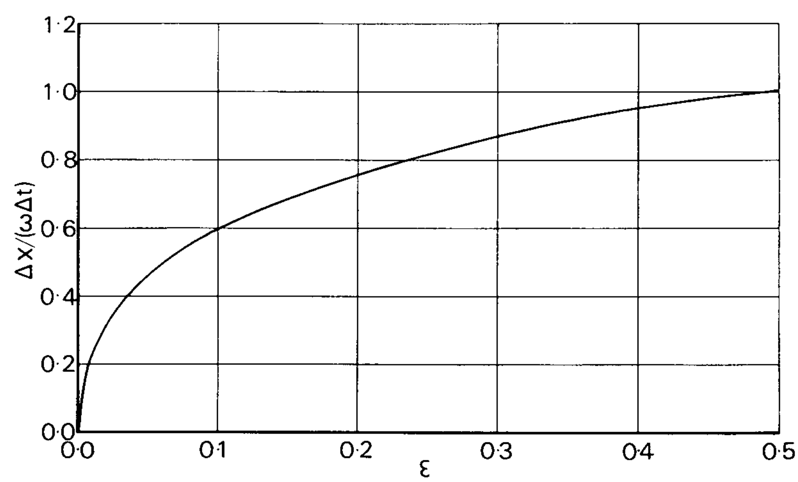


Fig 2.10 Curve for $\Delta x/(\omega\Delta t)$ *versus* ϵ (taken from Cunge (1969)).

2.16 Summary of methods

It is valuable to summarise the relationships between all the flood routing methods derived and referred to in this chapter (Figure 2.11).

Firstly it is anticipated that a method for solving the full Saint–Venant equations is likely to give the most accurate results for a flood routing study if features of the flow, such as flow over the flood plain, are correctly modelled, and there are sufficient data to determine the roughness coefficients with precision. The next class of flood routing method is obtained by ignoring the inertia terms in the dynamic equation. Without making any further approximation Equation 2.10 can be solved using, say, a

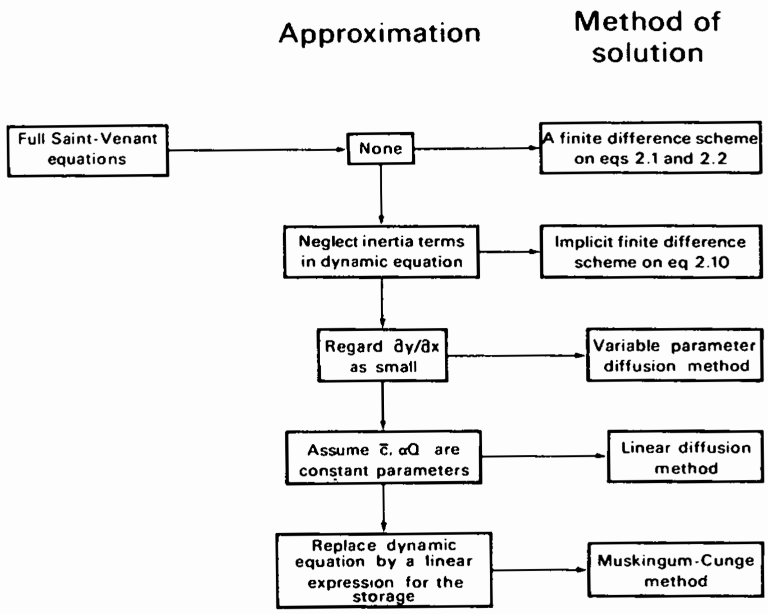


Fig 2.11 Summary of flood routing methods.

Crank–Nicholson type of implicit finite difference scheme. Such a method has not been considered here because of the need to know the roughness coefficient. The diffusion and kinematic wave methods essentially regard $\partial y/\partial x$ as small compared with the bottom slope, and $\bar{c}$ and α, or αQ, as either functions of Q or constant parameters for a given flood. The final class of methods assumes that the dynamic equation can be replaced by an algebraic relationship between the storage and the inflow and outflow to the reach. This is the basis of the Muskingum and similar storage routing methods.

2.17 Appendix: derivation of formula for the convection speed, ω (linear theory)

Hayami (1951) based his diffusion method on a linear convection-diffusion equation for the stage. The corresponding equation for the discharge is

$$\frac{\partial Q}{\partial t}+\omega\frac{\partial Q}{\partial x}=\mu\frac{\partial^2 Q}{\partial x^2} \tag{2.81}$$

where ω and μ are constant parameters. Consider the elementary flood wave solution

$$Q = Q_r \exp\left\{\left(\frac{\omega}{2\mu} - p_r\right)x\right\}\sin(\gamma_r t - q_r x) \tag{2.82}$$

where Q_r is the peak discharge at $x = 0$, $\gamma_r^2 Q_r$ is the curvature at the peak of the hydrograph at $x = 0$, and

$$\begin{pmatrix} p_r \\ q_r \end{pmatrix} = (2\mu)^{-\frac{1}{2}}\left\{\left[\left(\frac{\omega^2}{4\mu}\right)^2 + \gamma_r^2\right]^{\frac{1}{2}} \pm \frac{\omega^2}{4\mu}\right\}^{\frac{1}{2}}. \tag{2.83}$$

The observed speed of the flood peak is $\gamma_r/q_r = L/T_p$. In particular, when

$$\omega^2/4\mu \gg \gamma_r \tag{2.84}$$

as is the case for floods in British rivers, then

$$\frac{\gamma_r}{q_r} = \frac{L}{T_p} \simeq \omega + \frac{2\mu^2\gamma_r^2}{\omega^3}. \tag{2.85}$$

Similarly, with the condition in Equation 2.8,

$$p_r \simeq \frac{\omega}{2\mu} + \frac{\mu\gamma_r^2}{\omega^3}. \tag{2.86}$$

If μ is now defined by

$$\mu = \frac{\alpha Q_r}{L} \tag{2.87}$$

then the attenuation along the reach is given by

$$Q^* = Q_r\left[1 - \exp\left\{\left(\frac{\omega}{2\mu} - p_r\right)L\right\}\right] \simeq Q_r \frac{\mu\gamma_r^2 L}{\omega^3} = \frac{\alpha Q_r}{\omega^3} \cdot \gamma_r^2 Q_r. \tag{2.88}$$

So from Equations 2.85 and 2.88

$$\frac{L}{T_p} \simeq \omega + \frac{2\alpha Q^*}{L^2} \tag{2.89}$$

or

$$\omega \simeq \frac{L}{T_p} - \frac{2\alpha Q^*}{L^2}. \tag{2.90}$$

3 Comparison of flood routing methods

3.1 Methods considered

This chapter describes a comparison of several flood routing methods appropriate for use in British rivers. As explained in Chapter 1, attention is confined in this study to the simpler types of flood routing methods: those methods which are based on an accurate solution of the full Saint–Venant equations are ignored as being too complex for general use and so requiring the assistance of a research organisation. Three of the simpler types of flood routing methods are considered:

a Muskingum method, devised by McCarthy (1938) and improved by Cunge (1969).

b Linear diffusion method, formulated by Hayami (1951) and developed as a numerical method by Thomas & Wormleaton (1970, 1971).

c Variable parameter diffusion method, described in the previous chapter.

The concepts used in the derivation of the Muskingum method are basically hydrological, so the inclusion of the method may be disturbing to those who anticipate that attention should be confined to hydraulic flood routing methods. It happens, however, that the Muskingum method and similar storage routing methods have been used for a large number of years to route floods in rivers, and the method has been successfully applied in a number of cases; for example see McCarthy (1938) and Pitman & Midgley (1966). In addition, the improvement of the basic Muskingum method by Cunge, referred to in Chapter 1 and described in Section 2.15, essentially converts the Muskingum method from being hydrological in theory into a method based on hydraulic principles. This conversion arises in the numerical application of the original Muskingum equations. When these equations are written in finite difference form, a numerical error is introduced which acts as a diffusion on the basic solution. Cunge identified the magnitude of this error term with that of the diffusion term in the convection-diffusion equation as used by Hayami in the diffusion method. So, by a proper choice of the parameters in the Muskingum method, a good approximation can be obtained for the solution of the convection-diffusion equation.

There are two well documented methods which have not been included in the list above. These methods are the kinematic wave method suggested by Lighthill & Whitham (1955), and the graphical method of characteristics, described in detail by Chow (1959). The reason for not considering the kinematic wave method below is that this method can be regarded as a version of the linear diffusion method with the basic equation written in its characteristic form. Consequently the value of the kinematic wave method resides primarily in its use as a graphical method. However, the complexity of the method in comparison with, say, the graphical form of the Muskingum method, and the increasing availability of digital, or even analogue computers, makes it preferable to use the linear diffusion method. The method of characteristics is avoided, again because it is complicated to use, and because it is devised in terms of friction coefficients rather than the speeds of flood peaks. There is a marginal advantage in using the method of characteristics because the method includes the inertia terms in the dynamic equation. But more importantly, the method has the disadvantage that it is then tied to the dynamic wave speeds rather than the kinematic wave speed; see Lighthill & Whitham (1955, p. 291).

In view of the success of the variable parameter diffusion method described in the previous chapter, there exists the possibility that a version of the Muskingum method using similar variable parameters might also be a viable flood routing method, and an improvement on the original Muskingum method (Linsley, Kohler & Paulhus, 1958). However, a preliminary examination of a variable parameter Muskingum method has shown that the results tend to be somewhat inaccurate. It remains for future work to clarify the usefulness or otherwise of such a method.

To ensure consistency in the comparison of the three methods listed above, the same values for the speed and attenuation parameters are used, where relevant, in the tests below. Because these parameters are derived so that the methods predict accurately the value and time-of-arrival of the peak discharge for a flood at the downstream section, the comparison of the methods is based on the standard deviation of the predicted hydrograph from the recorded hydrograph. It should be observed that it is often the reverse procedure which is employed: namely that the standard deviation is first minimised and then the accuracy of the predicted peak discharge is noted. However, because most flood routing studies are aimed at an accurate prediction of peak values rather than the shape of a hydrograph, the former procedure is adopted in this study.

3.2 Comparison tests

The primary objective of the comparison tests described below is to determine the accuracy of each flood routing method under prescribed conditions, typical of those found in British rivers. As mentioned in Section 1.3, there are two factors which affect the accuracy for each method. The first factor is the neglect and simplification of terms in the full Saint–Venant equations by a particular method, and secondly, the assumptions inherent in the treatment of the storage and flow over the flood plain. In the first instance, the simulation of floods in uniform channels will isolate the significance of the neglected terms in the full equations. The magnitude of the error involved can be found by comparing results from the flood routing method with an accurate numerical solution of the full Saint–Venant equations. However, it is also important to determine how accurately the methods simulate floods in natural rivers. Such a test should also clarify the importance of the assumptions made about the interaction of the flow in the channel and over an irregular flood plain. So, the methods are applied to floods in the Rivers Wye, Nene and Eden. The Wye and the Nene have important flood plains. In particular a flood plain on the Wye some 20–30 km above Hereford contributes to an attenuation of a large flood at Belmont (Hereford) of up to 45% of the peak discharge for a large flood, measured 70 km upstream at Erwood. The Nene was chosen primarily because it has a large number of control structures such as weirs and locks. These structures play a significant part in controlling the smaller floods in the river. Unfortunately the only large flood in the Nene with reliable flow data is the snowmelt flood of 1947. Despite the fact that very little is known about the runoff from the catchment along the river, the flood routing methods are reasonably successful in simulating this flood. Finally, the Eden is an example of a shorter and steeper British river.

Throughout the following work four error parameters are used to compare the predicted with the recorded or 'exact' discharge hydrographs. These parameters are defined as follows:

Percentage error in the attenuation =

$$\left(\frac{\text{recorded peak discharge} - \text{predicted peak discharge}}{\text{recorded peak discharge}}\right) \times 100 \tag{3.1}$$

Percentage error in the speed of the flood peak =

$$\left(\frac{\text{recorded speed} - \text{predicted speed}}{\text{recorded speed}}\right) \times 100 \tag{3.2}$$

Percentage standard deviation =

$$\left[\frac{1}{2\mathrm{N}+1}\sum_{n=1}^{2\mathrm{N}+1}(\text{recorded discharge} - \text{predicted discharge})^2\right]^{\frac{1}{2}} \times \frac{100}{\text{average recorded discharge}} \tag{3.3}$$

Percentage mean deviation =

$$\left(\frac{\text{average recorded discharge} - \text{average predicted discharge}}{\text{average recorded discharge}}\right) \times 100 \tag{3.4}$$

where the average discharge is defined by

$$\text{average discharge} = \frac{1}{6\mathrm{N}}\sum_{n=1}^{\mathrm{N}}(Q_{\mathrm{J}}^{2n-1} + 4Q_{\mathrm{J}}^{2n} + Q_{\mathrm{J}}^{2n+1}). \tag{3.5}$$

Here 2N + 1 is the number of points for the downstream hydrograph produced by 2N time steps in each method, and J is the space label for the downstream boundary.

3.3 Flood routing in regular channels

The objective of this test is to isolate for each method the magnitude of the error due to the neglect or approximation of terms in the Saint–Venant equations. This is achieved by routing a synthetic flood in a uniform rectangular channel, 100 km long and 50 m wide, with a Manning's roughness coefficient of 0.035. Four channels with bottom slopes 2×10^{-3}, 10^{-3}, 0.5×10^{-3} and 0.25×10^{-3} are used in the test. The synthetic flood hydrograph at the upstream section of the reach is defined by

$$Q(t) = Q_{\text{base}} + Q_{\text{amp}}\left[\frac{t}{t_{\text{p}}}\exp\left(1 - \frac{t}{t_{\text{p}}}\right)\right]^{\beta} \tag{3.6}$$

where β is a parameter, t_{p} is the time to peak, Q_{base} is the base flow, and $Q_{\text{base}} + Q_{\text{amp}}$ is the peak discharge for the flood. The curvature at the peak of this hydrograph is $\beta Q_{\text{amp}}/T^2$, so β is directly proportional to the curvature at the peak of the upstream hydrograph. A typical extreme flood in the channel with bottom slope of 10^{-3} can be estimated to have a peak discharge of 900 $\text{m}^3\ \text{s}^{-1}$ with a base flow of 100 $\text{m}^3\ \text{s}^{-1}$. For the sake of consistency this flood is also used in the channels with the other bottom slopes. The time-to-peak of the flood is taken as 24 hours, and β is given the value 16. This gives a curvature at the peak of the hydrograph of $1.58 \times 10^{-6}\ \text{m}^3\ \text{s}^{-3}$. Using Equations 2.18 and 2.21, the speed–discharge curve can be derived from

$$Q = WyR^{2/3}s^{1/2}n^{-1} \tag{3.7}$$

and

$$\bar{c} = \frac{Q}{Wy}\left\{1+\frac{2}{3}\frac{y}{R}\frac{dR}{dy}\right\} \tag{3.8}$$

where

$$R = y(1+2y/W)^{-1}. \tag{3.9}$$

Figure 3.1 shows the speed–discharge curves calculated from the above equations for all four channels.

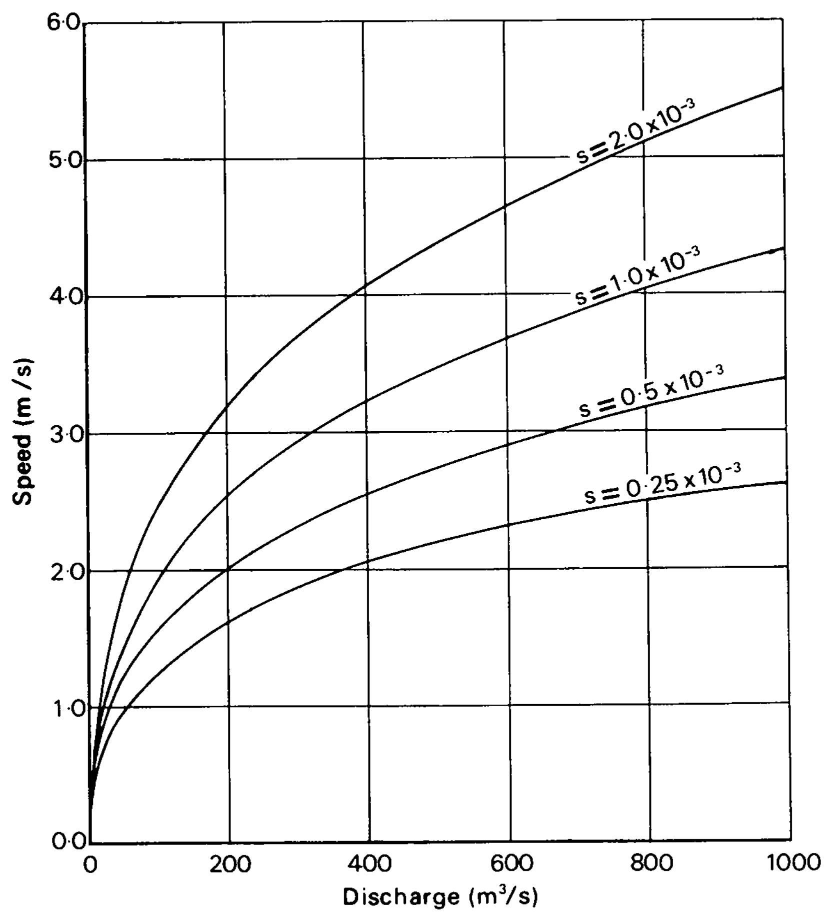


Fig 3.1 Speed–discharge curves for uniform channels.

In a detailed study of the various numerical techniques (including explicit, implicit and characteristic finite difference schemes) for solving the full Saint–Venant equations (Price, 1974), it was found that the explicit leapfrog method is the most accurate method of second order accuracy if a 'free' boundary condition is required at the downstream boundary and if the method is only applied to regular floods in uniform channels. For irregular channels the leapfrog method tends to become unstable and a characteristic method is more suitable. If a rating equation is available at the downstream boundary then the more versatile implicit method of Amein & Fang (1970) is to be preferred. Other results and conclusions from this secondary study also have an important bearing on the application of numerical methods to the solution of the full Saint–Venant equations for flood routing.

The results from the three simplified methods for routing the discharge hydrograph given by Equation 3.6 along the channel with bottom slope

10^{-3} were first used to determine which values of Δx and Δt should be adopted by each method to obtain the optimum accuracy for the hydrograph at the downstream section. In each case the predicted hydrographs were compared with the hydrograph produced by the explicit leapfrog method using $\Delta x = 5$ km and $\Delta t = 360$ s. Table 3.1 gives the values for

Table 3.1 Errors in predicted hydrographs for channel with slope 1.0×10^{-3}.

Recorded peak discharge = 878.7 m^3 s^{-1}, $\bar{Q}_p$ = 894 m^3 s^{-1}; recorded speed of flood peak = 4.07 m s^{-1}, ω = 4.06 m s^{-1}; average recorded discharge = 261 m^3 s^{-1}, $\alpha = 1.00 \times 10^6$.

Method	Δx (km)	Δt (s)	Error in peak discharge (%)	Error in speed of flood peak (%)	Standard deviation (%)	Mean deviation (%)
Muskingum–Cunge	10	720	−0.11	−0.80	27.1	−0.08
	10	1500	−0.08	−0.96	27.1	−0.08
	10	3600	−0.03	−1.57	27.0	−0.13
	10	18000	−2.87	−19.50	24.2	1.08
	100	720	−1.36	19.44	35.9	−0.08
	100	1800	−1.36	19.35	36.3	−0.08
	100	3600	−1.37	18.77	36.2	−0.13
	100	18000	0.27	4.22	29.4	1.24
Linear diffusion	5	720	−0.08	−1.07	27.0	−0.08
	5	1800	−0.06	−1.17	27.0	−0.08
	5	3600		Unstable		
	10	720	−0.06	−1.49	26.9	−0.08
	10	1800	−0.19	4.23	26.9	−0.08
	10	3600			26.9	−0.13
Variable parameter diffusion	5	720	−0.02	0.02	3.1	0.80
	5	1800	−0.02	−0.68	3.2	0.80
	5	3600		Unstable		
	10	720	0.02	−0.83	4.5	0.80
	10	1800	0.02	−0.92	4.7	0.78
	10	3600	−0.08	−1.03	5.0	0.71

the accuracy parameters together with the values of Δx and Δt used in each run for the methods. These parameters were calculated for the duration of the computer runs. The following conclusions can be made from Table 3.1.

Muskingum–Cunge method

The mean deviation is small, showing that the average predicted discharge compares well with the average recorded discharge for all values of Δx and Δt. Alternatively, the total volume is accurately predicted in each run. A close examination of the error in the speed of the flood peak shows that the error is zero when $\Delta x/\Delta t$ is approximately equal to the speed of the peak. This conclusion was also deduced theoretically by Cunge (1969). For a more precise definition of $\Delta x/\Delta t$ for optimum accuracy see Section 2.15. In addition, Table 3.1 shows that there is a greater latitude in the choice of Δt to preserve the accuracy of the peak discharge when Δx is smaller. The standard deviation is large for all the values of Δx and Δt tested, though a reduction in Δx generally leads to a reduction in the standard deviation.

Linear diffusion method

Again the mean deviation is small and about the same magnitude as for the Muskingum–Cunge method. Variations in Δx and Δt do not significantly affect the magnitude of the standard deviation, though the finite difference scheme is unstable if Δt is too large. It is preferable to keep $\Delta x/\Delta t$ greater than the wave speed (Equation 2.67). The predicted speed and peak discharge are reasonably accurate in each case.

Variable parameter diffusion method

Here the errors in the predicted speed, peak discharge and standard deviation are all significantly smaller than the other methods, though the error in the mean discharge is larger. Again the method is unstable if Δt is too large. A reduction in both Δx and Δt generally leads to an improvement in the accuracy of the method.

So, within the range of values tested the Muskingum–Cunge method is more accurate when $\Delta x/\Delta t$ is approximately equal to the wave speed. The linear diffusion method shows little variation in accuracy for different values of Δx and Δt, and the variable parameter diffusion method is more accurate when both Δx and Δt are small. In the remaining runs described in this section Δx = 100 km and Δt = 3600 s for the Muskingum–Cunge method, and Δx = 5 km and Δt = 720 s for the linear diffusion and variable parameter diffusion methods.

A comparison of the results from the three methods for the same flood in channels with bottom slopes 2×10^{-3}, 0.5×10^{-3} and 0.25×10^{-3} are given in Table 3.2. In addition, Figure 3.2 shows the recorded and the predicted hydrographs for each method in the channel with bottom slope 10^{-3}.

Method	Δx	Δt	Error in peak discharge (%)	Error in speed of flood peak (%)	Standard deviation (%)	Mean deviation (%)
			$s = 2\times10^{-3}$			
	Recorded peak discharge		= 892.2 m^3 s^{-1}		$\bar{Q}_p$ = 896 m^3 s^{-1}	
	Recorded speed of flood peak		= 5.25 m s^{-1}		ω = 5.25 m s^{-1}	
	Average recorded discharge		= 272 m^3 s^{-1}		α = 0.50×10^6	
MC	10	3600	0.79	−0.97	23.5	−0.25
LD	5	720	0.30	−0.47	23.6	−0.23
VPD	5	720	0.32	−0.30	1.1	−0.50
			$s = 1.0\times10^{-3}$			
	Recorded peak discharge		= 878.7 m^3 s^{-1}		$\bar{Q}_p$ = 894 m^3 s^{-1}	
	Recorded speed of flood peak		= 4.07 m s^{-1}		ω = 4.06 m s^{-1}	
	Average recorded discharge		= 261 m^3 s^{-1}		α = 1.00×10^6	
MC	10	3600	−0.03	−1.57	27.0	−0.13
LD	5	720	−0.08	−1.07	27.0	−0.08
VPD	5	720	−0.02	0.02	3.1	0.80
			$s = 0.5\times10^{-3}$			
	Recorded peak discharge		= 822.2 m^3 s^{-1}		$\bar{Q}_p$ = 861 m^3 s^{-1}	
	Recorded speed of flood peak		= 3.11 m s^{-1}		ω = 3.09 m s^{-1}	
	Average recorded discharge		= 251 m^3 s^{-1}		α = 2.00×10^6	
MC	10	3600	−0.92	−2.41	27.5	−0.13
LD	5	720	−0.73	−0.90	27.8	−0.00
VPD	5	720	−1.63	−5.70	1.4	1.61
			$s = 0.25\times10^{-3}$			
	Recorded peak discharge		= 700.0 m^3 s^{-1}		$\bar{Q}_p$ = 800 m^3 s^{-1}	
	Recorded speed of flood peak		= 2.50 m s^{-1}		ω = 2.48 m s^{-1}	
	Average recorded discharge		= 251 m^3 s^{-1}		α = 4.00×10^6	
MC	10	3600	−2.06	1.85	30.4	−0.01
LD	5	720	−0.91	7.45	31.8	−0.01
VPD	5	720	−9.17	−11.09	3.7	−0.06

MC Muskingum–Cunge,
LD linear diffusion,
VPD variable parameter diffusion.

Table 3.2 Errors in predicted hydrographs for regular channels.

Table 3.3 Predicted attenuation in regular channels.

α ($\times 10^6$)	$\bar{Q}$ ($m^3 s^{-1}$)	ω ($m s^{-1}$)	Curvature at peak ($m^3 s^{-1}$)	Predicted attenuation Equation 2.41 ($m^3 s^{-1}$)	Predicted attenuation Equation 2.42 ($m^3 s^{-1}$)	Recorded attenuation ($m^3 s^{-1}$)
0.5	900	5.25	1.58×10^{-6}	4.90	4.89	8
1.0	900	4.07	1.58×10^{-6}	21.09	20.85	11
2.0	900	3.09	1.58×10^{-6}	96.39	91.41	68
4.0	900	2.48	1.58×10^{-6}	372.91	305.30	200

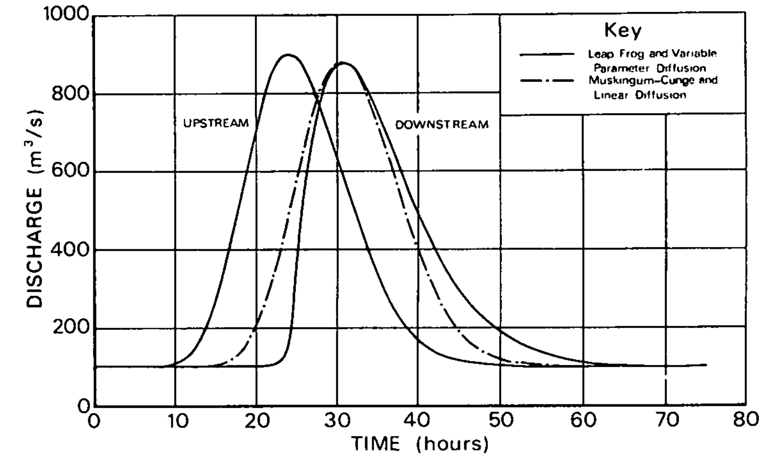


Fig 3.2 Recorded and predicted hydrographs for regular channel slope 10^{-3}.

It can therefore be concluded that the Muskingum–Cunge and linear diffusion methods are of similar accuracy, and that the variable parameter diffusion method generally predicts the shape of the hydrograph much more accurately than the other methods. From Figure 3.2 it is evident that the error in the shape of the predicted hydrograph for the first two methods is greatest near the foot of the wave in each case. So it can be anticipated that a benefit in using the variable parameter diffusion method instead of the Muskingum–Cunge and linear diffusion methods arises from a more accurate prediction of the shape of the hydrograph for the low flows occurring during a large flood. Notice that the prediction of the peak discharge by the variable parameter diffusion method is worse for the smaller slopes. This is possibly due to the neglect of terms in Equation 2.43.

The use of the formulae in Equations 2.41 and 2.42 to predict the attenuation of the peak discharge along the regular channels above produces the results shown in Table 3.3. Notice that the improved formula of Equation 2.42 is more accurate than the linearised formula of Equation 2.41 though both formulae are markedly inaccurate as the slope of the channel decreases. This highlights the need to route hydrographs using a numerical method when the attenuation is large. It is suggested that a numerical flood routing method should be used when Q^*/Q_p is greater than 0.1. For large slopes of the order of 2×10^{-3} it can be observed that the attenuation formula predicts an attenuation which is too large. This is because the convection terms, ignored in the derivation of the convection-diffusion equation, become important in this case; see Henderson (1963).

3.4 Flooding in the River Wye: Erwood to Belmont

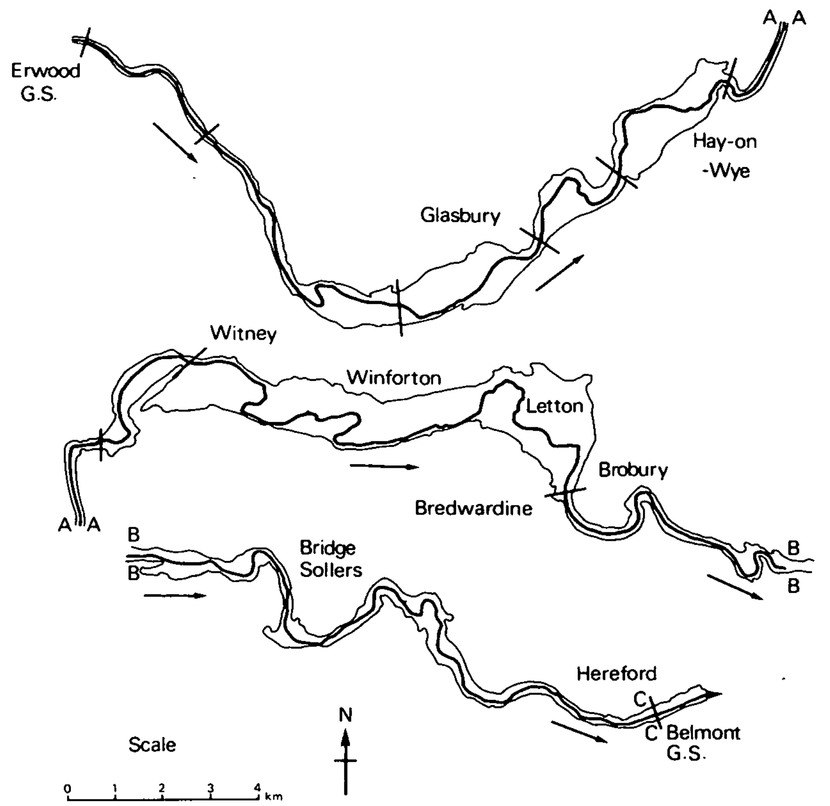


Fig 3.3 River Wye: Erwood to Belmont.

This reach of the River Wye is perhaps the most suitable of all reaches of British rivers in which to test the flood routing methods. As explained in Section 2.11 the recording stations at Erwood and Belmont have good quality rating curves, even for high flows, and there are more than 30 years of records at both stations. In addition, the reach, which is 69.75 km long, has no important tributary and the mean annual lateral inflow along the reach is about 14 $m^3 s^{-1}$. This value for the lateral inflow is small compared with the mean annual flood discharge at Belmont of 560 $m^3 s^{-1}$. Another important feature of the reach is the large flood plain between Bredwardine and Witney, some 20–30 km above Belmont; see Figure 3.3. This flood plain plays a crucial part in reducing peak discharges at Erwood by up to 45% at Belmont, and so gives important protection to the city of Hereford. The largest recorded flood in recent years occurred in December 1960 with a peak discharge of about 1200 $m^3 s^{-1}$ at Erwood. This value was reduced to 980 $m^3 s^{-1}$ at Belmont. A later flood, in December 1965, had a peak discharge at Erwood of 1080 $m^3 s^{-1}$. The corresponding value at Belmont was 620 $m^3 s^{-1}$.

Because of the quality and length of data at both gauging stations, it is an easy matter to extract the curve for L/T_p and $\bar{c}$. These curves, which are also presented in the previous chapter, are shown in Figure 3.4. The times of travel of the peak levels were extracted from stage data at Erwood and Belmont, and the curves for L/T_p and $\bar{c}$ were drawn using information about the peak discharges at the two stations. The attenuation parameter for the largest recorded flood (December, 1960) was calculated from Equation 2.45 using the area inundated by the flood along the reach. The data for this calculation were supplied by the Wye River Authority. See Table 3.4 for the numerical values for the parameters used in the

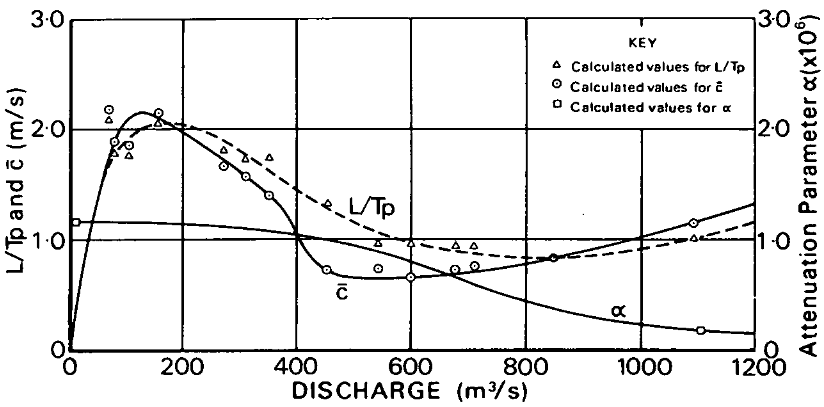


Fig 3.4 River Wye: speed and attenuation parameter for Erwood to Belmont.

calculation, and Figure 3.3 for the corresponding subdivisions of the reach. Similarly, Equation 2.46 was used to find the attenuation parameter for an inbank flood. The two values of α are 0.20×10^6 for the large overbank flood and 1.10×10^6 for the inbank flood. Observations of the river channel, particularly along much of the upper parts of the reach, show that

Table 3.4 Data used to calculate α for Erwood to Belmont, River Wye.

Length of subreach (km)	Flood plain area (km^2)	Bottom slope
4.5	1.23	2.0×10^{-3}
8.3	2.92	2.0×10^{-3}
3.0	1.74	0.8×10^{-3}
2.9	1.38	0.8×10^{-3}
4.5	2.52	0.8×10^{-3}
4.6	0.55	0.8×10^{-3}
3.5	1.28	0.8×10^{-3}
13.6	11.83	0.5×10^{-3}
24.9	5.12	0.6×10^{-3}
Total 69.8	Total 28.57	Average 0.88×10^{-3}

there is a considerable increase in the effective storage in the channel as the depth increases. Even though the average width of the channel appears to increase as the discharge increases, it was anticipated that the increase in the storage due to the irregularities in the channel width would probably lead to α being approximately constant for values of discharge less than the average bankfull discharge along the reach. A close examination of the discharge hydrographs at Belmont indicates that this discharge is about 400 $m^3\ s^{-1}$; see, in particular, Figure 3.6 below. This discharge can be compared with the bankfull discharge of 590 $m^3\ s^{-1}$ at Belmont. The curve for α was therefore drawn from $\alpha = 1.10 \times 10^6$ at $Q = 0$ to a value of 1.0×10^6 at $Q = 400\ m^3\ s^{-1}$ (Figure 3.4). Because α tends to zero, or a small finite value, as the discharge becomes very large, the curve for α was drawn from $Q = 400\ m^3\ s^{-1}$ through the point for $\alpha = 0.2 \times 10^6$ at $Q = 1080\ m^3\ s^{-1}$, so that this limiting condition on α was satisfied. In addition, because the flood plain is so irregular, the curve for α greater than 400 $m^3\ s^{-1}$ was drawn so that it varies slowly with discharge. If the flood plain had been relatively flat and not so irregular, the curve would have to be drawn so that α decreases more rapidly as the discharge increases.

The formulae for the attenuation of the peak discharge (Equations 2.41 and 2.42) were applied to a number of floods between Erwood and Belmont. Table 3.5 gives the corresponding predicted and recorded attenu-

Flood date	α ($\times 10^6$)	$\bar{Q}_p$ ($m^3 s^{-1}$)	ω ($m s^{-1}$)	Curvature at peak ($m^3 s^{-1}$)	Predicted attenuation Equation 2.41 ($m^3 s^{-1}$)	Predicted attenuation Equation 2.42 ($m^3 s^{-1}$)	Recorded attenuation ($m^3 s^{-1}$)
Jan. 1969	1.00	378	1.74	-0.747×10^{-6}	54	50	50
Dec. 1965	0.38	1077	0.80	-2.18×10^{-6}	1743	864	460
Dec. 1960	0.20	1210	0.98	-1.48×10^{-6}	381	327	235
Feb. 1950	0.93	626	0.95	-1.03×10^{-6}	699	421	194
Jan. 1948	0.33	815	0.93	-1.55×10^{-6}	518	383	260
Nov. 1939	0.80	710	0.92	-1.33×10^{-6}	966	528	260
Aug. 1939	1.00	536	1.38	-2.08×10^{-6}	424	293	173

Table 3.5 Predicted attenuation for floods in the River Wye between Erwood and Belmont.

ations. Generally the predicted attenuations are too large. The notable exception is the January 1969 flood, which has a predicted attenuation (Equation 2.41) of 14.8% of the peak discharge at Erwood. As has already been commented in Section 3.3, when Q^*/Q_p is of the order of 0.1 it can be anticipated that the predicted attenuation will be reasonably accurate.

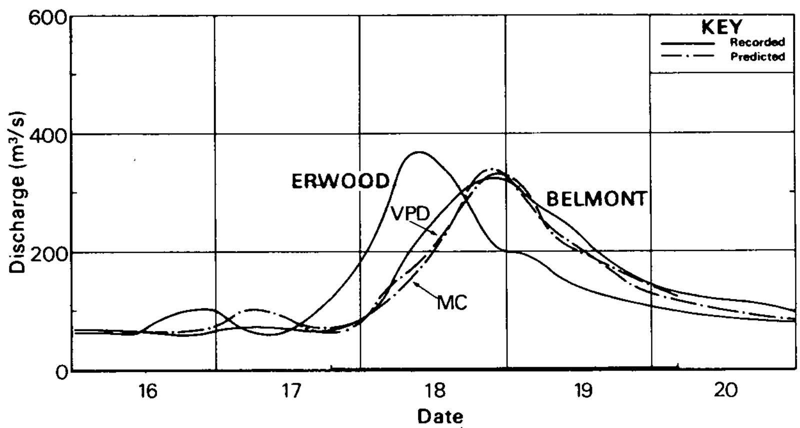


Fig 3.5 River Wye: flood of January 1969.

The flood routing methods were applied to four floods in the reach. The hydrographs predicted by the Muskingum–Cunge and variable parameter diffusion methods are shown in Figures 3.5–3.8, and the corresponding error parameters for all the methods are given in Table 3.6. The time interval during which the error parameters were calculated is indicated by a thick line along the time axis in the figures. Δx was taken as 6975 m in each method, whereas in the Muskingum–Cunge method Δt was 7200 s for all except the 1969 flood for which Δt was 3600 s, and in the remaining methods Δt was 1800 s. The cutoff discharge in the variable parameter diffusion method was 400 $m^3 s^{-1}$.

Both of the 1939 floods were affected by rain on the catchment along the reach; consequently the predicted hydrographs at Belmont show a marked deviation from the recorded hydrographs prior to the main part of the floods. However, it can readily be seen from the figures that the hydrographs predicted by the Muskingum–Cunge and variable parameter diffusion methods agree well with the recorded hydrographs. An examination of the standard deviations in Table 3.6 indicates that the variable parameter diffusion method is more accurate than the other methods, though all the methods predict the peak discharge, speed-of-flood peak, and the mean discharge to a similar degree of accuracy.

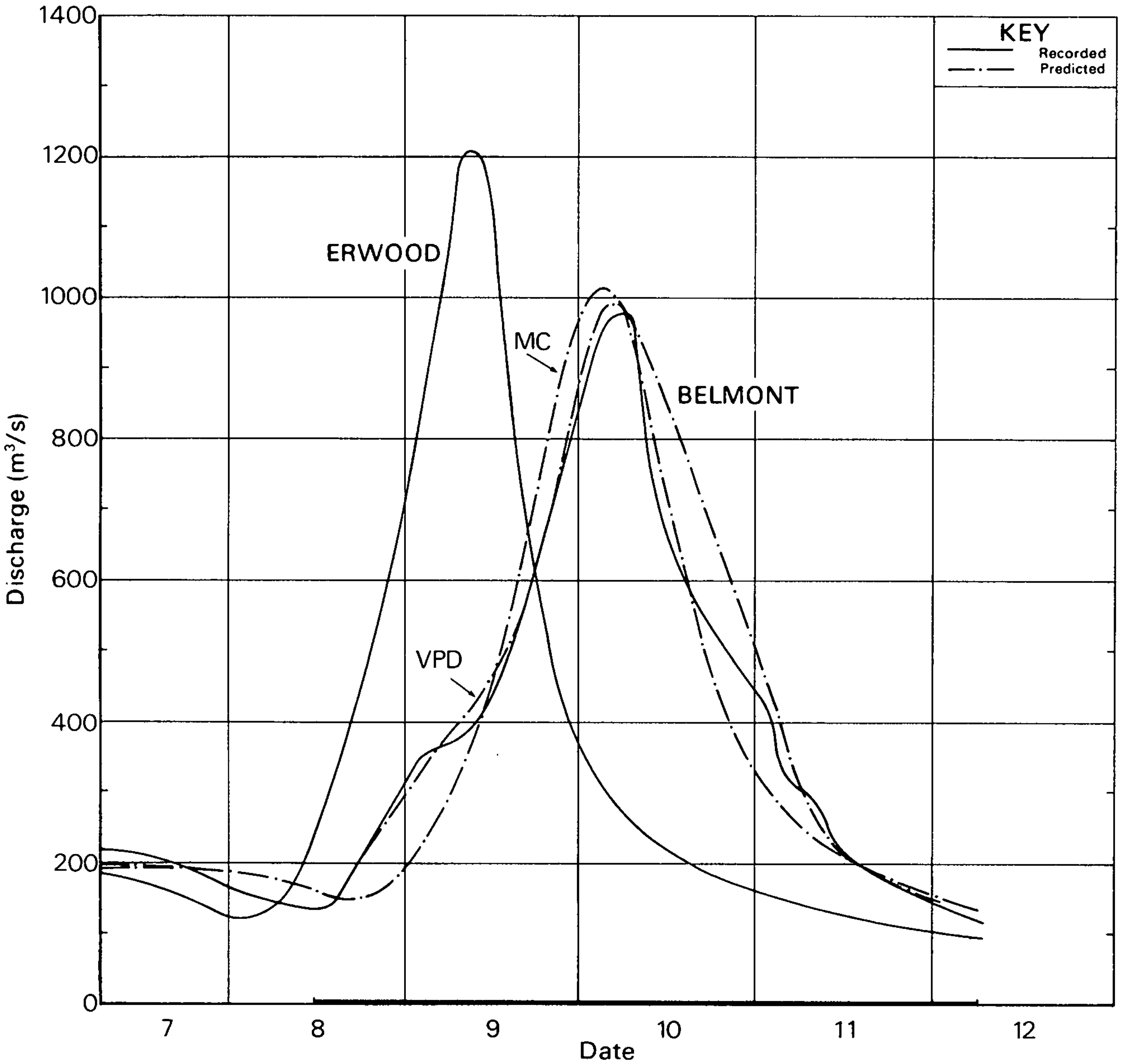


Fig 3.6 River Wye: flood of December 1960.

Fig 3.7 River Wye: flood of January 1948.

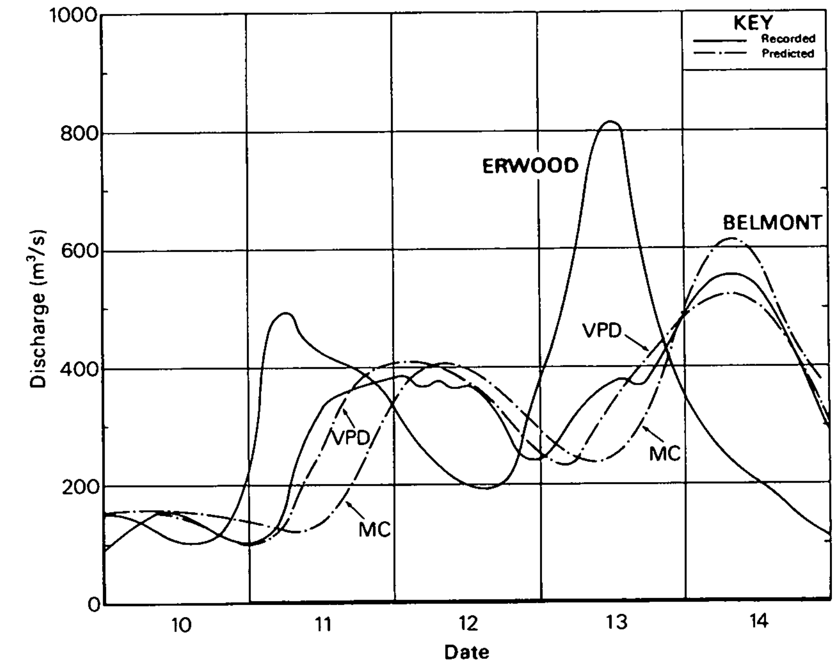


Fig 3.8 River Wye: flood of November 1939.

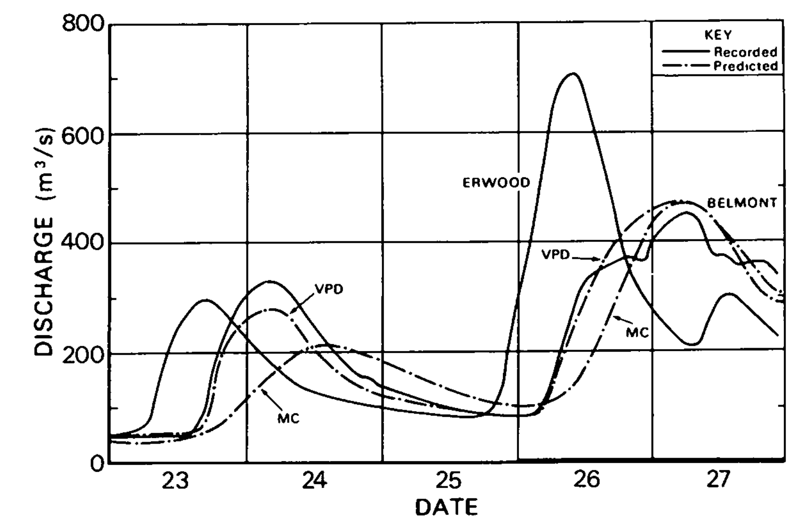


Table 3.6 Errors in predicted hydrographs at Belmont, River Wye.

MC Muskingum–Cunge,
LD linear diffusion,
VPD variable parameter diffusion.

Flood	Method	$\bar{Q}_p$ ($m^3 s^{-1}$)	ω ($m s^{-1}$)	α_p	Recorded peak discharge ($m^3 s^{-1}$)	Percentage error in predicted discharge	Recorded speed ($m s^{-1}$)	Percentage error in predicted speed	Average recorded discharge ($m^3 s^{-1}$)	Percentage standard deviation	Percentage mean deviation
Jan. 1969	MC	350	1.74	1.00×10^6	328	3.96	1.76	−2.948	164	10.4	3.09
	LD	350	1.74	1.00×10^6	328	3.35	1.76	−3.547	164	10.8	3.62
	VPD	—	—	—	328	−0.61	1.76	−11.094	164	11.2	3.70
Dec. 1960	MC	1090	0.98	0.20×10^6	975	3.69	1.01	1.950	294	18.12	−0.39
	LD	1090	0.98	0.20×10^6	975	6.67	1.01	7.229	295	19.71	2.05
	VPD	—	—	—	975	3.21	1.01	−1.035	295	16.10	−5.67
Jan. 1948	MC	710	0.93	0.33×10^6	555	12.43	0.95	1.013	373	21.23	6.01
	LD	710	0.93	0.33×10^6	555	11.35	0.95	3.334	373	20.90	6.14
	VPD	—	—	—	555	−6.25	0.95	3.027	373	8.64	1.68
Nov. 1939	MC	600	0.92	0.80×10^6	450	3.78	0.95	3.005	328	31.20	8.32
	LD	600	0.92	0.80×10^6	450	5.55	0.95	4.591	339	24.21	6.28
	VPD	—	—	—	450	4.44	0.95	6.084	339	13.62	−6.42

3.5 Flooding in the River Wye: Belmont to Redbrook

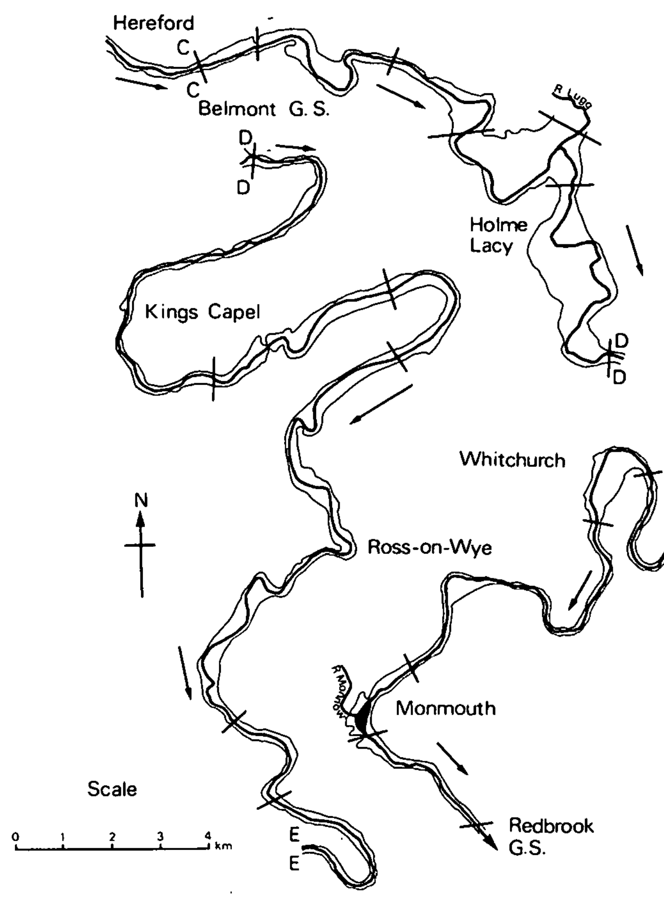


Fig 3.9 River Wye: Belmont to Redbrook.

The second reach of the River Wye follows on directly from Belmont down to Redbrook which is about 5 km downstream of Monmouth (Figure 3.9). The gauging station at Redbrook, like that at Belmont, has a good quality rating curve and the station has more than 30 years of records. Unlike the Erwood to Belmont reach however, this reach of the River Wye has two important tributaries: the River Lugg just downstream of Hereford and the River Monnow which joins the Wye at Monmouth. Unfortunately, the gauging station on the River Lugg upstream of the confluence is inaccurate for high flows, particularly as there can be a significant backwater effect from the Wye. Because of this difficulty it was decided to ignore the discrete lateral inflow due to the River Lugg and to suppose that the discharge from the tributary can be included as part of the lateral inflow uniformly distributed along the channel. Though this is a fairly crude assumption it does not appear to affect seriously the routing of a flood from Belmont down to Redbrook. This is primarily because the maximum discharge from the Lugg appears to be less than 10% of the peak discharge in the Wye for the floods considered below.

The influence of the River Monnow on flooding can be serious in the Monmouth area. However, because the confluence of the two rivers is near to Redbrook, the simplest procedure is to assume that the contribution to the discharge at Redbrook from the Monnow can be added to the discharge predicted by the flood routing method. Note, however, that in the diffusion methods this additional discharge should not be included in the calculations of the finite difference scheme itself but simply added to the predicted downstream value.

The River Monnow is gauged at Kentchurch, 40 km upstream of the confluence with the Wye. The hydrographs at Kentchurch were lagged by 3 hours to produce the relevant hydrographs at Monmouth.

The flood plain between Belmont and Redbrook is widest at the confluence with the River Lugg. Downstream of Ross-on-Wye, the river passes through hills and there is consequently very little flood plain. So, although the river channel is up to 90 m wide in places, most of the contribution to the attenuation parameter for an overbank discharge arises from the upper part of the reach.

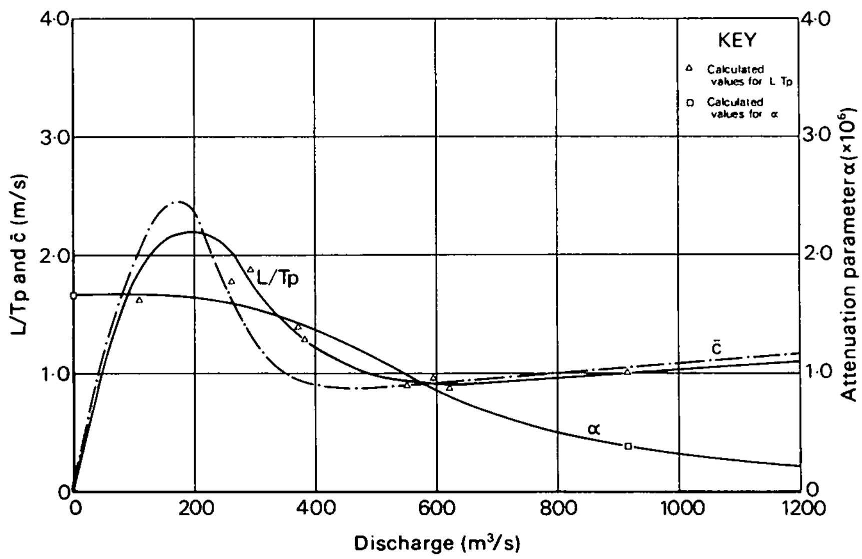


Fig 3.10 River Wye: speed and attenuation parameter for Belmont to Redbrook.

As for the Erwood to Belmont reach, the curves for L/T_p and $\bar{c}$ can readily be extracted from the records at Belmont and Redbrook (Figure 3.10). Because the peak of the flood in the Monnow generally reaches Monmouth before the flood peak in the Wye, no correction need be made to the peak discharge at Redbrook when calculating the average peak discharge along the reach to correlate with the values for L/T_p and $\bar{c}$. The attenuation parameter for the largest recorded flood (December 1960) was again calculated from data supplied by the Wye River Authority. An average channel width of 62 m was assumed when calculating the attenuation parameter for an inbank flood; see Figure 3.9 for the subdivisions of the reach used in the calculation of α. Table 3.7 gives the numerical values for quantities used in the calculations. For the flood of December 1960, $\alpha = 0.40 \times 10^6$. This corresponds to an average peak discharge along the reach of 910 $m^3 s^{-1}$. For an inbank flood, $\alpha = 1.66 \times 10^6$. Again the curve for α was drawn as a slowly varying function of discharge, passing through the point for $\alpha = 0.40 \times 10^6$ at $Q = 910$ $m^3 s^{-1}$, and tending to zero as the discharge became infinite (Figure 3.10).

Again, the formula for the attenuation of the peak discharge (Equation 2.41) was applied to the large overbank flood of December 1960. Here

$$Q^* = \frac{0.40 \times 10^6 \times 980 \times 1.51 \times 10^{-6}}{(1.17)^3} = 370 \text{ m}^3 \text{ s}^{-1} \qquad (3.10)$$

or 38 % of the upstream peak discharge. This compares with the recorded attenuation of about 260 $m^3 s^{-1}$, where allowance has been made for the lateral inflow. It follows that it is necessary to use the flood routing methods to route the complete discharge hydrograph so that the attenuation may be predicted more accurately.

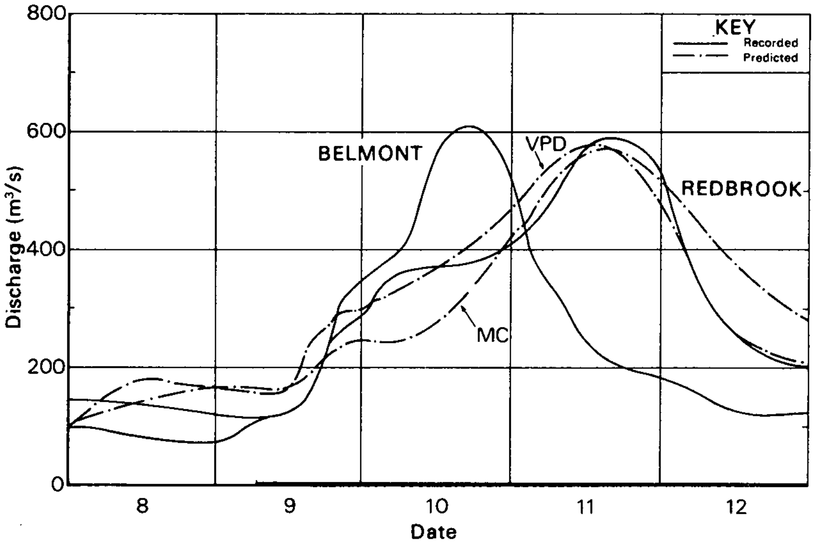


Fig 3.11 River Wye: flood of December 1965.

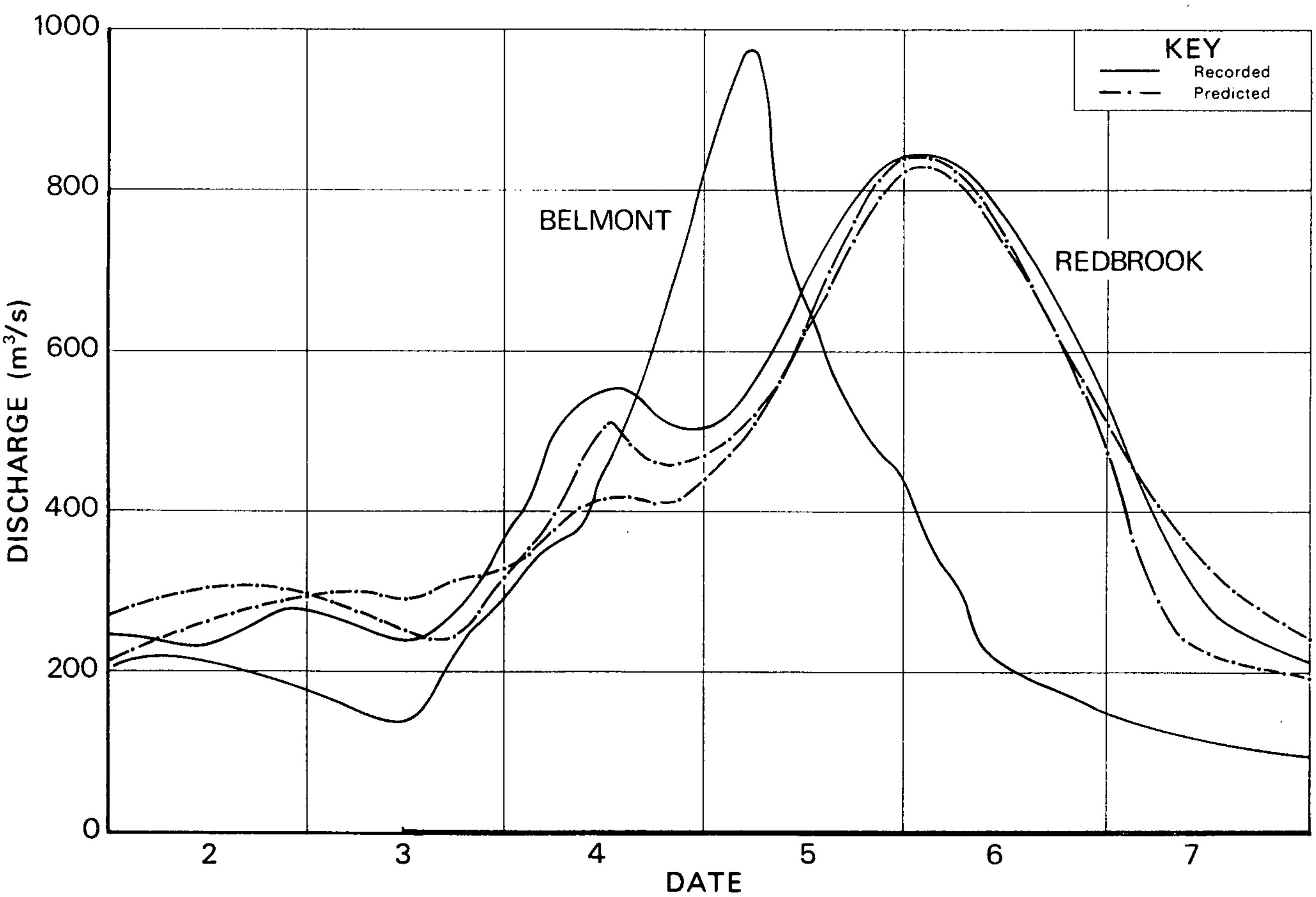


Fig 3.12 River Wye: flood of December 1960.

Length of subreach (km)	Flood plain area (km^2)	Bottom slope
1.5	0.34	0.4×10^{-3}
3.3	0.92	0.4×10^{-3}
3.5	1.04	0.5×10^{-3}
5.0	2.57	0.5×10^{-3}
6.6	3.14	0.4×10^{-3}
9.7	1.56	0.4×10^{-3}
4.1	2.23	0.4×10^{-3}
3.6	1.46	0.4×10^{-3}
12.5	3.89	0.4×10^{-3}
3.3	0.93	0.6×10^{-3}
9.4	1.09	0.4×10^{-3}
3.8	1.40	0.5×10^{-3}
8.0	0.77	0.5×10^{-3}
3.0	1.12	0.5×10^{-3}
3.3	0.31	0.5×10^{-3}
Total 80.5	Total 22.77	Average 0.46×10^{-3}

Table 3.7 Data used to calculate α for Belmont to Redbrook, River Wye.

Flood	Method	$\bar{Q}_p$ (m^3 s^{-1})	ω (m s^{-1})	α_p	Recorded peak discharge (m^3 s^{-1})	Percentage error in predicted discharge	Recorded speed (m s^{-1})	Percentage error in predicted speed	Average recorded discharge (m^3 s^{-1})	Percentage standard deviation	Percentage mean deviation
Dec. 1965	MC	595	0.92	0.87×10^6	585	7.51	0.94	8.64	354	19.08	−2.78
	LD	595	0.92	0.87×10^6	585	8.40	0.94	11.91	354	19.62	0.51
	VPD	—	—	—	585	−1.00	0.94	9.83	354	9.46	−3.51
Dec. 1960	MC	910	0.96	0.5×10^6	840	−0.60	1.00	2.46	416	15.07	5.94
	LD	910	0.96	0.5×10^6	840	0.10	1.00	5.53	416	13.60	8.00
	VPD	—	—	—	840	−0.10	1.00	8.71	416	10.84	7.52

MC Muskingum–Cunge, LD linear diffusion, VPD variable parameter diffusion.

Table 3.8 Errors in predicted hydrographs at Redbrook, River Wye.

The results of routing two floods along this reach of the Wye are shown in Figures 3.11 and 3.12 and Table 3.8. Again the peak discharges and times-of-arrival at Redbrook are accurately predicted, though there is some disagreement by all the methods at the front of the hydrographs. This is primarily due to an underestimate of the discharge hydrograph from the River Monnow. Despite the disagreement however, the error for the standard deviation is small for all the methods.

3.6 Flooding in the River Nene: Northampton to Wansford

The River Nene is one of the flatter British rivers and flows down through Northampton and Peterborough to the Wash. The river is navigable up to Northampton so there are a large number of locks and weirs along the river. These structures dominate all the low flows so that the hydrograph of a small flood at Northampton is completely distorted by the time the flood reaches Wansford (Figure 3.13). Inevitably such control structures make flood routing an extremely difficult problem. In addition, as the high flows, which are more amenable to a simple flood routing approach, are not particularly well gauged, the problem is made even more complex. The flood chosen for the routing exercise was the 1947 flood, which was largely generated by direct lateral runoff from melting snow. In this case it was recognised that as the snow was fairly uniformly distributed over the whole of the catchment above Wansford (Jamieson & Wilkinson, 1972)

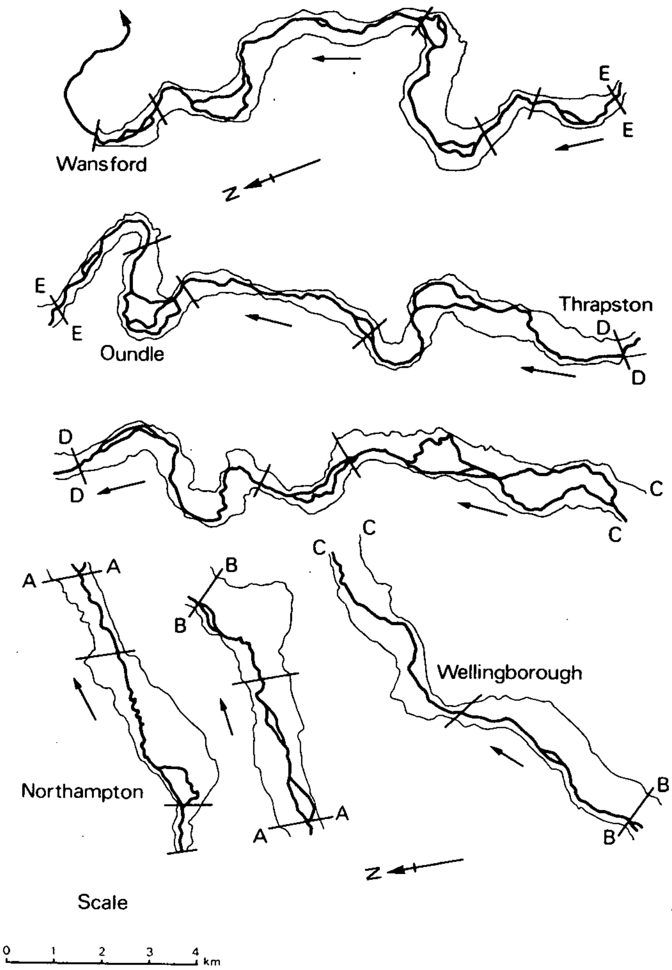


Fig 3.13 River Nene: Northampton to Wansford.

the lateral inflow along the reach during the main period of the flood could be regarded as approximately proportional to the instantaneous discharge at Northampton. This assumption gives a very simple measure of the variation in the lateral inflow with the meteorological conditions. A check on the total volume past the two gauging stations during the flood using discharge hydrographs supplied by the Welland and Nene River Authority showed that at any instant the lateral inflow (in $m^3\ s^{-1}\ km^{-1}$) could be taken as 2.74×10^{-5} times the corresponding discharge at Northampton. If this constant is taken as the ratio of the catchment area between Northampton and Wansford and the catchment area above Northampton, the corresponding value would be 2.22×10^{-5}. Although this value is less than the former value, the similarity of the values indicates the approximately uniform nature of the runoff over the whole catchment above Wansford under snowmelt conditions.

The attenuation parameter was calculated from data again supplied by the Welland and Nene River Authority (Table 3.9 and Figure 3.14). Some problems were raised in the calculation of the speed–discharge relationship, due to the shortage of data for large floods and the difficulty of correlating flood peaks at Northampton and Wansford for the smaller floods.

Because of the shortage of data it is evident that the more complicated variable parameter diffusion method has little advantage over the other

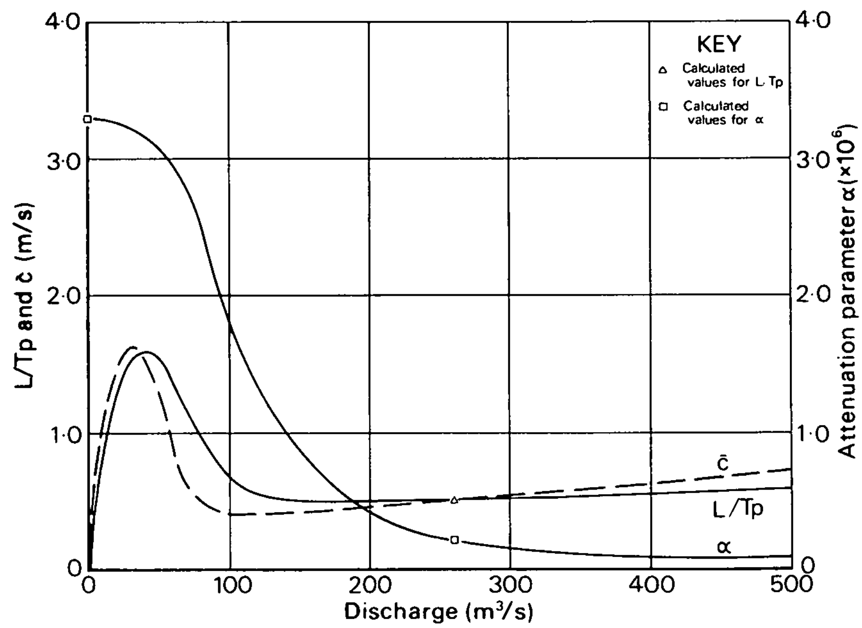


Fig 3.14 River Nene: speed and attenuation parameter for Northampton to Wansford.

Length of subreach (km)	Flood plain area (km^2)	Bottom slope
1.9	0.60	0.89×10^{-3}
3.8	3.88	0.89×10^{-3}
1.4	0.71	0.89×10^{-3}
3.9	3.23	0.89×10^{-3}
2.1	2.94	0.64×10^{-3}
16.8	6.11	0.64×10^{-3}
2.0	0.88	0.61×10^{-3}
7.0	3.20	0.61×10^{-3}
8.0	3.26	0.38×10^{-3}
4.9	1.50	0.38×10^{-3}
3.1	1.36	0.38×10^{-3}
3.7	0.73	0.38×10^{-3}
2.0	0.77	0.38×10^{-3}
1.8	0.39	0.38×10^{-3}
5.1	3.01	0.38×10^{-3}
7.2	3.50	0.38×10^{-3}
1.9	0.40	0.38×10^{-3}
Total 76.6	Total 36.07	Average 0.59×10^{-3}

Table 3.9 Data used to calculate α for Northampton to Wansford, River Nene.

Flood	Method	$\bar{Q}_p$ ($m^3 s^{-1}$)	ω ($m s^{-1}$)	α_p	Recorded peak discharge ($m^3 s^{-1}$)	Percentage error in predicted discharge	Recorded speed ($m s^{-1}$)	Percentage error in recorded speed	Average recorded discharge ($m^3 s^{-1}$)	Percentage standard deviation	Percentage mean deviation
Mar. 1947	MC	250	0.52	0.15×10^6	356	−12.82	0.52	−10.00	160	16.04	−0.65
	LD	250	0.52	0.15×10^6	356	−14.59	0.52	−18.18	159	15.80	−0.48
	VPD	—	—	—	356	−21.10	0.52	−18.18	159	21.71	−4.02

MC Muskingum–Cunge, LD linear diffusion, VPD variable parameter diffusion.

Table 3.10 Errors in predicted hydrographs at Wansford, River Nene.

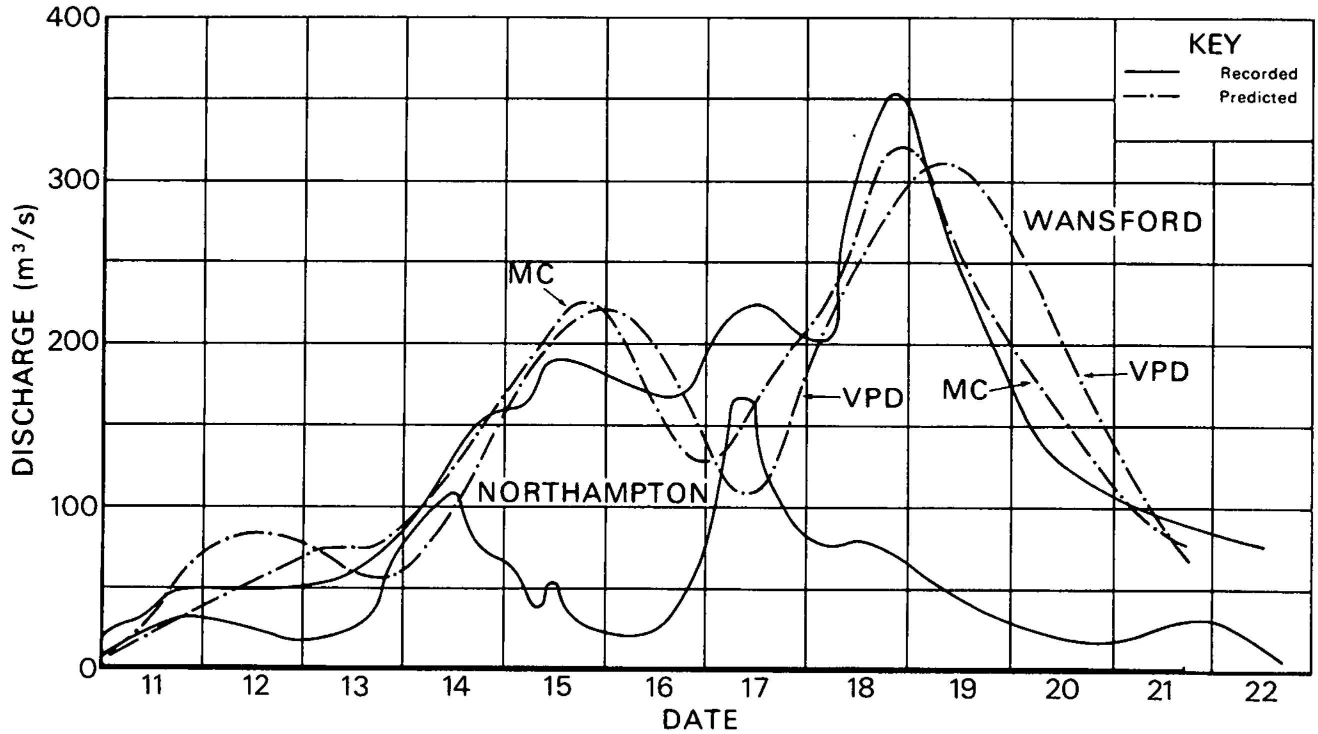


Fig 3.15 River Nene: flood of March 1947.

methods. This is reflected in the results of simulating the 1947 flood (Figure 3.15 and Table 3.10). However, despite the shortage of data there is a reasonable agreement between the predicted and recorded hydrographs at Wansford, particularly for the Muskingum–Cunge method.

3.7 Flooding in the River Eden: Temple Sowerby to Warwick Bridge

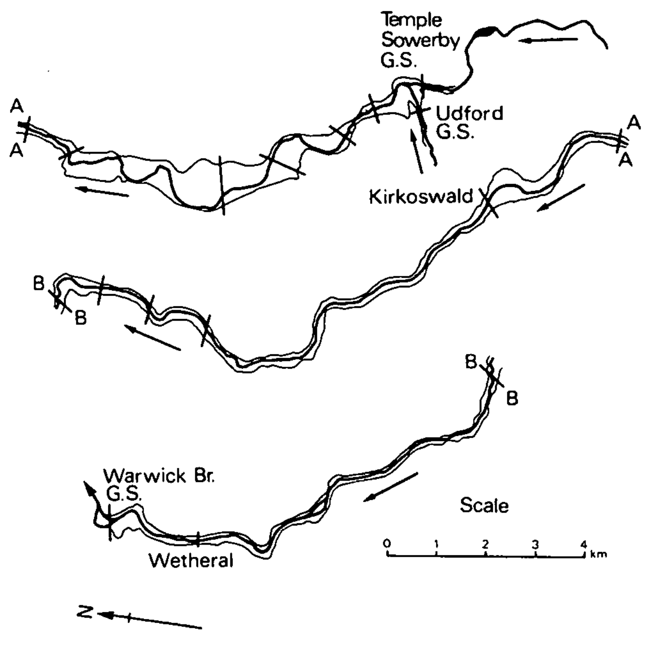


Fig 3.16 River Eden: Temple Sowerby to Warwick Bridge.

The River Eden has its source in the Pennines and flows down through a very picturesque valley, which in parts is fairly narrow, to Carlisle, and so into the Solway Firth (Figure 3.16). The river is fairly steep, and from Temple Sowerby to Warwick Bridge most of the flooding occurs just below the confluence with the River Eamont which is a tributary with a discharge of the same order as the Eden. The gauging stations at Temple Sowerby and Udford on the Eamont have rating equations of reasonable quality, though the river does overflow locally at both stations. Similarly, the station at Warwick Bridge is reliable until there is overbank flooding at about 560 $m^3 s^{-1}$. It is assumed by all the flood routing methods that the discharge from the Eamont can be added to that at Temple Sowerby to produce the relevant discharge hydrograph for the input to the reach.

Table 3.11 Data used to calculate α for Temple Sowerby to Warwick Bridge, River Eden.

Length of subreach (km)	Flood plain area (km^2)	Bottom slope
1.3	0.33	1.6×10^{-3}
0.8	0.09	1.6×10^{-3}
2.1	0.63	1.6×10^{-3}
1.2	0.69	1.5×10^{-3}
4.9	2.03	1.5×10^{-3}
1.1	0.15	1.3×10^{-3}
3.4	1.04	1.3×10^{-3}
8.0	0.65	2.6×10^{-3}
1.3	0.17	1.5×10^{-3}
0.9	0.07	1.5×10^{-3}
1.2	0.24	1.5×10^{-3}
8.8	0.85	2.0×10^{-3}
2.0	0.41	2.0×10^{-3}
Total 37.0	Total 7.35	Average 1.65×10^{-3}

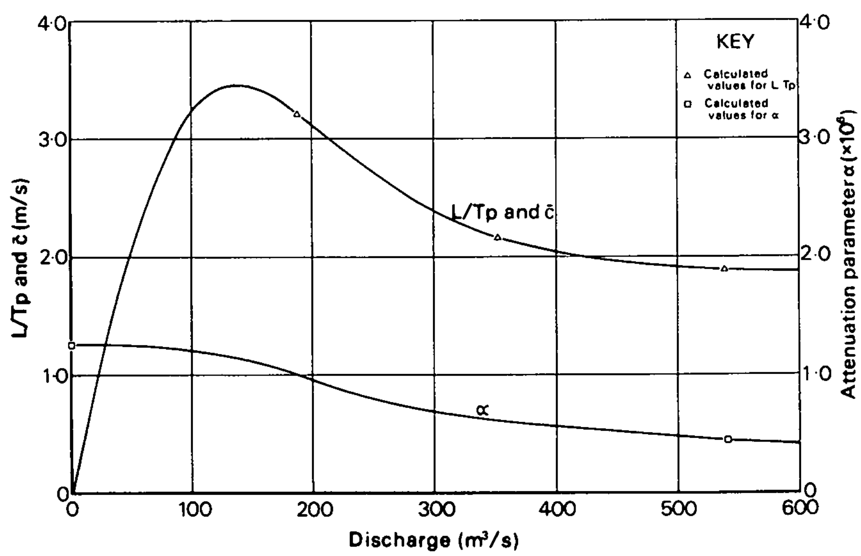


Fig 3.17 River Eden: speed and attenuation parameter for Temple Sowerby to Warwick Bridge.

The data used to calculate the attenuation parameter is given in Table 3.11, and Figure 3.16 shows the division of the river into the sub-reaches for the calculation. α and $\bar{c}$ are shown in Figure 3.17.

The results for the February 1967 flood in the River Eden are shown in Figure 3.18 and Table 3.12. Because the River Eden is one of the steeper

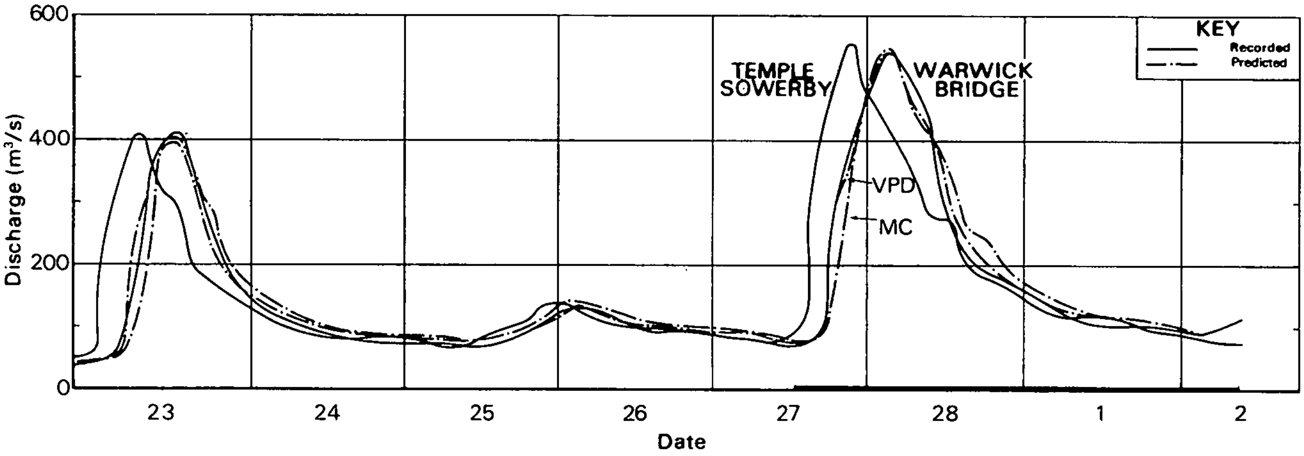


Fig 3.18 River Eden: flood of February 1967.

Flood	Method	$\bar{Q}_p$ ($m^3 s^{-1}$)	ω ($m s^{-1}$)	α_p	Recorded peak discharge ($m^3 s^{-1}$)	Percentage error in predicted discharge	Recorded speed ($m s^{-1}$)	Percentage error in predicted speed	Average recorded discharge ($m^3 s^{-1}$)	Percentage standard deviation	Percentage mean deviation
Feb. 1967	MC	540	1.90	0.35×10^5	543	0.93	1.93	9.88	153	15.17	−4.62
	LD	540	1.90	0.35×10^5	543	0.73	1.93	9.25	151	15.09	−4.61
	VPD	—	—	—	543	0.78	1.93	10.52	151	10.53	−4.95

MC Muskingum-Cunge, LD linear diffusion, VPD variable parameter diffusion.

Table 3.12 Errors in predicted hydrographs at Warwick Bridge, River Eden.

British rivers, there is little evidence of attenuation in this reach. So, again, the methods all predict the floods accurately, and there is little to distinguish the methods.

4 Strategy for flood routing

4.1 Final comparison of methods

Three major conclusions can be drawn from the results of the preceding chapter.

i Given sufficient data each of the methods predicts the total volume past the downstream section, the attenuation of the peak discharge, and the time of arrival of the flood peak within 10%.

ii Again, if the data for the river geometry and previous floods are sufficiently accurate, the standard deviation of results for a particular river and flood using the variable parameter diffusion method can be smaller by a factor of 0.25 or more than the standard deviation of results from the other three methods tested. Where there is difficulty in defining the data curves for $\bar{c}$ and α then the variable parameter diffusion method is no more accurate than the other methods.

iii There is little difference between the Muskingum–Cunge method and the linear diffusion method whether or not the data from the river are accurate.

It follows from the third conclusion that the Muskingum–Cunge method is preferable to the linear and non-linear diffusion methods. This is because the Muskingum–Cunge method has the advantages that it is very simple conceptually, it can be readily applied by desk calculation, and is much cheaper than the other methods when applied by computer. In addition, this method can include a tributary as a discrete lateral inflow, which the other methods cannot do in a simple way. A disadvantage with the Muskingum–Cunge method, and indeed with all the other simplified flood routing methods described in this volume, arises when there is a disturbance such as a tide affecting the flow in the river upstream of the downstream boundary.

A second disadvantage with the Muskingum–Cunge method is that it does not accurately predict the shape of the discharge hydrograph at the downstream boundary when there are large variations in the kinematic wave speed, such as due to the inundation of a large flood plain. If the convection speed and attenuation parameter can be accurately defined for the river then there is an advantage in using the variable parameter diffusion method which will give a better prediction of the shape of the hydrograph. If not, the Muskingum–Cunge method produces results as accurate as those produced by the other methods, and is preferable for the reasons of simplicity and ease of application as stated above.

With these conclusions it is now possible to outline how a flood routing problem should be approached.

4.2 Strategy for a flood routing problem

As in any engineering study, it is important to be clear in the first instance what the objective of the study is, including the type of information (attenuation of peak discharges, shape of hydrographs) and the accuracy required. Secondly, the type and quality of all the available data on previous floods in the river should be scrutinised. Where there are no data on the speed of flood peaks along the river there is little opportunity of being able to do an accurate flood routing exercise at all. However, other data, such as design hydrographs and peak discharges, can be refined using unit hydrograph theory or data from a neighbouring catchment. Where possible, the curves for the convection speeds, L/T_p and $\bar{c}$, and the attenuation parameter, α, versus discharge should be drawn (Section 5.1).

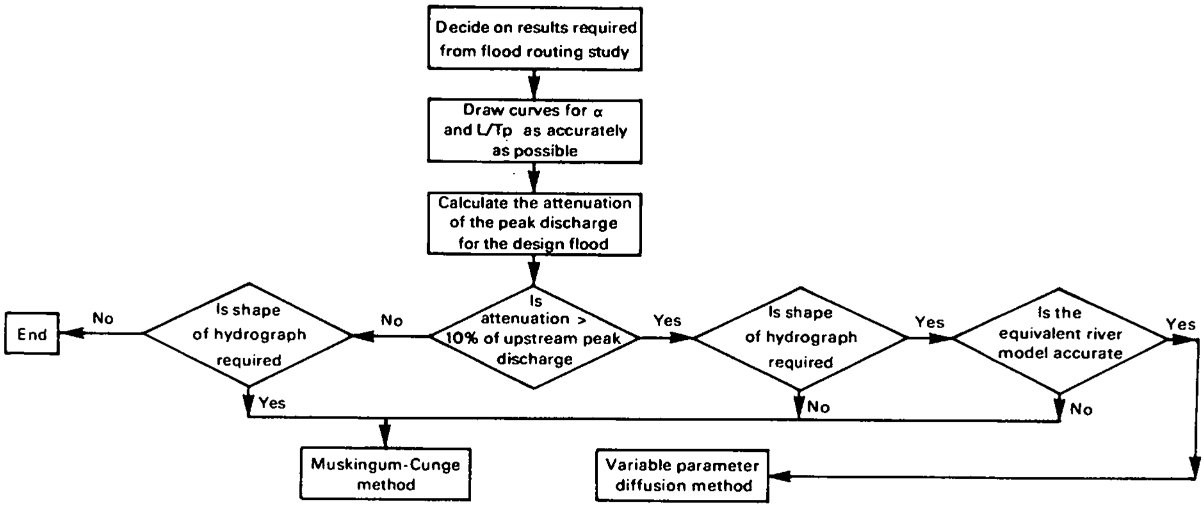


Fig 4.1 Strategy for flood routing.

Given the objective of the study and the curves, or at least some values for L/T_p and α, the formula for the attenuation of the peak discharge (Equation 2.41 or 5.5) should be used to calculate the magnitude of the attenuation along the river of a typical flood in the range being considered (Figure 4.1). This attenuation, as a ratio of the peak discharge at the upstream section of the reach, can be regarded as a parameter which indicates the magnitude of the flood routing problems. It is recommended that if the ratio is greater than 0.1, the Muskingum–Cunge method should be used to find the attenuation more accurately (Section 5.2). When the shape of the downstream hydrograph is important, it is sufficient to use the Muskingum–Cunge method to route the hydrograph, unless the equivalent river model is accurately defined and there is extensive inundation of an associated flood plain. In the latter case there are advantages in using the variable parameter diffusion method (Section 5.3).

4.3 Other applications for methods

The strategy in Section 4.2 has been outlined for a straightforward study of routing floods in a river. Obviously if the curves for $\bar{c}$ and α are accurate, they can be safely extrapolated to deal with larger floods than have previously been recorded. But the methods considered in this volume can also be used for other problems, such as the calibration of rating curves for downstream gauging stations, the improvement of flood warning systems, the operational management of upstream reservoirs, and the design of flood alleviation schemes.

Generally, low flows in smaller rivers can be accurately gauged using a control structure such as a weir. Alternatively, in the larger rivers, in-bank flows can be gauged by current metering at a stable cross-section. However, difficulties usually arise when gauging the higher flows if there is bypassing of the main channel with flow over the local flood plain or through a relief channel. In most cases the discharges for these high flows have to be obtained by extrapolating the rating curve calibrated from the low flows. It is suggested here that the flood routing methods presented in this volume can be used to check the rating curves for downstream gauging stations, and, if the upstream hydrographs for particular large floods are known to be accurate, the methods should provide an improvement to the curves for high discharges.

The benefit of using flood routing methods to improve a flood warning system arises from the increase in the accuracy of predicted flood levels at certain sites along the river, resulting in better use of resources and manpower when the flood arrives. For example, many flood warning systems in Britain are based on the correlation of levels recorded for previous floods at different stations along the river. Inevitably there is a certain range of error involved in the use of the correlations due to the different characteristics of each flood: a peaky flood at some upstream station will experience a larger attenuation along the reach to a particular site than a flood with a similar peak discharge and a smaller curvature at the peak. Similarly, flood levels in rivers with a complex network of tributaries, such as the Yorkshire Ouse or the Severn, can be difficult to predict using correlation curves. In these cases a flood routing method can provide a more accurate prediction of peak discharges and levels. There also exists the possibility of using the methods for detailed predictions during a flood, rather than just in the design of a particular flood warning scheme.

Although the flood routing methods have been discussed in the report primarily in relation to high flows off a given catchment, the methods can also be used to determine a strategy for operating upstream reservoirs. For example, several of the larger British rivers, such as the Dee and the Severn, have reservoirs designed to ensure proper control of flows at certain extraction points downstream. In particular, the reservoirs can be operated on a long-term basis to maintain flows above a certain minimum level, or to alleviate flooding (Jamieson & Wilkinson, 1972). Obviously the operating schedule for the reservoirs depends on a knowledge of the propagation and attenuation of flood waves in the river downstream of the reservoirs.

The value of using the flood routing methods to assist in the design of a flood alleviation scheme arises primarily from the definition of the equivalent river model. Suppose that there is some concern about the increase in peak discharge or levels downstream which a particular scheme may produce. Then the equivalent river model, on which all the methods described in this report are based, can be reconstructed for the river as it will be after the improvements have been made. This enables the results of routing floods in both equivalent river models to be compared, to find, in particular, the change in the attenuation along the reach of the peak discharge for a certain design flood. The techniques for constructing the new from the old equivalent model can be derived from the equations defining the speed and the discharge in terms of the water depth (Equations 2.49 and 2.50) and from the expression for the attenuation parameter (Equation 2.40).

Consider a simple example where the river channel is to be deepened to convey a larger discharge. Suppose that the curves for the speed and attenuation parameters are available for the river before improvements. Then Equations 2.49 and 2.50 are used first to specify a water depth over the flood plain for a particular discharge in the old model and secondly to find the new speed and discharge for the same depth over the flood plain with the new value for the bankfull depth, which has to be worked out beforehand from the flow which the new channel is to convey. In this way the new speed–discharge curve can be plotted, as can the attenuation parameter. The attenuation formula or the Muskingum–Cunge method is then used to compare the attenuation of a particular flood in both equivalent models.

If alterations are made to the flood plain by reducing the amount of area available for flooding, the attenuation parameter has to be recalculated using Equation 2.45. The discharge and speed for a given depth of flooding over the flood plain are again calculated from Equations 2.49 and 2.50, using adjusted values for $\overline{W}_f$ and $\bar{n}_f$ if necessary.

4.4 Further research

There still remains a considerable amount of basic research to be done on simplified flood routing methods. In particular, two lines of research should be pursued. The first is a closer study of the functional forms for $\bar{c}$ and α. There is a need to investigate the definition of $\bar{c}$ in Equation 2.48 more rigorously, and also to examine values of α for bankfull flows. In addition, there exists the possibility of using the $\bar{c}$ and α curves for a given reach to calculate a typical cross-section and roughness values (Kawecki, 1973). The second line of research is the development of a variable parameter Muskingum method, similar to the method proposed by Linsley, Kohler & Paulhus (1958). This latter method, if developed, may well be preferable to the variable parameter diffusion method in predicting the shape of a hydrograph.

In conclusion, mention should again be made of the considerable amount of world-wide research on the application to flood routing of numerical methods based on solutions of the full Saint–Venant equations. As yet, the use of such methods to simulate flooding in a river with extensive flood plains is at an early stage, but current developments both in this country and on the Continent hold considerable promise for the future.

5 Appendices

5.1 Attenuation of the peak discharge

Four quantities are required to be known before the attenuation of the peak discharge for a particular flood can be calculated. These quantities are the peak value and curvature at the peak of the upstream discharge hydrograph, together with the attenuation and speed parameters corresponding to an average value for the peak discharge along the reach. If the attenuation is being found for either the largest recorded overbank flood or a typical inbank flood, data are usually available to calculate the corresponding values for the attenuation and speed parameters. So, assuming that the first two quantities above are given or have been derived from the methods in Volume I it is a straightforward matter to calculate the attenuation for either of the floods. However, if the flood being studied is, say, a small overbank flood for which only the speed parameter can be directly calculated from records, or if a synthetic design flood is to be routed along the reach, then it is necessary to know the curves defining the attenuation and speed parameters as functions of discharge. In this latter case the problem resolves into one of defining the parametric curves.

So, two cases need to be considered, namely the calculation of the attenuation for the largest recorded flood or a typical inbank flood, and for a recorded intermediate flood or synthetic design flood.

Case I Attenuation of the peak discharges for the largest recorded flood and a typical inbank flood

A Calculate the attenuation parameter

i Define the flood plain as the known area inundated by the largest recorded flood, or as estimated from a survey map.

ii Divide the reach into a number of subreaches so that the geographical width of the flood plain in each subreach is approximately uniform. Where the flood is not, or is unlikely to be overbank, define the subreaches such that the slope of the channel is approximately uniform in each subreach.

iii For each subreach measure the length, L_m, of the channel, the average slope, s_m, of the channel, the plan area, P_m, of the flood plain, (including the plan area of the channel).

It is sufficient for most purposes to calculate s from the distance along the channel between the sections where the 25 ft contours cross the channel on an Ordnance Survey map.

iv For the whole reach calculate the length, L, of the channel, and an average width, $\overline{W}_c$, of the channel.

v Calculate the attenuation parameter, α_p, for the largest recorded flood from

$$\alpha_p = \tfrac{1}{2}\left\{\frac{1}{L}\sum_{m=1}^{M}\frac{P_m}{s_m^{1/3}}\right\}^{-3}\sum_{m=1}^{M}\frac{P_m^2}{L_m s_m^2} \tag{5.1}$$

and for a typical inbank flood from

$$\alpha_p = \frac{1}{2\overline{W}_c}\left\{\frac{1}{L}\sum_{m=1}^{M}\frac{L_m}{s_m^{1/3}}\right\}^{-3}\sum_{m=1}^{M}\frac{L_m}{s_m^2}\,. \tag{5.2}$$

B Calculate the convection speed

Extract the times of travel, T_p, of the peak of the largest recorded flood and the inbank flood from records. Define the speed by L/T_p.

C Calculate the curvature at the peak of the upstream discharge hydrograph

i Find the time-to-peak, t_p, of the flood hydrograph and locate the time-of-occurrence of the peak. Mark off two points on the hydrograph at a time interval, δt, either side of the peak, where δt is defined to the nearest hour by

$$\delta t = t_p/5. \tag{5.3}$$

δt need not however be greater than 3 hours.

ii Calculate the curvature at the peak from

$$\frac{d^2 Q_p}{dt^2} = \frac{Q_1 + Q_{-1} - 2Q_p}{(\delta t)^2} \tag{5.4}$$

where Q_p is the discharge at the peak and Q_1 and Q_{-1} are the two discharges either side of the peak.

D Calculate the attenuation of the peak discharge

i Use the formula

$$Q^* = \frac{\alpha_p}{(L/T_p)^3} Q_p \left| \frac{d^2 Q_p}{dt^2} \right|. \tag{5.5}$$

ii If $Q^*/Q_p > 0.1$, redefine Q^* by

$$Q^*_{new} = Q_p \left[1 - \exp\left(-\frac{Q^*}{Q_p} \right) \right] \tag{5.6}$$

and define ω_p by

$$\omega_p = \frac{L}{T_p} - \frac{2\alpha_p}{L^2} Q^*_{new}. \tag{5.7}$$

Recalculate the attenuation using Equation 5.5, with L/T_p replaced by ω_p, and Equation 5.6 if necessary.

Case 2 Attenuation of the peak discharge for an intermediate or a synthetic flood

A Define the curves for the attenuation and speed parameters as functions of discharge

i Calculate α_p for the largest recorded flood and for a typical inbank flood from Equations 5.1 and 5.2 above. In addition, calculate α_p for any other flood with the relevant data.

ii Calculate the attenuation of the peak discharge and the speed, ω_p, for the largest recorded flood and the inbank flood, as explained in Case 1.

iii Define the average peak discharge, $\bar{Q}_p$, for each flood by

$$\bar{Q}_p = Q_p - \tfrac{1}{2}Q^*. \tag{5.8}$$

iv Correlate the values of α_p and ω_p with the values of $\bar{Q}_p$. Plot the points on a graph.

v Estimate the curve for α. If the flood plain is fairly uniform along the river, it is probable that the curve for α will drop sharply as the discharge increases above an average bankfull value. Ensure that the curve tends asymptotically to zero as the discharge becomes infinite.

vi Extract the times of travel, T_p, for as many recorded floods as possible. Calculate values for ω_p using Equation 5.7 and the calculated values for the attenuation in each case. Use values for α_p read off the curve for the attenuation parameter.

vii Estimate the curve for $\omega(Q)$.

B Calculate the attenuation of the peak discharge

i Calculate the curvature at the peak of the upstream hydrograph as in Case 1.

ii Use the formula

$$Q^* = \frac{\alpha}{\omega^3} Q_p \left|\frac{d^2 Q_p}{dt^2}\right| \tag{5.9}$$

where α and ω are initially read off the corresponding curves at $Q = Q_p$.

iii Refine the estimate of Q^* as in Case 1, but this time adjusting the values of α and ω according to the value of the average peak discharge, $\bar{Q}_p$ (Equation 5.8).

5.2 Muskingum–Cunge method

This method can either be applied by desk calculation or by using a digital computer.

1 Desk version

A Calculate parameters

i Derive curves for the speed and attenuation parameters as described in Section 5.1, Case 2.

ii Record the upstream peak discharge, Q_p, of the flood to be routed, together with the corresponding value of the curvature at the peak of the discharge hydrograph.

iii Calculate the attenuation, Q^*, of the peak discharge along the reach, length L, for values of L/T_p and α corresponding to Q_p. If $Q^*/Q_p > 0.1$, correct Q^* using the equation

$$Q^*_{new} = Q_p\left\{1 - \exp\left(-\frac{Q^*}{Q_p}\right)\right\}. \tag{5.10}$$

iv Calculate a provisional value for the average peak discharge, $\bar{Q}_p$, along the reach from

$$\bar{Q}_p = Q_p - \tfrac{1}{2}Q^*_{new}. \tag{5.11}$$

(If the downstream peak discharge is known then define $\bar{Q}_p$ as the average of the peak discharges at each end of the reach.)

v Record the values of ω and α corresponding to $\bar{Q}_p$ and recalculate Q^* and a new value for $\bar{Q}_p$.

vi Calculate K and ε from

$$K = \frac{L}{\omega} \tag{5.12}$$

$$\varepsilon = \tfrac{1}{2} - \frac{\alpha \bar{Q}_p}{L^2 \omega}. \tag{5.13}$$

vii Read the value of $L/(\omega \Delta t)$ corresponding to this particular value of ε off the graph in Figure 2.11. Calculate Δt and round up to the nearest integral number of hours.

viii Calculate the four Muskingum parameters

$$C_1 = \frac{K\varepsilon + \frac{1}{2}\Delta t}{K(1-\varepsilon) + \frac{1}{2}\Delta t} \tag{5.14}$$

$$C_2 = \frac{\frac{1}{2}\Delta t - K\varepsilon}{K(1-\varepsilon) + \frac{1}{2}\Delta t} \tag{5.15}$$

$$C_3 = \frac{K(1-\varepsilon) - \frac{1}{2}\Delta t}{K(1-\varepsilon) + \frac{1}{2}\Delta t} \tag{5.16}$$

$$C_4 = \frac{Lq\Delta t}{K(1-\varepsilon) + \frac{1}{2}\Delta t} \tag{5.17}$$

where q is the lateral inflow/unit length.

B Calculate the outflow hydrograph

i Read off, or calculate from a stage discharge relationship, the values of the input discharge, $Q_{\mathrm{I}}^{\mathrm{n}}$, at intervals of Δt.

ii Assume an initial value, Q_{O}^{0}, for the outflow discharge equal to the value of the input discharge, Q_{I}^{0}, at the same time, and generate the rest of the outflow hydrograph, $Q_{\mathrm{O}}^{\mathrm{n}}$, using the recurrence formula

$$Q_{\mathrm{O}}^{\mathrm{n}+1} = C_1 Q_{\mathrm{I}}^{\mathrm{n}} + C_2 Q_{\mathrm{I}}^{\mathrm{n}+1} + C_3 Q_{\mathrm{O}}^{\mathrm{n}} + C_4. \tag{5.18}$$

Note The lateral inflow can be regarded as a function of t, in which case the lateral inflow hydrograph has to be specified at intervals of Δt.

2 Digital computer version

A Calculate parameters

i Derive $\bar{Q}_p$ and ω as in the desk version above.

ii Fix $\Delta x = L/10$ and determine K and ε from

$$K = \frac{\Delta x}{\omega} \tag{5.19}$$

$$\varepsilon = \tfrac{1}{2} - \frac{\alpha \bar{Q}_p}{L\omega \Delta x}. \tag{5.20}$$

iii Read the value of $\Delta x/(\omega \Delta t)$ corresponding to this particular value of

ε off the graph in Figure 2.11. Calculate Δt and round up to the nearest integral number of hours.

B Prepare card data

The program listed below reads the data in the following order. Eight cards containing the heading for the output from the program. Cards containing 11 integers (free I-format):

JX	the number of space nodes
LEND	the number of time steps
IQH	the number of data points for the upstream hydrograph
IQDNS	the number of data points for the downstream hydrograph
ITR1	the number of data points for the hydrograph from the first tributary
ITR2	the number of data points for the hydrograph from the second tributary
JT1	the space node for the confluence of the first tributary with the main channel
JT2	the space node for the confluence of the second tributary with the main channel
J1	the discharges along the reach are written at J1 intervals apart
L1	the discharges along the reach are written when the number of times steps is equal to a multiple of L1
L2	only the downstream discharge is written when the number of time steps is equal to a multiple of L2.

Cards containing 12 real numbers (free E-format):

DXLR	length of reach
DT	time step
DTQHYDRO	time interval between successive data points for each specified hydrograph
QINIT	initial discharge along the reach
QINB	base flow for the lateral inflow
QINA	amplitude of the variable part of the lateral inflow
TQIN	time when the peak lateral inflow occurs
TSQIN	timescale for the variable part of the lateral inflow
WSP	value of L/T_p
AP	value of α
QCON	value of $\bar{Q}_p$
TDEVN	time after which the error parameters are calculated.

Notes

i If there is no downstream hydrograph IQDNS = 0 and TDEVN is set to a value greater than the real time for routing the flood. Similarly, ITR1, ITR2, JT1 and JT2 should be set equal to zero if there are no tributaries.

ii The computer version above of the Muskingum–Cunge method can also be used in a desk calculation if JX is not too large.

The computer program below is written in FORTRAN IV. The program can be extended to deal with any number of different reaches in the same river.

```
C     Q1      - ARRAY FOR THE DISCHARGE ALONG THE REACH AT THE NEW TIME LEVEL.
C     Q2      - ARRAY FOR THE DISCHARGE ALONG THE REACH AT THE OLD TIME LEVEL.
C     DX      - SPACE INCREMENT.
C     DXLR    - LENGTH OF THE REACH.
C     JX      - NUMBER OF SPACE GRID POINTS.
C     X       - ARRAY FOR THE DISTANCES OF THE GRID POINTS ALONG THE CHANNEL.
C     DT      - TIME INCREMENT.
C     T       - TIME IN SECONDS.
C     TH      - TIME IN HOURS.
C     LEND    - NUMBER OF TIME STEPS.
C     TITLE   - ARRAY WHICH STORES DATA FOR THE HEADING OF THE OUTPUT.
C     J1, L1 AND L2 REFER TO WHEN AND HOW THE RESULTS ARE WRITTEN.

C     WSP     - WAVE SPEED.
C     AP      - ATTENUATION PARAMETER.
C     QCON    - DISCHARGE CONSTANT.
C     C1, C2, C3 AND C4 ARE PARAMETERS IN THE MUSKINGUM METHOD.
C     DTQHYDRO  TIME INTERVAL IN HOURS BETWEEN THE DATA POINTS FOR THE INPUT AND
C               OTHER DISCHARGE HYDROGRAPHS.
C     QHYDRO  - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C               AT THE UPSTREAM SECTION.
C     IQH     - NUMBER OF DATA POINTS FOR THE UPSTREAM HYDROGRAPH.
C     QTRIB1  - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C               AT THE FIRST TRIBUTARY.
C     ITR1    - NUMBER OF DATA POINTS FOR THE HYDROGRAPH AT THE FIRST TRIBUTARY.
C     JT1     - POSITION OF THE INPUT DISCHARGE FROM THE FIRST TRIBUTARY.
C     QTRIB2  - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C               AT THE SECOND TRIBUTARY.
C     ITR2    - NUMBER OF DATA POINTS FOR THE HYDROGRAPH AT THE SECOND TRIBUTARY.
C     JT2     - POSITION OF THE INPUT DISCHARGE FROM THE SECOND TRIBUTARY.
C     QINIT    - INITIAL DISCHARGE FOR THE FLOW THROUGHOUT THE REACH.
C     QINB    - BASE VALUE FOR THE LATERAL INFLOW.
C     QINA    - AMPLITUDE OF THE FUNCTION FOR THE LATERAL INFLOW.
C     TQIN    - TIME WHEN THE LATERAL INFLOW TAKES ITS MAXIMUM VALUE.
C     TSCQIN  - TIME SCALE FOR THE LATERAL INFLOW FUNCTION.
C     QDNS    - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C               AT THE DOWNSTREAM END OF THE REACH.
C     IQDNS   - NUMBER OF DATA POINTS FOR THE DOWNSTREAM HYDROGRAPH.

C     TDEVN   - TIME IN HOURS WHEN THE CALCULATIONS OF THE ERROR PARAMETERS ARE
C               BEGUN.
C     TOTDIS  - AVERAGE RECORDED DISCHARGE AFTER THE TIME TDEVN.
C     TOTD    - AVERAGE PREDICTED DISCHARGE AFTER THE TIME TDEVN.
C     DISDIF  - DIFFERENCE BETWEEN THE AVERAGE RECORDED AND PREDICTED DISCHARGES
C               AS A PERCENTAGE OF THE RECORDED DISCHARGES.
C     DEVN    - ULTIMATELY RECORDS THE STANDARD DEVIATION OF THE PREDICTED
C               DISCHARGE AS A PERCENTAGE OF THE AVERAGE RECORDED DISCHARGE.

C*****************************************************************

      MASTER FLOODS1

      REAL Q1(50),Q2(50),X(50)
      COMMON DX,DT,JX,DXLR,LEND,TH,J1,L1,L2,TITLE(60),
     1JT1,JT2,QTRIB1(200),QTRIB2(200),QHYDRO(200),DTQHYDRO,
     2QINIT,QINB,QINA,TQIN,TSCQIN,AP,WSP,QCON,
     4      QDNS(200),TDEVN

C  FIRST READ IN THE APPROPRIATE DATA AND WRITE IT OUT FOR EASY REFERENCE.

      CALL DATIN

C  NEXT CALCULATE VARIOUS CONSTANTS AND PARAMETERS FOR USE BELOW.

         DX=DXLR/FLOAT(JX-1)
         DK=DX/WSP
         EPSILON=0.5*(1.0-QCON*AP*2.0/(DX*WSP*DXLR))
         CC=DK*(1.0-EPSILON)+DT*0.5
         C1=(DK*EPSILON+DT*0.5)/CC
         C2=(DT*0.5-DK*EPSILON)/CC
         C3=(DK*(1.0-EPSILON)-DT*0.5)/CC
         C4=DT*DX/CC
         JXM1=JX-1
         T=0.0

C  THE FOLLOWING VARIABLES ARE ONLY USED WHEN A RECORDED DOWNSTREAM
C      HYDROGRAPH IS AVAILABLE FOR COMPARISON.   ALL SUCCEEDING
C      STATEMENTS BEGINNING IN COLUMN 13 REFER TO THIS CASE ONLY, AND
C      SHOULD NOT BE INCLUDED OTHERWISE.

            DEVN=0.0
            TOTDIS =0.0
            TOTD=0.0
            QP1=QINIT
            QP2=QINIT
            QDNP2=QINIT
            QDNP1=QINIT
            JCOUNT=0
            ICOUNT=0

C  DEFINE X AND THE INITIAL VALUES OF Q1 QND Q2

      DO 2 J=1,JX
         X(J)=DX*FLOAT(J-1)
         Q1(J), Q2(J)=QINIT
 2    CONTINUE

C  BEGIN THE MAIN TIME LOOP
```

```
      DO 14 L=1,LEND
         T=T+DT
         TH=T/3600.0

            QP2=QP1
            QP1=Q1(JX)
            QDNP2=QDNP1
            QDNP1=QDN
            QDN=FF(QDNS,TH,DTQHYDRO)

C  UPDATE THE VALUES FOR THE DISCHARGE

      DO 4 J=1,JX
         Q2(J)=Q1(J)
 4    CONTINUE

C  FIND THE VALUE OF Q1(1) FROM THE INPUT HYDROGRAPH AND ADD THE
C     CONTRIBUTION FROM THE TRIBUTARY, IF ANY

         QTOT=0.0
      IF(JT1.EQ.1) QTOT=FF(QTRIB1,TH,DTQHYDRO)
      Q1(1)=FF(QHYDRO,TH,DTQHYDRO)+QTOT

C  THE MAIN CALCULATIONS FOR Q1 ARE PERFORMED IN THE FOLLOWING LOOP

      DO 6 J=2,JX
C  QTOT REFERS TO THE INFLOW DISCHARGE FROM TRIBUTARIES AND LATERAL
C     RUNOFF.  UNDER SNOW-MELT CONDITIONS IT MAY BE PREFERABLE TO TAKE
C     THE LATERAL INFLOW AS PROPORTIONAL TO THE DISCHARGE AT THE
C     UPSTREAM BOUNDARY
         TX=(TH-TQIN)/TSCQIN
         QTOT=QINA*EXP(-TX*TX)+QINB
C        QTOT=Q1(1)*CONST/DXLR
      IF(J.EQ.JT1) QTOT=QTOT+FF(QTRIB1,TH,DTQHYDRO)/DX
      IF(J.EQ.JT2) QTOT=QTOT+FF(QTRIB2,TH,DTQHYDRO)/DX
         Q1(J)=C1*Q2(J-1)+C2*Q1(J-1)+C3*Q2(J)+C4*QTOT
 6    CONTINUE

C  PERFORM VARIOUS SUMS ON THE ERROR PARAMETERS

            IF(TH.LT.TDEVN) GO TO 8
            DEVN=DEVN+(Q1(JX)-QDN)*(Q1(JX)-QDN)
            ICOUNT=ICOUNT+1
            JCOUNT=JCOUNT+1
            IF(JCOUNT.NE.2) GO TO 8
            TOTD=TOTD+2.0*DT*(Q1(JX)+QP1+QP2)/3.0
            TOTDIS=TOTDIS+2.0*DT*(QDN+QDNP1+QDNP2)/3.0
            JCOUNT=0

C  NOW WRITE OUT THE RESULTS - IF IT IS TIME

 8    IF(L.NE.L1*(L/L1)) GO TO 12
      WRITE(3,301) (TH,X(J),Q1(J),J=1,JXM1,J1)
 301  FORMAT((20X,F6.2,9X,F8.1,8X,F7.2))
 10   WRITE(3,302) TH,X(JX),Q1(JX),QDN
 302  FORMAT((20X,F6.2,9X,F8.1,2(8X,F7.2)))
      WRITE(3,303)
 303  FORMAT(1H ,/)
      GO TO 14
 12   IF(L.EQ.L2*(L/L2)) GO TO 10
 14   CONTINUE

C  FINALLY, WORK OUT THE ERROR PARAMETERS

            TOTDIS=TOTDIS/((TH-TDEVN)*3600.0)
            TOTD=TOTD/((TH-TDEVN)*3600.0)
            DISDIF=(1.0-TOTD/TOTDIS)*100.0
            DEVN=SQRT(DEVN/FLOAT(ICOUNT))*100.0/TOTDIS
            WRITE(3,304) TOTDIS,DISDIF,DEVN
 304        FORMAT(34H      AVERAGE RECORDED DISCHARGE = ,F7.2,
     1      9H   CUMECS/
     2      56H      DIFFERENCE BETWEEN RECORDED AND PREDICTED AVERAGE D,
     3      11HISCHARGE = ,F6.2,1HX/
     4      26H      STANDARD DEVIATION = ,F6.2,1HX//)
      STOP
      END

C ****************************************************************

      SUBROUTINE DATIN

C  THIS SUBROUTINE READS IN THE RELEVANT DATA, WRITES IT OUT FOR EASY
C     REFERENCE, AND PROVIDES A HEADING FOR THE RESULTS

      COMMON DX,DT,JX,DXLR,LEND,TH,J1,L1,L2,TITLE(60),
     1JT1,JT2,QTRIB1(200),QTRIB2(200),QHYDRO(200),DTQHYDRO,
     2QINIT,QINB,QINA,TQIN,TSCQIN,AP,WSP,QCON,
     4      QDNS(200),TDEVN

      READ(1,100) (TITLE(I),I=1,60)
 100  FORMAT(10A8/5A8)
      READ(1,101) JX,LEND,IQH,IQDNS,ITR1,ITR2,JT1,JT2,J1,L1,L2
 101  FORMAT(11I0)
      READ(1,102) DXLR,DT,DTQHYDRO,QINIT,QINB,QINA,TQIN,TSCQIN,WSP,AP,
     1QCON
     2      ,TDEVN
 102  FORMAT(13E0.0)

      WRITE(3,300) (TITLE(I),I=1,60)
 300  FORMAT(1H ,(15A8))
```

```
C     WRITE(3,301) JX,LEND,IQH,IQDNS,ITR1,ITR2,JT1,JT2,J1,L1,L2
C300  FORMAT - FOR WRITE STATEMENT
C     WRITE(3,302) DXLR,DT,DTQHYDRO,QINIT,QINB,QINA,TQIN,TSCQIN,WSP,AP,
C    1QCON
C    2        ,TDEVN
C302  FORMAT - FOR WRITE STATEMENT

C  READ IN THE DISCHARGE HYDROGRAPHS

      READ(2,200) (QHYDRO(I),I=1,IQH)
      IF(ITR1.NE.0) READ(2,200) (QTRIB1(I),I=1,ITR1)
      IF(ITR2.NE.0) READ(2,200) (QTRIB2(I),I=1,ITR2)
            READ(2,200) (QDNS(I),I=1,IQDNS)
 200  FORMAT(41X,F9.2)
      WRITE(3,306)
 306  FORMAT(1H ,21X,4HTIME,10X,8HDISTANCE,6X,10HCALCULATED,6X,9HPROTOTY
     1PE/1H ,20X,5H(HRS),13X,3H(M),9X,9HDISCHARGE,6X,9HDISCHARGE/)
      RETURN
      END

C ************************************************************************

      FUNCTION FF(Q,T,DT)

C  THIS FUNCTION INTERPOLATES USING A CUBIC POLYNOMIAL

      REAL Q(200)

         L=INT(T/DT)+1
      IF(L.LE.1) L=2
         FRI=T/DT-FLOAT(L-1)
         A=(Q(L+2)-3.0*(Q(L+1)-Q(L))-Q(L-1))/6.0
         B=(Q(L-1)-2.0*Q(L)+Q(L+1))*0.5
         C=(-Q(L+2)+6.0*Q(L+1)-3.0*Q(L)-2.0*Q(L-1))/6.0
         D=Q(L)
      FF=((A*FRI+B)*FRI+C)*FRI+D
      RETURN
      END
      FINISH

*************************************************************************
```

5.3 Variable parameter diffusion method

A Calculate parameters

i Derive curves for the speed, L/T_p, and attenuation parameters as described in Section 5.1.

ii Construct the function $\bar{c}(Q)$ from the equation

$$\bar{c} = \omega + Q^* \frac{d}{dQ}\left(\frac{L}{T_p}\right) \tag{5.17}$$

where $d(L/T_p)/dQ$ is measured graphically from the curve for L/T_p and $\bar{c}$ is defined for each recorded flood using known values for Q^*.

iii Digitalise the functions for $\bar{c}$ and α at 10 $m^3\,s^{-1}$ or 20 $m^3\,s^{-1}$ intervals.

iv Set $\Delta x = L/10$, where L is the length of the reach, and define Δt by

$$\Delta t = \Delta x/\bar{c}_{ave} \tag{5.18}$$

where $\bar{c}_{ave}$ is an average value for $\bar{c}$ over the range of discharge anticipated for the flood. Choose Δt as a convenient fraction of an hour.

B Prepare card data

The program listed below reads the data in the following order. Eight cards containing the heading for the output from the program. Cards containing 14 integers (free I-format):

JX, LEND, KEND, IWSP, IQH, IQDNS, ITR1, ITR2, JT1, JT2, J1, L1, L2, NPAR.

These integers are all explained in Part *B* of 5.2, except for

KEND maximum number of iterations allowed in the solution of the non-linear equations

IWSP number of data points defining each of the functions of $\bar{c}$ and α

NPAR = 1 if the coefficients of the smoothed cubics through the data points for $\bar{c}$ and α are to be calculated
= 0 if the coefficients are to be read.

Cards containing 12 real numbers (free E-format)
DXLR, DT, EQ, DTQHYDRO, QERROR, QINIT, QINB, QINA, TQIN, TSCQIN, QCF, TDEVN.
Again, these numbers are all explained in Part B of 5.2, except for

EQ interval between successive data points for $\bar{c}$ and α
QERROR maximum error allowed in the iteration procedure
QCF cut-off discharge to simulate drainage off the flood plain and to stabilise the finite difference solution technique.

Cards containing the data defining $\bar{c}$ in free F-format and α in free E-format.

Again, the computer program below for the variable parameter diffusion method is written in FORTRAN IV.

```
C    Q1      - ARRAY FOR THE DISCHARGE ALONG THE REACH AT THE NEW TIME LEVEL.
C    Q2      - ARRAY FOR THE DISCHARGE ALONG THE REACH AT THE OLD TIME LEVEL.
C    DX      - SPACE INCREMENT.
C    DXLR    - LENGTH OF THE REACH.
C    JX      - NUMBER OF SPACE GRID POINTS.
C    X       - ARRAY FOR THE DISTANCES OF THE GRID POINTS ALONG THE CHANNEL.
C    DT      - TIME INCREMENT.
C    LEND    - NUMBER OF TIME STEPS.
C    T       - TIME IN SECONDS
C    TH      - TIME IN HOURS.
C    KEND    - MAXIMUM NUMBER OF ITERATIONS ALLOWED.
C    QERROR  - MAXIMUM ERROR ALLOWED IN THE NEWTON ITERATION PROCESS.
C    EQ      - INCREMENT IN THE DISCHARGE BETWEEN SUCCESSIVE VALUES OF THE DATA
C              FOR THE WAVE SPEED AND THE ATTENUATION PARAMETER.
C    TITLE   - ARRAY WHICH STORES DATA FOR THE HEADING OF THE OUTPUT.
C    J1, L1 AND L2 REFER TO WHEN AND HOW THE RESULTS ARE WRITTEN.

C    DTQHYDRO  TIME INTERVAL IN HOURS BETWEEN THE DATA POINTS FOR THE INPUT AND
C              OTHER DISCHARGE HYDROGRAPHS.
C    QHYDRO  - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C              AT THE UPSTREAM SECTION.
C    IQH     - NUMBER OF DATA POINTS FOR THE UPSTREAM HYDROGRAPH.
C    QTRIB1  - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C              AT THE FIRST TRIBUTARY.
C    ITR1    - NUMBER OF DATA POINTS FOR THE HYDROGRAPH AT THE FIRST TRIBUTARY.
C    JT1     - POSITION OF THE INPUT DISCHARGE FROM THE FIRST TRIBUTARY  (0 OR 1)
C    QTRIB2  - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C              AT THE SECOND TRIBUTARY.
C    ITR2    - NUMBER OF DATA POINTS FOR THE HYDROGRAPH AT THE SECOND TRIBUTARY.
C    JT2     - POSITION OF THE INPUT DISCHARGE FROM THE SECOND TRIBUTARY (0 OR JX)
C    QINIT   - INITIAL DISCHARGE FOR THE FLOW THROUGHOUT THE REACH.
C    QINB    - BASE VALUE FOR THE LATERAL INFLOW.
C    QINA    - AMPLITUDE OF THE FUNCTION FOR THE LATERAL INFLOW.
C    TQIN    - TIME WHEN THE LATERAL INFLOW TAKES ITS MAXIMUM VALUE.
C    TSCQIN  - TIME SCALE FOR THE LATERAL INFLOW FUNCTION.
C    QCF     - CUT-OFF DISCHARGE FOR DRAINAGE OFF THE FLOOD PLAIN TO BE INCLUDED.
C    INDIC   - AN INDICATOR WHICH IS 0 WHEN THE DISCHARGE AT A SECTION IS BELOW
C              BANK-FULL AND INCREASING, AND 1 WHEN THE DISCHARGE HAS PREVIOUSLY
C              EXCEEDED BANK-FULL AND YET IS BELOW BANK-FULL AND DECREASING.
C    QDNS    - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C              AT THE DOWNSTREAM END OF THE REACH.
C    IQDNS   - NUMBER OF DATA POINTS FOR THE DOWNSTREAM HYDROGRAPH.
C    TDEVN   - TIME IN HOURS WHEN THE CALCULATIONS OF THE ERROR PARAMETERS ARE
C              BEGUN.
C    TOTDIS  - AVERAGE RECORDED DISCHARGE AFTER THE TIME TDEVN.
C    TOTD    - AVERAGE PREDICTED DISCHARGE AFTER THE TIME TDEVN.
C    DISDIF  - DIFFERENCE BETWEEN THE AVERAGE RECORDED AND PREDICTED DISCHARGES
C              AS A PERCENTAGE OF THE RECORDED DISCHARGES.
C    DEVN    - ULTIMATELY RECORDS THE STANDARD DEVIATION OF THE PREDICTED
C              DISCHARGE AS A PERCENTAGE OF THE AVERAGE RECORDED DISCHARGE.

C    ****************************************************************

      MASTER FLOODS2

C  THIS PROGRAM SOLVES A GENERALISED CONVECTION-DIFFUSION EQUATION FOR
C     FLOOD ROUTING (VARIABLE PARAMETER DIFFUSION METHOD)

      REAL P1(50),P2(50),B1(50),B2(50),B3(50),R(50),BB(50),RR(50)
      COMMON DX,DT,JX,LEND,DXLR,KEND,TH,J1,L1,L2,QERROR,TITLE(60)
     1,EQ,JT1,JT2,QTRIB1(200),QTRIB2(200),QHYDRO(200),DTQHYDRO,
     2QINIT,QINB,QINA,TSCQIN,TQIN,QCF,CCF,ACF,INDIC(50),Q1(50),Q2(50),
     3AP(100),WSP(100),COEFAP(3,30),COEFWS(3,30),X(50)
     4      ,TDEVN,QDNS(200)

C  FIRST READ IN THE RELEVANT DATA AND PRINT IT OUT FOR EASY REFERENCE

      CALL DATIN
```

```
C  DEFINE VARIOUS CONSTANTS USED DURING THE CALCULATIONS

         JXM1=JX-1
         JXM2=JX-2
         DX=DXLR/(JX-1)
         DTX=0.25*DT/DX
         DXSQ=0.5*DT/(DX*DX)
         QJX=QINIT
         QB=QCF
         I=QB/(EQ*3)+1
         CCF=(COEFWS(3,I)*QB+COEFWS(2,I))*QB+COEFWS(1,I)
         ACF=DXSQ*((COEFAP(3,I)*QB+COEFAP(2,I))*QB+COEFAP(1,I))

C  STATEMENTS BEGINNING IN COLUMN 13 ARE CONCERNED WITH THE CALCULATION
C     OF THE ERROR PARAMETERS DEVN, TOTDIS AND DISDIF

            TOTDIS=0.0
            DEVN=0.0
            TOTD=0.0
            QP1=QINIT
            QP2=QINIT
            QDNP1=QINIT
            QDNP2=QINIT
            JCOUNT=0
            ICOUNT=0

C  NEXT, GENERATE THE INITIAL VALUES OF Q1, Q2, X AND INDIC.

      DO 2 J=1,JX
         X(J)=DX*(J-1)
         INDIC(J)=0
         Q1(J), Q2(J)=QINIT
 2    CONTINUE

C  BEGIN THE MAIN TIME LOOP

         T=0.0
      DO 32 L=1,LEND
         T=T+DT
         TH=T/3600.0

C  EVALUATE THE PROTOTYPE DISCHARGE AT THE DOWNSTREAM BOUNDARY

            QP2=QP1
            QP1=QJX
            QDNP2=QDNP1
            QDNP1=QDN
            QDN=FF(QDNS,TH,DTQHYDRO)

C  UPDATE Q1, Q2 AND INDIC

      DO 4 J=1,JX
         QQ=Q2(J)
      IF(Q1(J).GT.QCF) INDIC(J)=1
         Q2(J)=Q1(J)
         Q1(J)=2.0*Q1(J)-QQ
 4    CONTINUE

C DEFINE THE ARRAYS P1 AND P2 TO STORE FUNCTIONS FOR USE IN LOOP 14

      DO 6 J=2,JXM1
         P1(J)=Q2(J+1)-Q2(J-1)
         P2(J)=Q2(J+1)-2.0*Q2(J)+Q2(J-1)
      IF(P1(J).LE.0) INDIC(J)=0
 6    CONTINUE

C  CALCULATE THE UPSTREAM VALUE FOR Q1.  INCLUDE THE DISCHARGE FROM THE
C     TRIBUTARY, IF THERE IS ONE

         QTOT=0.0
      IF(JT1.EQ.1) QTOT=FF(QTRIB1,TH,DTQHYDRO)
      Q1(1)=FF(QHYDRO,TH,DTQHYDRO)+QTOT

C  CALCULATE THE NEW VALUE FOR THE DISCHARGE AT THE DOWNSTREAM BOUNDARY
C     IN THE MODEL

      CALL QDOWN

C  NOW FIND THE VALUES FOR THE REST OF THE ARRAY Q1
C  USE THE NEWTON ITERATION PROCESS TO SOLVE THE NON-LINEAR SIMULTANEOUS
C     EQUATIONS

      DO 22 K=1,KEND
      DO 14 J=2,JXM1
         DQP=Q1(J+1)-Q1(J-1)+P1(J)
         QQ=0.5*(Q1(J)+Q2(J))
      IF(INDIC(J).EQ.1) GO TO 10
 8    QB=QQ
         I=QB/(EQ*3)+1
         C=(COEFWS(3,I)*QB+COEFWS(2,I))*QB+COEFWS(1,I)
         DC=2.0*COEFWS(3,I)*QB+COEFWS(2,I)
         ADXSQ=DXSQ*((COEFAP(3,I)*QB+COEFAP(2,I))*QB+COEFAP(1,I))
         DADXSQ=DXSQ*(2.0*COEFAP(3,I)*QB+COEFAP(2,I))
      GO TO 12
 10   IF(Q2(J).GT.QCF) GO TO 8
         C=CCF
         ADXSQ=ACF
         DC,DADXSQ=0.0
 12   DDQP=Q1(J+1)-2.0*Q1(J)+Q1(J-1)+P2(J)
C  PUT IN AN ARTIFICIAL FUNCTION FOR THE LATERAL INFLOW.
```

```
C        IF A SNOW-MELT FLOOD IS BEING SIMULATED IT MAY BE PREFERABLE
C        TO DEFINE QTOT AS SOME CONSTANT TIMES Q1(1)
         TX=(TH-TQIN)/TSCQIN
         QTOT=QINA*EXP(-TX*TX)+QINB
         D1=DQP*DTX-QTOT*DT
         R(J)=Q1(J)-Q2(J)+D1*C-QQ*ADXSQ*DDQP
         B1(J)=-DTX*C-ADXSQ*QQ
         B2(J)=1.0+DC*0.5*D1-DADXSQ*0.5*QQ*DDQP-ADXSQ*(0.5*DDQP-QQ*2.0)
         B3(J)=DTX*C-ADXSQ*QQ
 14   CONTINUE
         B1(2), B3(JXM1) = 0.0

C  A GAUSSIAN ELIMINATION PROCEDURE AVOIDS THE USE OF EXCESSIVE STORAGE
C     IN THE COMPUTER

         BB(2)=B2(2)
         RR(2)=R(2)
      DO 16 J=3,JXM1
         BB(J)=B2(J)-B3(J-1)*B1(J)/BB(J-1)
         RR(J)=R(J)-RR(J-1)*B1(J)/BB(J-1)
 16   CONTINUE

         DQ=RR(JXM1)/BB(JXM1)
         Q1(JXM1)=Q1(JXM1)-DQ
         DQM=ABS(DQ)
         JQ=JXM1
      DO 20 JJ=3,JXM1
         J=JX-JJ+1
         DQ=(RR(J)-B3(J)*DQ)/BB(J)
      IF(ABS(DQ).LE.DQM) GO TO 18
         DQM=ABS(DQ)
         JQ=J
 18   Q1(J)=Q1(J)-DQ
 20   CONTINUE

C  CONTINUE THE ITERATION UNLESS THE ACCURACY CONDITION IS SATISFIED,
C     OR THERE HAVE BEEN TOO MANY ITERATIONS

      IF(DQM.LE.QERROR) GO TO 24
 22   CONTINUE
      WRITE(3,300) JQ,DQM
 300  FORMAT(20H TOO MANY ITERATIONS,5X,13HVALUE OF J = ,I3/
    1    44H MAXIMUM VALUE OF THE DISCHARGE INCREMENT = ,F7.4/)
 24   CONTINUE

C  DEFINE QJX AS THE TOTAL DISCHARGE AT THE DOWNSTREAM BOUNDARY,
C     INCLUDING A TRIBUTARY IF THERE IS ONE

         QTOT=0.0
      IF(JT2.EQ.JX) QTOT=FF(QTRIB2,TH,DTQHYDRO)
         QJX=Q1(JX)+QTOT

C  CALCULATE VARIOUS ERROR PARAMETERS

            IF(TH.LT.TDEVN) GO TO 26
            DEVN=DEVN+(QJX-QDN)*(QJX-QDN)
            ICOUNT=ICOUNT+1
            JCOUNT=JCOUNT+1
            IF(JCOUNT.NE.2) GO TO 26
            TOTDIS=TOTDIS+2.0*DT*(QDN+QDNP1+QDNP2)/3.0
            TOTD=TOTD+2.0*DT*(QJX+QP1+QP2)/3.0
            JCOUNT=0

C  WRITE OUT THE RESULTS  - IF IT IS TIME

 26   IF(L.NE.L1*(L/L1)) GO TO 30
      WRITE(3,301) (TH,X(J),Q1(J),J=1,JX,J1)
 301  FORMAT((20X,F6.2,9X,F8.1,8X,F7.2))
 28   WRITE(3,302) TH,X(JX),QJX,QDN
 302  FORMAT((20X,F6.2,9X,F8.1,2(8X,F7.2)))
      WRITE(3,303)
 303  FORMAT(1H ,/)
      GO TO 32
 30   IF(L.EQ.L2*(L/L2)) GO TO 28
 32   CONTINUE
            TOTDIS=TOTDIS/((TH-TDEVN)*3600.0)
            TOTD=TOTD/((TH-TDEVN)*3600.0)
            DISDIF=(1.0-TOTD/TOTDIS)*100.0
            DEVN=SQRT(DEVN/FLOAT(ICOUNT))*100.0/TOTDIS
            WRITE(3,304) TOTDIS,DISDIF,DEVN
 304        FORMAT(34H     AVERAGE RECORDED DISCHARGE = ,F7.2,
    1       9H   CUMECS/
    2       56H     DIFFERENCE BETWEEN RECORDED AND PREDICTED AVERAGE D,
    3       11HISCHARGE = ,F6.2,1HX/
    4       26H     STANDARD DEVIATION = ,F6.2,1HX//)
      STOP
      END

C ********************************************************************

      SUBROUTINE DATIN

C  THIS SUBROUTINE READS IN THE RELEVANT DATA, WRITES IT OUT FOR EASY
C     REFERENCE, AND CALCULATES THE COEFFICIENTS, COEFAP AND COEFWS,
C     DEFINING THE CURVES FOR THE ATTENUATION PARAMETER, AP, AND THE
C     WAVE SPEED, WSP.

      REAL QX(10),WX(10),APX(10),COEF(5)
      COMMON DX,DT,JX,LEND,DXLR,KEND,TH,J1,L1,L2,QERROR,TITLE(60)
     1,EQ,JT1,JT2,QTRIB1(200),QTRIB2(200),QHYDRO(200),DTQHYDRO,
```

```
     2QINIT,QINB,QINA,TSCQIN,TQIN,QCF,CCF,ACF,INDIC(50),Q1(50),Q2(50),
     3AP(100),WSP(100),COEFAP(3,30),COEFWS(3,30),X(50)
     4      ,TDEVN,QDNS(200)

      READ(1,100) (TITLE(I),I=1,60)
 100  FORMAT(10A8/5A8)
      READ(1,101) JX,LEND,KEND,IWSP,IQH,ITR1,ITR2,JT1,JT2,J1,L1,L2,NPAR
     1      ,IQDNS
 101  FORMAT(14I0)
      READ(1,102) DXLR,DT,EQ,DTQHYDRO,QERROR,QINIT,QINB,QINA,TQIN,TSCQIN
     1,QCF
     2      ,TDEVN
 102  FORMAT(12E0.0)

C  WRITE OUT THE DATA USED IN RUNNING THE PROGRAM

      WRITE(3,300) (TITLE(I),I=1,60)
 300  FORMAT(1H ,(15A8))
C     WRITE(3,301) JX,LEND,KEND,IWSP,IQH,ITR1,ITR2,JT1,JT2,J1,L1,L2,NPAR
C    1      ,IQDNS
C  INSERT AN APPROPRIATE FORMAT STATEMENT 301
C     WRITE(3,302) DXLR,DT,EQ,DTQHYDRO,QERROR,QINIT,QINB,QINA,TQIN,
C    1TSCQIN,QCF
C    2      ,TDEVN
C  INSERT AN APPROPRIATE FORMAT STATEMENT 302

C  NEXT, EITHER READ IN THE DATA FOR THE WAVE SPEED AND ATTENUATION
C     PARAMETERS TO CALCULATE THE COEFFICIENTS FOR THE SMOOTHED
C     QUADRATIC CURVES OR READ IN THE COEFFICIENTS.

        IWSPD3=IWSP/3
      IF(NPAR.EQ.0) GO TO 10
      READ(1,103) (WSP(I),I=1,IWSP)
 103  FORMAT(16F5.3)
      READ(1,104) (AP(I),I=1,IWSP)
 104  FORMAT(8E0.0)
      WRITE(3,303) EQ,(WSP(I),I=1,IWSP)
 303  FORMAT(1H ,25X,27HDATA FOR THE WAVE SPEED AT ,F3.0,16H CUMEC INTER
     1VALS/(1H ,10(1PE8.2,3X)))
      WRITE(3,304)
 304  FORMAT(1H ,//)
      WRITE(3,305) EQ,(AP(I),I=1,IWSP)
 305  FORMAT(1H ,20X,38HDATA FOR THE ATTENUATION PARAMETER AT ,F3.0,16H
     1CUMEC INTERVALS/(1H ,10(1PE8.2,3X)))

C  THE FOLLOWING PROCEDURE SMOOTHS THE DATA FOR THE SPEED AND ATTENUATION
C     PARAMETERS.  THIS IS DONE BY FITTING A QUADRATIC CURVE THROUGH 4
C     NEIGHBOURING POINTS.

      DO 8 I=1,IWSPD3
      DO 2 M=1,4
         I3=3*(I-1)+M
         QX(M)=EQ*(I3-1)
         WX(M)=WSP(I3)
         APX(M)=AP(I3)/DXLR
 2    CONTINUE
      CALL FIT(QX,WX,COEF)
      DO 4 J=1,3
         COEFWS(J,I)=COEF(J)
 4    CONTINUE
      CALL FIT(QX,APX,COEF)
      DO 6 J=1,3
         COEFAP(J,I)=COEF(J)
 6    CONTINUE
 8    CONTINUE

C  WRITE OUT THE COEFFICIENTS TO A CARD FILE OR TO THE LINE PRINTER
C     (CHANGE THE CHANNEL NUMBER IN THE LATTER CASE)

      WRITE(4,400) ((COEFWS(J,I),COEFAP(J,I),J=1,3),I=1,IWSPD3)
      GO TO 12

 10   PAUSE (RP)
      READ(1,400) ((COEFWS(J,I),COEFAP(J,I),J=1,3),I=1,IWSPD3)
 400  FORMAT(6E12.6)

C  READ IN THE DISCHARGE HYDROGRAPHS AT THE UPSTREAM AND DOWNSTREAM ENDS
C      OF THE REACH, TOGETHER WITH THE HYDROGRAPHS FOR THE TWO
C     TRIBUTARIES

 12   READ(2,200) (QHYDRO(I),I=1,IQH)
      IF(ITR1.NE.0) READ(2,200) (QTRIB1(I),I=1,ITR1)
      IF(ITR2.NE.0) READ(2,200) (QTRIB2(I),I=1,ITR2)
            READ(2,200) (QDNS(I),I=1,IQDNS)
 200  FORMAT(41X,F9.2)

C  WRITE THE HEADINGS FOR THE OUTPUT DATA FROM THE CALCULATIONS

      WRITE(3,306)
 306  FORMAT(1H ,21X,4HTIME,10X,8HDISTANCE,6X,10HCALCULATED,6X,9HPROTOTY
     1PE/1H ,20X,5H(HRS),13X,3H(M),9X,9HDISCHARGE,6X,9HDISCHARGE/)
      RETURN
      END

C ********************************************************************

      SUBROUTINE QDOWN

C  THIS SUBROUTINE CALCULATES THE DISCHARGE AT THE DOWNSTREAM BOUNDARY
C     IN THE MODEL
```

```
      DIMENSION XX(10),QX(10),COEFQ(5)
      COMMON DX,DT,JX,LEND,DXLR,KEND,TH,J1,L1,L2,QERROR,TITLE(60)
     1,EQ,JT1,JT2,QTRIB1(200),QTRIB2(200),QHYDRO(200),DTQHYDRO,
     2QINIT,QINB,QINA,TSCQIN,TQIN,QCF,CCF,ACF,INDIC(50),Q1(50),Q2(50),
     3AP(100),WSP(100),COEFAP(3,30),COEFWS(3,30),X(50)
     4       ,TDEVN,QDNS(200)

C  FIRST FIT A QUADRATIC THROUGH THE LAST FOUR POINTS AT THE OLD TIME
C     LEVEL

         JXM1=JX-1
         JXM2=JX-2
         COEFQ(1)=Q2(JX)
         CCEFQ(2)=0.5*(4.0*Q2(JXM1)-Q2(JXM2)-3.0*Q2(JX))
         COEFQ(3)=0.5*(Q2(JXM2)-2.0*Q2(JXM1)+Q2(JX))

C  NOW ITERATE TO FIND THE CORRECT VALUE OF Q2 AT THE FOOT OF THE
C     CHARACTERISTIC.  THE WEGSTEIN ITERATION PROCEDURE IS USED HERE

         DELTX=0.0
         QP=Q2(JX)
         QBM2=QP
         QB=QP

      DO 12 K=1,KEND
         DQP=-(COEFQ(2)+2.0*DELTX*COEFQ(3))
      IF(DQP.LT.0.0) INDIC(JX)=0
      IF(INDIC(JX).EQ.1) GO TO 4
 2    QC=QB
         I=QB/(EQ*3)+1
         CC=(COEFWS(3,I)*QC+COEFWS(2,I))*QC+COEFWS(1,I)
      GO TO 6
 4    IF(QB.GT.QCF) GO TO 2
         QC=QCF
         CC=CCF
 6    DELTX=CC*DT/DX
         Q=COEFQ(1)+DELTX*(COEFQ(2)+DELTX*COEFQ(3))
      IF(ABS(Q-QP).LE.QERROR) GO TO 14
      IF(K.EQ.1) GO TO 8
      IF(K.LT.KEND) GO TO 10
      WRITE(3,300)
 300  FORMAT(15H ERROR IN QDOWN)
      STOP
 8    QB=Q
         QBM1=Q
         QP=Q
      GO TO 12
 10   QB=Q-(Q-QP)*(Q-QBM1)/(Q-QP-QBM1+QBM2)
         QBM2=QBM1
         QBM1=QB
         QP=Q
 12   CONTINUE

C  FINALLY, CALCULATE THE NEW DOWNSTREAM VALUE FOR Q (Q1)

 14   TX=(TH-TQIN)/TSCQIN
         QQ=QINA*EXP(-TX*TX)+QINB
      IF(INDIC(JX)) 0,0,16
         ADX=((COEFAP(3,I)*QC+COEFAP(2,I))*QC+COEFAP(1,I))/(DX*DX)
      GO TO 18
 16   ADX=ACF*2/DT
 18   A=2.0*COEFQ(3)
      Q1(JX)=Q+(CC*QQ+Q*ADX*A)*DT
      RETURN
      END

C *************************************************************************

      FUNCTION FF(Q,T,DT)

C  THIS FUNCTION FITS A CUBIC SPLINE THROUGH THE FOUR DATA POINTS IN THE
C     NEIGHBOURHOOD OF THE POINT AT WHICH THE DEPENDENT VARIABLE IS
C     REQUIRED

      REAL  Q(200)
         L=INT(T/DT)+1
      IF(L.LE.1) L=2
         FRI=T/DT-FLOAT(L-1)
        A=(Q(L+2)-3.0*(Q(L+1)-Q(L))-Q(L-1))/6.0
        B=(Q(L-1)-2.0*Q(L)+Q(L+1))*0.5
        C=(-Q(L+2)+6.0*Q(L+1)-3.0*Q(L)-2.0*Q(L-1))/6.0
        D=Q(L)
      FF=((A*FRI+B)*FRI+C)*FRI+D
      RETURN
      END

C *************************************************************************

      SUBROUTINE FIT(X,Y,E)

C  THIS SUBROUTINE FITS A QUADRATIC CURVE THROUGH FOUR POINTS USING THE
C     LEAST SQUARES PROCEDURE.

      DIMENSION X(4),Y(4),E(4)
      DIMENSION A(4),B(4),C(4),D(4)

         C11=(X(2)-X(4))*(X(2)-X(1))
         C21=(X(3)-X(4))*(X(3)-X(1))
         C1=1.
         C2=C11+C21
```

```
      C3=1.
      B1=X(4)
      B2=X(2)*C11+X(3)*C21
      B3=X(1)
      A1=X(4)*X(4)
      A2=X(2)*X(2)*C11+X(3)*X(3)*C21
      A3=X(1)*X(1)
      D1=Y(4)
      D2=Y(2)*C11+Y(3)*C21
      D3=Y(1)
      B2=B2-B1*A2/A1
      C2=C2-C1*A2/A1
      D2=D2-D1*A2/A1
      B3=B3-B1*A3/A1
      C3=C3-C1*A3/A1-C2*B3/B2
      D3=D3-D1*A3/A1-D2*B3/B2
      E(1)=D3/C3
      E(2)=(D2-C2*E(1))/B2
      E(3)=(D1-B1*E(2)-C1*E(1))/A1
   RETURN
   END
   FINISH

***********************************************************************
```

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